Chapter ML:II (continued)

II. Machine Learning Basics

- Regression
- Concept Learning: Search in Hypothesis Space
- Concept Learning: Search in Version Space
- Evaluating Effectiveness

Concept Learning: Search in Hypothesis Space

A Learning Task

Given is a set D of examples: days that are characterized by the six features “Sky”, “Temperature”, “Humidity”, “Wind”, “Water”, and “Forecast”, along with a statement (in fact: a feature) whether or not our friend will enjoy her favorite sport.

Example	Sky	Temperature	Humidity	Wind	Water	Forecast	EnjoySport
1	sunny	warm	normal	strong	warm	same	yes
2	sunny	warm	high	strong	warm	same	yes
3	rainy	cold	high	strong	warm	change	no
4	sunny	warm	high	strong	cool	change	yes

- What is the concept behind “EnjoySport” ?
- What are possible hypotheses to formalize the concept “EnjoySport” ?
  Similarly: What are the elements of the set or class “EnjoySport” ?

Remarks:

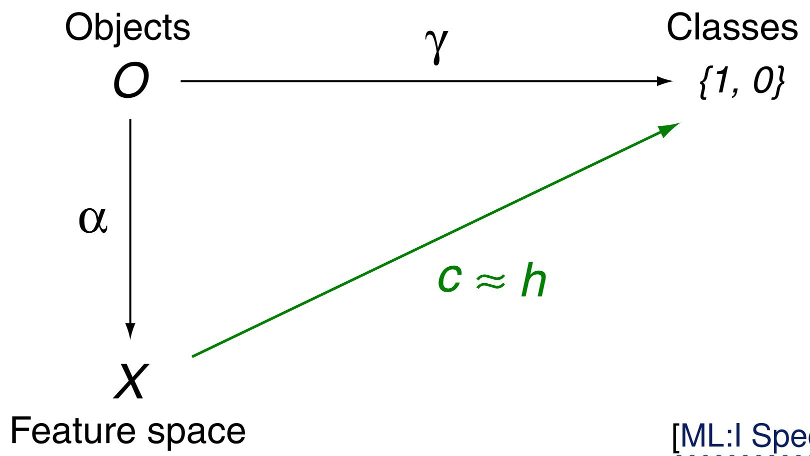
- Domains of the features in the learning task:

Sky	Temperature	Humidity	Wind	Water	Forecast
sunny	warm	normal	strong	warm	same
rainy	cold	high	weak	cool	change
cloudy					

- A hypothesis is a finding or an insight gained by inductive reasoning. Within concept learning tasks, hypotheses are used to capture the target concept.

- A hypothesis cannot be inferred or proved by deductive reasoning. Rather, a hypothesis is justified inductively, by its capability to represent (= to explain) a given set of observations, which are called examples here.

Concept Learning: Search in Hypothesis Space



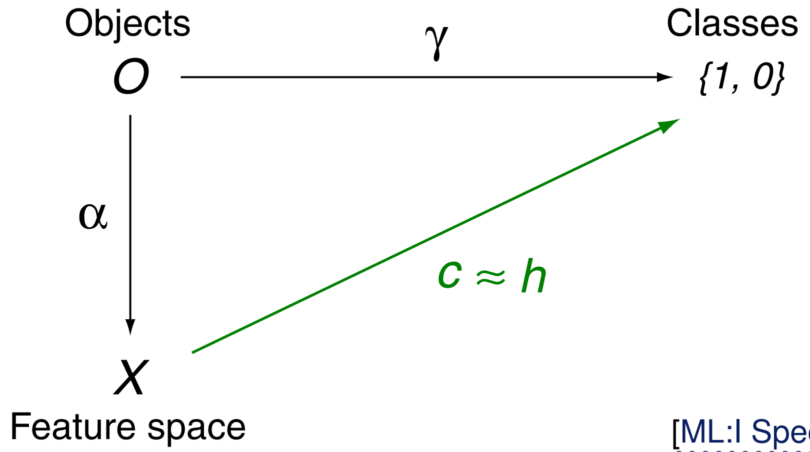
[ML:I Specification of Learning Problems]

Definition 1 (Concept, Hypothesis, Hypothesis Space)

A concept is a subset of an object set O and hence determines a subset of the feature space $X = \alpha(O)$. Concept learning is the approximation of the ideal classifier $c : X \to \{0, 1\}$ by a function h, where c is defined as follows:

$$c(\mathbf{x}) = \begin{cases} 1 & \text{if } \alpha^{-1}(\mathbf{x}) \text{ belongs to the concept} \\ 0 & \text{otherwise} \end{cases}$$

Concept Learning: Search in Hypothesis Space



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$$c(\mathbf{x}) = \begin{cases} 1 & \text{if } \alpha^{-1}(\mathbf{x}) \text{ belongs to the concept} \\ 0 & \text{otherwise} \end{cases}$$

The approximation function $h : X \to \{0, 1\}$ is called hypothesis here. A set H of hypotheses among which h is searched is called hypothesis space.

Concept Learning: Search in Hypothesis Space

Usually, an example set D, $D = \{(\mathbf{x}_1, c(\mathbf{x}_1)), \ldots, (\mathbf{x}_n, c(\mathbf{x}_n))\}$, contains positive ($c(\mathbf{x}) = 1$) and negative ($c(\mathbf{x}) = 0$) examples. [Learning Task]

Definition 2 (Hypothesis-Fulfilling, Consistency)

An example $(\mathbf{x}, c(\mathbf{x}))$ fulfills a hypothesis h iff $h(\mathbf{x}) = 1$. A hypothesis h is consistent with an example $(\mathbf{x}, c(\mathbf{x}))$ iff $h(\mathbf{x}) = c(\mathbf{x})$.

A hypothesis h is consistent with a set D of examples, denoted as *consistent*(h, D), iff:

$$\forall (\mathbf{x}, c(\mathbf{x})) \in D : \; h(\mathbf{x}) = c(\mathbf{x})$$

Remarks:

- The string “Iff” or “iff” is an abbreviation for “If and only if”, which means “necessary and sufficient”. It is a textual representation for the logical biconditional, also known as material biconditional or iff-connective. The respective symbol is “$\leftrightarrow$”. [Wolfram] [Wikipedia]

- The following terms are used synonymously: target concept, target function, ideal classifier.

- The fact that a hypothesis is consistent with an example can also be described the other way round: an example is consistent with a hypothesis.

- Given an example $(\mathbf{x}, c(\mathbf{x}))$, notice the difference between (1) hypothesis-fulfilling and (2) being consistent with a hypothesis. The former asks for $h(\mathbf{x}) = 1$, disregarding the actual target concept value $c(\mathbf{x})$. The latter asks for the identity between the target concept $c(\mathbf{x})$ and the hypothesis $h(\mathbf{x})$.

- The consistency of h can be analyzed for a single example as well as for a set D of examples. Given the latter, consistency requires for all elements in D that $h(\mathbf{x}) = 1$ iff $c(\mathbf{x}) = 1$. This is equivalent with the condition that $h(\mathbf{x}) = 0$ iff $c(\mathbf{x}) = 0$ for all $\mathbf{x} \in D$.

- Learning means to determine a hypothesis $h \in H$ that is consistent with D.

Concept Learning: Search in Hypothesis Space

A Learning Task (continued)

Structure of a hypothesis h:

1. conjunction of feature-value pairs
2. three kinds of values: literal, ? (wildcard), $\perp$ (contradiction)

A hypothesis for EnjoySport [Learning Task] : $\langle$ *sunny, ?, ?, strong, ?, same* $\rangle$

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1. conjunction of feature-value pairs
2. three kinds of values: literal, ? (wildcard), $\perp$ (contradiction)

A hypothesis for EnjoySport [Learning Task] : $\langle$ *sunny, ?, ?, strong, ?, same* $\rangle$

Definition 3 (Maximally Specific / General Hypothesis)

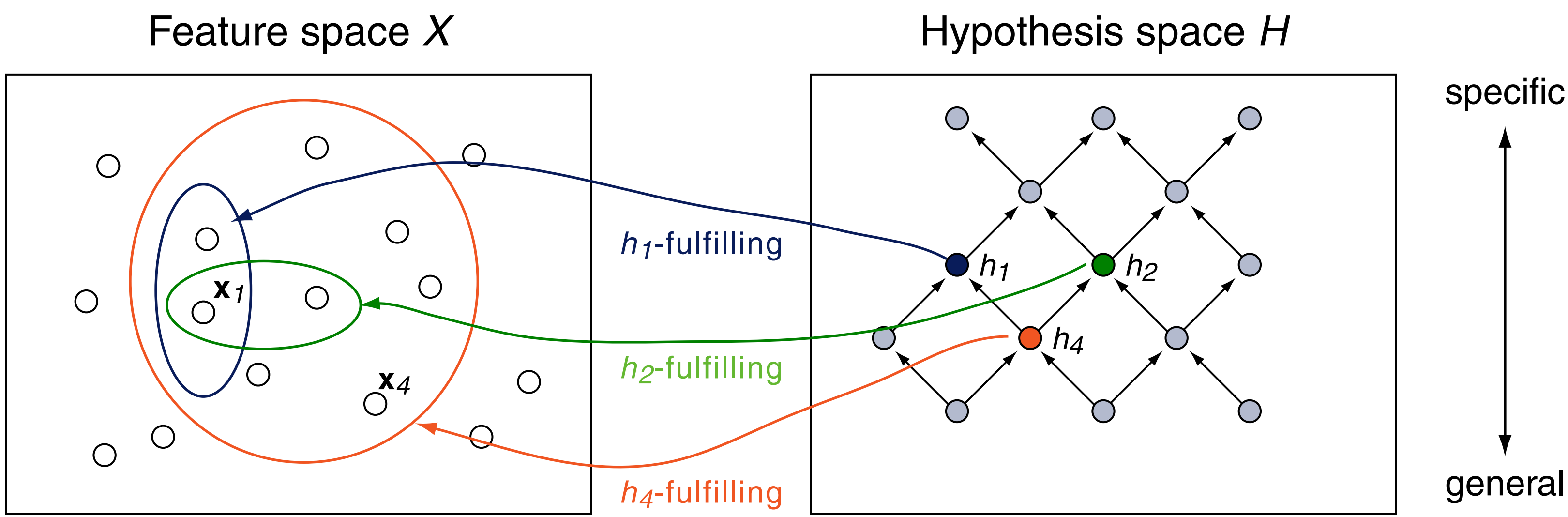
The hypotheses $s_0(\mathbf{x}) \equiv 0$ and $g_0(\mathbf{x}) \equiv 1$ are called maximally specific and maximally general hypothesis respectively. No $\mathbf{x} \in X$ fulfills s_0, and all $\mathbf{x} \in X$ fulfill g_0.

Maximally specific / general hypothesis in the example [Learning Task] :

- $s_0 = \langle \perp, \perp, \perp, \perp, \perp, \perp \rangle$ (never enjoy sport)
- $g_0 = \langle$ *?, ?, ?, ?, ?, ?* $\rangle$ (always enjoy sport)

Concept Learning: Search in Hypothesis Space

Order of Hypotheses



$\mathbf{x}_1 = ($*sunny, warm, normal, strong, warm, same*$)$ $\quad h_1 = \langle$ *sunny, ?, normal, ?, ?, ?* $\rangle$

$h_2 = \langle$ *sunny, ?, ?, ?, warm, ?* $\rangle$

$\mathbf{x}_4 = ($*sunny, warm, high, strong, cool, change*$)$ $\quad h_4 = \langle$ *sunny, ?, ?, ?, ?, ?* $\rangle$

Concept Learning: Search in Hypothesis Space

Order of Hypotheses

Definition 4 (More General Relation)

Let X be a feature space and let h_1 and h_2 be two boolean-valued functions with domain X. Then function h_1 is called more general than function h_2, denoted as $h_1 \geq_g h_2$, iff:

$$\forall \mathbf{x} \in X : \ (\ h_2(\mathbf{x}) = 1 \ \text{ implies } \ h_1(\mathbf{x}) = 1 \ )$$

h_1 is called stricly more general than h_2, denoted as $h_1 >_g h_2$, iff:

$$(h_1 \geq_g h_2) \ \text{ and } \ (h_2 \not\geq_g h_1)$$

Concept Learning: Search in Hypothesis Space

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$$(h_1 \geq_g h_2) \;\; \text{and} \;\; (h_2 \not\geq_g h_1)$$

About the maximally specific / general hypothesis:

- s_0 is minimum and g_0 is maximum with regard to $\geq_g$: no hypothesis is more specific wrt. s_0, and no hypothesis is more general wrt. g_0.
- We will consider only hypothesis spaces that contain s_0 and g_0.

Remarks:

- If h_1 is more general than h_2, then h_2 can also be called being more specific than h_1.
- $\geq_g$ and $>_g$ are independent of a target concept c. They depend only on the fact that examples fulfill a hypothesis, i.e., whether $h(\mathbf{x}) = 1$. They require not that $c(\mathbf{x}) = 1$.
- The $\geq_g$-relation defines a partial order on the hypothesis space H : $\geq_g$ is reflexive, anti-symmetric, and transitive. The order is *partial* since (unlike in a total order) not all hypothesis pairs stand in the relation. [Wikipedia partial, total]

  I.e., we are given hypotheses h_i, h_j, for which neither $h_i \geq_g h_j$ nor $h_j \geq_g h_i$ holds, such as the hypotheses h_1 and h_2 in the hypothesis space.

Remarks: (continued)

- The semantics of the implication, in words "a implies b", denoted as $a \to b$, is as follows. $a \to b$ is true if either (1) a is true and b is true, or (2) if a is false and b is true, or (3) if a is false and b is false—in short: "if a is true then b is true as well", or, "the truth of a implies the truth of b".

- "$\to$" can be understood as "causality connective": Let a and b be two events where a is a cause for b. If we interpret the occurrence of an event as true and its non-occurrence as false, we will observe only occurrence combinations such that the formula $a \to b$ is true. The connective is also known as material conditional, material implication, material consequence, or simply, implication or conditional.

- Note in particular that the connective "$\to$" does not stand for "entails", which would be denoted as either $\Rightarrow$ or $\models$. Logical entailment (synonymously: logical inference, logical deduction, logical consequence) allows to infer or to prove a fact. Consider for instance Definition 4: From the fact $h_2(\mathbf{x}) = 1$ we cannot infer or prove the fact $h_1(\mathbf{x}) = 1$.

- In Definition 4 the implication specifies a condition that is to be fulfilled by the definiendum (= the thing to be defined). The implication is used to check whether or not a thing belongs to the set of things specified by the definiens (= the expression that defines):
  Each pair of functions, h_1, h_2, is a thing that belongs to the set of things specified by the definition of the $\geq_g$-relation (i.e., stands in the $\geq_g$-relation) if and only if the implication $h_2(\mathbf{x}) = 1 \to h_1(\mathbf{x}) = 1$ is true for all $\mathbf{x} \in X$.

Remarks: (continued)

- In a nutshell: distinguish carefully between "α requires β", denoted as $\alpha \rightarrow \beta$, on the one hand, and "from α follows β", denoted as $\alpha \Rightarrow \beta$, on the other hand. $\alpha \rightarrow \beta$ is considered as a sentence from the *object language* (language of discourse) and stipulates a computing operation, whereas $\alpha \Rightarrow \beta$ is a sentence from the *meta language* and makes an assertion *about* the sentence $\alpha \rightarrow \beta$, namely: "$\alpha \rightarrow \beta$ is a tautology".

- Finally, consider the following sentences from the object language, which are synonymous: "$\alpha \rightarrow \beta$", "α implies β", "if α then β", "α causes β", "α requires β", "α is sufficient for β", "β is necessary for α".

Concept Learning: Search in Hypothesis Space

Inductive Learning Hypothesis

“Any hypothesis found to approximate the target function well over a sufficiently large set of training examples will also approximate the target function well over other unobserved examples.”

[p.23, Mitchell 1997]

Concept Learning: Search in Hypothesis Space

Find-S Algorithm

```
1. h = s_0  // h is a maximally specific hypothesis in H.
2. FOREACH (x, c(x)) ∈ D DO
     IF c(x) = 1 THEN  // Use only positive examples.
       IF h(x) = 0 DO
         h = min_generalization(h, x)  // Relax hypothesis h wrt. x.
       ENDIF
     ENDIF
   ENDDO
3. return(h)
```

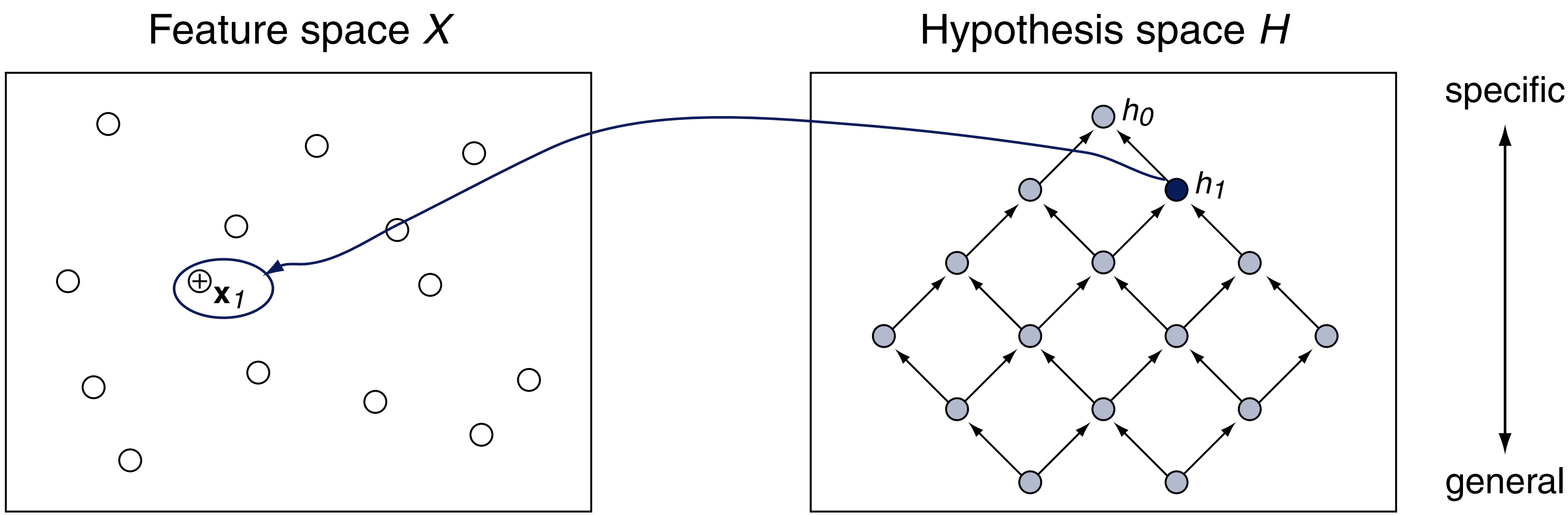
Remarks:

- Another term for “generalization” is “relaxation”.
- The function *min_generalization*$(h, \mathbf{x})$ returns a hypothesis h' that is minimally generalized wrt. h and that is consistent with $(\mathbf{x}, 1)$. Denoted formally: $h' \geq_g h$ and $h'(\mathbf{x}) = 1$ and there is no h'' with $h' >_g h'' \geq_g h$ and $h''(\mathbf{x}) = 1$.
- For more complex hypothesis structures the relaxation of h given $\mathbf{x}$, *min_generalization*$(h, \mathbf{x})$, may not be unique. In such a case one of the alternatives has to be chosen.
- If a hypothesis h needs to be relaxed towards some h' with $h' \notin H$, the maximally general hypothesis $g_0 \equiv 1$ can be added to H.
- Similar to *min_generalization*$(h, \mathbf{x})$, a function *min_specialization*$(h, \mathbf{x})$ can be defined, which returns a minimally specialized, consistent hypotheses for negative examples.

Concept Learning: Search in Hypothesis Space

Find-S Algorithm

See the example set D for the concept *EnjoySport*.



$h_0 = s_0 = \langle \perp, \perp, \perp, \perp, \perp, \perp \rangle$

$\mathbf{x}_1 = ($*sunny, warm, normal, strong, warm, same*$)$ $\quad h_1 = \langle$ *sunny, warm, normal, strong, warm, same* $\rangle$

Concept Learning: Search in Hypothesis Space

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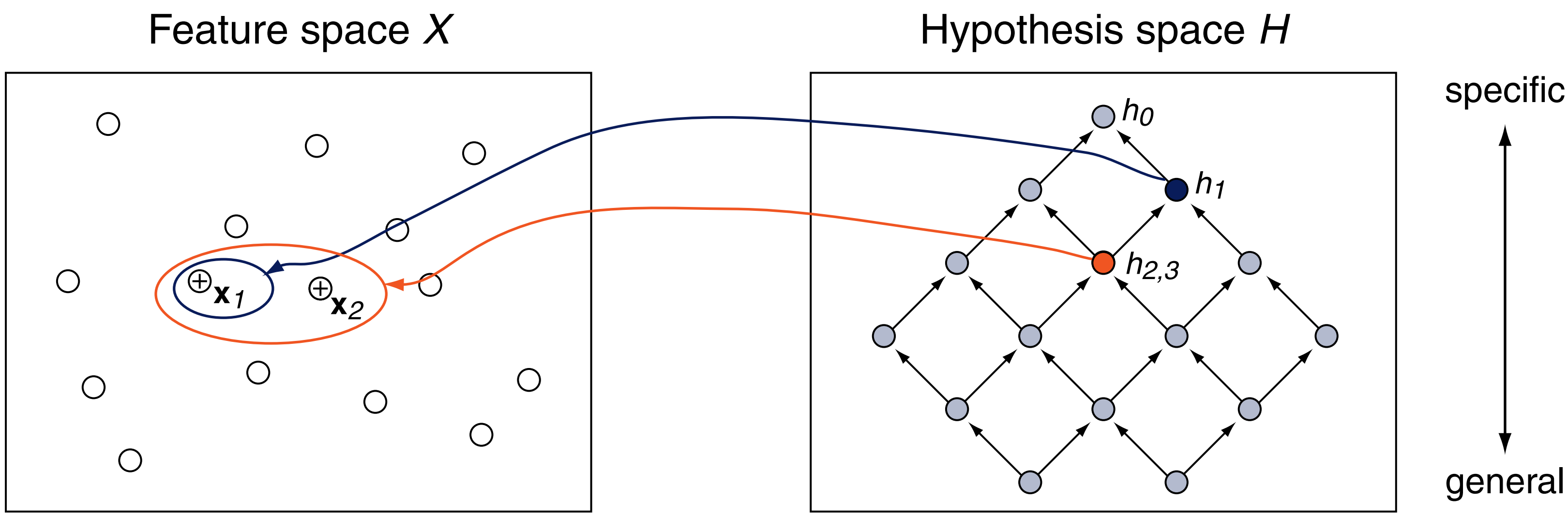

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Concept Learning: Search in Hypothesis Space

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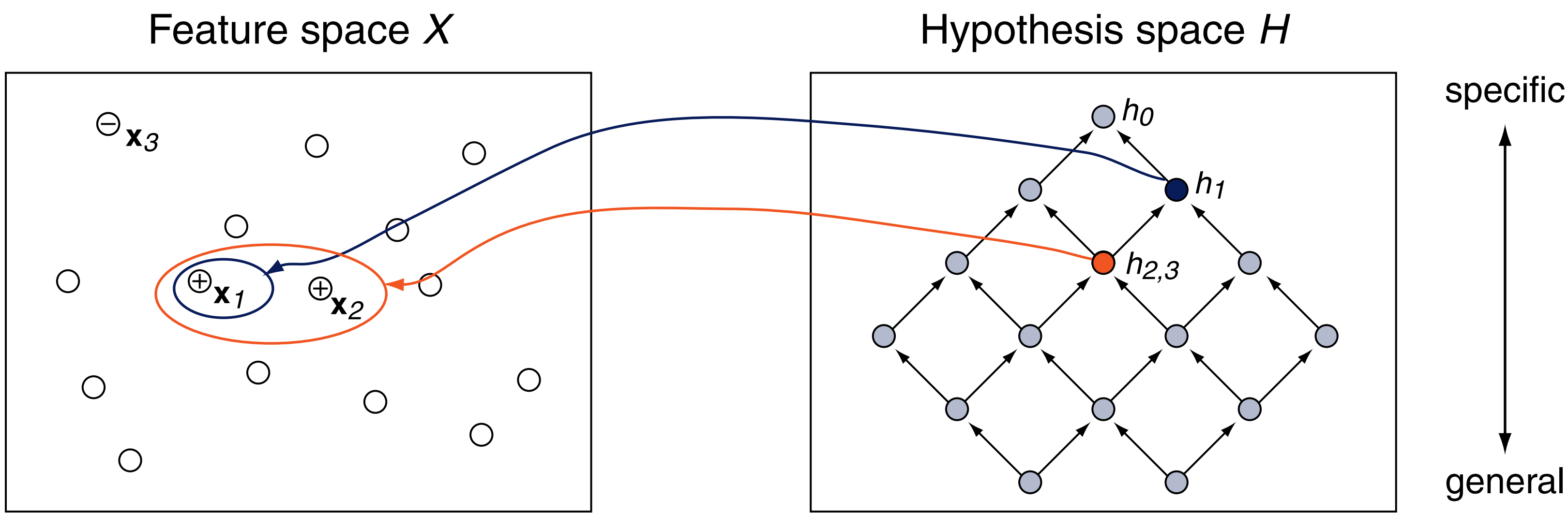

$h_0 = s_0 = \langle\ \bot, \bot, \bot, \bot, \bot, \bot\ \rangle$

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$\mathbf{x}_3 = ($*rainy, cold, high, strong, warm, change*$)$ $\quad h_3 = \langle$ *sunny, warm, ?, strong, warm, same* $\rangle$

Concept Learning: Search in Hypothesis Space

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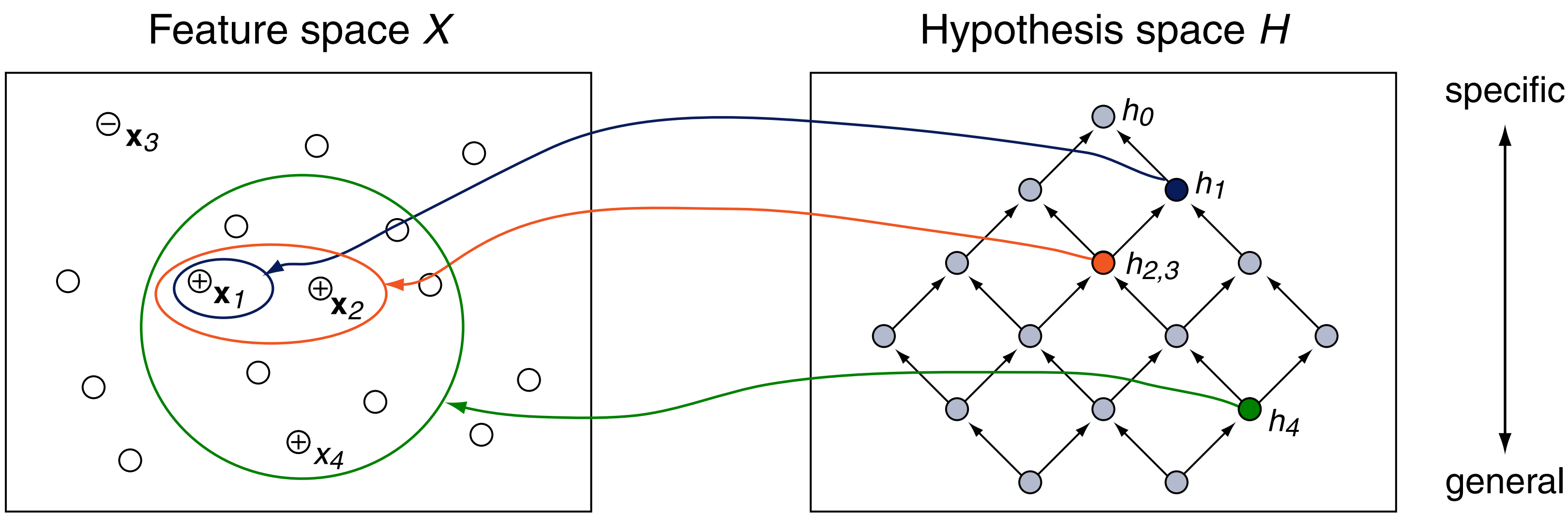

$h_0 = s_0 = \langle \perp, \perp, \perp, \perp, \perp, \perp \rangle$

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Concept Learning: Search in Hypothesis Space

Discussion of the Find-S Algorithm

1. Did we learn the only concept—or are there others?
2. Why should one pursuit the maximally specific hypothesis?
3. What if several maximally specific hypotheses exist?
4. Inconsistencies in the example set D remain undetected.
5. An inappropriate hypothesis structure or space H remains undetected.

Concept Learning: Search in Version Space

Definition 5 (Version Space)

The version space $V_{H,D}$ of an hypothesis space H and a example set D is comprised of all hypotheses $h \in H$ that are consistent with a set D of examples:

$$V_{H,D} = \{h \mid h \in H \ \wedge \ (\ \forall(\mathbf{x}, c(\mathbf{x})) \in D : \ h(\mathbf{x}) = c(\mathbf{x}) \ ) \ \}$$

Concept Learning: Search in Version Space

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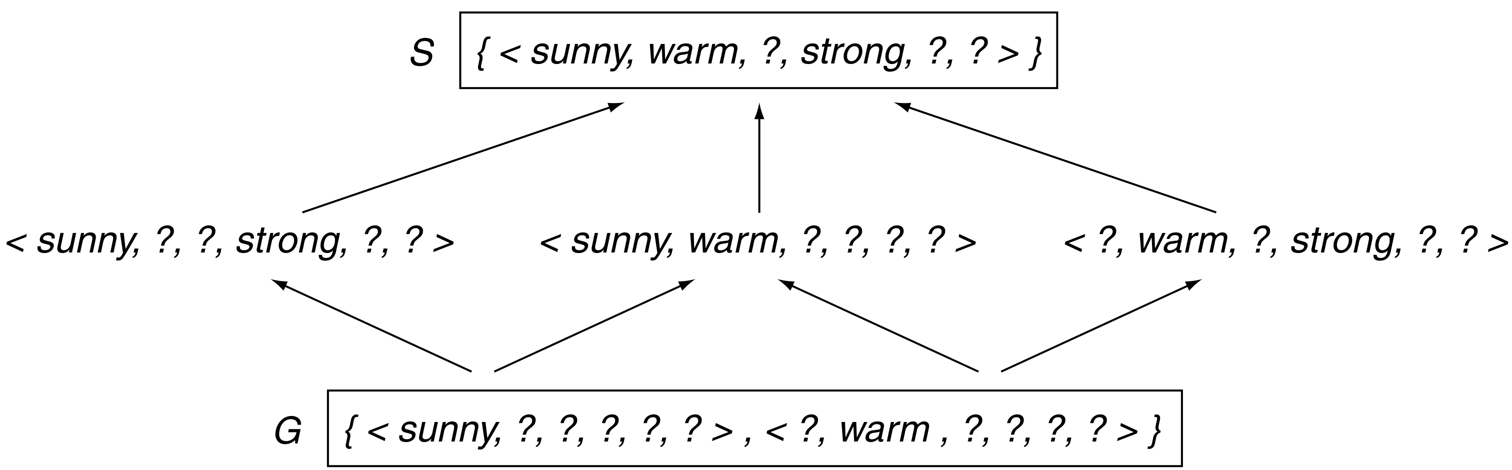
Illustration of $V_{H,D}$ for the example set D:



Remarks:

- The term “version space” reflects the fact that $V_{H,D}$ represents the set of all consistent versions of the target concept that are encoded in D.
- A naive approach for the construction of the version space is the following: (1) enumeration of all members of H, and, (2) elimination of those $h \in H$ for which $h(\mathbf{x}) \neq c(\mathbf{x})$ holds. This approach presumes a finite hypothesis space H and is feasible only for toy problems.

Concept Learning: Search in Version Space

Definition 6 (Boundary Sets of a Version Space)

Let H be hypothesis space and let D be set of examples. Then, based on the $\geq_g$-relation, the set of maximally general hypotheses, G, is defined as follows:

$$\{g \mid g \in H \;\wedge\; \textit{consistent}(g, D) \;\wedge\; (\nexists g' : g' \in\; H \;\wedge\; g' >_g\; g \;\wedge\; \textit{consistent}(g', D))\; \}$$

Similarly, the set of maximally specific (i.e., minimally general) hypotheses, S, is defined as follows:

$$\{s \mid s \in H \;\wedge\; \textit{consistent}(s, D) \;\wedge\; (\nexists s' : s' \in\; H \;\wedge\; s >_g\; s' \;\wedge\; \textit{consistent}(s', D))\; \}$$

Concept Learning: Search in Version Space

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Theorem 7 (Version Space Representation)

Let X be a feature space and let H be a set of boolean-valued functions with domain X. Moreover, let $c : X \to \{0, 1\}$ be a target concept and let D be a set of examples of the form $(\mathbf{x}, c(\mathbf{x}))$. Then, based on the $\geq_g$-relation, each member of the version space $V_{H,D}$ lies in between two members of G and S respectively:

$$V_{H,D} = \{h \mid h \in H \ \wedge \ (\ \exists g \in G \ \ \exists s \in S : \ g \geq_g h \geq_g s \ ) \ \}$$

Concept Learning: Search in Version Space

Candidate Elimination Algorithm [Mitchell 1997]

1. Initialization: $G = \{g_0\},\ S = \{s_0\}$

2. If $\mathbf{x}$ is a positive example

   - Remove from G any hypothesis that is not consistent with $\mathbf{x}$
   - For each hypothesis s in S that is not consistent with $\mathbf{x}$
     - Remove s from S
     - Add to S all minimal generalizations h of s such that
       1. h is consistent with $\mathbf{x}$ and
       2. some member of G is more general than h
     - Remove from S any hypothesis that is less specific than another hypothesis in S

Concept Learning: Search in Version Space

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       2. some member of G is more general than h
     - Remove from S any hypothesis that is less specific than another hypothesis in S
3. If $\mathbf{x}$ is a negative example
   - Remove from S any hypothesis that is not consistent with $\mathbf{x}$
   - For each hypothesis g in G that is not consistent with $\mathbf{x}$
     - Remove g from G
     - Add to G all minimal specializations h of g such that
       1. h is consistent with $\mathbf{x}$ and
       2. some member of S is more specific than h
     - Remove from G any hypothesis that is less general than another hypothesis in G

Remarks:

- The basic idea of Candidate Elimination is as follows:
  - A maximally specific hypothesis $s \in S$ restricts the positive examples in first instance. Hence, s must be relaxed (= generalized) with regard to each positive example that is not consistent with s.
  - Conversely, a maximally general hypothesis $g \in G$ tolerates the negative examples in first instance. Hence, g must be constrained (= specialized) with regard to each negative example that is not consistent with g.

Concept Learning: Search in Version Space

Candidate Elimination Algorithm (pseudo code)

```
1. G = {g_0} // G is the set of maximally general hypothesis in H.
   S = {s_0} // S is the set of maximally specific hypothesis in H.

2. FOREACH (x, c(x)) ∈ D DO
     IF c(x) = 1 THEN // x is a positive example.
       FOREACH g ∈ G DO IF g(x) ≠ 1 THEN G = G \ {g} ENDDO
       FOREACH s ∈ S DO
         IF s(x) ≠ 1 THEN
           S = S \ {s}, S^+ = min_generalizations(s, x)
           FOREACH s ∈ S^+ DO IF (∃g ∈ G : g ≥_g s) THEN S = S ∪ {s} ENDDO
           FOREACH s ∈ S DO IF (∃s' ∈ S : s' ≠ s ∧ s ≥_g s') THEN S = S \ {s} ENDDO
       ENDDO
     ELSE // x is a negative example.



     ENDIF
   ENDDO

3. return(G, S)
```

Concept Learning: Search in Version Space

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       FOREACH g ∈ G DO IF g(x) ≠ 1 THEN G = G \ {g} ENDDO
       FOREACH s ∈ S DO
         IF s(x) ≠ 1 THEN
           S = S \ {s},  S+ = min_generalizations(s, x)
           FOREACH s ∈ S+ DO IF (∃g ∈ G : g ≥_g s) THEN S = S ∪ {s} ENDDO
           FOREACH s ∈ S DO IF (∃s' ∈ S : s' ≠ s ∧ s ≥_g s') THEN S = S \ {s} ENDDO
       ENDDO
     ELSE  // x is a negative example.
       FOREACH s ∈ S DO IF s(x) ≠ 0 THEN S = S \ {s} ENDDO
       FOREACH g ∈ G DO
         IF g(x) ≠ 0 THEN
           G = G \ {g},  G- = min_specializations(g, x)
           FOREACH g ∈ G- DO IF (∃s ∈ S : g ≥_g s) THEN G = G ∪ {g} ENDDO
           FOREACH g ∈ G DO IF (∃g' ∈ G : g' ≠ g ∧ g' ≥_g g) THEN G = G \ {g} ENDDO
       ENDDO
     ENDIF
   ENDDO

3. return(G, S)
```

Concept Learning: Search in Version Space

Candidate Elimination Algorithm (illustration)

$\boxed{\{ <\perp, \perp, \perp, \perp, \perp, \perp> \}}$ S_0

$\boxed{\{ <?, ?, ?, ?, ?, ?> \}}$ $G_0,$

Concept Learning: Search in Version Space

Candidate Elimination Algorithm (illustration)

$\{ < \perp, \perp, \perp, \perp, \perp, \perp > \}$ S_0

↓

$\{$ < *sunny, warm, normal, strong, warm, same* > $\}$ S_1

$\{ < ?, ?, ?, ?, ?, ? > \}$ $G_0, G_1,$

$\mathbf{x}_1 = ($*sunny, warm, normal, strong, warm, same*$)$ *EnjoySport*$(\mathbf{x}_1) = 1$

Concept Learning: Search in Version Space

Candidate Elimination Algorithm (illustration)

$\{ < \bot, \bot, \bot, \bot, \bot, \bot > \}$ S_0

↓

{ < *sunny, warm, normal, strong, warm, same* > } S_1

↓

{ < *sunny, warm, ?, strong, warm, same* > } S_2,

{ < ?, ?, ?, ?, ?, ? > } G_0, G_1, G_2

$\mathbf{x}_1 = ($*sunny, warm, normal, strong, warm, same*$)$ *EnjoySport*$(\mathbf{x}_1) = 1$

$\mathbf{x}_2 = ($*sunny, warm, high, strong, warm, same*$)$ *EnjoySport*$(\mathbf{x}_2) = 1$

Concept Learning: Search in Version Space

Candidate Elimination Algorithm (illustration)

{ < ⊥, ⊥, ⊥, ⊥, ⊥, ⊥ > } S_0

↓

{ < *sunny, warm, normal, strong, warm, same* > } S_1

↓

{ < *sunny, warm, ?, strong, warm, same* > } S_2, S_3

{ < *sunny, ?, ?, ?, ?, ?* >, < *?, warm, ?, ?, ?, ?* >, < *?, ?, ?, ?, ?, same* > } G_3

↑

{ < *?, ?, ?, ?, ?, ?* > } G_0, G_1, G_2

$\mathbf{x}_1 = ($*sunny, warm, normal, strong, warm, same*$)$ *EnjoySport*$(\mathbf{x}_1) = 1$

$\mathbf{x}_2 = ($*sunny, warm, high, strong, warm, same*$)$ *EnjoySport*$(\mathbf{x}_2) = 1$

$\mathbf{x}_3 = ($*rainy, cold, high, strong, warm, change*$)$ *EnjoySport*$(\mathbf{x}_3) = 0$ [Feature domains] [Algorithm]

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Candidate Elimination Algorithm (illustration)

$\{ < \bot, \bot, \bot, \bot, \bot, \bot > \}$ S_0

{ < sunny, warm, normal, strong, warm, same > } S_1

{ < sunny, warm, ?, strong, warm, same > } S_2, S_3

{ < sunny, warm, ?, strong, ?, ? > } S_4

< sunny, ?, ?, strong, ?, ? > *< sunny, warm, ?, ?, ?, ? >* *< ?, warm, ?, strong, ?, ? >*

{ < sunny, ?, ?, ?, ?, ? >, < ?, warm, ?, ?, ?, ? > } G_4

{ < sunny, ?, ?, ?, ?, ? >, < ?, warm, ?, ?, ?, ? >, < ?, ?, ?, ?, ?, same > } G_3

{ < ?, ?, ?, ?, ?, ? > } G_0, G_1, G_2

$\mathbf{x}_1 = (\textit{sunny, warm, normal, strong, warm, same})$ $\textit{EnjoySport}(\mathbf{x}_1) = 1$

$\mathbf{x}_2 = (\textit{sunny, warm, high, strong, warm, same})$ $\textit{EnjoySport}(\mathbf{x}_2) = 1$

$\mathbf{x}_3 = (\textit{rainy, cold, high, strong, warm, change})$ $\textit{EnjoySport}(\mathbf{x}_3) = 0$ [Feature domains] [Algorithm]

$\mathbf{x}_4 = (\textit{sunny, warm, high, strong, cool, change})$ $\textit{EnjoySport}(\mathbf{x}_4) = 1$

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Discussion of the Candidate Elimination Algorithm

1. What about selecting examples from D according to a certain strategy?
   Keyword: active learning

2. What are partially learned concepts and how to exploit them?
   Keyword: ensemble classification

3. The version space as defined here is “biased”. What does this mean?

4. Will Candidate Elimination converge towards the correct hypothesis?

5. When does one end up with an empty version space?

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Question 1: Selecting Examples from D

S { < *sunny, warm, ?, strong, ?, ?* > }

< *sunny, ?, ?, strong, ?, ?* >   < *sunny, warm, ?, ?, ?, ?* >   < *?, warm, ?, strong, ?, ?* >

G { < *sunny, ?, ?, ?, ?, ?* > , < *?, warm , ?, ?, ?, ?* > }

An example from which we can “maximally” learn:

$\mathbf{x}_7 = ($*sunny, warm, normal, light, warm, same*$)$

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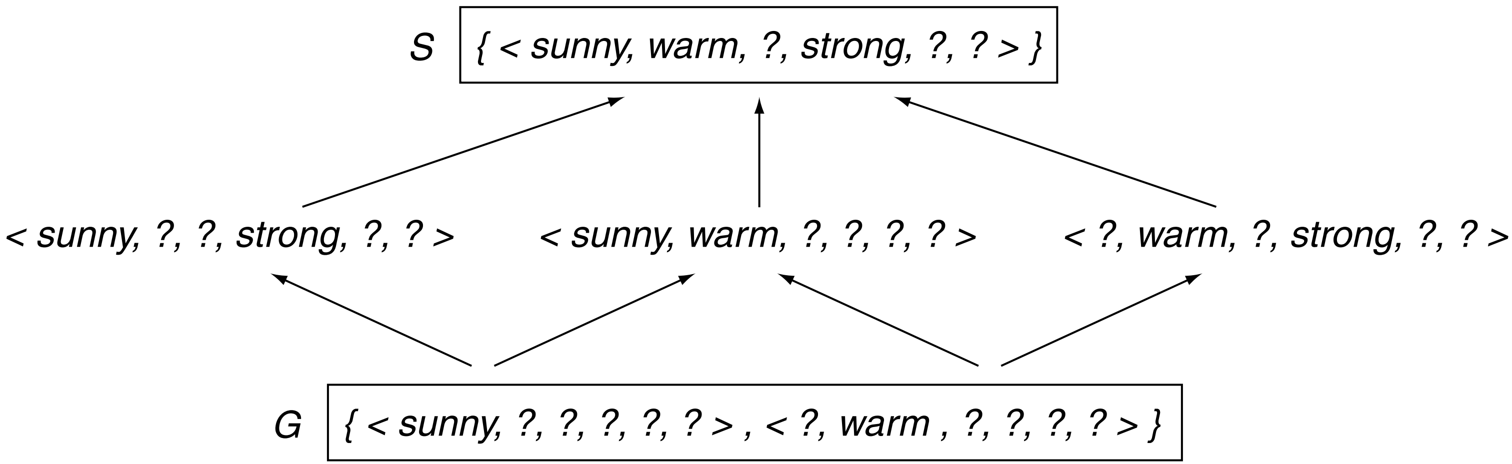
Question 1: Selecting Examples from D



An example from which we can “maximally” learn:

$\mathbf{x}_7 = ($*sunny, warm, normal, light, warm, same*$)$

Discussion:

Irrespective the value of $c(\mathbf{x}_7)$, the example $(\mathbf{x}_7, c(\mathbf{x}_7))$ will be consistent with three of the six hypotheses. It follows:

- If *EnjoySport*$(\mathbf{x}_7) = 1$ $\quad S$ can be further generalized.
- If *EnjoySport*$(\mathbf{x}_7) = 0$ $\quad G$ can be further specialized.

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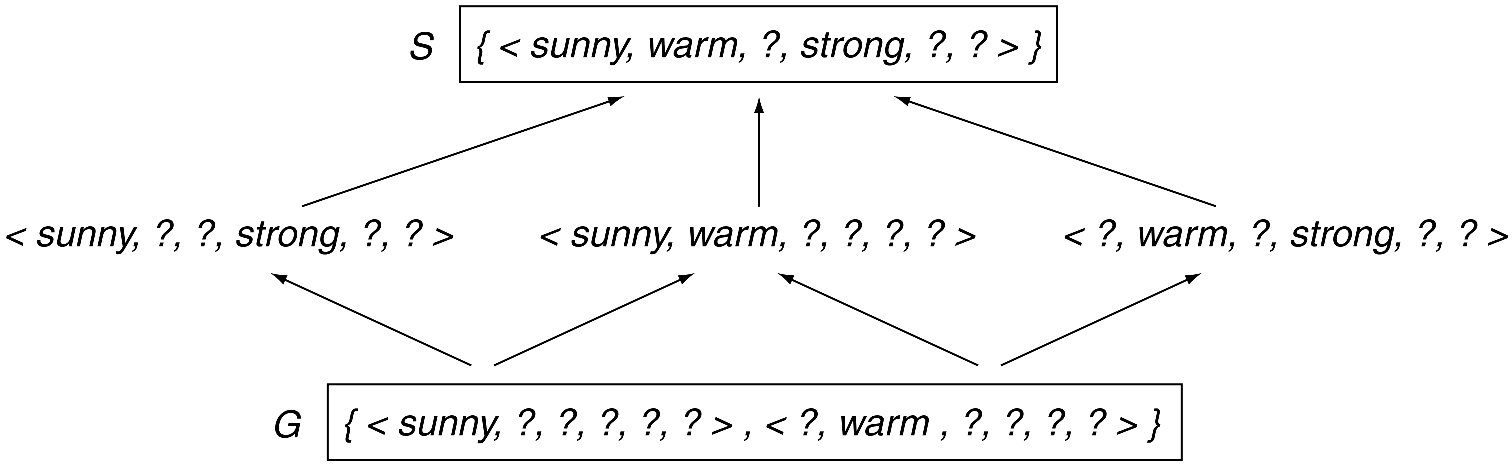
Question 2: Partially Learned Concepts

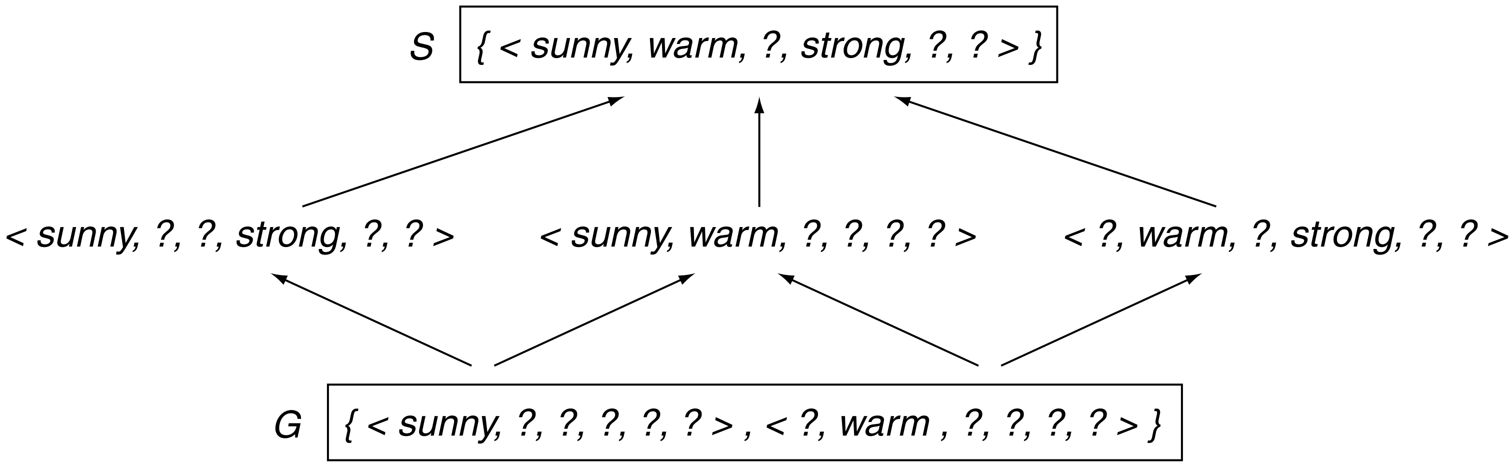


Combine the six classifiers in the version space:

Example	Sky	Temperature	Humidity	Wind	Water	Forecast	EnjoySport
5	sunny	warm	normal	strong	cool	change	
6	rainy	cold	normal	light	warm	same	
7	sunny	warm	normal	light	warm	same	
8	sunny	cold	normal	strong	warm	same	

Concept Learning: Search in Version Space

Question 2: Partially Learned Concepts



Combine the six classifiers in the version space:

Example	Sky	Temperature	Humidity	Wind	Water	Forecast	EnjoySport
5	sunny	warm	normal	strong	cool	change	6+ : 0–
6	rainy	cold	normal	light	warm	same	
7	sunny	warm	normal	light	warm	same	
8	sunny	cold	normal	strong	warm	same	

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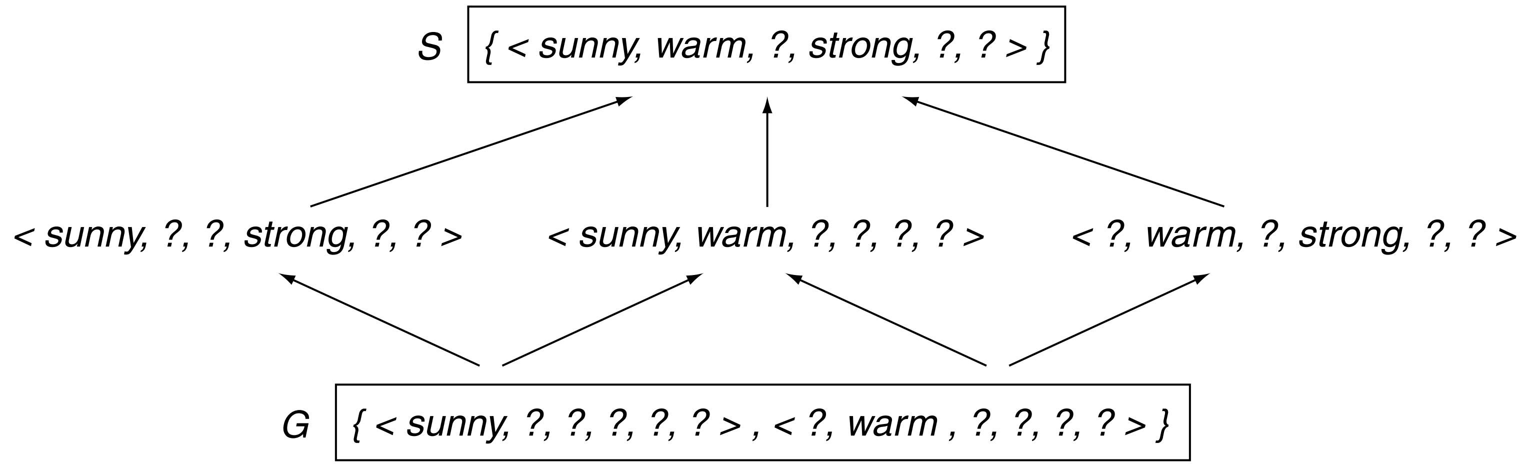
Question 2: Partially Learned Concepts



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Example	Sky	Temperature	Humidity	Wind	Water	Forecast	EnjoySport
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6	rainy	cold	normal	light	warm	same	0+ : 6–
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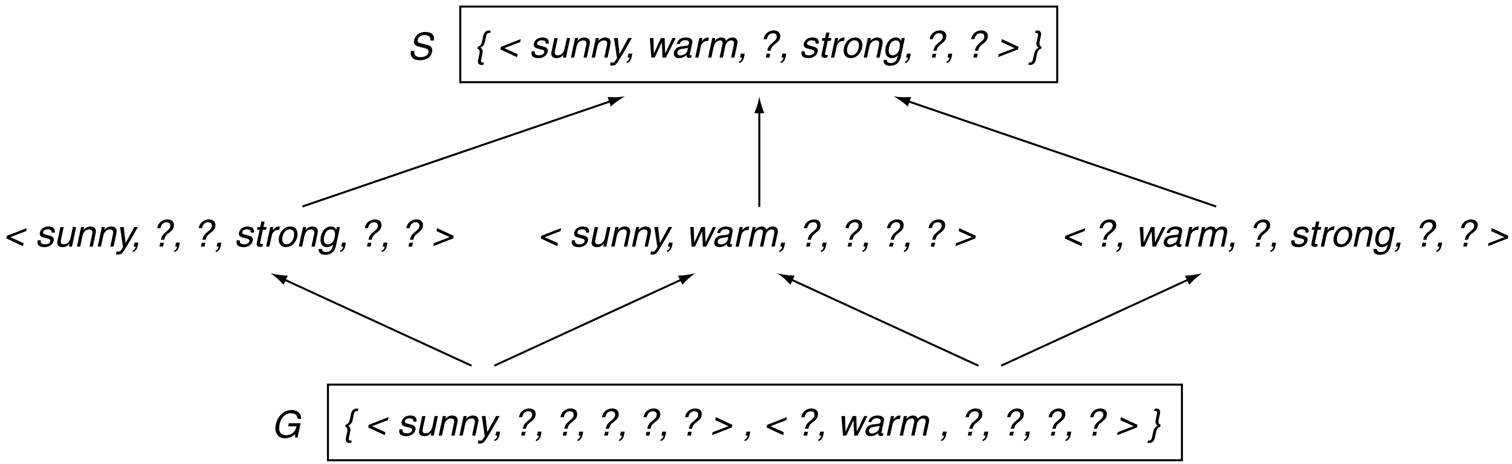

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7	sunny	warm	normal	light	warm	same	3+ : 3–
8	sunny	cold	normal	strong	warm	same	

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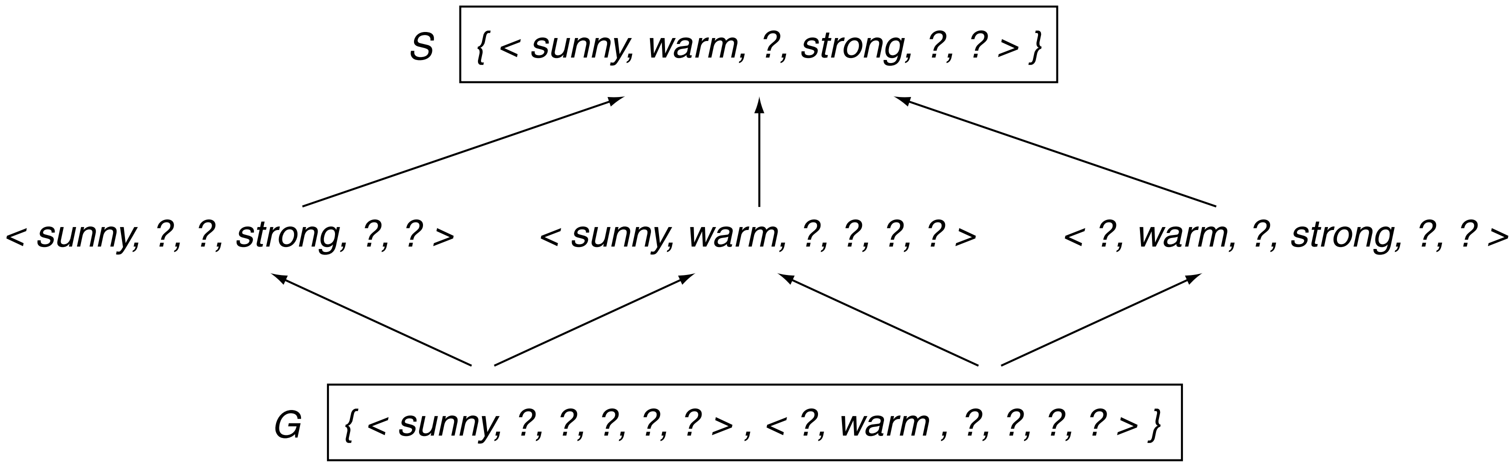
Question 2: Partially Learned Concepts



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Example	Sky	Temperature	Humidity	Wind	Water	Forecast	EnjoySport
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6	rainy	cold	normal	light	warm	same	0+ : 6–
7	sunny	warm	normal	light	warm	same	3+ : 3–
8	sunny	cold	normal	strong	warm	same	2+ : 4–

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Question 3: Inductive Bias

A new set of training examples D :

Example	Sky	Temperature	Humidity	Wind	Water	Forecast	EnjoySport
9	sunny	warm	normal	strong	cool	change	yes
10	cloudy	warm	normal	strong	cool	change	yes

➜ $S = \{ \langle$ *?, warm, normal, strong, cool, change* $\rangle \}$

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A new set of training examples D :

Example	Sky	Temperature	Humidity	Wind	Water	Forecast	EnjoySport
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➜ $S = \{ \langle$ *?, warm, normal, strong, cool, change* $\rangle \}$

⋮

11	rainy	warm	normal	strong	cool	change	no

➜ $S = \{ \}$

Discussion:

- What assumptions about the target concept are met a-priori by the learner?

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Question 3: Inductive Bias

A new set of training examples D :

Example	Sky	Temperature	Humidity	Wind	Water	Forecast	EnjoySport
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➜ $S = \{ \langle$ *?, warm, normal, strong, cool, change* $\rangle \}$

⋮

11	rainy	warm	normal	strong	cool	change	no

➜ $S = \{ \}$

Discussion:

- What assumptions about the target concept are met a-priori by the learner?

➜ Consequence: The hypothesis space H may be designed to contain more complex concepts, e.g., $\langle$ *sunny, ?, ?, ?, ?, ?* $\rangle \vee \langle$ *cloudy, ?, ?, ?, ?, ?* $\rangle$.

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Question 3: Inductive Bias (continued)

- In a binary classification problem the unrestricted (= unbiased) hypothesis space contains $|\mathcal{P}(X)| \equiv 2^{|X|}$ elements.
- A learning algorithm that considers all possible hypotheses as equally likely makes no a-priori assumption with regard to the target concept.
- A learning algorithm without a-priori assumptions has no "inductive bias".

> *"The policy by which a* [learning] *algorithm generalizes from observed training examples to classify unseen instances is its inductive bias.* [...] *Inductive bias is the set of assumptions that, together with the training data, deductively justify the classification by the learner to future instances."*
>
> [p.63, Mitchell 1997]

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→ A learning algorithm without inductive bias has no directive to classify unseen examples. Put another way: the learner cannot *generalize*.

→ A learning algorithm without inductive bias will only *memorize*.

Which algorithm (Find-S, Candidate Elimination) has a stronger inductive bias?