# HW # 2: Due Tues 9/10 by 11:59pm ET

Graded

2 Hours, 49 Minutes Late

**Student**

Ivan Wang

**Total Points**

**13.25 / 15 pts**

**Question 1**

## Problem 1

**2.75** / 3 pts

✔ **– 0.25 pts** part (c): construct and *interpret*...

**Question 2**

## Problem 2

**3** / 3 pts

✔ **– 0 pts** Correct

**Question 3**

## Problem 3

**3** / 3 pts

✔ **– 0 pts** Correct

**Question 4**

## Problem 4

**2** / 2 pts

✔ **– 0 pts** Correct

**Question 5**

## Problem 5

**2.5** / 4 pts

✔ **– 1.5 pts** part (d) not submitted.

Question assigned to the following page: 1

Problem 1.

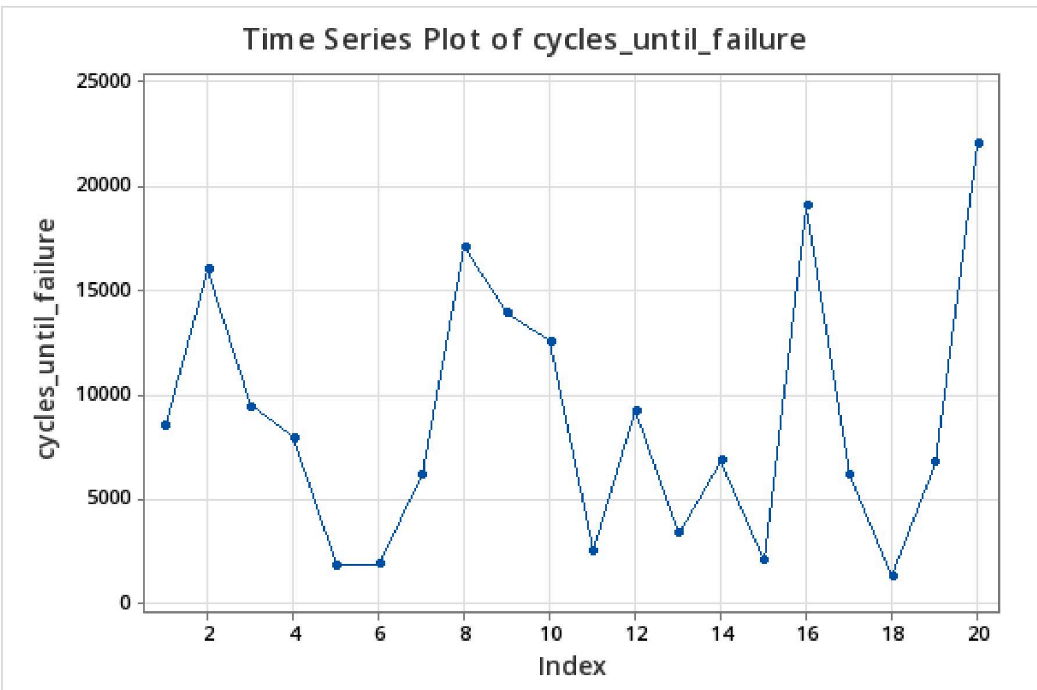
Use the variable `cycles_until_failure` in `hw_data.xlsx` for this question, which gives the number of cycles until failure for a certain aluminum component.

(a) Determine the sample average and the sample standard deviation.

Statistics

| Variable | N | N* | Mean | SE Mean | StDev | Minimum | Q1 | Median | Q3 | Maximum |
|---|---|---|---|---|---|---|---|---|---|---|
| cycles_until_failure | 20 | 0 | 8750.25 | 1383.03 | 6185.10 | 1334 | 2773.25 | 7416 | 13571.8 | 21997 |

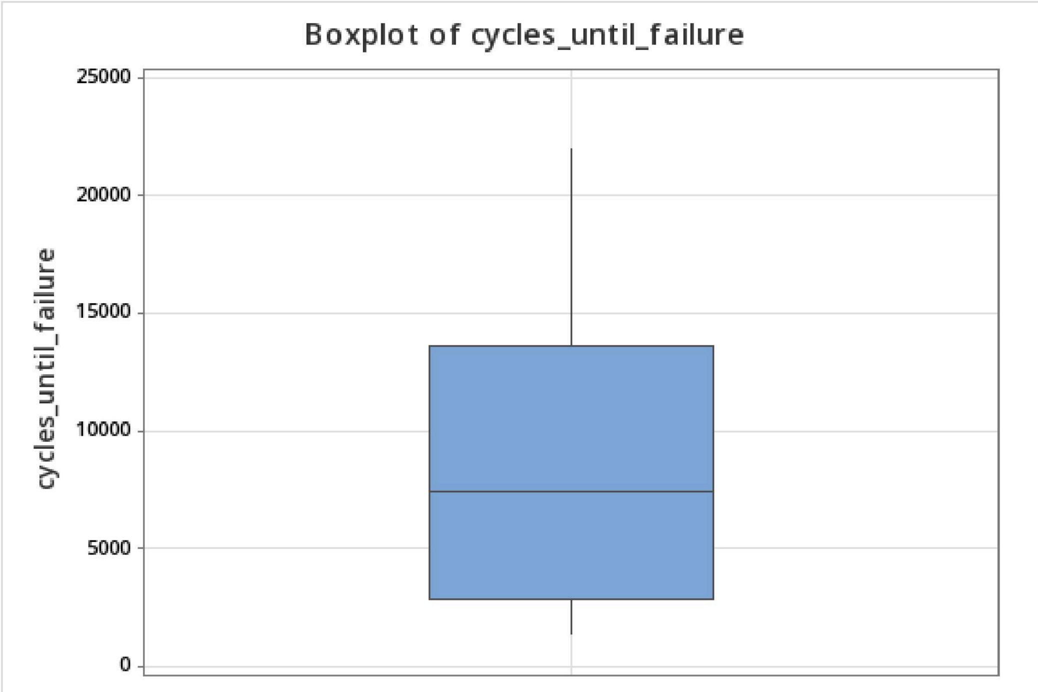
(b) Assuming the data is in time order, construct and interpret a marginal (or time-series) plot.




The only noticeable trend is that the time series would occasionally dip to around the same cycle until failure point and then the graph would increase.

(c) Construct and interpret a boxplot for this data. Label all the important quantities on the boxplot.

Questions assigned to the following page: 1 and 2




```
Q1 = 2773.25
Median = 7416
Q3 = 13571.8
IQRange = 10798.5
Whiskers to: 1334, 21997
N = 20
```

Problem 2.
A production process operates with 5% nonconforming output. Each hour, a sample of 20 units is taken and the number of nonconforming units counted.

(a) What is the expected number of nonconforming items in a sample?
E[x] = 0.05 x 20 = 1

(b) Suppose the process is stopped if two or more nonconforming items are found in a sample.The quality control technician must search for the cause before production can be resumed. Determine the probability that production is stopped.

Questions assigned to the following page: 2 and 3

$n = 20$

Probability of Success $(p) = 0.05$

Number of Successes $(K) = P(X \le 2)$

$$\text{Pmf: } f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\text{mean: } \mu = np$$

$$\text{Variance: } \sigma^2 = np(1-p)$$

$$P(X=0) = \binom{20}{0} (0.05)^0 (0.95)^{20} = (0.95)^{20} = 0.3585$$

$$P(X=1) = \binom{20}{1} (0.05)^1 (0.95)^{19} = 20(0.05)(0.95)^{19} = 0.3774$$

$$P(X<2) = 0.3585 + 0.3774 = 0.7359$$

$$P(X>2) = 1 - P(X<2) = 1 - 0.7359 = 0.2641$$

Problem 3.

The tensile strength of a metal part is normally distributed with mean 125 pounds and standard deviation 5 pounds.

(a) Suppose the lower specification limit for this part is 115 pounds tensile strength. If one million parts are produced, how many parts do we expect to NOT meet the minimum required tensile strength?

$$Z = \frac{x-\mu}{\sigma} \Rightarrow Z = \frac{115-125}{5} = \frac{-10}{5} = -2$$

$$P(Z<-2) = 0.0228$$

Expected Number $= 1{,}000{,}000 \times 0.0228 = 22{,}800$

Questions assigned to the following page: 3 and 4

(b) Suppose we decide to offer a warranty on this part. If the tensile strength is less than or equal to x, the part will be replaced. If the expectation is that we will have to replace 0.25% of parts under the warranty, determine x.

$\mu = 125$ $\quad\quad$ $0.25\% = 0.0025$

$\sigma = 5$ $\quad\quad$ Z-score $\approx -2.807$

$x = \mu + Z \times \sigma$

$x = 125 + (-2.807) \times 5 = 125 - 14.035 = 110.965$

Problem 4.
Provide a short description of each of the steps in the DMAIC process.

DMAIC is a structured, five-step problem-solving procedure that can be used to successfully complete projects by proceeding through and implementing solutions that are designed to solve root cause of quality and process and to ensure best practices are permanent and reusable.

Define - Identity business improvement opportunity and establish customer requirements, project charter, and build team.

Measure - Determine what to measure, data collection, validate measure systems, and determine sigma performance level.

Analyze - Analyze data to understand reasons for variation and identify potential root causes, investigate root cause hypotheses.

Improve - Generate and quantify potential solutions, evaluate and select final solution, verify and gain approval for final solution.

Control - Develop ongoing process management plans, mistake proof process, monitor and control critical process characteristics, develop out of control action plans.

Question assigned to the following page: 5

Problem 5.
A manufacturer of calculators offers a one-year warranty. If the calculator fails for any reason during this period, it is replaced. Suppose the time to failure (in years) is exponentially distributed with probability density function (pdf) $f(x) = 0.25e^{-0.25x}$
For $x > 0$.

(a) What percentage of calculators will need to be replaced according to the warranty?

Exponential with mean = 1

| x | P( X ≤ x ) |
|---|---|
| 0.25 | 0.221199 |

Approximately 22.1199% of calculators need to be replaced under the one-year warranty.

(b) Each calculator costs \$10 to manufacture and is sold for \$30. Suppose 100,000 calculators are produced and sold. Warranty replacements are manufactured as needed at a cost of \$10 per calculator. Note that warranty replacement calculators are not sold, but rather given to the customers whose calculators have failed. Determine the expected profit.

Question assigned to the following page: 5

Number of failed calculators = 100,000 × 0.2212 = 22,120

Revenue = 100,000 × 30 = 3,000,000

Manufacturing cost = 100,000 × 10 = 1,000,000

Replacement cost = 22,120 × 10 = 221,200

Expected Profit = 3,000,000 − 1,000,000 − 221,200

= 1,778,800

(c) What is the expected lifetime of a calculator?

$$E(x) = \frac{1}{\lambda} \qquad \lambda = 0.25$$

$$E(x) = \frac{1}{0.25} = 4 \text{ years}$$

(d) Some process improvements are made, and the expected lifetime computed in (c) doubles. This leads to a revised pdf describing the time to failure for the calculators. Using the revised pdf, redo parts (a) and (b) to see the effect of the improvements.