HW # 12: Due Thurs 12/5 by 11:59pm ET

Graded

Student

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Total Points

10.5 / 15 pts

Question 1

Problem 1

5 / 7 pts

✔ **– 1 pt** part (d): answer of 203.33 is incorrect, and not exactly clear what you're trying to accomplish with the supporting details. Need to consider the proportion of items expected to be scrap and rework in order to determine the optimal mean.

✔ **– 1 pt** part (e): refer to the formula in the Ch. 8 notes for the confidence interval for C_{pk}. Your method assumes the standard deviation of $\widehat{C}_{pk}$ is $\frac{\widehat{\sigma}}{\sqrt{n}}$, which isn't the case.

Question 2

Problem 2

3 / 4 pts

✔ **– 1 pt** part (b): refer to the formula in the Ch. 8 notes for the confidence interval for C_p. Your method assumes the standard deviation of $\widehat{C}_p$ is $\frac{1}{\sqrt{n}}$, which isn't the case.

Question 3

Problem 3

2.5 / 4 pts

✔ **– 1.5 pts** part (a): for this test, refer to the table in the Ch. 8 notes to obtain the sample size n and the critical value C.

Question assigned to the following page: 1

Problem 1. A process is in statistical control, and a recent sample of size n = 20 resulted in μ = x = 201 and σˆ = 1.75. Specification limits are set at 202 ± 4. Assume the quality characteristic of interest is normally distributed.

(a) Compute the capability ratios ˆCp and ˆCpk.

(a)

$n = 20$

$\mu = \bar{x} = 201$

$\hat{\sigma} = 1.75$

$USL = 206$

$LSL = 198$

$$\hat{C}_p = \frac{206-198}{6(1.75)} = 0.7619047619$$

$$\hat{C}_{pk} = \min\left(\frac{206-201}{3(1.75)}, \frac{201-198}{3(1.75)}\right)$$

$$0.95238095238 \qquad 0.57142857142$$

$$\hat{C}_{pk} = 0.57142857142$$

b) Which capability ratio from part (a) is more appropriate to measure process capability in this situation?
Provide a reason to support your answer.

Cpk is more appropriate to measure process capability in this situation because it uses the process mean location to the specification limit, while Cp considers the spread of the process.

(c) Items that are produced above the upper specification limit must be scrapped, while items that are below the lower specification limit can be reworked. What proportion of the process output is scrap and what proportion is rework?

Question assigned to the following page: 1

(c) $\frac{USL - \hat{\mu}}{\hat{\sigma}} = \frac{206 - 201}{1.75} = 2.85714285714$

$P(\text{scrap}) = P(Z > 2.857) = 0.0021$

$\frac{LSL - \hat{\mu}}{\hat{\sigma}} = \frac{198 - 201}{1.75} = -1.71428571429$

$P(\text{rework}) = P(Z < -1.714) = 0.0433$

(d) Because scrap is more expensive than rework, the process should be centered so that the process mean
is closer to the lower specification limit than to the upper specification limit.
If scrap is twice as
expensive as rework, determine the optimal process mean.

Question assigned to the following page: 1

(d) $$\frac{\mu^* - LSL}{USL - \mu^*} = 2$$

$$\mu^* - LSL = 2(USL - \mu^*)$$

$$\Rightarrow \mu^* - 198 = 2(206 - \mu^*)$$

$$\mu^* - 198 = 412 - 2\mu^*$$

$$+2\mu^* \; +198 \qquad +198 \; +2\mu^*$$

$$3\mu^* = 610$$

$$\mu^* = 203.33$$

(e) Determine an approximate 95% confidence interval estimate for Cpk.

Questions assigned to the following page: 1 and 2

e) $\hat{C}_{pk} \pm Z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}$

$= 0.571 \pm 1.96 \cdot \frac{1.75}{\sqrt{20}}$

$\Rightarrow (-0.19597131628, 1.33797131628)$

Problem 2. A process is in-control with $x = 105$, $R = 3.95$, and $n = 5$. The process specifications are at 105 ± 6. The quality characteristic of interest is normally distributed. Use $\hat{\mu} = x$ and $\hat{\sigma} = R/d_2$ to estimate the process mean and standard deviation, respectively.

(a) Estimate process capability using an appropriate capability ratio.

Question assigned to the following page: 2

(a) $\bar{X} = 105$

$R = 3.95$

$n = 5$

$$\hat{\sigma} = \frac{R}{d_2} = \frac{3.95}{2.326}$$

$$= 1.69819432502$$

$USL = 111$

$LSL = 99$

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} \Rightarrow \frac{111-99}{6(1.698)}$$

$$= 1.17785630153$$

(b) Using n = 5, determine a 95% confidence interval estimate for C_p.

Questions assigned to the following page: 2 and 3

(b) $CI = \hat{C_p} \pm Z_{1-\alpha/2} \cdot \sqrt{\frac{1}{n}}$

$1.178 \pm 1.96 \cdot \sqrt{\frac{1}{5}}$

$(0.30146135282,\ 2.05453864718)$

Problem 3.

Suppose you were asked to perform a test of the hypotheses

$$H_0:\ C_p = 1.5$$
$$H_1:\ C_p > 1.5$$

Using $C_p(\text{Low}) = 1.5$, $C_p(\text{High}) = 2.325$, and $\alpha = \beta = 0.05$.

(a) Determine the sample size n and the critical value C that should be used to perform this test.

Question assigned to the following page: 3

(a) $\Delta = C_p(\text{High}) - C_p(\text{low}) = 2.325 - 1.5 = 0.825$

$$n = \left(\frac{1.645 + 1.645}{0.825/0.2}\right)^2 = 0.63612708907$$

$$C(\text{critical}) = C_p(\text{low}) + \Delta$$

$$= 1.5 + 0.825$$

(b) Suppose a sample of the appropriate size is collected and the capability ratio is estimated to be $\hat{C}_p = 1.9$. What is the conclusion of the test?

Question assigned to the following page: 3

(b) $\hat{C_p} = 1.9$

Critical value $C = 2.235$

Therefore, $\hat{C_p} = 1.9 < C(\text{critical}) = 2.235$,

we fail to reject H_0.

There is no significant evidence that the process capability > 1.5 at the 5% significance level.