STA301 Name Ivan Wang CLASS NUMBER W-150

Test 2 Fall 2022

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SHOW ALL WORK ON ALL PROBLEMS

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1. (9 points) Determine the following:

a) $P(.46 < Z < 1.61)$

$.9463 - .6772 = 0.2691$

ANSWER 0.2691

b) Determine the value k so that $P(k < Z) = .063$

0.937 .063

-1.53

ANSWER 1.53

c) Determine $P[(.75 < Z) | (Z < 1.8)]$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ $\frac{P(.75 < Z < 1.8)}{P(Z < 1.8)} = \frac{.9641 - .7734}{.9641}$

$= \frac{0.1907}{0.9641} =$

ANSWER 0.197801058

2. (6 points) Determine the mean of the random variable X that has pdf $f(x) = \begin{cases} x & 0 < x < 1 \\ .1 & 4 \le x \le \cancel{8}\, 9 \\ 0 & otherwise \end{cases}$

$\frac{1}{3} + 3.25$

$E(x) = \int x f(x)$ $\boxed{E(x) = 3.58333333333}$

$\int_{x=0}^{x=1} x \cdot x \, dx + \int_{x=4}^{x=9} x \cdot .1 \, dx$ $\int_{x=4}^{x=9} (.1)(x)\, dx$

$\int_{x=0}^{x=1} x^2 \, dx \Rightarrow \frac{x^3}{3} \Big|_{x=0}^{x=1} = \frac{1}{3}$ $\frac{.1x^2}{2} \Big|_{x=4}^{x=9}$

$\frac{(.1)(9)^2}{2} - \frac{(.1)(4)^2}{2}$

$4.05 - 0.8 = 3.25$

ANSWER 3.583333333

3. Given the probability density function for X is $f(x)=\begin{cases}6x^5 & 0<x<1\\ 0 & \text{otherwise}\end{cases}$,

(4 points) Determine $P(.6 < X)$ (4-decimal places)

$$\int_{x=.6}^{x=1} 6x^5\,dx \Rightarrow 6\int_{x=.6}^{x=1} x^5\,dx$$

$$6\left(\frac{x^6}{6}\right)\Big|_{x=.6}^{x=1}$$

$$=6\left(\tfrac{1}{6}-0.046656\right)$$
$$=0.720064$$

-2

ANSWER 0.7201

$\frac{f'g-fg'}{2g}$

4. (4 points) Given the random variable X has CDF $F(x)=\begin{cases}0 & x\le 1\\ 1-\frac{1}{x^4} & 1<x\end{cases}$. Determine the pdf of X.

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$F(x) \xrightarrow{\text{derittive}} f(x)$ $\binom{2}{2}$

$f'(x)=1-\frac{1}{x^4}$

$0-\left(\frac{0-4x^3}{2x}\right)$

$\frac{4x^3}{2x}=2x^2$

$$f(x)=\begin{cases}0 & x\le 1\\ 2x^2 & 1<X\end{cases}$$

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ANSWER

X

$\cancel{0}$ x^{-4}

$0-$

5. (6 points) Given the pdf for X is $f(x)=\begin{cases}5x^4 & 0<x<1\\ 0 & otherwise\end{cases}$, ~~determine the variance of X~~. (4-decimal places)

Determine $E\left[\frac{1}{X^3}\right]$

$E(x)=\int x\, f(x)$

$$5\int_{x=0}^{x=1}\frac{1}{x^3}\cdot x^4\,dx \Rightarrow 5\int_{x=0}^{x=1} x\,dx$$

$$=5\left(\frac{x^2}{2}\right)\Big|_{x=0}^{x=1}$$

$$=5\left(\tfrac{1}{2}-0\right)$$

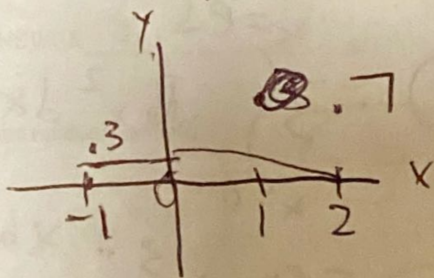
$$=2.5$$

ANSWER 2.5000

6. (6 points) Find the value k so that $f(x)$ is a pdf for a continuous random variable. $f(x)=\begin{cases}.3 & -1<x<0\\ .6+kx & 0\le x\le 2\\ 0 & otherwise\end{cases}$

$1-.3=.7$

$$\int_{x=0}^{x=2} .6+kx\,dx$$



$$.6x+\frac{kx^2}{2}\Big|_{x=0}^{x=2} = \left(.6(2)+\frac{k(2)^2}{2}\right)-(0)$$

$1.2+2k$

$$1.2+2k=.7$$
$$-1.2 \qquad -1.2$$

$$\frac{2k}{2}=\frac{-0.5}{2} \qquad k=-0.25$$

ANSWER -0.25

7. (8 points) Given $F(x)=\begin{cases}0 & x\le 0\\ \frac{x^2}{10} & 0<x<2\\ \frac{x}{5} & 2\le x\le 5\\ 1 & 5<x\end{cases}$

a) Determine $P(1.8\le X\le 7)$

$F(7)-F(1.8)$

$=1-\frac{(1.8)^2}{10} \Rightarrow 1-\frac{3.24}{10}$

$1-0.324$

ANSWER 0.676

b) Determine the value k so that $P(X\le k)=.65$

$\frac{x}{5}=.65$

$x=3.25$

ANSWER 3.25

8. (4-points) Find the values a and b, so that $F(x)$ is a CDF for a continuous random variable.

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$F(x)=\begin{cases}0 & x<a\\ \ln(x)-2 & a\le x\le b\\ 1 & b<x\end{cases}$

$\int_{x=a}^{x=b} \ln(x)-2\,dx$

$\ln(a)-2=0$

$\ln(a)=2$

$a=e^2$

$\ln(b)-2=1$

$\ln(b)=3$

$b=e^3$

$\frac{1}{x}-2x\Big|_{x=a}^{x=b}=\left(\frac{1}{b}-2b\right)-\left(\frac{1}{a}-2a\right)$

$E(x)=\int_{x=a}^{x=b} x\ln(x)-2\,dx$

$\frac{x^2}{2}\cdot\frac{1}{x}-2x\Big|_{x=a}^{x=b}=\left(\frac{b^2}{2}\cdot\frac{1}{b}-2b\right)-\left(\frac{a^2}{2}\cdot\frac{1}{a}-2a\right)$

$a=0 \quad b=1$

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9. (5-points) Determine k so that $f(x,y,z)=\dfrac{x+yz}{k}$ *for* $x=1,2,3$ $y=1,2$ *and* $z=0,1$ is a pmf.

$$\frac{(1)+(1)(0)}{k}+\frac{(2)+(1)(0)}{k}+\frac{(3)+(1)(0)}{k}+\frac{(1)+(2)(0)}{k}$$

$$+\frac{(2)+(2)(0)}{k}+\frac{(3)+(2)(0)}{k}+\frac{(1)+(1)(1)}{k}+\frac{(2)+(1)(1)}{k}+\frac{(3)+(1)(1)}{k}$$

$$+\frac{(1)+(2)(1)}{k}+\frac{(2)+(2)(1)}{k}+\frac{(3)+(2)(1)}{k}=\frac{1}{k}+\frac{2}{k}+\frac{3}{k}+\frac{1}{k}+\frac{2}{k}+\frac{3}{k}+\frac{2}{k}+\frac{3}{k}+\frac{4}{k}+\frac{3}{k}+\frac{4}{k}+\frac{5}{k}$$

ANSWER 33

$$\frac{33}{k}=1$$

10. (8-points) Suppose that the joint probability density function of the jointly continuous random variables X and Y is $f_{X,Y}(x,y)=\begin{cases} ky^2 & \text{on the given region} \\ 0 & \text{otherwise}\end{cases}$

Determine the value of k.

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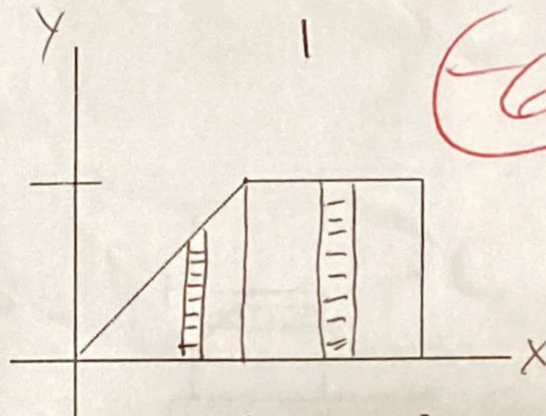
1



$$\int_{x=0}^{x=2}\int_{y=x}^{y=1} ky^2\,dy\,dx$$

$$k\int_{y=x}^{y=1} y^2\,dy \Rightarrow k\left(\frac{y^3}{3}\right)\Big|_{y=x}^{y=1}$$

$$k\left(\frac{1}{3}-\frac{x^3}{3}\right)$$

$$k\int_{x=0}^{x=2}\frac{1}{3}-\frac{x^3}{3}\,dx$$

$$k\left(\frac{1}{3}x-\frac{x^4}{12}\right)\Big|_{x=0}^{x=2} \Rightarrow k\left(\frac{1}{3}(2)-\frac{2^4}{12}\right)-(0)$$

$$k\left(\frac{2}{3}-\frac{16}{12}\right)$$

$$k\left(\frac{8}{12}-\frac{16}{12}\right)=1$$

$$k\left(-\frac{8}{12}\right)=1$$

$$-\frac{12}{8}k\left(-\frac{8}{12}\right)=1$$

$$\boxed{k=0.5}$$

11. (4-points) Given that $V[X]=4.1$, $V[Y]=2.5$ and $Cov[X,Y]=1.2$, determine $V[3X-2Y]$.

$$V(3x-2Y) = V(3x) - V(2Y) - 2\,Cov(X,Y)$$

$$3^2V(X) - 2^2V(Y) - 2cov(X,Y)$$

$$9(4.1) - 4(2.5) - 2(1.2) \Rightarrow 36.9 - 10 - 2.4$$

$$= 24.5$$

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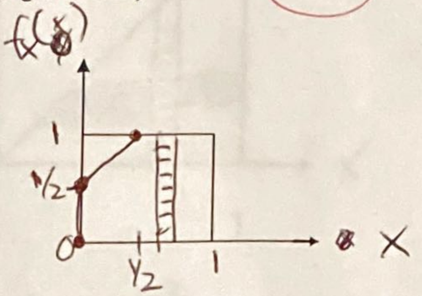
12. The joint pdf of X and Y is $f_{X,Y}(x,y) = \begin{cases} x+y & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$ (region below)

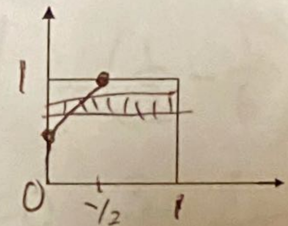


a) (5-points) Determine $f_X(x)$

$$\int_{y=0}^{y=1} x+y \, dy$$

$$xy + \frac{y^2}{2}\Big|_{y=0}^{y=1} \Rightarrow x + \frac{1}{2}$$

b) (5-points) Determine $f_Y(y)$



$$\int_{x=0}^{x=1} x+y \, dx$$

$$\frac{x^2}{2} + yx\Big|_{x=0}^{x=1} \Rightarrow \frac{1}{2} + y$$

c) (2-points) Determine $f_{Y|x}(y)$

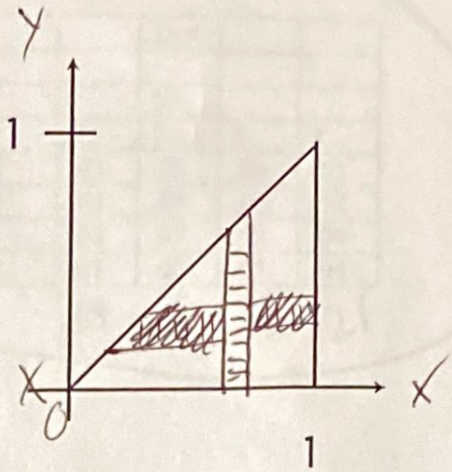
$$f_{X,Y}(x,y) = \frac{x+y}{x+\frac{1}{2}}$$

$$f_X(x) =$$

13. (6-points) Suppose that the joint probability density function of the jointly continuous random variables X and Y is

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{x} & \text{On Support} \\ 0 & \text{otherwise} \end{cases}$$

Determine $E[XY]$



$$E[XY] = \int_{x=0}^{x=1}\int_{y=x}^{y=1} XY \cdot \frac{1}{X}\, dy\, dX$$

$$\int_{y=x}^{y=1} Y\,dy \Rightarrow \frac{Y^2}{2}\Big|_{y=x}^{y=1} = \frac{1}{2} - \frac{x^2}{2}$$

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$$\int_{x=0}^{x=1} \frac{1}{2} - \frac{x^2}{2}\, dx \Rightarrow \frac{1}{2}x - \frac{x^3}{6}\Big|_{x=0}^{x=1} = \left(\frac{1}{2} - \frac{1}{6}\right) - 0$$

$$\frac{3}{6} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$E(XY) = 0.33333333$

14. (4-points) Let X and Y be continuous random variables with joint **CDF** (not the pdf)

$F(x,y) = \frac{1}{250}(20xy - x^2y - xy^2)$ for $0 \le x \le 5$ and $0 \le y \le 5$

$\frac{1}{250}(20(4)(3) - 4^2(3) - 4 \cdot 3^2) = 0.624$

$-\frac{1}{250}(20(2)(2) - 2^2 \cdot 2 - 2 \cdot 2^2) = 0.256$

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$\boxed{0.368}$

Determine $P[(2 \le X \le 4) \cap (2 \le Y \le 3)]$ $= 0.624 - 0.256$

$\cancel{\frac{1}{250}((20(2)(2) - 2^2(2) - (2)(2^2)) + (20(3)(2) - 3^2(2) - (3)(2^2)) + (20(4)(2) - 4^2(2) - 4(2)^2)}$

$\cancel{+ (20(2)(3) - 2^2(3) - (2)(3^2)) + (20(3)(3) - 3^2(3) - (3)(3^2)) + (20(4)(3) - 4^2(3) - 4(3)^2)}$

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$\cancel{\frac{1}{250}((80-8-8) + (120-18-12) + (160-32-16) + (120-12-18) + (180-27-27) + (240-48-36))}$

$\cancel{\frac{1}{250}(64 + 90 + 112 + 90 + 126 + 156) = 2.952}$

15. The joint pmf for random variables X, Y, and Z is given below: Determine each of the following making sure that your **Final Answer is in Table Format.**

		Z=2			
			X		
		0	1	2	3
	1	.01	.02	.03	.04
Y	2	.02	.03	.04	.05
	3	.01	.01	.02	.02
	4	.03	.02	.01	.01
		0.07	0.08	.1	.12

			Z=4		
			X		
		0	1	2	3
	1	.03	.05	.04	.01
Y	2	.02	.04	.06	.08
	3	.03	.07	.02	.08
	4	.01	.02	.03	.04
		.09	.18	.15	.21

a) (4-points) $f_X(x)$

X	$f_X(x)$
0	0.16
1	0.26
2	0.16
3	0.33

b) (4-points) $f_{X,Y}(x,y)$

		X			
		0	1	2	3
Y	~~0~~				
	1	0.04	0.07	0.07	0.05
	2	0.04	0.07	0.10	0.13
	3	0.04	0.08	0.04	0.10
	4	0.04	0.04	0.04	0.05

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c) (4-points) $f_{X,Y|Z=4}(x,y)$

		X			
		0	1	2	3
Y	1	.03	.05	.04	.01
	2	.02	.04	.06	.08
	3	.03	.07	.02	.08
	4	.01	.02	.03	.04

		X			
		0	1	2	3
Y	1	$\frac{.03}{.63}$	$\frac{.05}{.63}$	$\frac{.04}{.63}$	$\frac{.01}{.63}$
	2	$\frac{.02}{.63}$	$\frac{.04}{.63}$	$\frac{.06}{.63}$	$.08/.63$
	3	$\frac{.03}{.63}$	$\frac{.07}{.63}$	$\frac{.02}{.63}$	$\frac{.08}{.63}$
	4	$\frac{.01}{.63}$	$\frac{.02}{.63}$	$\frac{.05}{.63}$	$\frac{.04}{.63}$

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