HW # 11: Due Tues 11/19 by 11:59pm ET

Graded

Student

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Total Points

9.25 / 15 pts

Question 1

Problem 1

1.75 / 4 pts

- ✔ **− 1.75 pts** part (a): missing the stead state limits for case (i) and missing the limits for $i = 1, 2, 3$ for case (ii).
- ✔ **− 0.5 pts** part (b): "the control limits increase" is not correct, since the UCL increases, but the LCL decreases. Also increasing λ will increase the ARL.

Question 2

Problem 2

3.5 / 6 pts

- ✔ **− 0.5 pts** part (a); value of MR-bar is incorrect
- ✔ **− 1.5 pts** part (b): computed limits are incorrect (it appears you're substituting value of L for λ) and do not match the limits in the Minitab chart. The Minitab chart is also incorrect.
- ✔ **− 0.5 pts** part (d): computed value of z_2 is incorrect

Question 3

Problem 3

4 / 5 pts

- ✔ **− 0.5 pts** part (a): Minitab chart is not correct as there limits should not increase around samples 77-80, and the limits labeled should be the "steady-state" limits.
- ✔ **− 0.5 pts** part (b): a bit off, as what you call M_3 is actually M_4, what you call M_4 is actually M_5, and so on.

Question assigned to the following page: 1

Problem 1. Consider a process with target 25 and process standard deviation 8.

(a) Recall that the control limits for an EWMA-control chart depend on i(the sample number). As $i \to \infty$, the control limits approach the steady-state limits. For the following combinations of λ and L, determine the control limits and center line that should be used for $i = 1, 2, 3$, as well as the steady-state limits: (i) $\lambda = 0.15$, $L = 2.8$, (ii) $\lambda = 0.25$, $L = 2.8$.

(b) Based on part (a), for a fixed L, how does changing λ appear to affect the control limits? What does this mean in terms of ARL_0, the average run length for an in-control process?

Question assigned to the following page: 1

(a)

$$UCL = \mu_0 + L \cdot \sigma \cdot \sqrt{\frac{\lambda}{2-\lambda} \cdot \left(1-(1-\lambda)^{2i}\right)}$$

$$CL = \mu_0$$

$$LCL = \mu_0 - L \cdot \sigma \cdot \sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)}$$

$$UCL = \mu_0 + L \cdot \sigma \cdot \sqrt{\frac{\lambda}{2-\lambda}}$$

$$CL = \mu_0$$

$$LCL = \mu_0 - L \cdot \sigma \cdot \sqrt{\frac{\lambda}{2-\lambda}}$$

$\mu_0 = 25$, $\lambda = 0.15$, $L = 2.8$

$\sigma = 8$

$\underline{i=1}$:

$$UCL = 25 + 2.8 \times 8 \times \sqrt{\frac{0.15}{2-0.15}\left(1-(1-0.15)^{2\times 1}\right)} = 28.36$$

$$CL = 25$$

$$LCL = 25 - 2.8 \times 8 \times \sqrt{\frac{0.15}{2-0.15}\left(1-(1-0.15)^{2\times 1}\right)} = 21.64$$

$\underline{i=2}$:

$$UCL = 25 + 2.8 \times 8 \times \sqrt{\frac{0.15}{2-0.15}\left(1-(1-0.15)^{2\times 2}\right)} = 29.41$$

$$CL = 25$$

$$LCL = 25 - 2.8 \times 8 \times \sqrt{\frac{0.15}{2-0.15}\left(1-(1-0.15)^{2\times 2}\right)} = 20.59$$

$\underline{i=3}$:

$$UCL = 25 + 2.8 \times 8 \times \sqrt{\frac{0.15}{2-0.15}\left(1-(1-0.15)\right)^{2\times 3}} = 30.03$$

$$CL = 25$$

$$LCL = 25 - 2.8 \times 8 \times \sqrt{\frac{0.15}{2-0.15}\left(1-(1-0.15)^{2\times 3}\right)} = 19.97$$

$$UCL = 25 + 2.8 \times 8 \times \sqrt{\frac{0.25}{2-0.25}} = 33.47$$

$$CL = 25$$

$$LCL = 25 - 2.8 \times 8 \times \sqrt{\frac{0.25}{2-0.25}} = 16.53$$

(b) As λ increases, the control limits also increases because $\sqrt{\frac{\lambda}{2-\lambda}}$ increases as λ increases.

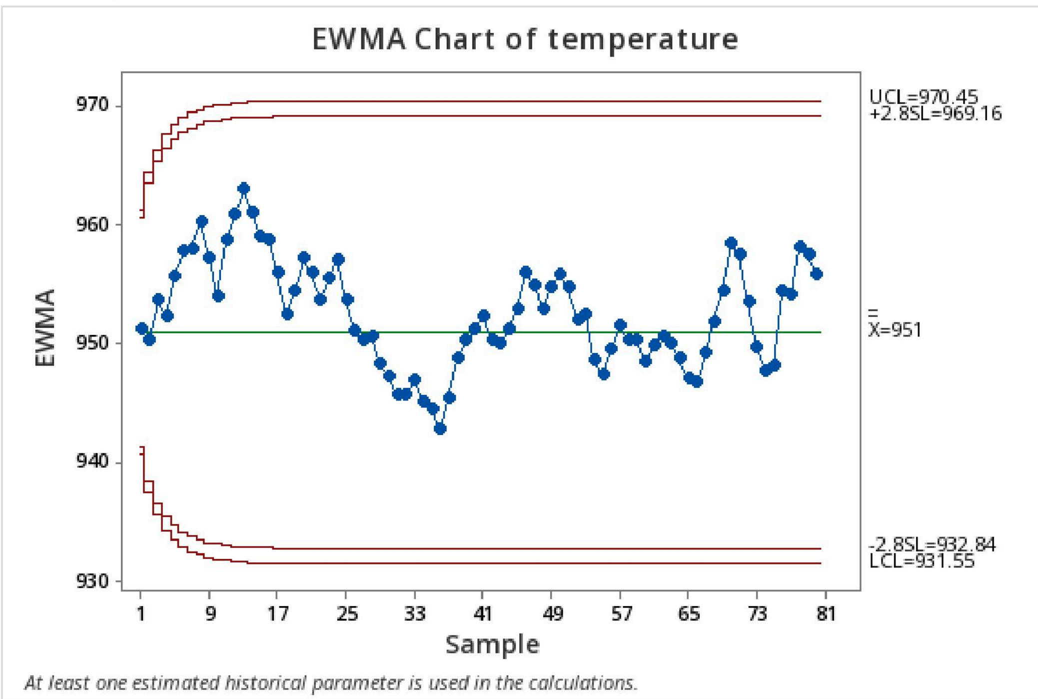
- Increasing λ will decrease the ARL_0 because there of the number of point of out of control as the chart becomes more sensitive to out of change.

Question assigned to the following page: 2

Problem 2.
Temperature readings from a chemical process (in degrees Celsius) were taken every two min-utes. The data is contained in column W of the `hw_data.xlsx` file. Suppose the target value is $\mu_0 = 951$.

(a) Estimate the process standard deviation using $\sigma = MR/d_2$.
$\sigma = 22.772606383$

(b) Show how to compute the steady-state limits using $\lambda = 0.15$ and $L = 2.8$. Then use Minitab to create the control chart (include the chart, and make sure your computed limits agree with Minitab's).



Question assigned to the following page: 2

a) $\overline{MR} = 25.6875$ $\qquad \hat{\sigma} = \frac{25.6875}{1.128} = 22.77266383$

$d_2 = 1.128$

b) $UCL = \mu_0 + L \cdot \sigma \cdot \sqrt{\frac{\lambda}{2-\lambda}}$

$CL = \mu_0$

$LCL = \mu_0 - L \cdot \sigma \cdot \sqrt{\frac{\lambda}{2-\lambda}}$

$\lambda = 0.15$

$L = 2.8$

$\hat{\sigma} = 22.77266383$

$\mu_0 = 951$

$UCL = 951 + 2.8 \cdot 22.77266383 \cdot \sqrt{\frac{2.8}{2-2.8}} =$

969.1564331

$CL = 951$

$LCL = 932.8435669$

(c) Based on the chart in part (b), is the process in-control? If not, identify the sample number and the value of the exponentially weighted moving average for each out-of-control point.

There doesn't seem to be any points that fall outside the process in-control limits.

(d) Show how to compute the values of z_0, z_1, z_2, and z_3 using the appropriate formulas.

Questions assigned to the following page: 2 and 3

$\boxed{z_0, z_1, z_2, z_3}$

$x_1 = 953$

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}$$

$$z_0 = \mu_0 = 951$$

$$z_1 = 0.15 \times 953 + 0.85 \times 951 = 951.3$$

$$z_2 = 0.15 \times 945 + 0.85 \times 951.3 = 90.355$$

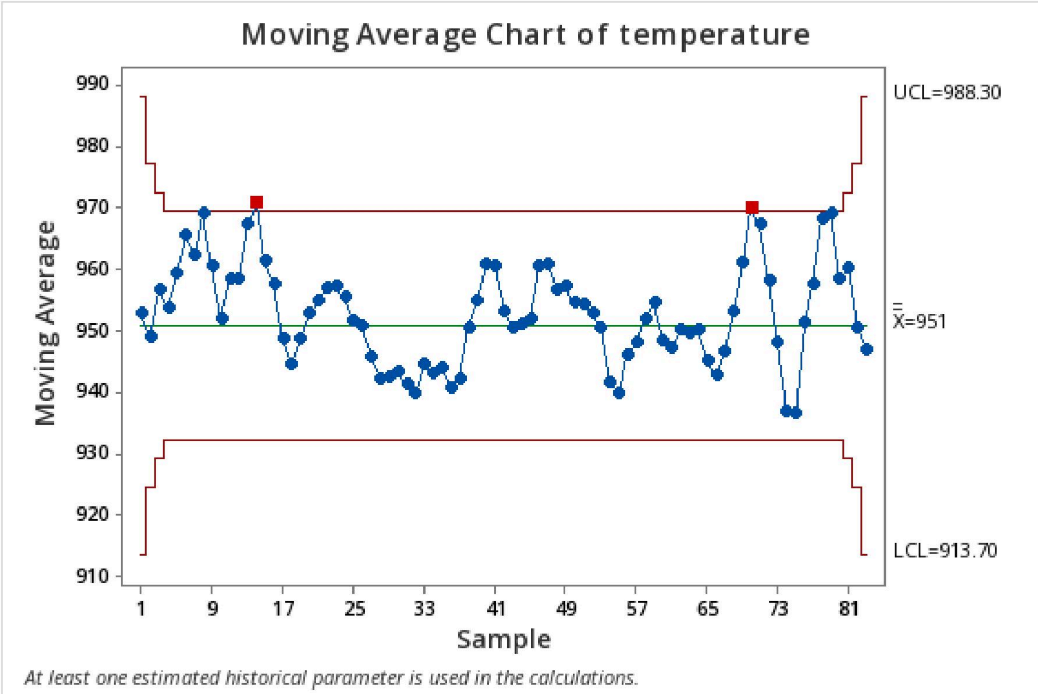
$$z_3 = 0.15 \times 972 + 0.85 \times 90.355 = 222.60175$$

Problem 3.

Temperature readings from a chemical process (in degrees Celsius) were taken every two min-utes. The data is contained in column W of the `hw_data.xlsx` file. Suppose the target value is $\mu_0 = 951$.

(a) Show how to compute the center line and the 3-sigma control limits for the span- 4 moving average control chart (remember that there are 4 sets of limits for a span- 4 chart!). Include a copy of the Minitab chart to support your calculations.

Question assigned to the following page: 3


Moving Average Chart of temperature
990
980
970
960
950
940
930
920
910
Moving Average
1
9
17
25
33
41
49
57
65
73
81
Sample
UCL=988.30
$\bar{\bar{X}}$=951
LCL=913.70
At least one estimated historical parameter is used in the calculations.

Question assigned to the following page: 3

$$UCL = \mu_0 + L \cdot \frac{\sigma}{\sqrt{i}}$$

$$CL = \mu_0$$

$$LCL = \mu_0 - L \cdot \frac{\sigma}{\sqrt{i}}$$

$\mu_0 = 951$

$\sigma = 14.44533563$

$L = 3$

$w = 4$

$$UCL = \mu_0 + L \cdot \frac{\sigma}{\sqrt{w}}$$
$$CL = \mu_0$$
$$LCL = \mu_0 - L \cdot \frac{\sigma}{\sqrt{w}}$$

$i=1$:

$$UCL = 951 + 3 \cdot \frac{14.44}{\sqrt{1}} = 994.335$$
$$CL = 951$$
$$LCL = 951 - 3 \cdot \frac{14.44}{\sqrt{1}} = 907.665$$

$i=2$:

$$UCL = 951 + 3 \cdot \frac{14.44}{\sqrt{2}} = 981.687$$
$$CL = 951$$
$$LCL = 951 - 3 \cdot \frac{14.44}{\sqrt{2}} = 920.313$$

$i=3$:

$$UCL = 951 + 3 \cdot \frac{14.44}{\sqrt{3}} = 976.500$$
$$CL = 951$$
$$LCL = 951 - 3 \cdot \frac{14.44}{\sqrt{3}} = 925.492$$

$i=4$:

$$UCL = 951 + 3 \cdot \frac{14.44}{\sqrt{4}} = 972.667$$
$$CL = 951$$
$$LCL = 951 - 3 \cdot \frac{14.44}{\sqrt{4}} = 929.333$$

$$UCL = 951 + 3 \cdot \frac{14.44}{\sqrt{4}} = 972.67$$
$$CL = 951$$
$$LCL = 951 - 3 \cdot \frac{14.44}{\sqrt{4}} = 929.33$$

(b) Show how to compute the moving averages M_3, M_4, M_5, and M_6.

Question assigned to the following page: 3

$$M_i = \frac{x_{i-1} + x_i + x_{i+1} + x_{i+2}}{4}$$

$$M_3 = \frac{953 + 945 + 972 + 945}{4} = \frac{3815}{4} = 953.75$$

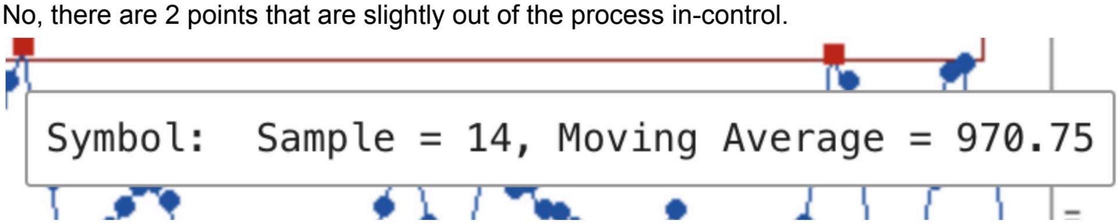
$$M_4 = \frac{945 + 972 + 945 + 975}{4} = \frac{3837}{4} = 959.25$$

$$M_5 = \frac{972 + 945 + 975 + 970}{4} = \frac{3862}{4} = 965.5$$

$$M_6 = \frac{945 + 975 + 970 + 954}{4} = \frac{3844}{4} = 962.25$$

(c) Is the process in-control? If not, identify the sample number and the value of the moving average for each out-of-control point.

No, there are 2 points that are slightly out of the process in-control.



And