# Chapter IR:IV

## IV. Retrieval Models

- Overview of Retrieval Models
- Empirical Models
- Boolean Retrieval
- Vector Space Model
- Probabilistic Models
- Binary Independence Model
- Okapi BM25
- Hidden Variable Models
- Latent Semantic Indexing
- Explicit Semantic Analysis
- Generative Models
- Language Models
- Divergence From Randomness
- Combining Evidence
- Web Search
- Learning to Rank

# Empirical Models [Probabilistic Models] [Hidden Variable Models] [Generative Models]

Basic empirical retrieval models abstract over a document $d \in D$ by treating it as a "bag of words" comprising the index terms derived from $d$.

A document representation $\mathbf{d}$ is composed of weighted index terms of $d$.

Discriminating factors of empirical models:

1. Term weighting method to compute the weight $w_i$ of an index term $t_i$.
2. Construction method of the query representation $\mathbf{q}$.
3. Computation method of the relevance function $\rho(\mathbf{q}, \mathbf{d})$.
4. Composition method of the result set $R$.

# Boolean Retrieval

## Retrieval Model $\mathcal{R} = \langle \mathbf{D}, \mathbf{Q}, \rho \rangle$ [Generic Model] [VSM] [BIM] [BM25] [LSI] [ESA] [LM]

### Document representations $\mathbf{D}$.

The set of index terms $T = \{t_1, \ldots, t_m\}$ is composed of nouns as lemmatized word stems.

The representation $\mathbf{d}$ of a document $d$ is a function from $T$ to $\{0, 1\}$, where $\mathbf{d}(t_i) = 1$ is interpreted as "term $t_i$ present in $d$", and $\mathbf{d}(t_i) = 0$ as "term $t_i$ absent from $d$".

### Query representations $\mathbf{Q}$.

A query representation $\mathbf{q}$ corresponds to a logical formula with alphabet $\Sigma = T$, where the logical operators $\wedge$, $\vee$, $\neg$, and brackets can be used.

### Relevance function $\rho$.

The document representation $\mathbf{d}$ of a document $d$ induces an interpretation $\mathcal{I}_\mathbf{d}$ for $\mathbf{q}$, yielding $\rho(\mathbf{q}, \mathbf{d}) = \mathcal{I}_\mathbf{d}(\mathbf{q})$.

If $\rho(\mathbf{q}, \mathbf{d}) = 1$, the document $d$ is an element of the result set $R$.

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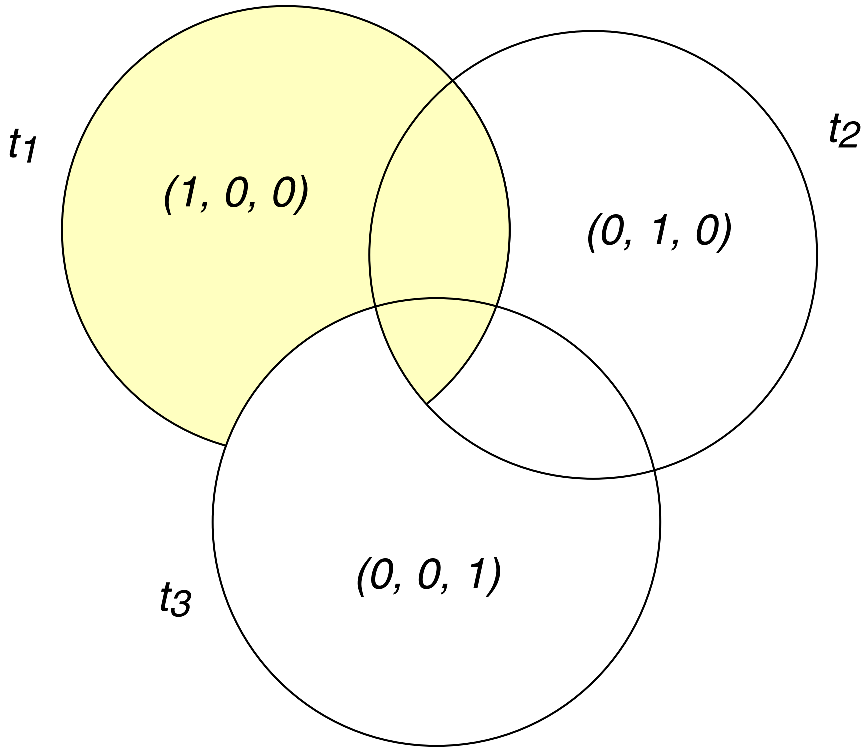
# Boolean Retrieval

## Relevance Function $\rho$




What is the query illustrated?

# Boolean Retrieval

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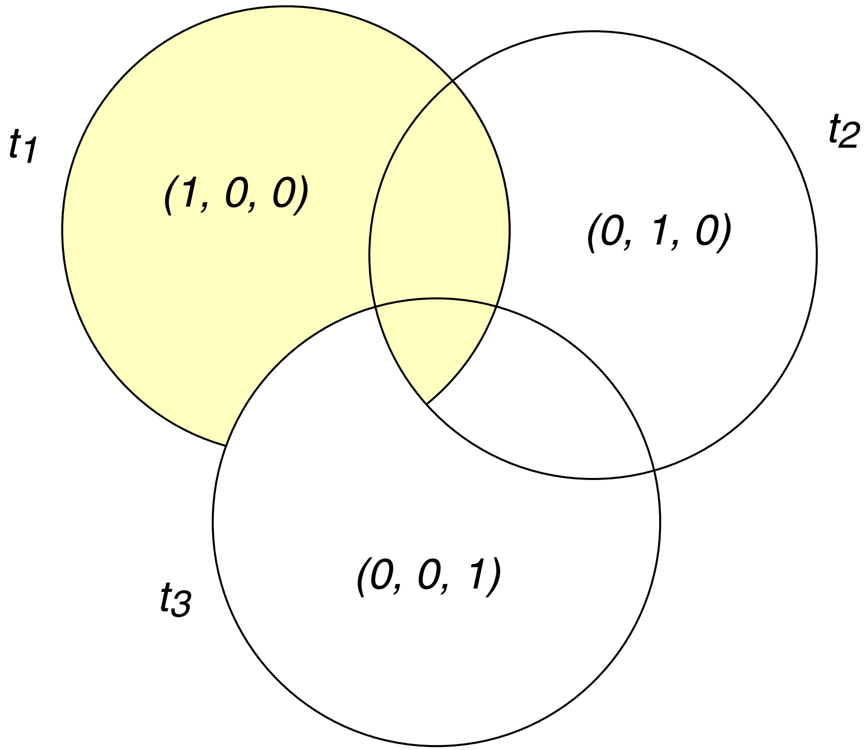



What is the query illustrated?

$$\mathbf{q} \;=\; t_1 \wedge (t_2 \vee \neg t_3) \;\equiv\; (t_1 \wedge \neg t_2 \wedge \neg t_3) \vee (t_1 \wedge t_2 \wedge \neg t_3) \vee (t_1 \wedge t_2 \wedge t_3)$$

# Boolean Retrieval

## Example

Document representation:

$$\mathbf{d} = \{\ (\texttt{chrysler}, 1),\ (\texttt{deal}, 1),\ (\texttt{usa}, 1),\ (\texttt{china}, 0),\ (\texttt{cat}, 0),\ (\texttt{sales}, 1),\ (\texttt{dog}, 0),\ \ldots\ \}$$

Query representation:

$$\begin{aligned}\mathbf{q} &= \texttt{usa} \wedge (\texttt{dog} \vee \neg\texttt{cat}) \\ &\equiv (\texttt{usa} \wedge \texttt{dog}) \vee (\texttt{usa} \wedge \neg\texttt{cat}) \\ &\equiv (\texttt{usa} \wedge \neg\texttt{dog} \wedge \neg\texttt{cat}) \vee \\ &\quad\ (\texttt{usa} \wedge \texttt{dog} \wedge \neg\texttt{cat}) \vee \\ &\quad\ (\texttt{usa} \wedge \texttt{dog} \wedge \texttt{cat})\end{aligned}$$

Induces interpretation:

$\mathcal{I}_{\mathbf{d}}(\mathbf{q}) = 1$, since $\mathcal{I}_{\mathbf{d}}(\texttt{usa}) = 1, \mathcal{I}_{\mathbf{d}}(\texttt{dog}) = 0$, and $\mathcal{I}_{\mathbf{d}}(\texttt{cat}) = 0$.

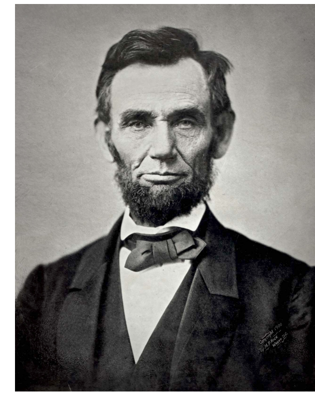
Remarks:

- The symbol "$\equiv$" denotes "is logically equivalent with".
- What does logical equivalence mean?
- A Boolean query in disjunctive normal form can be answered straightforward using an inverted index in parallel for each conjunction.
- A Boolean query in canonical disjunctive normal form will retrieve each document only once.

# Boolean Retrieval

## Query Refinement: “Searching by Numbers”

Best practice in Boolean retrieval: (re)formulate queries until the number of documents retrieved is manageable. Example: pages about President Lincoln.



1. `lincoln`
   Result: many pages about cars, places, people

2. $\texttt{president} \land \texttt{lincoln}$
   A result: “Ford Motor Company today announced that Darryl Hazel will succeed Brian Kelley as <u>president</u> of <u>Lincoln</u> Mercury.”

3. $\texttt{president} \land \texttt{lincoln} \land \neg\texttt{automobile} \land \neg\texttt{car}$
   Not in result: “<u>President Lincoln</u>’s body departs Washington in a nine-<u>car</u> funeral train.”

4. $\texttt{president} \land \texttt{lincoln} \land \neg\texttt{automobile} \land \texttt{biography} \land \texttt{life} \land \texttt{birthplace} \land \texttt{gettysburg}$
   Result: $\emptyset$

5. $\texttt{president} \land \texttt{lincoln} \land \neg\texttt{automobile} \land (\texttt{biography} \lor \texttt{life} \lor \texttt{birthplace} \lor \texttt{gettysburg})$
   Top result might be: “<u>President</u>’s Day – Holiday activities – crafts, mazes, word searches, . . . ’The <u>Life</u> of Washington’ Read the entire book online! Abraham <u>Lincoln</u> Research Site”

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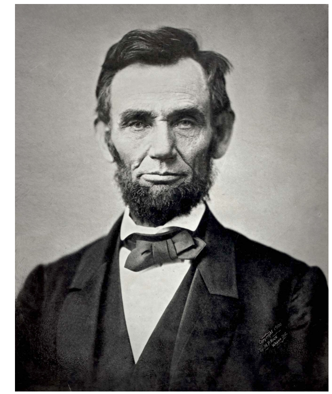
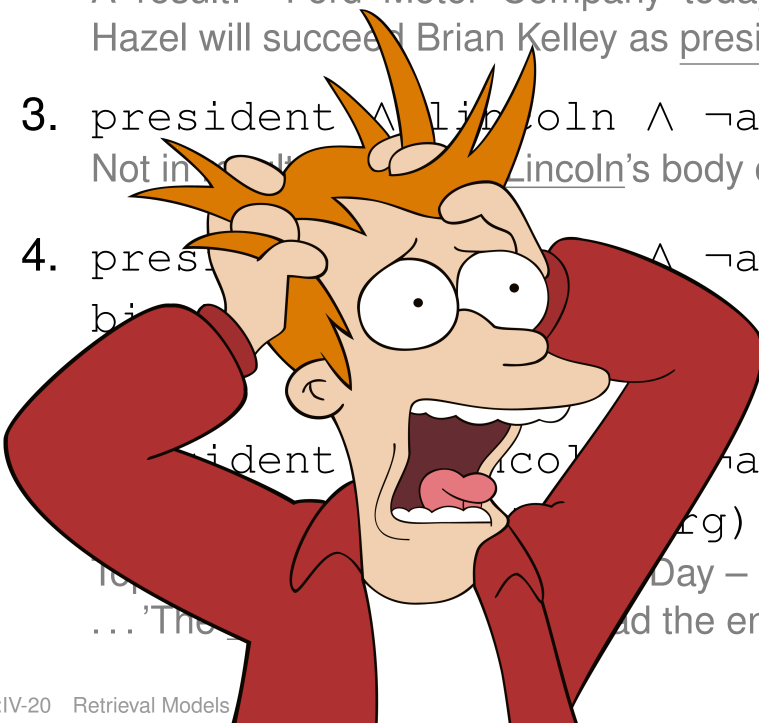


1. `lincoln`
   Result: many pages about cars, places, people

2. `president ∧ lincoln`
   A result: “Ford Motor Company today announced that Darryl Hazel will succeed Brian Kelley as president of Lincoln Mercury.”

3. `president ∧ lincoln ∧ ¬automobile ∧ ¬car`
   Not in … Lincoln’s body departs Washington in a nine-car funeral train.”

4. `pres… ∧ ¬automobile ∧ biography ∧ life ∧`
   `bi…`



WAAAAHHH

`…ident … col… ¬automobile ∧ (biography ∨ life ∨`
`…rg)`

To… Day – Holiday activities – crafts, mazes, word searches, … ’The … d the entire book online! Abraham Lincoln Research Site”

# Boolean Retrieval
## Discussion

Advantages:

- Precision: in principle, any subset of documents from a collection can be designated by a Boolean query
- as in data retrieval, other fields are possible (e.g., date, document type, etc.)
- simple, efficient implementation

Disadvantages:

- retrieval effectiveness depends entirely on the user
- cumbersome query formulation (e.g., expertise required)
- no possibility to weight query terms
- no ranking; binary relevance scoring is too restrictive for most practical purposes (exceptions: systematic reviews, patent prior art, legal cases)
- the size of the result set is difficult to be controlled

# Vector Space Model

## Retrieval Model $\mathcal{R} = \langle \mathbf{D}, \mathbf{Q}, \rho \rangle$ [Generic Model] [Boolean Retrieval] [BIM] [BM25] [LSI] [ESA] [LM]

### Document representations $\mathbf{D}$.

The set of index terms $T = \{t_1, \ldots, t_m\}$ is typically composed of the word stems of the vocabulary of a document collection, excluding stop words.

The representation $\mathbf{d}$ of a document $d$ is a $|T|$-dimensional vector, where the $i$-th vector component of $\mathbf{d}$ corresponds to a term weight $w_i$ of term $t_i \in T$, indicating its importance for $d$. Various term weighting schemes have been proposed.

### Query representations $\mathbf{Q}$.

A query representation $\mathbf{q}$ is constructed like a document representation.

### Relevance function $\rho$.

Document representations and query representations are interpreted as points in a vector space spanned by unit vectors for each term in $T$, assuming their orthogonality.

Distance and similarity functions defined for vector spaces serve as relevance functions $\rho$. The Euclidean distance and the cosine similarity are important examples.

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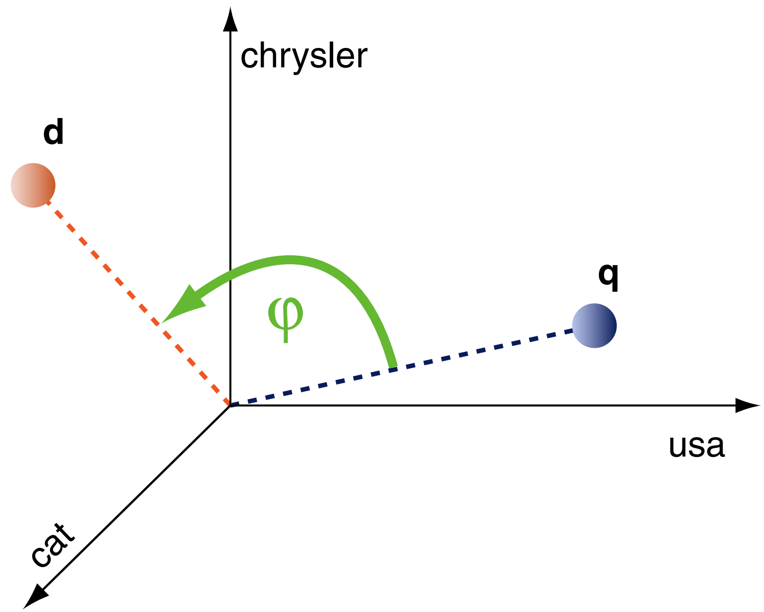
# Vector Space Model

## Relevance Function $\rho$: Cosine Similarity



# Vector Space Model

## Relevance Function $\rho$: Cosine Similarity

The scalar product $\mathbf{a}^T\mathbf{b}$ between two $n$-dimensional vectors $\mathbf{a}$ and $\mathbf{b}$, where $\varphi$ denotes the angle between them, is defined as follows:

$$\begin{aligned} \mathbf{a}^T\mathbf{b} &= ||\mathbf{a}|| \cdot ||\mathbf{b}|| \cdot \cos(\varphi) \\ \Leftrightarrow \quad \cos(\varphi) &= \frac{\mathbf{a}^T\mathbf{b}}{||\mathbf{a}|| \cdot ||\mathbf{b}||}, \end{aligned}$$

where $||\mathbf{x}||$ denotes the L2 norm of vector $\mathbf{x}$:

$$||\mathbf{x}|| = \left( \sum_{i=1}^{n} x_i^2 \right)^{1/2}$$

Let $\rho(\mathbf{q}, \mathbf{d}) = \cos(\varphi)$ the relevance function of the vector space model.

# Vector Space Model

## Example

$$\mathbf{d} = \begin{pmatrix} \texttt{chrysler} & w_1 \\ \texttt{usa} & w_2 \\ \texttt{cat} & w_3 \\ \texttt{dog} & w_4 \\ \texttt{mouse} & w_5 \end{pmatrix} = \begin{pmatrix} \texttt{chrysler} & 1 \\ \texttt{usa} & 4 \\ \texttt{cat} & 3 \\ \texttt{dog} & 7 \\ \texttt{mouse} & 5 \end{pmatrix}$$

$$\mathbf{d}' = \begin{pmatrix} \texttt{chrysler} & 0.1 \\ \texttt{usa} & 0.4 \\ \texttt{cat} & 0.3 \\ \texttt{dog} & 0.7 \\ \texttt{mouse} & 0.5 \end{pmatrix}, \qquad \mathbf{q}' = \begin{pmatrix} \texttt{chrysler} & 0.5 \\ \texttt{usa} & 0.5 \\ \texttt{cat} & 0.5 \\ \texttt{dog} & 0.5 \\ \texttt{elephant} & 0.5 \end{pmatrix}$$

The angle $\varphi$ between $\mathbf{d}'$ and $\mathbf{q}'$ is about $41°$, $\cos(\varphi) \approx 0.75$.

# Vector Space Model

## Term Weighting: *tf* · *idf* [BIM Relevance Function]

To compute the weight $w$ for a term $t$ from document $d$ under the vector space model, the most commonly employed term weighting scheme $\omega(t)$ is *tf* · *idf*:

- *tf*$(t, d)$ denotes the normalized *term frequency* of term $t$ in document $d$.
  The basic idea is that the importance of term $t$ is proportional to its frequency in document $d$. However, $t$'s importance does not increase linearly: the raw frequency must be normalized.

- *df*$(t, D)$ denotes the *document frequency* of term $t$ in document collection $D$.
  It counts the number of documents that contain $t$ at least once.

- *idf*$(t, D)$ denotes the *inverse document frequency*:

$$\mathit{idf}(t, D) = \log \frac{|D|}{\mathit{df}(t, D)}$$

  The importance of term $t$ in general is inversely proportional to its document frequency.

A term weight $w$ for term $t$ in document $d \in D$ is computed as follows:

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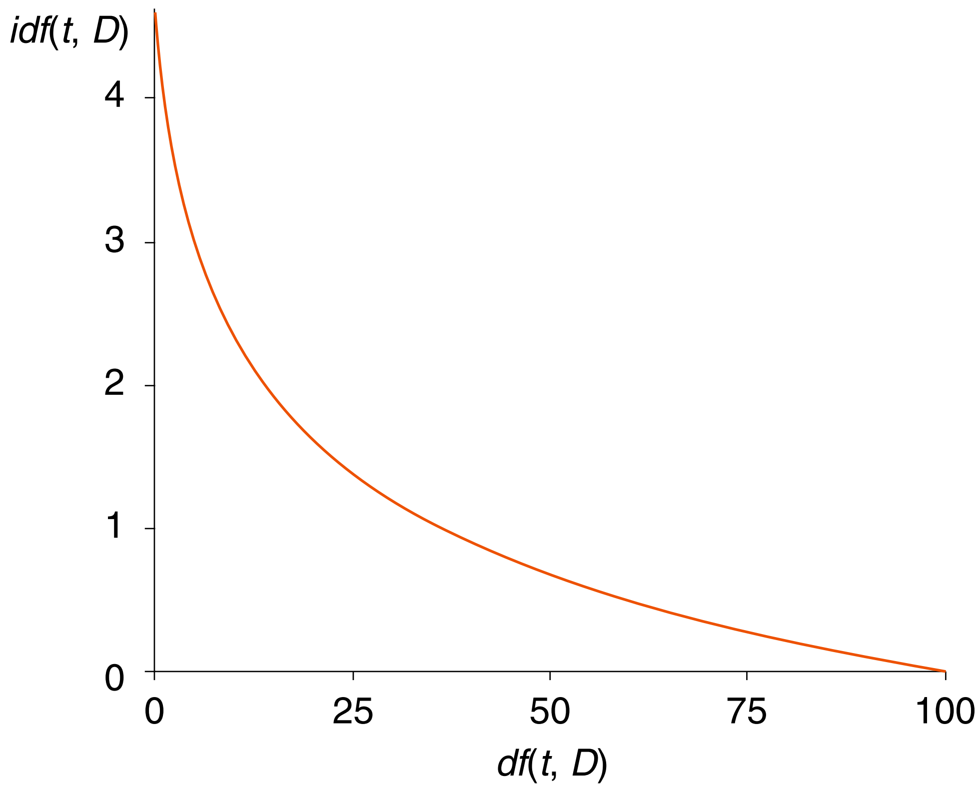
# Vector Space Model

## Term Weighting: *tf* · *idf*

Plot of the function $\mathit{idf}(t, D) = \log \frac{|D|}{\mathit{df}(t, D)}$ for $|D| = 100$.



Remarks:

- Term frequency weighting was invented by Hans Peter Luhn: "There is also the probability that the more frequently a notion and combination of notions occur, the more importance the author attaches to them as reflecting the essence of his overall idea." [Luhn 1957]

- The importance of a term $t$ for a document $d$ is not linearly correlated with its frequency. Several normalization factors have been proposed [Wikipedia]:
  - $\mathit{tf}(t,d)/|d|$
  - $1+\log(\mathit{tf}(t,d))$ for $\mathit{tf}(t,d)>0$
  - $k+(1-k)\dfrac{\mathit{tf}(t,d)}{\max_{t'\in d}(\mathit{tf}(t',d))}$, where $k$ serves as smoothing term; typically $k=0.4$

- Inverse document frequency weighting was invented by Karen Spärck Jones: "it seems we should treat matches on non-frequent terms as more valuable than ones on frequent terms, without disregarding the latter altogether. The natural solution is to correlate a term's matching value with its collection frequency." [Spärck Jones 1972]

- Spärck Jones gives little theoretical justification for her intuition. Given the success of *idf* in practice, over the decades, numerous attempts at a theoretical justification have been made. A comprehensive overview has been compiled by [Robertson 2004].

- For example, interpreting the term $\frac{|D|}{\mathit{df}(t,D)}$ as inverse of the probability $P_{\mathit{df}}(t)=\frac{\mathit{df}(t,D)}{|D|}$ of $t$ occurring in a random document in $D$ yields $\mathit{idf}(t,D)=\log\frac{|D|}{\mathit{df}(t,D)}=-\log P_{\mathit{df}}(t)$. Logarithms fit relevance functions $\rho$ since both are additive, yielding the interpretation: "The less likely (on a random basis) it is that a given combination of terms occurs, the more likely it is that a document containing this combination is relevant to the question." [Robertson 1972]

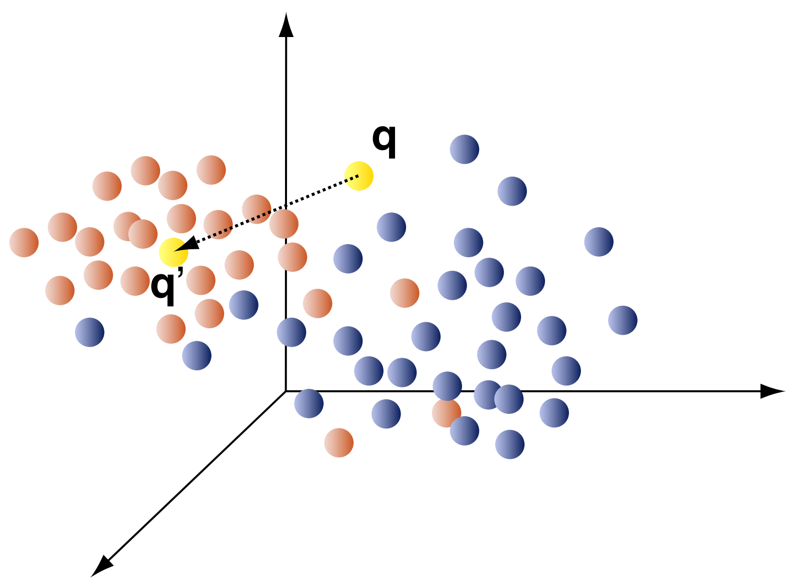
# Vector Space Model

## Query Refinement: Relevance Feedback

Given a result set $R$ for a query $q$, and subsets $R^+ \subseteq R$ and $R^- \subseteq R$ of relevant and irrelevant documents, where $R^+ \cap R^- = \emptyset$, the query representation $\mathbf{q}$ can be refined using Rocchio's update formula:

$$\mathbf{q}' = \quad \alpha \cdot \mathbf{q} \quad + \quad \beta \cdot \frac{1}{|R^+|} \sum_{d^+ \in R^+} \mathbf{d}^+ \quad - \quad \gamma \cdot \frac{1}{|R^-|} \sum_{d^- \in R^-} \mathbf{d}^-,$$

where $\alpha$, $\beta$, and $\gamma$ adjust the impact of original query and (ir)relevant documents.



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where $\alpha$, $\beta$, and $\gamma$ adjust the impact of original query and (ir)relevant documents.

Observations:

- Terms not in query $q$ may get added; often a limit is imposed (say, 50).
- Terms may accrue negative weight; such weights are set to 0.
- Moves the query vector closer to the centroid of relevant documents.
- Works well if relevant documents cluster; less suited for multi-faceted topics.

Relevance feedback can be obtained directly from the user, indirectly through user interaction, or automatically assuming the top-retrieved documents as relevant.

 

# Vector Space Model

## Discussion

Advantages:

- Severely improved retrieval performance compared to Boolean retrieval
- Partial query matching: not all query terms need to be present in a document for it to be retrieved
- The relevance function $\rho$ defines a ranking among retrieved documents with respect to their computed similarity to the query

Disadvantages:

- Index terms are assumed to occur independent of one another