Chapter IR:III

III. Text Transformation

- ❑ Text Statistics
- ❑ Parsing Documents
- ❑ Information Extraction
- ❑ Link Analysis

Link Analysis
Hyperlinks

The web is a network of documents induced by hyperlinks:

> This web page `is perhaps the most famous example there ever was.`

Hyperlinks refer readers of a web page to another. There can be but one reason for adding a hyperlink to a web page:

> The author believes the linked page important to be reachable.

A hyperlink is usually attached to, or in the vicinity of a text or an image found on a web page that explains the linked page's relevance (e.g., by summarizing it).

These properties of hyperlinks can be exploited for web search.
Never trust user input:

- Omit hyperlinks that can be created by users of the linked web page.
- Omit hyperlinks that originate from malicious pages.
- Omit hyperlinks that are added by default to a web page.

Link Analysis

Anchor Text

The web is a network of documents induced by hyperlinks (HTML source code):

```
<a href="http://www.example.com">This web page</a> is perhaps
             the most famous example there ever was.
```

The text enclosed by an HTML anchor element is called anchor text. It forms the clickable part of a hyperlink, redirecting to the URL given in the `href` attribute.

Anchor texts, and optionally their surrounding passages (e.g., sentence or paragraph) are used as an additional source of index terms for the linked page.

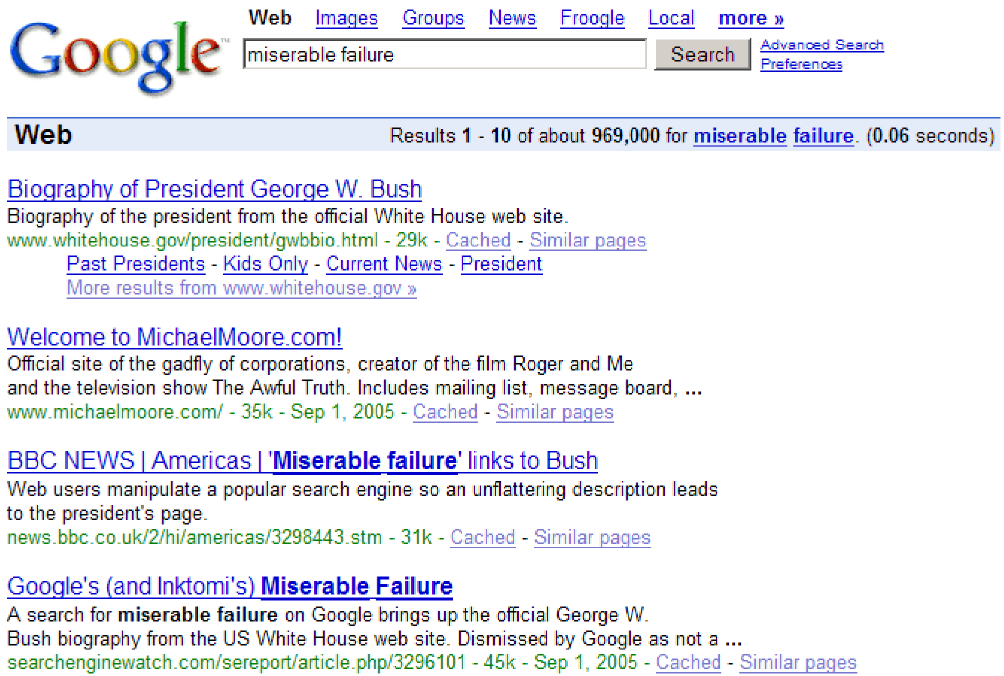
Anchor texts provide for index terms not necessarily found on the linked web page, severely improving retrieval performance.

Never trust user input: This may be misused (e.g., to give web pages a bad name).

An anchor text processing pipeline will include a customized stop word list, including words such as `page`, `here`, `click`.

Remarks:

- The term Google bomb refers to the practice of causing a website to rank highly in web search engine results for irrelevant, unrelated or off-topic search terms by linking heavily.



[Wikipedia]

Link Analysis

PageRank [Brin 1998]

Links between web pages may be used to gauge web page importance: The more links point to a web page, the more important it must be.

Naive importance measure for a web page A:

$$\textit{importance}(A) = |\{B \mid (B \text{ is a web page}) \text{ and } (B \text{ links to } A)\}|$$

Problems:

- every link counts equally much
- every web page can have an arbitrary number of links to other web pages

Desirable properties:

- the importance of A should depend on that of pages linking to it
- the importance of B should be partitioned to the pages it links to, not multiplied

➜ Meet the random surfer model

Link Analysis

PageRank: Random Surfer Model [Brin 1998]

Link Analysis

PageRank: Random Surfer Model [Brin 1998]

The PageRank of web page A is the probability that a random surfer will look at A.

Random surfing:

1. Open a random web page
2. Choose $\alpha \in [0, 1]$ at random
3. If $\alpha < \lambda$: go to Step 1
4. If the current page has no links: go to Step 1
5. Else: follow a random link on the current page, then go to Step 2

Observations:

- Random surfing has the Markov property.
- Steps 2–4 ensure the surfer does not get stuck, and that every page has a non-zero chance of being visited.
- Empirically, $\lambda = 0.15$.

Link Analysis

PageRank: Definition [Brin 1998]

Given a page u, its PageRank is computed as follows:

$$PR(u) = \lambda \cdot \frac{1}{n} + (1-\lambda) \cdot \sum_{v \in B_u} \frac{PR(v)}{L_v},$$

where n is the number of web pages, B_u is the set of pages linking to u, and L_v the number of outgoing links on page v.

Algebraic formulation: Let $\mathbf{T}$ denote the matrix of page transition probabilities, so that the probability of transitioning from page i to j is given by:

$$\mathbf{T}_{ij} = \lambda \cdot \frac{1}{n} + (1-\lambda)\frac{1}{L_i} \qquad \text{for } L_i > 0\text{, otherwise} \qquad \mathbf{T}_{ij} = \lambda \cdot \frac{1}{n}.$$

Then $\mathbf{r}$ is the vector of page probabilities at time t of executing the random surfing process when repeatedly multiplying it with $\mathbf{T}$:

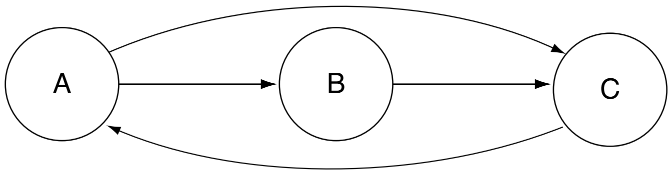
$$\mathbf{r} \cdot \mathbf{T}^t$$

As $t \to \infty$, $\mathbf{r}$ yields the PageRanks for all pages, which corresponds to the principal eigenvector of $\mathbf{T}$.

Since T is stochastic, irreducible, and aperidodic, this process converges.

Link Analysis

PageRank: Example



$$\mathbf{T} = \begin{bmatrix} 0.05 & 0.475 & 0.475 \\ 0.05 & 0.05 & 0.9 \\ 0.9 & 0.05 & 0.05 \end{bmatrix}$$

$t = 0$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [1, 0, 0]$

$t = 1$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.05, 0.475, 0.475]$

$t = 2$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.454, 0.071, 0.475]$

$t = 3$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.454, 0.243, 0.303]$

$t = 5$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.432, 0.181, 0.387]$

$t = 10$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.389, 0.212, 0.399]$ [calculator]

Assume $\lambda = 0.15$. The initialization of $\mathbf{r}$ can also be chosen uniformly distributed, or based on previously computed PageRanks.

Link Analysis

Algorithm: IterativePageRank

Input: $G = (P, L)$. Web graph with pages P and links L.
λ. Random jump probability.

Output: I. Approximate PageRanks for all pages in P.

```
# Initialization of I
I, R = vectors of length |P|
FOREACH i ∈ [1, |P|] DO
    I[i] = 1/|P|
ENDDO

# Update loop
WHILE NOT converged(I, R) DO
|
ENDDO

RETURN(I)
```

Link Analysis

Algorithm: IterativePageRank

Input: $G = (P, L)$. Web graph with pages P and links L.
λ. Random jump probability.

Output: I. Approximate PageRanks for all pages in P.

```
# Update loop
WHILE NOT converged(I, R) DO
  # Reinitialization of R
  FOREACH i ∈ [1, |P|] DO
    R[i] = 1/|P|
  ENDDO

  # Update step
  FOREACH p ∈ P DO
|
  ENDDO

  I = R
ENDDO
```


Link Analysis

Algorithm: IterativePageRank

Input: $G = (P, L)$. Web graph with pages P and links L.
λ. Random jump probability.

Output: I. Approximate PageRanks for all pages in P.

```
# Update step
FOREACH p ∈ P DO
  Q = { q | q ∈ P and (p, q) ∈ L }
  IF |Q| > 0 THEN
    FOREACH q ∈ Q DO
      R[q] = R[q] + (1 − λ) · I[p]/|Q|
    ENDDO
  ELSE
    FOREACH p ∈ P DO
      R[p] = R[p] + (1 − λ) · I[p]/|P|
    ENDDO
  ENDIF
ENDDO
```

Link Analysis

PageRank: Convergence

Convergence is typically checked with

$$||R - I|| < \tau,$$

where $||\cdot||$ denotes the L_1 or L_2 norm, and τ is a threshold.

The choice of τ depends on the number n of documents, since $||R - I||$ (for a fixed numerical precision) increases with n. The larger τ, the faster convergence is reached. Optionally, $||R - I||/n$ can be used instead.

The number of iterations required to converge is roughly in $O(\log n)$. [Page 1999]

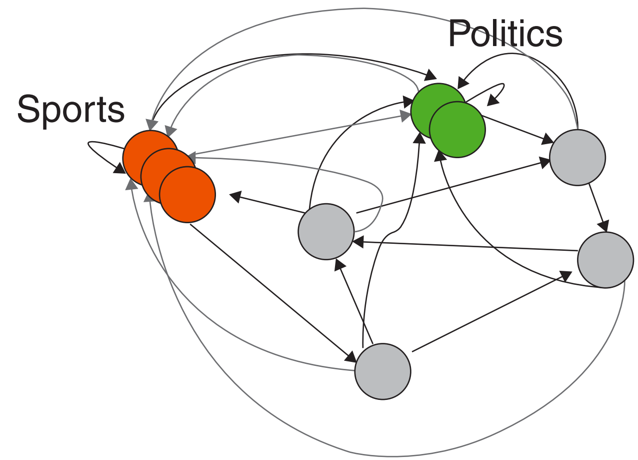
Counterintuitively, the PageRank algorithm does not converge faster when initialized with the PageRanks from a previously converged run compared to a uniform initialization. This is partly due to the rapid pace at which the web evolves.

[Meyer 2004]

Link Analysis
PageRank: Variants

The PageRank algorithm can be applied to web graphs at different levels of granularity:

- Web pages
- Websites
  Combining all pages hosted under a domain allows for computing the importance of websites as a whole.
- Topic-specific clusters
  Categorizing web page by topic, or clustering them induces a web graph between categories / clusters. This allows for computing PageRanks within and across categories / clusters.



- Personalized PageRank
  Based on topic-specific PageRanks, a user may provide personal interests which can be applied as normalized weights onto each topic's PageRank vector.

