Chapter IR:XI

Web Technology
Overview

- ❑ Internet
- ❑ World Wide Web
- ❑ Addressing
- ❑ HTTP
- ❑ HTML
- ❑ Web Graph

Web Technology

Internet

Definition 1 (Internetwork, Internet [Wikipedia]**)**

The Internet is a global system of interconnected computer networks. A computer network connects computers (hosts) via a specific technology to allow data exchange.

Web Technology

Internet

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The Internet is a global system of interconnected computer networks. A computer network connects computers (hosts) via a specific technology to allow data exchange.

1958   Founding of the Defense Advanced Research Project Agency DARPA in reaction to the Soviet Union taking the lead in the space race.

1969   First version of the ARPANET works with 4 hosts. The goal is to decentralize military networks, rendering them resilient to attacks.

1973   35 hosts connected, including trans-atlantic ones in England and Norway.

1989   More than 150,000 hosts. ARPANET is shut down. The NSFNET is now called INTERNET.

1991   Dawn of the World Wide Web.

1992   More than 1 million hosts. The Internet Society is founded.

2013   More than 1 billion hosts.

Web Technology

Internet

Definition 1 (**Internetwork, Internet** [Wikipedia])

The Internet is a global system of interconnected computer networks. A computer network connects computers (hosts) via a specific technology to allow data exchange.

Internet Domain Survey Host Count
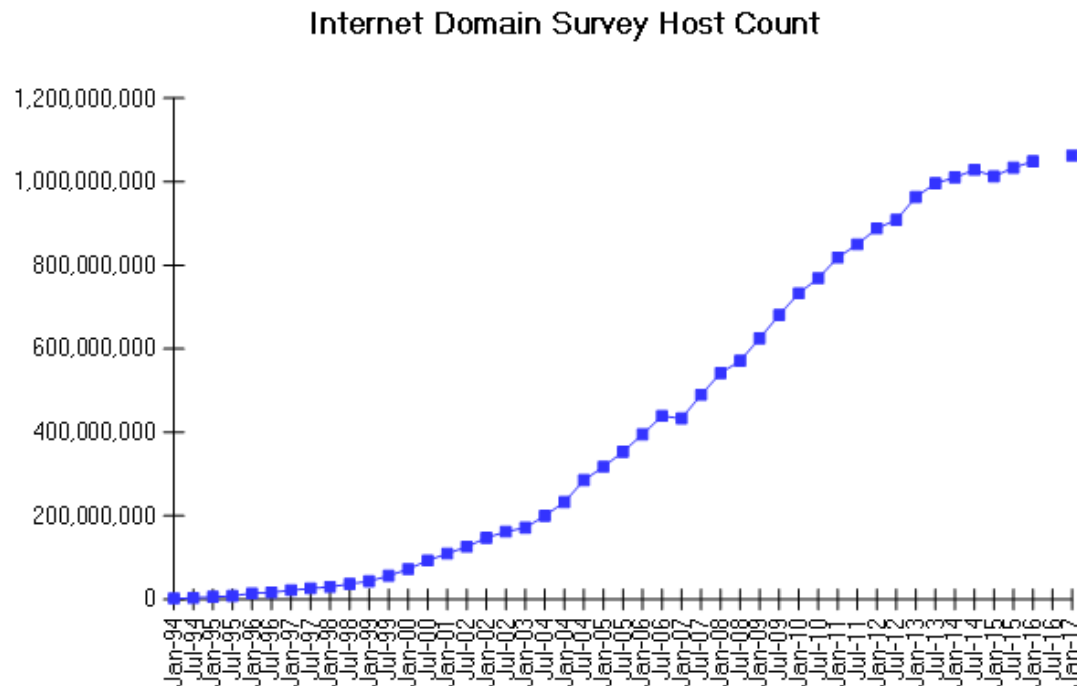
Source: Internet Systems Consortium (www.isc.org)

[Internet Systems Consortium, www.isc.org]

Web Technology
World Wide Web

Definition 2 **(World Wide Web, WWW** [Wikipedia]**)**

The World Wide Web is a network of documents, located by Uniform Resource Locators (URLs), connected by hypertext links, and accessible via the Internet.

Web Technology
World Wide Web

Definition 2 (World Wide Web, WWW [Wikipedia]**)**

The World Wide Web is a network of documents, located by Uniform Resource Locators (URLs), connected by hypertext links, and accessible via the Internet.

1945   Vannevar Bush envisions Memex, the first hypertext-like system.

1963   Ted Nelson coins the term hypertext. In the following decades, many hypertext systems are proposed and developed.

1990   Tim Berners-Lee develops the first web client, invents HTML, and implements the first web server, calling his system "WorldWideWeb". CERN's phone book is the first application.

1993   CERN agrees to allow anyone to use Web protocol and code royalty-free.

1994   623 web servers. The World Wide Web Consortium W3C is founded.

2017   1.760.630.795 Websites (unique Hostnames) [internetlivestats]
         174.463.315 Websites (active)
           6.271.146 Web-Servers (Web-Facing Computers)

Web Technology

World Wide Web

Definition 2 (World Wide Web, WWW [Wikipedia])

The World Wide Web is a network of documents, located by Uniform Resource Locators (URLs), connected by hypertext links, and accessible via the Internet.

Total number of websites (logarithmic scale)
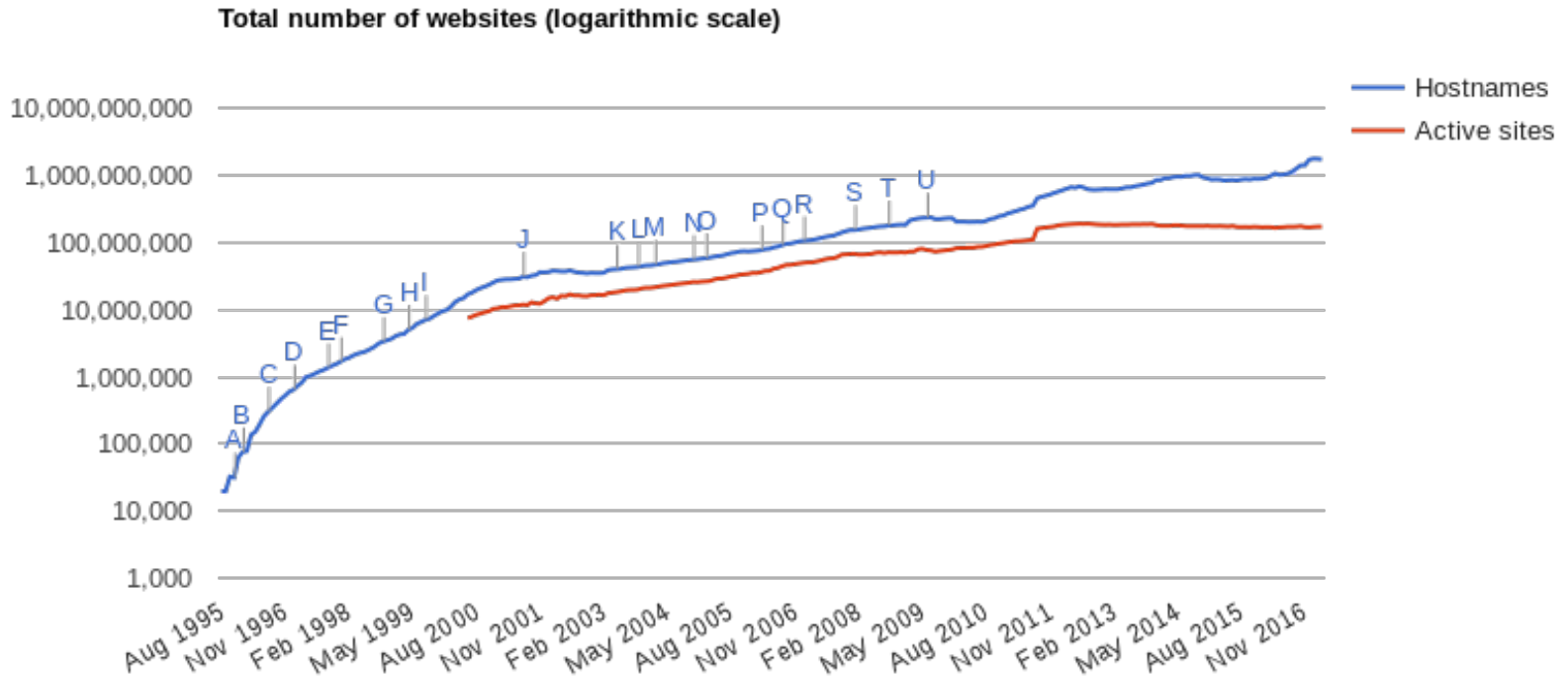
[www.netcraft.com]

Web Technology
Addressing

The Internet Protocol (IP) is the principal communications protocol in the Internet. It delivers data packets between hosts, found solely based on their IP addresses. Two major versions are in use, IPv4 and IPv6:

- ❏ IPv4 addresses are 32 bit (4 byte) integers, denoted as sequence of 4 decimals, separated by points.

$$\texttt{141.54.1.11/16}$$

- ❏ IPv6 addresses are 128 bit (16 byte) integers, denoted as sequence of 8 hexadecimals, separated by colons.

$$\texttt{2001:0db8:85a3:08d3:1319:8a2e:0370:7344/64}$$

- ❏ IP addresses divide into address prefix and address suffix.

- ❏ The prefix (network ID) identifies the physical network.

- ❏ The suffix (host ID) identifies the computer within the netowrk ID's network.

- ❏ The prefix length is determined via CIDR notation (legacy: subnet mask).

Web Technology
Addressing

The domain name system (DNS) resolves hostnames to their hosts' IP addresses.

➡ DNS servers form a distributed database

First version:

- ❑ All name-address pairs were collected in a central master file, which was distributed via FTP to other servers.

- ❑ Does not scale, since local organization becomes impossible.

Current version:

- ❑ Hierarchical organization via organizational partitioning (.com, .edu, .gov, .mil, etc.) as well as geographic partitioning (.de, .uk, .fr, etc.)

- ❑ The suffix following the last dot is called top level domain, TLD. List of current and new TLDs.
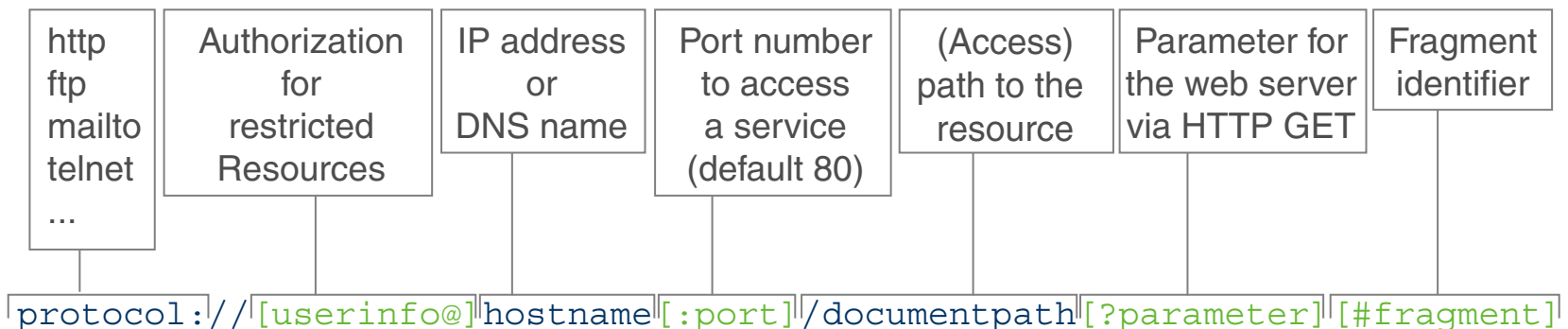
Web Technology
Addressing

| URI | = | **Prefix** (Information system) | **Suffix** (Location and identification) |

URI ← URL, URN
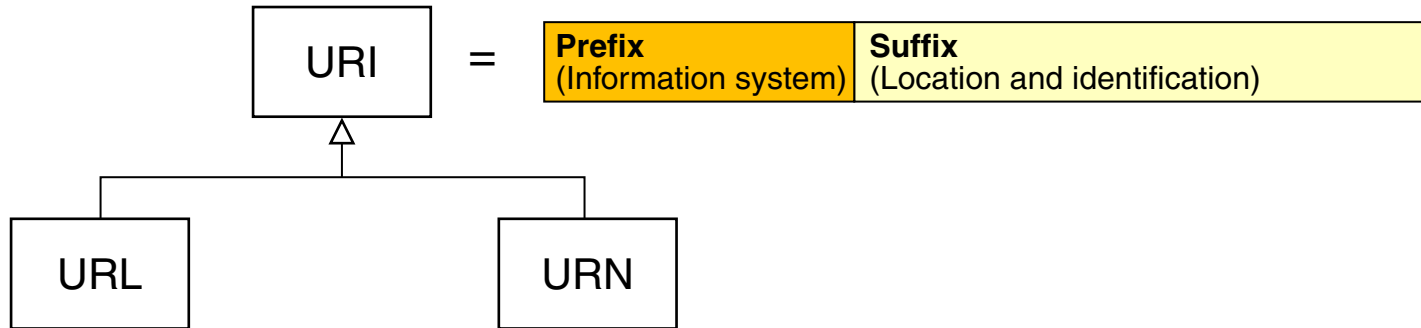
A Uniform Resource Identifier (URI) provides a simple and extensible means for identifying a resource. It can be further classified as a locator, a name, or both.
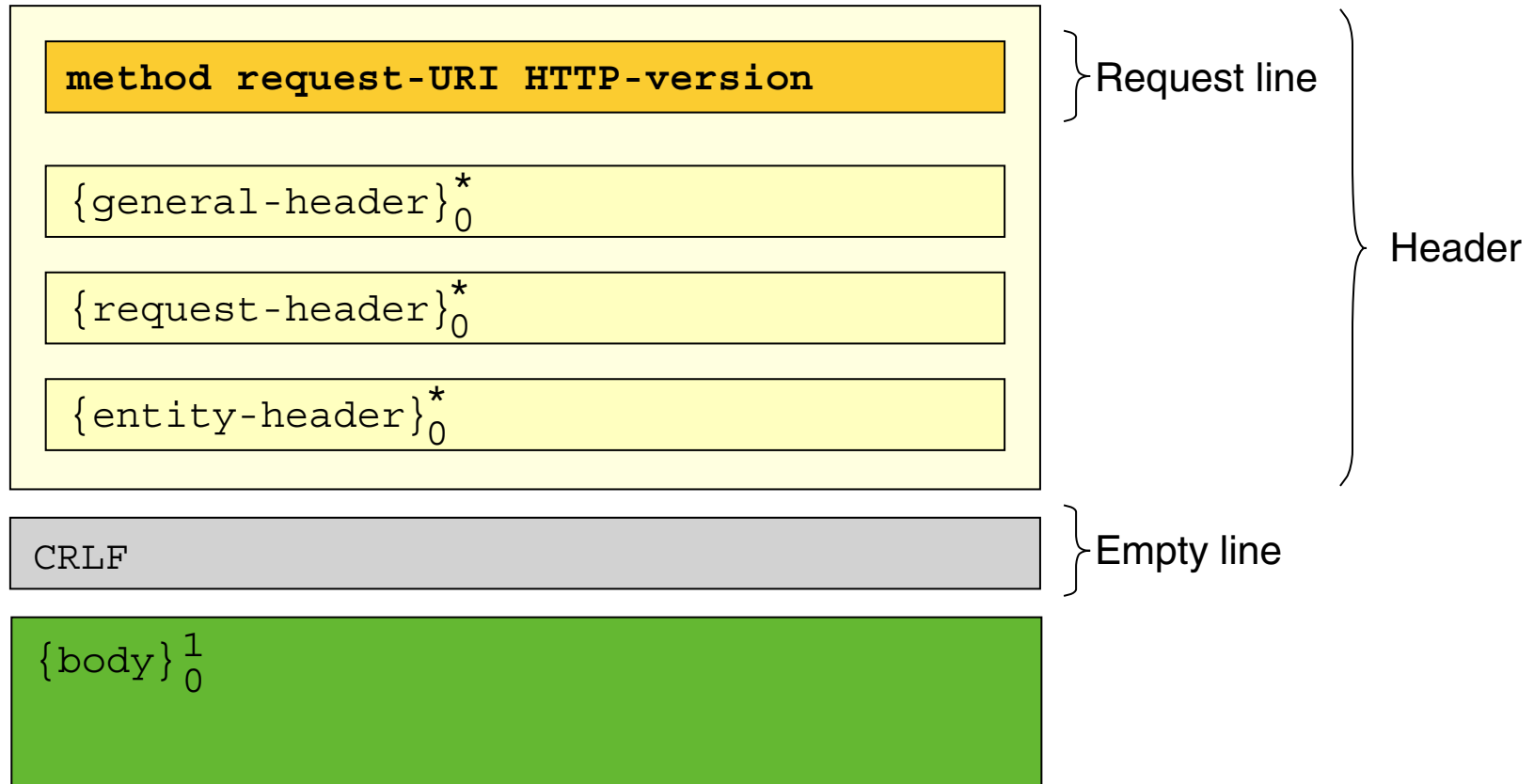
The term "Uniform Resource Locator" (URL) refers to URIs that, in addition to identifying a resource, provide a means of locating the resource by describing its primary access mechanism (e.g., its network "location").   [RFC 3986]

| http ftp mailto telnet ... | Authorization for restricted Resources | IP address or DNS name | Port number to access a service (default 80) | (Access) path to the resource | Parameter for the web server via HTTP GET | Fragment identifier |

`protocol://[userinfo@]hostname[:port]/documentpath[?parameter][#fragment]`

Web Technology
Hypertext Transfer Protocol (HTTP)

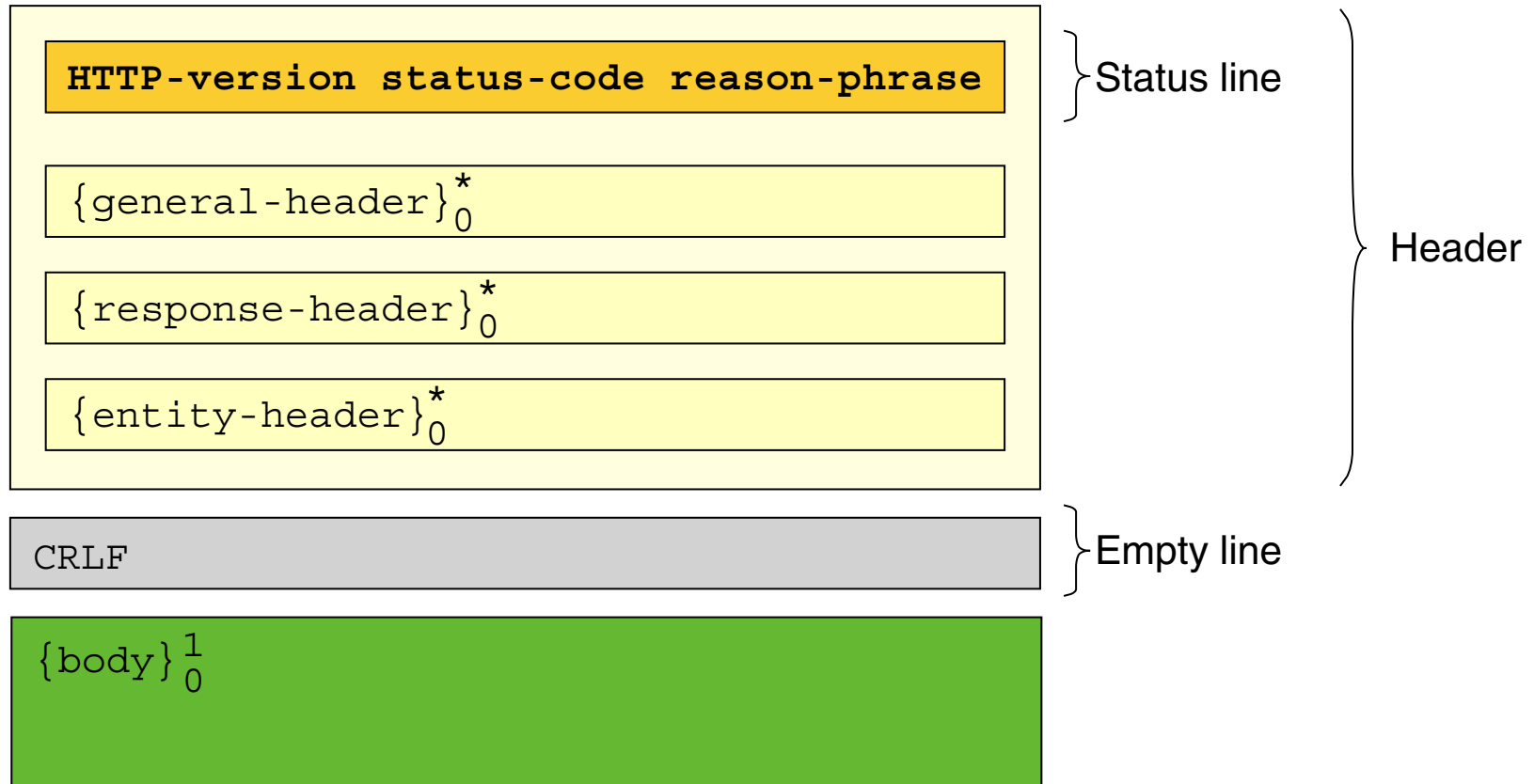|	
method request-URI HTTP-version	Request line
${\rm \{general\mbox{-}header\}}_0^*$	
${\rm \{request\mbox{-}header\}}_0^*$	Header
${\rm \{entity\mbox{-}header\}}_0^*$	
CRLF	Empty line
${\rm \{body\}}_0^1$	

Example for a request line: `GET www.example.com/index.html HTTP/1.0`

Methods: GET, POST, HEAD, PUT, DELETE, OPTIONS, TRACE, CONNECT

Web Technology

Hypertext Transfer Protocol (HTTP)

HTTP-version status-code reason-phrase	Status line

$\{general-header\}_0^*$

$\{response-header\}_0^*$

$\{entity-header\}_0^*$

Header

CRLF — Empty line

$\{body\}_0^1$

Example for a status line: `HTTP/1.0 200 OK`

Status codes: 200 OK, 301 Moved permanently, 304 Not modified, 404 Not found, 500 Internal server error, etc.

Web Technology
Hypertext Transfer Protocol (HTTP)

```
HTTP/1.1 200 OK
```
Status line

```
Date: Tue, 15 Apr 2014 19:17:35 GMT
```
General header

```
Server: Apache/2.2.12 (Unix) DAV/1.0.3
    PHP/4.3.10 mod_ssl/2.8.16 OpenSSL/0.9.7c
```
Response header

```
Last-Modified: Sat, 22 Mar 2014 14:11:21 GMT
ETag: "205e812-1479-42402789"
```
Entity header

```
Accept-Ranges: bytes
```
Response header

```
Content-Length: 5241
```
Entity header

```
Connection: close
```
General header

```
Content-Type: text/html; charset=utf-8
```
Entity header

```
<!DOCTYPE html>
<html lang="de-DE" xmlns="http://www.w3.org/...
<head>
<base href="http://www.uni-weimar.de">
<title>Bauhaus-Universit&auml;t Weimar</title>
...
```

Web Technology

Hypertext Markup Language (HTML)

HTML
document

Document type	

Head element

Body element
$\{ \text{Anchor element} \}_0^*$
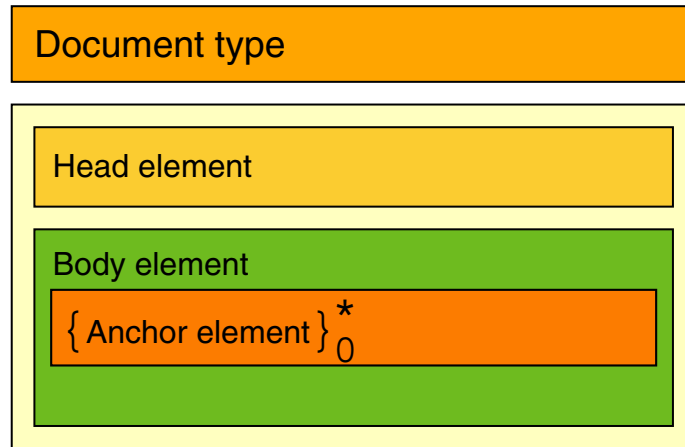
```
<!DOCTYPE html>

<html>
  <head>
    ...
  </head>
  <body>
    ...
    <a href="URI">link</a>
    ...
  </body>
</html>
```

❑ The `<html>` element represents the document root. [W3C REC 4.1]

❑ The `<head>` element represents meta data. [W3C REC 4.2]

❑ The `<body>` element represents document contents. [W3C REC 4.3]

❑ The `<a>` (anchor) element represents a hyperlink. [W3C REC 4.5.1]

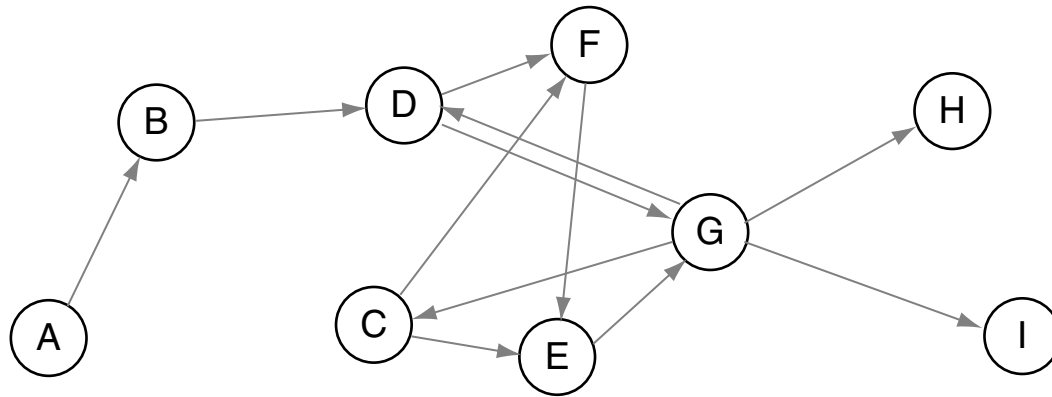|	
`<a href="URI">`	Target defined by *URI*
`Ex.: <a href="#Identifier">`	Target element with id attribute within document
`Ex.: <a href="Path#Identifier">`	Target in relative document (i.e., same web server)

Web Graph

Web Graph

Topology of the Web [Broder 2000]

Web Graph
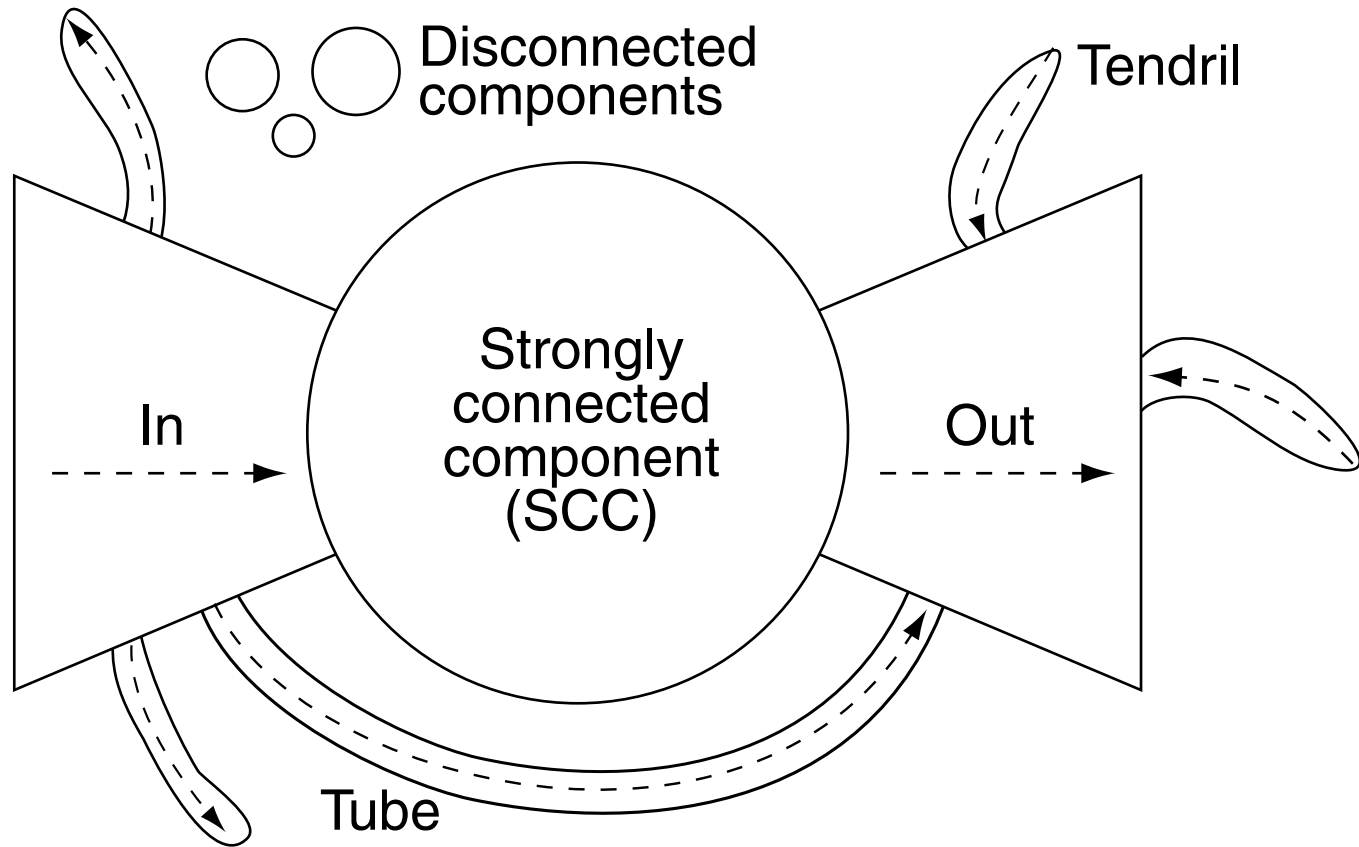
- The fraction of web pages with i in-links is proportional to $1/i^{2.1}$ (power law).
- SCC, In, Out, and the tendrils are of roughly equal size (SCC a little larger).
- Probability of a path between random pages is 0.24, the expected length 16.

Web Crawling
Crawling Hypertext

The web can be crawled by traversing the web graph.

A basic crawling algorithm:

Algorithm: BFSCrawler
Input: seeds (list of URLs to start with),
       store (document store)

1: frontier ← seeds
2: **while** frontier is not empty **do**
3:    url ← frontier.dequeue()
4:    page ← getURL(url)
5:    store.add(url, page)
6:    **for each** linkedUrl in page **do**
7:       **if** not store.has(linkedUrl) **then**
8:          frontier.enqueue(linkedUrl)
9:       **end if**
10:   **end for**
11: **end while**

Properties:

- Implements breadth-first search

- URLs are handled first in, first out

- Resilient to loops in the web graph

- Recrawls same pages if URL is different (e.g., http vs. https)

- Crawls site's pages in a row if linked consecutive on a page

- Does not recrawl pages

→ Domain-specific adjustments necessary

Web Crawling

Requirements [Manning 2008]

- ❑ **Selectivity**

  Strategies and policies for choosing web pages to download. This particularly includes the identification of malicious web pages and the prioritization of high-quality web pages.

- ❑ **Politeness**

  Adherence to explicit restrictions to download web pages. Prevention of unnecessary load on a web server.

- ❑ **Freshness**

  Strategies for recrawling web pages to obtain the most recent version as soon as possible.

- ❑ **Efficiency**

  Optimal utilization of available network bandwidth and hardware.

- ❑ **Scalability**

  Linear increase of efficiency by adding resources.

- ❑ **Extensibility**

  Modular architecture for easy integration and adaptation to new data formats and crawling strategies.

Web Crawling
Selectivity

A crawler implements a selection policy determining which pages are crawled.

- ❑ **Crawl seeds** (Where to begin?)

  Initialization of a crawler frontier with previously collected URLs, so-called seeds.

- ❑ **Crawl target** (What to crawl?)

  Simple answer: Everything. More specifically: Every document for which a search engine's user might search ("Where was that document again?"). For web search engines, only few exceptions apply. In general, predicting universal non-usefulness of documents is difficult.

- ❑ **Crawl priority** (What first?)

  Web pages that are *predictably* more important to the search engine's users than others. Web sites may be judged as a whole. In particular, pages comprising high-quality content.

- ❑ **Crawl filtering** (What to avoid?)

  "Spider traps", and web pages from web sites whose owners harbor malicious intents toward the search engine, or its users, such as spam pages.

Remarks:

❑ Crawling is also called spidering, as in a spider wandering the web.

❑ A spider trap is a web page that (un)intentionally serves dynamically generated pages that comprise links to other dynamically generated pages ad infinitum. Examples: number pages referring to successive numbers, date pages referring to successive dates, hierarchy pages referring to deeper levels.

Web Crawling

Malicious pages try to manipulate the search engine in pursuit of goals that are at odds with those of a search engine. The only way to do so is at crawling time.

❑ Cloaking

  Serving different pages to a crawler than to a human user for a given URL.

❑ Mirroring

  Reuse of another web page's contents, either duplicating or paraphrasing it.

❑ Keyword stuffing

  Including keywords or text into a web page which did not occur organically during web page creation; possibly as hidden text.

❑ Doorway page

  Pages redirecting human users to another page, or asking them to click to proceed, otherwise comprising only little, specifically chosen content for indexing.

❑ Link farm

  Networks of pages/sites from different domains linking to each other to boost their predicted importance. They can be automatically generated or emerge manually via link exchange.

*SEO: Search Engine Optimization

Web Crawling

A crawler which crawls third party resources must implement a politeness policy to comply with their owner's wishes and needs.

❑ Server load conservation
A web server hosts many pages and may have many users. Requesting all pages at once, or in a short time frame, may overload the server, affecting other users. Politeness dictates to leave significant headroom in terms of server resources to other users.

❑ Copyright compliance (Robots Exclusion Protocol)
Web site owners may not wish for parts or all of their pages to be indexed. Politeness dictates compliance with this wish. The robots exclusion protocol provides a standard interface to communicate such directives. Example:

```
User-agent: *
Disallow: /
Allow: /public/

User-agent: FavoredCrawler
Disallow:

Sitemap: http://www.example.com/sitemap.xml.gz
```
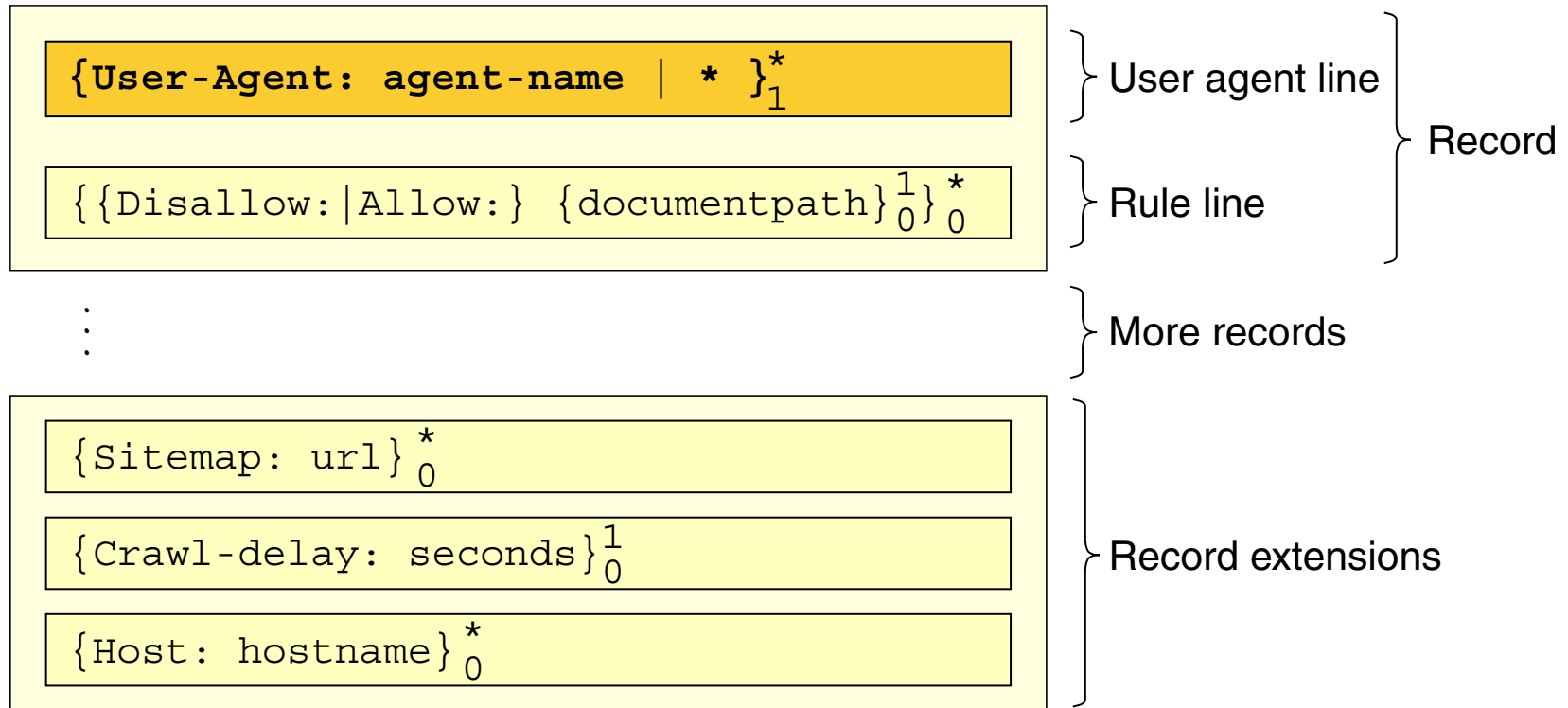
FavoredCrawler may access everything, whereas every other crawler is disallowed to crawl anything, except for document paths starting with /public/.

Web Crawling

Politeness: Robots Exclusion Protocol (robots.txt)

$$\{\texttt{User-Agent: agent-name} \mid * \}_1^*$$ — User agent line

$$\{\{\texttt{Disallow:|Allow:}\} \ \{\texttt{documentpath}\}_0^1\}_0^*$$ — Rule line

} Record

⋮ — More records

$$\{\texttt{Sitemap: url}\}_0^*$$

$$\{\texttt{Crawl-delay: seconds}\}_0^1$$

$$\{\texttt{Host: hostname}\}_0^*$$

} Record extensions

❑ The robots.txt must be at the document root of a domain:

  `www.example.com/robots.txt`

❑ The RFC has not been passed, but it is a de-facto standard.

❑ Extensions to records are interpreted only by some crawlers.

Remarks:

- The User-Agent line corresponds to the HTTP request header. A list of bots and the user agent lines they use. The `agent-name` is supposed to be just the name of the bot, not including extraneous information.

- The Sitemap extension was introduced as a kind of *Robots Inclusion Protocol*, telling crawlers with a standardized XML file where to look for pages, including ones that are not linked from anywhere else, and meta data such as last modification date, change frequency, priority, and alternative versions of a page, e.g., in different languages.

- The Host extension lets the crawler know which hostname is preferred for a site in cases the same site can be reached via multiple hostnames.

- Other means to disallow crawlers include adding response headers to HTTP requests:
  `X-Robots-Tag: [agent-name:] noindex, nofollow`,
  adding meta elements in a web page's header:
  `<meta name="robots" content="noindex,nofollow">`,
  adding the `rel="nofollow"` attribute to anchor elements:
  `<a href="URI" rel="nofollow">`,
  adding class attributes to arbitrary elements:
  `<div class="robots-noindex robots-nofollow">Text.</div>`,
  enclosing plain text with (crawler-dependent) structured comments:
  `<p>Do index this text. <!--googleoff: all-->Don't index this text.<!--googleon: all--></p>`,
  allowing for crawler-specific rules by replacing `robots` with agent names.

Web Crawling
Politeness: Crawling Algorithm Revisited

A polite crawling algorithm:

Algorithm: PoliteCrawler
Input: seeds (list of URLs to start with),
store (document store)

```
 1: frontier ← seeds
 2: while frontier is not empty do
 3:     site ← frontier.nextSite()
 4:     url ← site.dequeue()
 5:     if site.permitsCrawl(url) then
 6:         page ← getURL(url)
 7:         store.add(url, page)
 8:         for each linkedUrl in page do
 9:             if not store.has(linkedUrl) then
10:                 site.enqueue(linkedUrl)
11:             end if
12:         end for
13:     end if
14: end while
```

Web Crawling
Freshness

Previously crawled web pages may be revised or deleted. A crawler for a web search engine implements a recrawl policy to ensure that the most recent version of a web page is indexed.

❑ Update checking

Web servers may report `Last-Modified:<date>` as a response header, indicating the last time the page under a given URL changed. A HTTP HEAD request will reveal the date without having to download the entire page again. Otherwise, update checking requires a recrawl, and a comparison of the most recent version with the previous one.

❑ Update prediction

The crawler needs to learn to predict page updates to not waste resources on update checking, nor missing updates. Learning a web page's update probability is done by scheduling update checks, adjusting the recrawl delay based on whether a page changed since its last recrawl. Adjustments may be done via binary search, or time series analyses.

Examples:

❑ Stock market pages change frequently, and their most recent versions are what people will be searching for.

❑ News pages are updated as news unfold. People search for current events.

Web Crawling
Freshness: Metric

We call a crawled web page fresh, if it corresponds to the version currently online, and stale otherwise.

A simple freshness metric (also called "freshness") is the fraction of stale pages in a crawl at a given point in time. The smaller the fraction, the fresher the crawl.

Optimizing for this freshness metric is problematic:

❑ Why?

Web Crawling

We call a crawled web page fresh, if it corresponds to the version currently online, and stale otherwise.

A simple freshness metric (also called "freshness") is the fraction of stale pages in a crawl at a given point in time. The smaller the fraction, the fresher the crawl.

Optimizing for this freshness metric is problematic:

- ❏ Recrawling requires resources, and a crawler has only a limited amount.

- ❏ Allocating resources to recrawls will optimize freshness, if and only if, web pages are recrawled ordered from low to high update probability.

- ❏ Web pages with a change rate below the minimum recrawl delay (e.g., to conserve server load) will never be fresh.

- ❏ Hence, frequently changing web pages may not be crawled.

- ❏ Important pages often have a high change rate.

- ➜ Optimizing for this freshness metric contradicts the need for freshness.

Web Crawling
Freshness: Age Metric

An idea for a better metric:
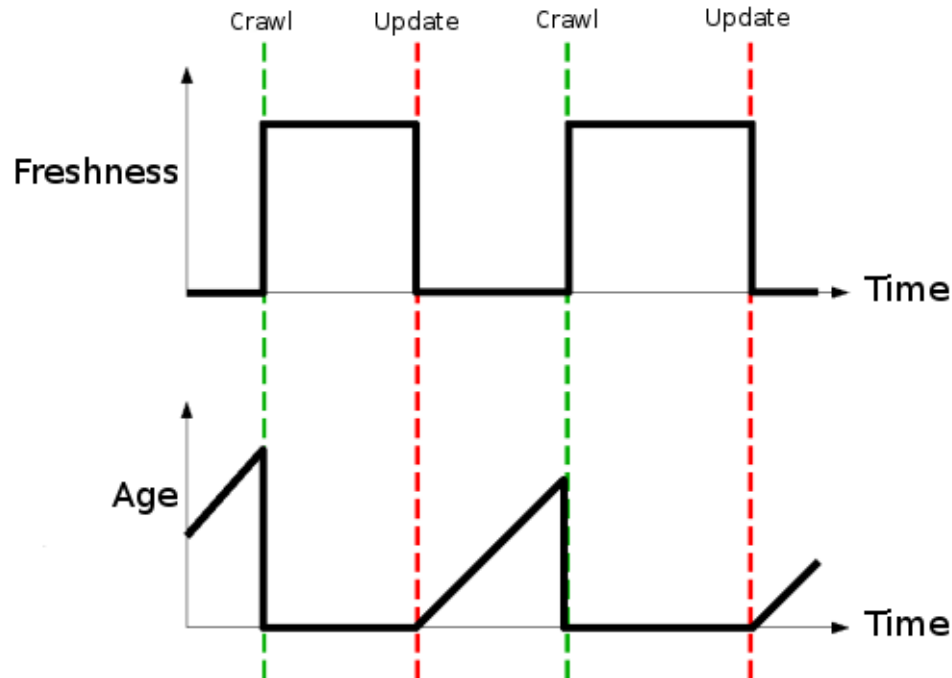
- ❏ Freshness is completely lost after an update.
- ❏ Age constantly grows after an update.
- ❏ This allows for measuring the urgency of an update.
- ❏ Problem: we do not know when an update occurs, so we have to guess.

Web Crawling

Freshness: Age Metric [Cho and Garcia-Molina 2003]

Let λ denote the change rate of a page (i.e., λ changes per day). We calculate the expected age of a page at time t (e.g., t days) after it was last crawled as follows:

$$\text{Age}(\lambda, t) = \int_0^t P(\text{page changed at time } x)(t - x)dx,$$

where $(t - x)$ measures age at recrawl time t and change time $x < t$. Mutiplying by the change probability at time x, and integrating over it yields the expected value.

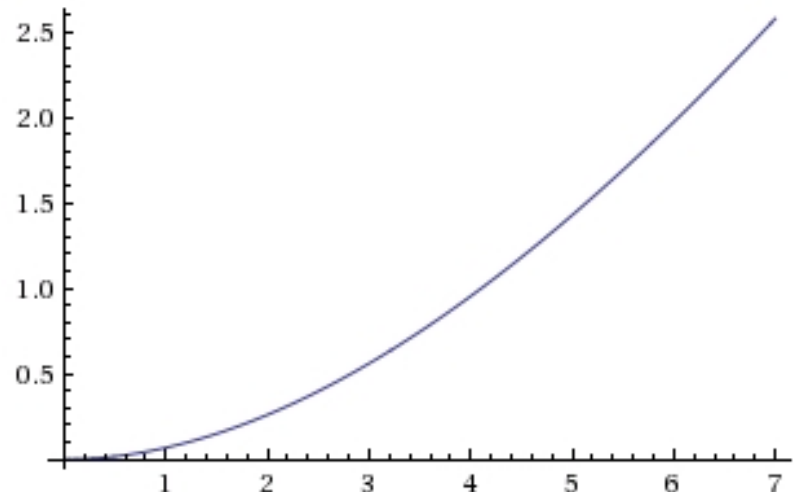# Web Crawling
Freshness: Age Metric [Cho and Garcia-Molina 2003]

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Assuming that web page updates are governed by a Poisson process, the probability P at change rate λ can be expressed with $\lambda e^{-\lambda x}$, yielding:

$$\text{Age}(\lambda, t) = \int_0^t \lambda e^{-\lambda x}(t - x)dx$$

$$= \frac{\lambda t + e^{-\lambda t} - 1}{\lambda}.$$

For example, let $\lambda = 1/7$ (e.g., one change a week):

$$\text{Age}(1/7, 7) = 2.6$$

denotes the expexted age of a page when recrawling it at $t = 7$.

Web Crawling

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$$= \frac{\lambda t + e^{-\lambda t} - 1}{\lambda}.$$

Web Crawling

Observations:

- ❑ The second derivative of Age is $\lambda e^{-\lambda t}$.
- ❑ For $\lambda > 0$, the second derivative is always positive.
- ❑ Age increases at an accellerating rate.
- ❑ Age also increases with the change rate λ.
- ❑ The older a page and the more frequently it changes, the higher its Age.

→ Optimizing for low overall Age prioritizes page recrawls more accurately.

Web Crawling
Efficiency and Scalability

A crawler should utilize its resources efficiently, and be scalable linearly by adding more resources. Key bottlenecks:

❑ Internet connection utilization

- Downloads incur communication costs: DNS lookup, IP connection, data transfer.
- At 1MB/s bandwidth and 20KB web page size, 50 pages per second can be crawled.
- 80ms connection latency + 20ms data transfer yields 100ms per page request.
- Downloading 50 pages consecutively takes 5 seconds (underutilization).
- 5 parallel connections are needed to maximize utilization.
- This presumes the bandwidth of web servers is the same as ours. However, this is not the case; typically, the bandwidth of a crawler is faster than that of a web server.

❑ Politeness policy (e.g., crawl delays)

- Recall that we want to maximize utilization of the crawlers internet connection, not that of the connections of web servers.
- Downloads from a web site are restricted by crawl delays (say, 30 seconds).
- At 50 pages/s download rate, we need pages from 1500 sites in the frontier.

In a distributed crawler, each crawl node is assigned a specific web site so that no two crawl nodes download simultaneously from the same site.

Remarks:

❑ Suppose sites are "more distant" with a local bandwidth of 1MB/s, a server bandwidth of only 100KB/s, and 500ms latency. How many connections are needed then?

   – 100KB/s bandwidth means 200ms for data transfer of 20KB

   – Results in 700ms per page request (200ms data transfer + 500ms latency)

   – 50 pages times 700ms yields 35 seconds

   – 35 connections are needed to transfer 50 pages per second

❑ Hash functions are used to assign hostnames to crawl nodes at random.

Web Crawling
Extensibility

The web evolves at a rapid pace. In particular, web pages have turned from static HTML pages to dynamic pieces of software, compiled and executed within a browser. Crawlers must be adaptable to new technology.

Web pages/sites notoriously difficult to crawl (called the "deep web"):

❑ Private sites
  – No incoming links, or may require login with a valid account
  – Example: paywalled news pages (which still want to get indexed)

❑ Form results
  – Sites that can be reached only after entering some data into a form
  – Example: flight ticket selling (enter destination + dates etc.)

❑ Dynamic pages
  – Pages that generate content via JavaScript or other client technologies
  – Crawler needs to execute scripts (e.g., Google does so)
  – Significantly slows down the whole process

Link Analysis
Hyperlinks

The web is a network of documents induced by hyperlinks:

> <u>This web page</u> `is perhaps the most famous example`
> `                    there ever was.`

Hyperlinks refer readers of a web page to another. There can be but one reason for adding a hyperlink to a web page:

> The author believes the linked page is important to be reachable.

A hyperlink is usually attached to, or in the vicinity of a text or an image found on a web page that explains the linked page's relevance (e.g., by summarizing it).

These properties of hyperlinks can be exploited for web search.
But never trust user input:

- ❏ Omit hyperlinks that can be created by users of the linked web page.
- ❏ Omit hyperlinks that originate from malicious pages.
- ❏ Omit hyperlinks that are added by default to a web page.

Link Analysis
Anchor Text

The web is a network of documents induced by hyperlinks (HTML source code):

```
<a href="http://www.example.com">This web page</a> is perhaps
            the most famous example there ever was.
```

The text enclosed by an HTML anchor element is called anchor text. It forms the clickable part of a hyperlink, redirecting to the URL given in the `href` attribute.

Anchor texts, and optionally their surrounding passages (e.g., sentence or paragraph) are used as an additional source of index terms for the linked page.

Anchor texts provide for index terms not necessarily found on the linked web page, severely improving retrieval performance.
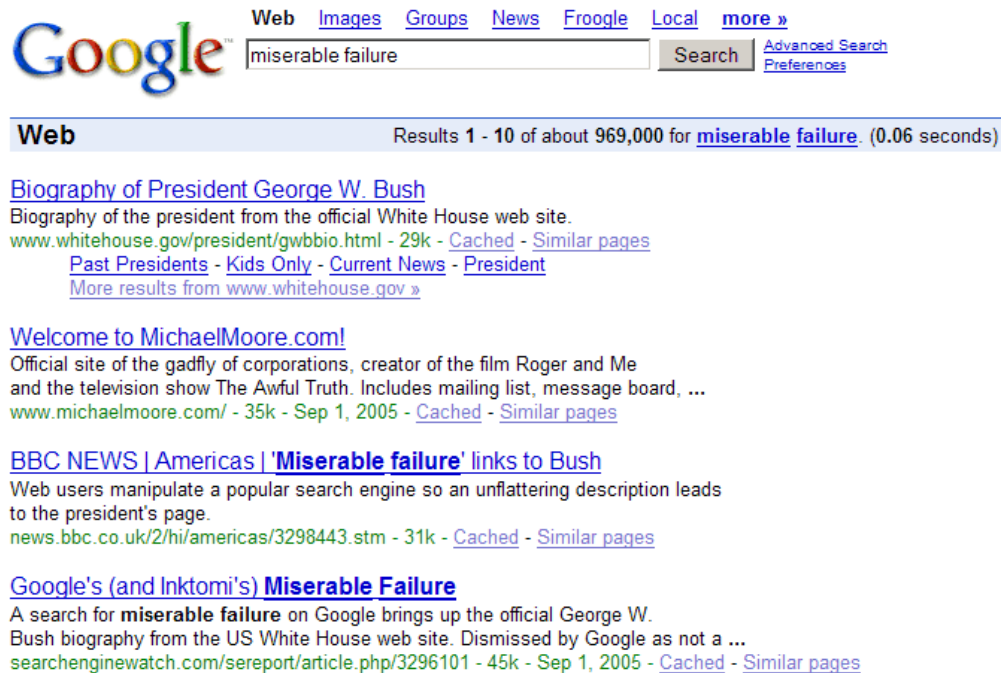
Never trust user input: This may be misused (e.g., to give web pages a bad name).

An anchor text processing pipeline will include a customized stop word list, including words such as `page`, `here`, `click`.

Remarks:

❑ The term Google bomb refers to the practice of causing a website to rank highly in web search engine results for irrelevant, unrelated or off-topic search terms by linking heavily.

Link Analysis

PageRank [Brin 1998]

Links between web pages may be used to gauge web page importance: The more links point to a web page, the more important it must be.

Naive importance measure for a web page A:

$$importance(A) = |\{B \mid (B \text{ is a web page}) \text{ and } (B \text{ links to } A)\}|$$

Problems:

❑ every link counts equally much
❑ every web page can have an arbitrary number of links to other web pages

Desirable properties:

❑ the importance of A should depend on that of pages linking to it
❑ the importance of B should be partitioned to the pages it links to, not multiplied

➜ Meet the random surfer model

Link Analysis
PageRank: Random Surfer Model [Brin 1998]

The PageRank of web page A is the probability that a random surfer will look at A.

Random surfing:

1. Open a random web page

2. Choose $\alpha \in [0, 1]$ at random

3. If $\alpha < \lambda$: go to Step 1

4. If the current page has no links: go to Step 1

5. Else: follow a random link on the current page, then go to Step 2

Observations:

❑ Random surfing has the Markov property.

❑ Steps 2–4 ensure the surfer does not get stuck, and that every page has a non-zero chance of being visited.

❑ Empirically, $\lambda = 0.15$.

Link Analysis

PageRank: Definition [Brin 1998]

Given a page u, its PageRank is computed as follows:

$$PR(u) = \lambda \cdot \frac{1}{n} + (1 - \lambda) \cdot \sum_{v \in B_u} \frac{PR(v)}{L_v},$$

where n is the number of web pages, B_u is the set of pages linking to u, and L_v the number of outgoing links on page v.

Algebraic formulation: Let $\mathbf{T}$ denote the matrix of page transition probabilities, so that the probability of transitioning from page i to j is given by:

$$\mathbf{T}_{ij} = \lambda \cdot \frac{1}{n} + (1 - \lambda)\frac{1}{L_i} \qquad \text{if } i \text{ links } j, \text{ otherwise} \qquad \mathbf{T}_{ij} = \lambda \cdot \frac{1}{n}.$$

Then $\mathbf{r}$ is the vector of page probabilities at time t of executing the random surfing process when repeatedly multiplying it with $\mathbf{T}$:
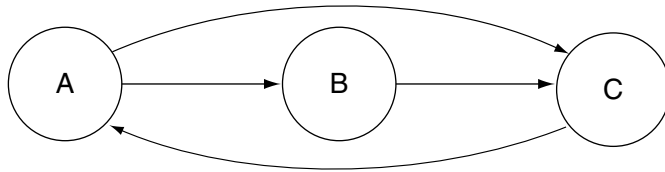
$$\mathbf{r} \cdot \mathbf{T}^t$$

As $t \to \infty$, $\mathbf{r}$ yields the PageRanks for all pages, which corresponds to the principal eigenvector of $\mathbf{T}$.

Since T is stochastic, irreducible, and aperidodic, this process converges.

Link Analysis
PageRank: Example

$$\mathbf{T} = \begin{bmatrix} 0.05 & 0.475 & 0.475 \\ 0.05 & 0.05 & 0.9 \\ 0.9 & 0.05 & 0.05 \end{bmatrix}$$

$t = 0$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [1, 0, 0]$

$t = 1$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.05, 0.475, 0.475]$

$t = 2$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.454, 0.071, 0.475]$

$t = 3$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.454, 0.243, 0.303]$

$t = 5$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.432, 0.181, 0.387]$

$t = 10$: $\quad \mathbf{r} \cdot \mathbf{T}^t = [0.389, 0.212, 0.399]$ [calculator]

Assume $\lambda = 0.15$. The initialization of $\mathbf{r}$ can also be chosen uniformly distributed, or based on previously computed PageRanks.

Link Analysis

Algorithm: IterativePageRank

Input: $G = (P, L)$. Web graph with pages P and links L.

                λ. Random jump probability.

Output: I. Approximate PageRanks for all pages in P.

```
1.  # Initialization of I
2.  I, R = vectors of length |P|
3.  FOREACH i ∈ [1, |P|] DO
4.      I[i] = 1/|P|
5.  ENDDO


6.  # Update loop
7.  WHILE NOT converged(I, R) DO
    |
26. ENDDO


27. RETURN(I)
```

Link Analysis

Algorithm: IterativePageRank

Input: $G = (P, L)$. Web graph with pages P and links L.

             λ. Random jump probability.

Output: I. Approximate PageRanks for all pages in P.

```
 6.  # Update loop
 7.  WHILE NOT converged(I, R) DO
 8.    # Reinitialization of R
 9.    FOREACH i ∈ [1, |P|] DO
10.       R[i] = 1/|P|
11.    ENDDO

12.    # Update step
13.    FOREACH p ∈ P DO
 |
24.    ENDDO

25.    I = R
26.  ENDDO
```

Line 10: $R[i] = 1/|P|$

Line 9: $i \in [1, |P|]$

Link Analysis

Algorithm: IterativePageRank

Input: $G = (P, L)$. Web graph with pages P and links L.
          λ. Random jump probability.

Output: I. Approximate PageRanks for all pages in P.

```
12.  # Update step
13.  FOREACH p ∈ P DO
14.    Q = { q | q ∈ P and (p,q) ∈ L }
15.    IF |Q| > 0 THEN
16.      FOREACH q ∈ Q DO
17.        R[q] = R[q] + (1 − λ) · I[p]/|Q|
18.      ENDDO
19.    ELSE
20.      FOREACH q ∈ P DO
21.        R[q] = R[q] + (1 − λ) · I[p]/|P|
22.      ENDDO
23.    ENDIF
24.  ENDDO
```

Link Analysis
PageRank: Convergence

Convergence is typically checked with

$$||R - I|| < \tau,$$

where $|| \cdot ||$ denotes the L_1 or L_2 norm, and τ is a threshold.

The choice of τ depends on the number n of documents, since $||R - I||$ (for a fixed numerical precision) increases with n. The larger τ, the faster convergence is reached. Optionally, $||R - I||/n$ can be used instead.

The number of iterations required to converge is roughly in $O(\log n)$. [Page 1999]

Counterintuitively, the PageRank algorithm does not converge faster when initialized with the PageRanks from a previously converged run compared to a uniform initialization. This is partly due to the rapid pace at which the web evolves.

[Meyer 2004]

Link Analysis

PageRank: Variants

The PageRank algorithm can be applied to web graphs at different levels of granularity:

- ❏ Web pages

- ❏ **Websites**
  Combining all pages hosted under a domain allows for computing the importance of websites as a whole.

- ❏ **Topic-specific clusters**
  Categorizing web pages by topic, or clustering them induces a web graph between categories / clusters. This allows for computing PageRanks within and across categories / clusters.

- ❏ **Personalized PageRank**
  Based on topic-specific PageRanks, a user may provide personal interests which can be applied as normalized weights onto each topic's PageRank vector.