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Test 1 (Retake) Fall 2022

All answers are to be to 4-decimal places! Only Answers in the Answer Box will count.

Remember the Formula: No Work = No Credit

All problems worked through to a numerical answer unless otherwise stated

1. Given the random variable X with pmf given below.

x	-1	0	1	1.5	2.5	4
f (x)	.1	.15	.15	.1	.15	.35

Answer

7.0125

Determine $E[X^2]$. Show work.

$E(X^2) = (-1)^2(.1) + (0)^2(.15) + (1)^2(.15) + (1.5)^2(.1) + (2.5)^2(.15) + (4)^2(.35)$

2. If A and B are independent events, $P(A) = .38$, and $P(B) = .26$, determine $P(A \cup B)$. Show work.

Answer

.5412

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$.38 + .26 - .0988$
$= .5412$

3. Determine the value of *c* that makes the table below a pmf.

x	1	2	3	4	5	6	7	8
f(x)	.16	.05	.06	*c*	4c	.02	.22	.14

$1 = .16 + .05 + .06 + c + 4c + .02 + .22 + .14$

Answer

.07

$1 = .65 + 5c$
$.35 = 5c$
$c = .07$

4. The CDF of a random variable is given below. Answer each question. **INCLUDE CDF NOTATION!**

x	F(x)
0	.0011
1	.0126
2	.0348
3	.0495
4	.0709
5	.1006
6	.1211
7	.2280
8	.3352
9	.3504
10	.4603
11	.5496
12	.6243
13	.7304
14	.8217
15	.8311
16	.8735
17	.8977
18	.9203
19	.9280
20	.9502
21	.9732
22	1.000

a) Determine $P(X<15)$

14 15 16

CDF Notation: $F(14)$

Final Probability Answer: .8217

b) Determine $P(X=4)$

.0709 − .0495 = .0214

CDF Notation: $F(4)-F(3)$

Final Probability Answer: .0214

c) Determine $P(9 \le X \le 20)$

.9502 − .3352

CDF Notation: $F(20)-F(8)$

Final Probability Answer: .615

d) Determine $P[\,(2<X<12) \mid (6 \le X)\,]$ (Show Work)

Final Probability Answer: .514~~8~~

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$P(A|B) = P\left(\frac{A \cap B}{B}\right)$

$P[(2<X<12) \mid (6 \le X)]$

$= \dfrac{(F(11)-F(2))(1-F(6))}{1-F(6)}$

1 2 3 11 12 13 5 6 7

$F(11)-F(2)$ $1-F(6)$

11 12 13

1 2 3

$\Rightarrow \dfrac{(.5496-.0348)(1-.1211)}{(1-.1211)}$

$\boxed{=.5148}$

5. An urn contains 7 white, 5 red and 3 blue chips. A person selects 4 chips without replacement.

Determine P(The fourth chip is blue | The first 2 were white). **(Show work. Final answer must be in decimal form.)**

WWNB

$$\frac{4!}{2!1!1!}\left(\frac{7}{15}\cdot\frac{6}{14}\cdot\frac{5}{13}\cdot\frac{3}{12}\right)$$

$= .2307692308$

Answer

.2307692308

6. Suppose we have a random variable X such that $E[X]=6$ and $Var[X]=98$.

a) Determine $E[25X+5]$.

$E(25x)+E(5)$

$25E(x)+5$

$=25(6)+5 = 155$

Answer

155

b) Determine $E[X^2+3X]$

$Var(x)=E[x^2]-(E[x])^2$

$E[x^2]=Var(x)-(E[x])^2$

$=98-6^2 \Rightarrow 62$

$E[x^2]=62$

$E[x^2]+3E[x]$

$62+3(6)=80$

Answer

80

-2

7. If $Var[X]=8$ (Variance = 8), determine $Var[5-3X]$

$Var(5)-Var(3x)$

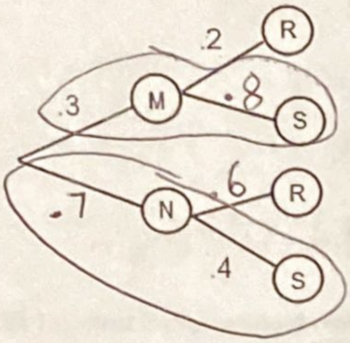
$0-3^2Var(x)$

$=9(8)$

Answer

72

8. The first choice that a person makes is between M and N. The probabilities are given in the tree diagram below. The second choice a person makes is between R and S. Determine the following probabilities. Write in your final decimal answer.



a) P(S) Answer: .52

$(.7)(.4)+(.3)(.8)$

b) P(R | N) Answer: .6

c) P(M | R) Answer: .125

$P(M|R) = \frac{P(M\cap R)}{P(R)}$

$= \frac{.06}{.48} = .125$

d) $P(N \cup S)$ Answer: .94

$P(N\cup S) = P(N) + P(S) - P(N\cap S)$

$.7 + .52 - .28$

9. There are two urns. The first urn has 2 black balls and 1 white ball inside and there are 2 black balls and 2 white balls in the second Urn. You ask your friend to randomly choose an Urn and pick 2 balls from that urn without replacement. **(SHOW ALL WORK.)**

urn1 WW + WB

urn2 WW + WB

9a) What is the probability that the first ball is white?

Urn WB

$\boxed{.8333333333}$

P(white either urn 1st) $= (\frac{1}{3})(\frac{2}{4})$

$(\frac{1}{3})(\frac{2}{4}) + (\frac{2}{2})(\frac{2}{3})$

white urn1, white urn2, Black urn1, Black urn2

$\frac{2!}{1!1!}$ ~~$((\frac{1}{3})(\frac{2}{4}) + (\frac{2}{2})(\frac{2}{3}))$~~

9b) What is the probability that the second ball is black given that the first one was white?

Urn → WB

$\boxed{.8333333333}$

P(white 1st) $= (\frac{1}{3})(\frac{2}{4})$

P(second black) $= (\frac{2}{2})(\frac{2}{3})$

$(\frac{1}{3})(\frac{2}{4}) + (\frac{2}{2})(\frac{2}{3})$

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10. An urn contains 5 Blue Chips numbered 1 through 5 and 4 Red Chips numbered 1 through 4.

a) We select 2 chips **with replacement**. Determine the probability that the numbers on the two chips match.

$B_M \; R_m$ $5+4=9$

$= \frac{2!}{1!1!}\left(\frac{4}{9}\right)\left(\frac{4}{9}\right)$

$= .3950617284$

$\boxed{.3950617284}$

-4

b) We select 2 chips **without replacement**. Determine the probability that the numbers on the two chips match.

$B_M \; R_M$ $M = 1-4$

$= \frac{2!}{1!1!}\left(\frac{4}{9}\right)\left(\frac{4}{8}\right)$

$= .4444444444$

$\boxed{.4444444444}$

c) We select 2 chips **without replacement**.
Determine P(The Number on the Second Chip is a 5 | The First Chip is Blue)

Blue

1 2 3 4 5

$= \frac{\left(\frac{5}{9} \cdot \frac{1}{8}\right)}{\left(\frac{5}{9}\right)}$

$= \frac{2!}{1!1!} \frac{\left(\frac{5}{9} \cdot \frac{1}{8}\right)}{(5/9)}$

$\boxed{\cancel{.125}\ .25}$

d) We select 2 chips without replacement. Determine the probability that the first chip is blue, the second is red and at least one of the numbers is a 3.

All ~~possible~~ combinations $(B_3 r_3) + (B_3 r) + (B r_3)$

$= \left(\frac{1}{9}\right)\left(\frac{1}{8}\right) + \left(\frac{1}{9}\right)\left(\frac{4}{8}\right) + \left(\frac{5}{9}\right)\left(\frac{1}{8}\right)$

$= \cancel{.1388888889}$

$\frac{2!}{1!1!}\left(\frac{1}{9}\right)\left(\frac{1}{8}\right) + \frac{2!}{1!1!}\left(\frac{1}{9}\right)\left(\frac{4}{8}\right) + \frac{2!}{1!1!}\left(\frac{5}{9}\right)\left(\frac{1}{8}\right)$

$= .2777777778$

$\boxed{\cancel{.1388888889}\ .2777777778}$

11. The probability of success for some experiment is .8. The experiment will be performed repeatedly. Determine P(The 6th success will be on the 10th experiment). **Decimal Answer.**

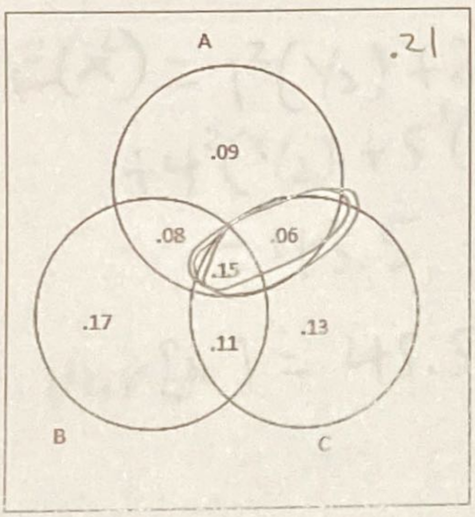
S S S S S F F F F | S

$$\frac{9!}{5!\,4!}\left(\frac{8}{10}\cdot\frac{8}{10}\cdot\frac{8}{10}\cdot\frac{8}{10}\cdot\frac{8}{10}\cdot\frac{2}{10}\cdot\frac{2}{10}\cdot\frac{2}{10}\cdot\frac{2}{10}\cdot\frac{8}{10}\right)$$

$= .0528482304$

$\boxed{.0528482304}$

12. Use the diagram to determine the probabilities:



a) $P(B^C)$ $\boxed{.49}$

b) $P(A \cup C)$ $\boxed{.62}$

c) $P[B|(A\cap C)]$ $\boxed{.714285714}$

$P(B^c) = .13 + .06 + .09 + .21$
$= .49$

$P(A\cup C) = .13 + .11 + .15 + .06 + .08 + .09 = .62$

$$P[B|(A\cap C)] = \frac{P(B\cap(A\cap C))}{P(A\cap C)} = \frac{.15}{.21} = .714285714$$

13. A die is rolled. If we roll a 1, 2, 3, or 4, we will toss 20 coins. If we roll a 5 or 6, we will toss 30 coins. Let X count the number of heads tossed. $P(\text{head}) = 1/2$

The mean of a Binomial is $\mu = np$ and the variance of a Binomial is $\sigma^2 = np(1-p)$.

a) Determine $E[X]$.

x	1	2	3	4	5	6	
$f(x)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$1/2$	$1/2$	

$\boxed{10.5}$

$E(x) = 1(1/2) + 2(1/2) + 3(1/2) + 4(1/2) + 5(1/2) + 6(1/2)$

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b) Determine $Var[X]$.

$Var(x) = E[X^2] - (E[X])^2$

$\boxed{-64.75}$

$E(x^2) = 1^2(1/2) + 2^2(1/2) + 3^2(1/2)$
$+ 4^2(1/2) + 5^2(1/2) + 6^2(1/2)$
$= 45.5$

$Var[x] = 45.5 - (10.5)^2$