HW # 4: Due Tues 9/24 by 11:59pm ET

Graded

Student

Ivan Wang

Total Points

11.25 / 15 pts

Question 1

Problem 1

3 / 4 pts

✔ **– 1 pt** part (b): (i) upper-bound is slightly off. (ii) no *interval* is provided. (iii) description of checking if p-value is in the interval is incorrect.

Question 2

Problem 2

3.5 / 4 pts

✔ **– 0.5 pts** part (c): whether or not the p-value is in the interval is irrelevant. You just need to check whether the *hypothesized* value of the unknown parameter (in this case $p_1 - p_2 = 0$) is in the interval.

Question 3

Problem 3

2 / 4 pts

✔ **– 1.5 pts** parts (a)-(b): need to use the "paired t-test" here since the samples are not independent since each piece of metal contributes two sample values, one in the steel sample and one in the diamond sample.

✔ **– 0.5 pts** part (b): interval provided does not follow from Minitab output, and should be a one-sided confidence interval since the test in part (a) is one-sided. Also, it's irrelevant whether or not the p-value is in the interval. Need to check whether or not the hypothesized value of the parameter is in the interval (in this case, that's $\mu_{\text{diff}} = \mu_1 - \mu_2 = 0$).

Question 4

Problem 4

2.75 / 3 pts

✔ **– 0.25 pts** part (b): answer is slightly off, looks like you started with 0.9866 and replaced with 0.9873, but I was unable to replicate your answer with either of these. You should use 0.9973^{25} which is 0.9346 rather than 0.9335

Question assigned to the following page: 1

Recommended Reading Assignment: Second half of Chapter 4 in the textbook.

Directions: Submit through Gradescope by the due date and time. Show all work (if applicable) to be eligible for full credit. Unless otherwise specified, use an $\alpha = 0.05$ level of significance for all hypothesis tests.

Problem 1.
Two different hardening processes, saltwater quenching and oil quenching, are used on samples of a particular type of metal alloy. The data are contained in the file `hw_data.xlsx` (use the `saltwater` and `oil` variables). Assume that hardness is normally distributed and assume equal variances.

(a) Test the hypothesis that the mean hardness for the saltwater quenching process is lower than the mean hardness for the oil quenching process. Include the value of the test statistic, the p -value, and a well-written conclusion.
Worksheet 1

Two-Sample T-Test and CI: saltwater, oil

Two-Sample T-Test and CI: saltwater, oil

Method

μ_1: population mean of saltwater

μ_2: population mean of oil

Difference: $\mu_1 - \mu_2$

Equal variances are not assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
saltwater	12	147.25	5.34	1.5
oil	12	149.50	4.96	1.4

Estimation for Difference

Difference	90% Upper Bound for Difference
-2.25	0.54

Test

Null hypothesis $\quad H_0: \mu_1 - \mu_2 = 0$

Questions assigned to the following page: 1 and 2

Alternative hypothesis	$H_1: \mu_1 - \mu_2 < 0$

T-Value	DF	P-Value
-1.07	21	0.149

1) (a) Ho: $\cancel{\mu_{salt} \leq \mu_{oil}}$ $\mu_{Salt} \geq \mu_{oil}$

Ha: $\cancel{\mu_{salt} > \mu_{oil}}$ $\mu_{salt} < \mu_{oil}$

Since our p-value = 0.148 > 0.05, we fail to reject Ho. There is not enough evidence that the mean hardness for the saltwater quenching process is lower than the oil quenching process.

(b) Suppose that we decide to use an α = 0.10 level of significance instead of the usual α = 0.05. Determine an appropriate confidence interval for the difference in mean hardness that could be used to test the hypotheses in part (a). Explain how the interval can be used to test the hypotheses, and provide the conclusion of the test.

Estimation for Difference

Difference	90% Upper Bound for Difference
-2.25	0.54

The confidence interval can test the hypothesis if the P-value is contain inside the confidence interval then you failed to reject Ho.
If Ho value is not contained in the confidence interval then reject Ho.

Problem 2.

Two processes are used to produce forgings used in an aircraft wing assembly. Of 60 forgings selected from Process 1, 6 do not conform to the strength specifications, whereas of 100 forgings selected from Process 2, 25 are nonconforming.

(a) Estimate the fraction nonconforming for each process.

Question assigned to the following page: 2

Process 1: $\hat{p}_1 = \frac{6}{60}$

Process 2: $\hat{p}_2 = \frac{25}{100}$

(b) Test the hypothesis that the two processes have identical fractions nonconforming. Include the value of the test statistic, the p-value, and a well-written conclusion.

Test

Null hypothesis	$H_0: p_1 - p_2 = 0$
Alternative hypothesis	$H_1: p_1 - p_2 \neq 0$

Method	Z-Value	P-Value
Normal approximation	-2.32	0.020
Fisher's exact		0.023

The test based on the normal approximation uses the pooled estimate of the proportion (0.19375).

Since P-value = 0.020 < 0.05, we reject Ho. There is enough evidence that the two processes do not have identical fractions nonconforming.

(c) Assuming the same conditions as in part (b), determine a 95% confidence interval for the difference in fraction nonconforming between the two processes. Explain how the interval supports your answer to part (b).

Estimation for Difference

Difference	*95% CI for Difference*
-0.15	*(-0.263864, -0.036136)*

CI based on normal approximation

Since 0.020 is not included in the 95% CI, it supports that conclusion that the two processes do not have identical fractions nonconforming.

Questions assigned to the following page: 3 and 4

Problem 3.
The manufacturer of hardness testing equipment uses steel-ball indenters to penetrate metal that is being tested. However, the manufacturer thinks it would be better to use a diamond indenter so that all types of metal can be tested. Because of differences between the two types of indenters, it is suspected that the two methods will produce different hardness readings. The metal specimens to be tested are large enough so that two indentions can be made on each piece of metal. Therefore, the manufacturer uses both indenters on each specimen and compares the hardness readings. The data are contained in the file `hw_data.xlsx` (use the `steel_ball` and `diamond` variables).

(a) Test the hypothesis that the mean hardness readings are the same for steel-ball and diamond indenters versus the alternative that the diamond indenters yield greater hardness readings. Include the value of the test statistic, the p-value, and a well-written conclusion.

Test

Null hypothesis	H_0: $\mu_1 - \mu_2 = 0$
Alternative hypothesis	H_1: $\mu_1 - \mu_2 < 0$

T-Value	DF	P-Value
-0.86	19	0.201

Since P-Value = 0.201 > 0.05, we fail to reject Ho. This means that there is not enough evidence to conclude that diamond indenter produces greater hardness readings than steel-ball reading.

(b) Construct an appropriate 95% confidence interval that can be used to support your answer to part (a).

Estimation for Difference

Difference	95% Upper Bound for Difference
-2.73	2.77

CI: (0.04, 5.5): since the P-Value is in the CI, we can conclude that there is not enough evidence to conclude that diamond indenter produces greater hardness readings than steel-ball reading.

Problem 4.
Consider an x-control chart for an in-control normally distributed process. With 3-sigma control limits, recall that the probability of a point falling outside the control limits when the process is in-control is approximately 0.0027. The goal of this problem is to demonstrate that even if the process is operating in-control and the probability of a single point plotting outside the control limits is small, we still expect some points to plot outside the control limits, especially when plotting many points.

(a) Assuming independence of samples, what is the probability that at least one of the next 5 points plotted on the control chart will fall outside the control limits?

Question assigned to the following page: 4

(b) Assuming independence of samples, what is the probability that at least one of the next 25 points plotted on the control chart will fall outside the control limits?

ⓐ

$P(\text{in control limits}) = 1 - P(\text{outside control limits}) = 1 - 0.0027 = 0.9973$

$P(\text{5 within limits}) = (0.9973)^5 = 0.9866$

$P(\text{at least one falls outside control limits}) = 1 - 0.9866 = 0.0134$

ⓑ $P(\text{25 within limits}) = (0.9866)^{25} = 0.9335$

$P(\text{at one outside}) = 1 - 0.9335 = 0.0665$