# HW # 6: Due Tues 10/8 by 11:59pm ET

Graded

2 Hours, 9 Minutes Late

**Student**

Ivan Wang

**Total Points**

**10 / 15 pts**

**Question 1**

## Problem 1

**5** / 9 pts

- ✔ **– 0 pts** part (a): while you will arrive at the same answer in the case of equal sample sizes, you are not computing $\overline{\overline{x}}$ correctly. You are taking the mean of each column and averaging those, whereas you should instead take the mean of each row and average those.
- ✔ **– 1.5 pts** part (b): here you need to compute $\widehat{\sigma} = \frac{\overline{s}}{c_4}$ and $\widehat{\sigma} = \frac{s_p}{c_4}$, where the special $N$ is used in the second case.
- ✔ **– 0.5 pts** part (c): missing the $c_4$ from the calculations, which is why your computed limits disagree with Minitab's.
- ✔ **– 2 pts** part (d): missing the calculations.

**Question 2**

## Problem 2

**5** / 6 pts

- ✔ **– 0.5 pts** part (c): mostly the right idea, except you're using the standard deviation of $X$ rather than the standard deviation of $\overline{x}$. Since this question deals with a value of $\overline{x}$ plotting inside the control limits, need to use the distribution of $\overline{x}$.
- ✔ **– 0.5 pts** part (a): mostly correct, except double-check your calculation of the 1.28

Question assigned to the following page: 1

**Problem 1.** Suppose 20 random samples of size n = 5 were collected from some normally distributed process. The data is contained in columns G through K in the `hw_data.xlsx` file. The process mean μ and the process standard deviation σ are assumed to be unknown.

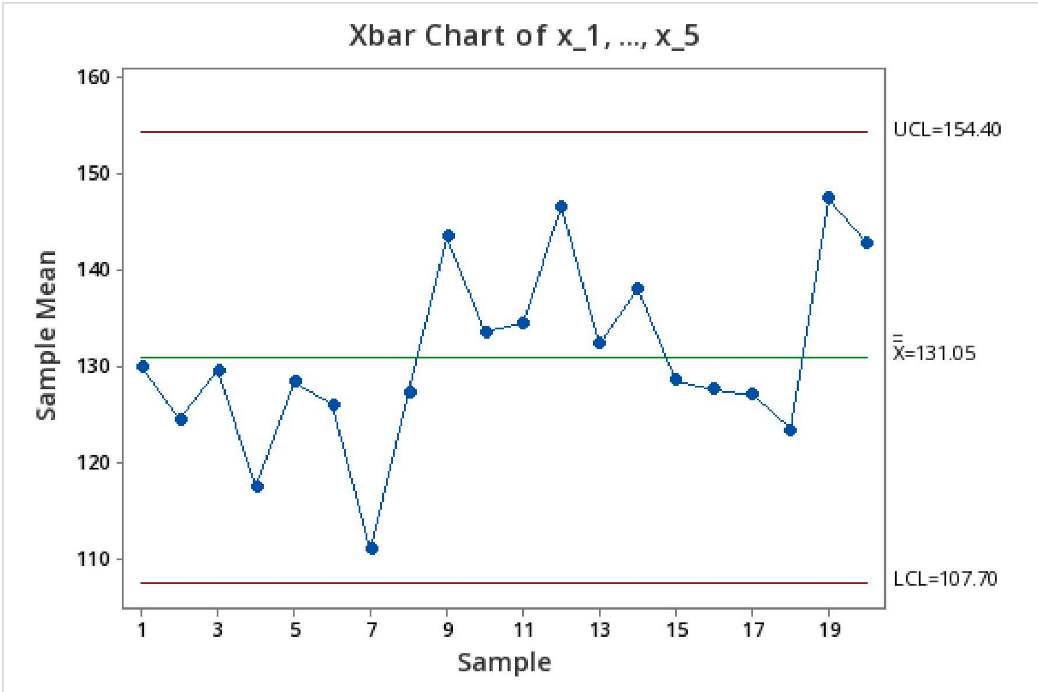
(a) Use the data to estimate the process mean.

Statistics

| Variable | N | N* | Mean | SE Mean | StDev | Minimum | Q1 | Median | Q3 | Maximum |
|---|---|---|---|---|---|---|---|---|---|---|
| x_1 | 20 | 0 | 129.43 | 3.07032 | 13.7309 | 102.8 | 118.975 | 131.2 | 139.35 | 149.3 |
| x_2 | 20 | 0 | 124.16 | 4.47432 | 20.0097 | 84.3 | 111.1 | 125 | 138.825 | 160.2 |
| x_3 | 20 | 0 | 135.58 | 3.76144 | 16.8217 | 105 | 124.05 | 131.2 | 150.7 | 165.1 |
| x_4 | 20 | 0 | 133.845 | 3.23534 | 14.4689 | 109 | 120.8 | 136.2 | 146.85 | 155 |
| x_5 | 20 | 0 | 132.24 | 5.17061 | 23.1237 | 92.3 | 112.2 | 135 | 150.8 | 173.2 |

(129.43 + 124.16 + 135.58 + 133.84 + 132.24)/5 = 131.05

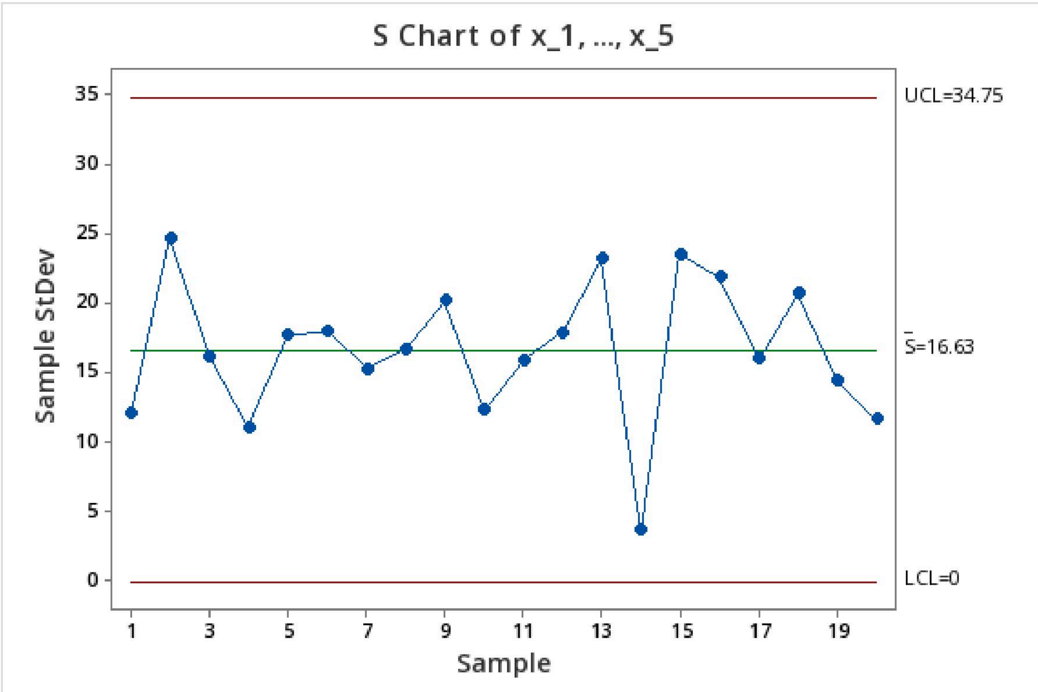
Question assigned to the following page: 1



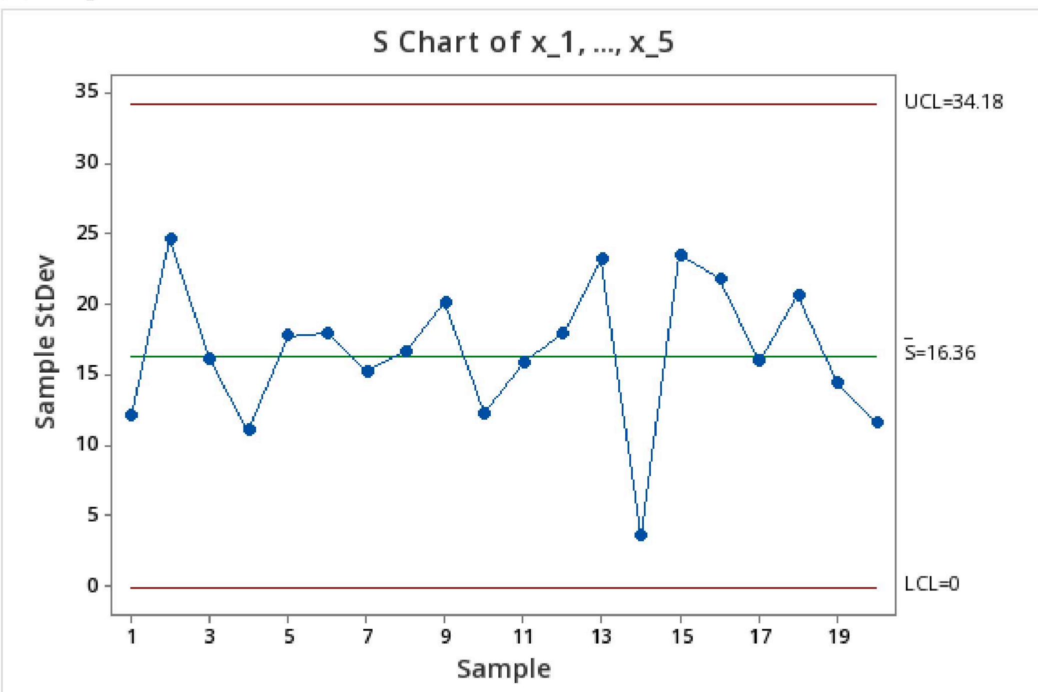

(b) Use the data to estimate the process standard deviation using (i) the s -bar method and (ii) the pooled- s method.

Question assigned to the following page: 1

(i) the s -bar method




(ii) the pooled- s method



Question assigned to the following page: 1

(c) Use the appropriate formulas to compute the center line and the 3-sigma control limits for an x -control chart using the s-bar method. Then use Minitab to create this control chart (include the chart, and make sure your computed limits agree with Minitab's).

$$UCL = \hat{\mu} + 3 \cdot \frac{\bar{s}}{\sqrt{n}}$$

$$CL = \hat{\mu}$$

$$LCL = \hat{\mu} - 3 \cdot \frac{\bar{s}}{\sqrt{n}}$$

$n = 5$

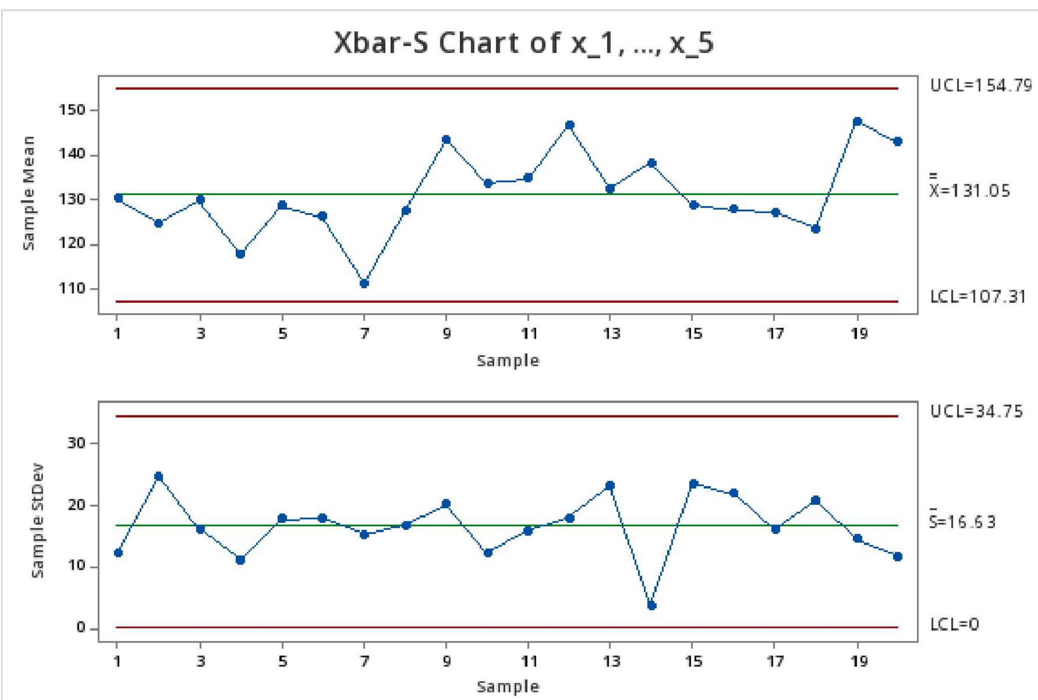
$\hat{\mu} = 131.05$

$\bar{s} = 16.6345$

$UCL = 153.367523663$

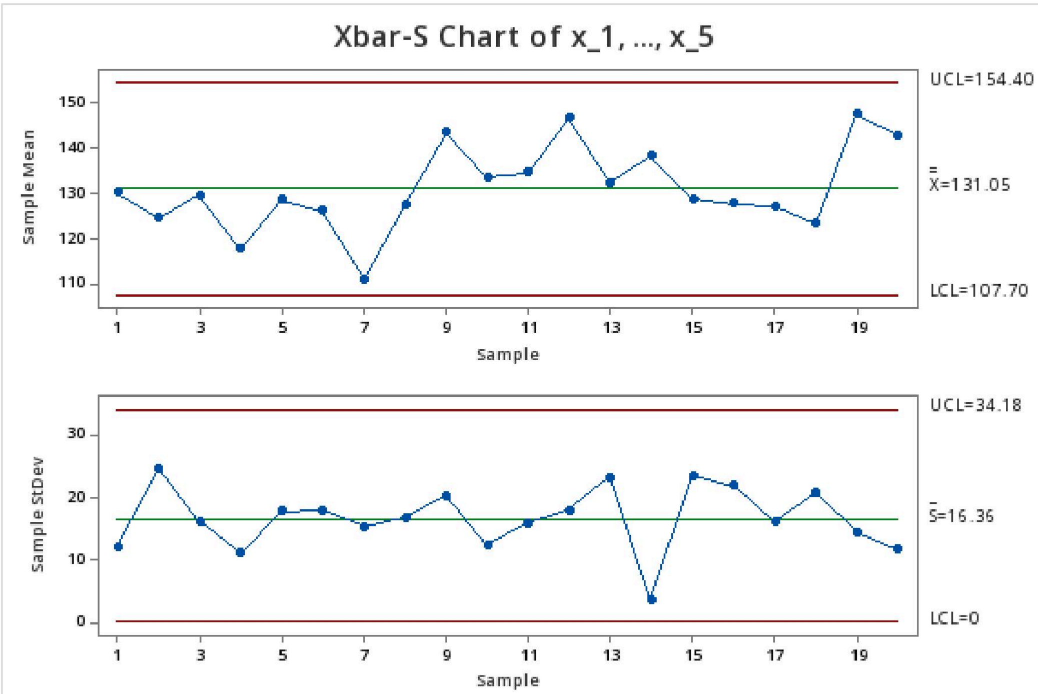
$CL = 131.05$

$LCL = 108.732476337$



Question assigned to the following page: 1

(d) Repeat part (c) using the pooled- s method.



Question assigned to the following page: 2

**Problem 2**. An x -control chart produced using the R -bar method with R = 9 has the following control limits:

X−chart LCL = 360, CL = 363, UCL = 366

The sample size is n = 10, the chart indicates the process average is in-control, and the process is assumed to be normally distributed.

(a) What is the α -risk (i.e., level of significance) associated with the x -chart?

a

Standard Error of Mean (SEM) $= \dfrac{R}{d_2 \cdot \sqrt{n}}$ $\quad d_2 = 3.078$

$$= \frac{9}{3.078 \cdot \sqrt{10}} = 1.28$$

LCL: $\dfrac{360-363}{1.28} = -2.34$

UCL: $\dfrac{366-363}{1.28} = 2.34$

$P(Z<-2.34) + P(Z>2.34) = 0.0096 + 0.0096$
$= 0.0192$

Question assigned to the following page: 2

(b) Specifications on this quality characteristic are 362 ± 6. What proportion of items produced will meet specifications?

(b) $362 \pm 6 = (356, 368)$

$$\sigma = \frac{\bar{R}}{d_2} = \frac{9}{3.078} = 2.93$$

$$LSL: \frac{356-368}{\sigma} = \frac{-7}{2.93} = -2.39$$

$$USL: \frac{368-363}{\sigma} = \frac{5}{2.93} = 1.71$$

- For LSL: $P(Z < -2.39) = 0.0084$
- For USL: $P(Z < 1.71) = 0.9564$

$$P(LSL < X < USL) = 0.9564 - 0.0084 = 0.9480$$

(c) Suppose the process means shifts to 361. What is the probability that the shift will not be detected on the first sample following the shift?

Question assigned to the following page: 2

$$Z_{LCL}: \frac{360-361}{\sigma} = \frac{-1}{2.93} = -0.34$$

$$Z_{UCL}: \frac{366-361}{\sigma} = \frac{5}{2.93} = 1.71$$

- For $Z_{LCL}$: $P(Z < -0.34) \approx 0.3662$
- For $Z_{UCL}$: $P(Z < 1.71) \approx 0.9564$

$P(Z_{LCL} < X < Z_{UCL}) = 0.9564 - 0.3662 = 0.5902$

59% of the shift will not be detected on the first sample follows the shift.