IR/chapter 18

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Rank: Equal to number of linearly independent rows.

How to calculate rank?
→ use row-echelon method

For a diagonal matrix, rank is equal to number of non-zero diagonal entries.

Eigens

Given:- square matrix C (M x M)

$C\vec{x} = \lambda \vec{x}$

λ — eigen value, $\vec{x}$ — eigen vector

$(C - \lambda I)\vec{x} = 0$ — (1)

solve $|C - \lambda I| = 0$ to get all eigen values

Put each each eigen value in equation and solve for eigen vector $\vec{x}$

Calculator:- (eigen values)

For a 3x3 matrix A

$\lambda^3 - trace(A).\lambda^2 + trace(adjA).\lambda - det(A) = 0$

$adjA = A^{-1} * |A|$

or

$trace(adjA)$ = sum of minors of diagonal entries

Matrix Decomposition :-

→ Eigen Decomposition :-

$S = U \Lambda U^{-1}$

S: M x M matrix

U: cols are eigen vectors of S

Λ: diagonal matrix, diagonal entries are eigen vals in decreasing order

$$S = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_M \\ | & | & & | \end{bmatrix} \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \cdots & \\ & & & \lambda_M \end{bmatrix} U^{-1}$$

→ Symmetric diagonalization theorem :-

$S = Q \Lambda Q^T$

S: square and symmetric M x M

Q: cols are normalized (unit length, real) eigenvectors of S

Q^T: same

$$\left\{ \begin{array}{l} \text{Symmetric } S \Rightarrow S^T = S \\ Q^{-1} = Q^T \end{array} \right\}$$

Singular Value Decomposition (SVD) :-

Let r be the rank of $M \times N$ matrix C

M = number of terms

N = number of documents

C is term – document matrix

$$\underset{(M \times N)}{C} = \underset{(M\times M)}{U}\ \underset{(M\times N)}{\Sigma}\ \underset{(N\times N)}{V^T}$$

$$\underset{(M\times N)}{C} = \underset{(M\times M)}{\begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_M \\ | & | & & | \end{bmatrix}} \underset{(M\times N)}{\begin{bmatrix} \sqrt{\lambda_1} & 0 & \cdots & 0 \\ 0 & \sqrt{\lambda_2} & 0 \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ \vdots & \vdots & & \sqrt{\lambda_N} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}} \underset{(N\times N)}{\begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_N \\ | & | & & | \end{bmatrix}^T}$$

Eigenvectors of CC^T (orthonormal)

Diagonal matrix with eigenvalues of C^TC in ↓ing order

Eigenvectors of C^TC (orthonormal)

$$CC^T = U\Sigma^2U^T \qquad C^TC = V\Sigma^2V^T$$

CC^T is a term co-occurrence matrix. Entry (i,j) is a measure of overlap b/w ith and jth term. Entry (i,j) is the number of documents in which both term i and j occur, in case of term-doc matrix.

Eigenvalues of CC^T are same as eigenvalues of C^TC

$\sigma_i = \sqrt{\lambda_i}$ (Singular values of C)

$UU^T = I$; $VV^T = I$

$\boxed{v_i = \dfrac{1}{\sigma_i} C^T u_i}$