Chapter IR:V

V. Evaluation

Training and Testing

Statistical Hypothesis Testing

Claim:

- System 1 is better than System 2 because it achieves 0.61 MAP, 0.13 more than System 2.

What would you reply to this claim?

Training and Testing

Statistical Hypothesis Testing

Claim:

- System 1 is better than System 2 because it achieves 0.61 MAP, 0.13 more than System 2.

Supporting data:

	Average Precision		**Mean**
	Topic 1	Topic 2	
System 1	0.78	0.44	0.61
System 2	0.52	0.44	0.48
Difference	$+0.26$	± 0.00	$+0.13$

What would you reply to this data?

Training and Testing

Statistical Hypothesis Testing

Claim:

- System 1 is better than System 2 because it achieves 0.61 MAP, 0.13 more than System 2.

Supporting data:

	Average Precision		Mean
	Topic 1	Topic 2	
System 1	0.78	0.44	0.61
System 2	0.52	0.44	0.48
Difference	$+0.26$	± 0.00	$+0.13$

Rebuttal:

- That was just luck.
- With more topics, the gains and losses may even out.

➜ Although better on a specific topic, System 1 is not really shown more effective than System 2.

Training and Testing

Statistical Hypothesis Testing

	Average Precision						Mean
	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6	
System 1	0.78	0.44	0.54	0.62	0.45	0.22	0.51
System 2	0.52	0.44	0.55	0.32	0.12	0.13	0.35
Difference	$+0.26$	± 0.00	-0.01	$+0.30$	$+0.33$	$+0.09$	$+0.16$

Given these results, determine whether they have been obtained by chance.

Training and Testing

Statistical Hypothesis Testing

	Average Precision						Mean
	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6	
System 1	0.78	0.44	0.54	0.62	0.45	0.22	0.51
System 2	0.52	0.44	0.55	0.32	0.12	0.13	0.35
Difference	$+0.26$	± 0.00	-0.01	$+0.30$	$+0.33$	$+0.09$	$+0.16$

Given these results, determine whether they have been obtained by chance.

Null hypothesis:

- Average precision values of both systems are drawn from the same underlying probability distribution.
- The differences observed arise from the natural variation of that distribution.

→ The differences are randomly distributed.

Employ a test statistic to compute the probability p of observing the differences if the null hypothesis were true. If the p value is small, the null hypothesis may be false.

Typically, $p < 0.05$ suffices to claim that the differences are statistically significant.

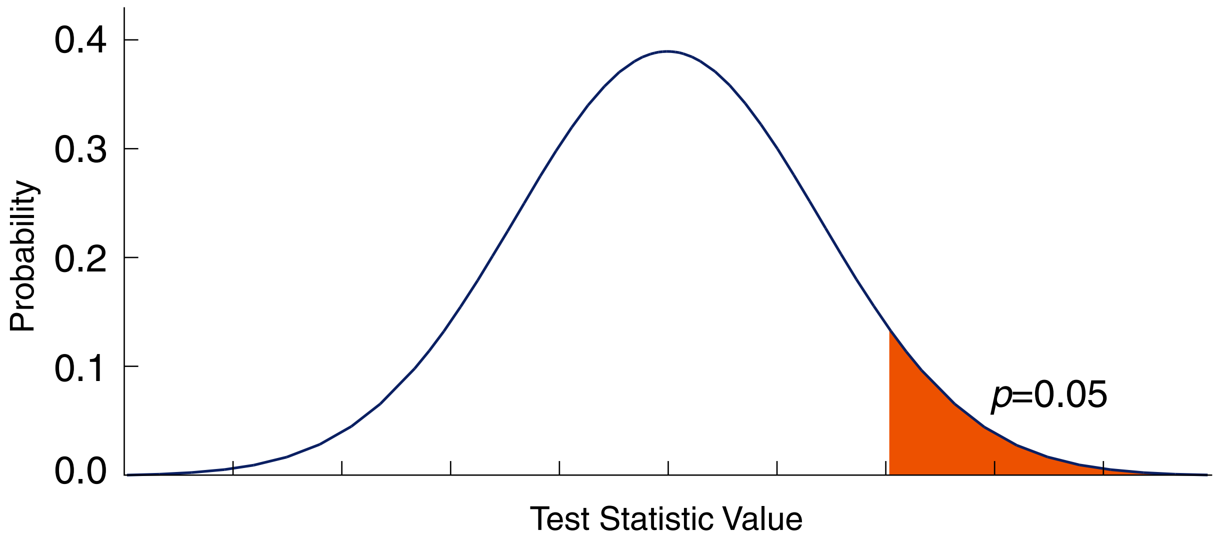
Training and Testing

Statistical Hypothesis Testing

	Average Precision						Mean
	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6	
System 1	0.78	0.44	0.54	0.62	0.45	0.22	0.51
System 2	0.52	0.44	0.55	0.32	0.12	0.13	0.35
Difference	$+0.26$	± 0.00	-0.01	$+0.30$	$+0.33$	$+0.09$	$+0.16$

Given these results, determine whether they have been obtained by chance.

Illustration:



Remarks:

- Rejecting the null hypothesis based on a small p value does not necessarily mean we can accept the opposing hypothesis as true.

Training and Testing

Statistical Hypothesis Testing: Sign Test

	Average Precision						Mean
	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6	
System 1	0.78	0.44	0.54	0.62	0.45	0.22	0.51
System 2	0.52	0.44	0.55	0.32	0.12	0.13	0.35
Difference	$+0.26$	± 0.00	-0.01	$+0.30$	$+0.33$	$+0.09$	$+0.16$
Sign	$+$	$=$	$-$	$+$	$+$	$+$	n/a

Training and Testing

Statistical Hypothesis Testing: Sign Test

	Average Precision						Mean
	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6	
System 1	0.78	0.44	0.54	0.62	0.45	0.22	0.51
System 2	0.52	0.44	0.55	0.32	0.12	0.13	0.35
Difference	$+0.26$	± 0.00	-0.01	$+0.30$	$+0.33$	$+0.09$	$+0.16$
Sign	$+$	$=$	$-$	$+$	$+$	$+$	n/a

Procedure:

- Sign $+$ denotes System 1 $>$ System 2, $-$ the opposite, and $=$ a tie.
- Test statistic: number m of $+$ signs.

Null hypothesis:

- Disregarding $=$, the probability of $+$ and $-$ is equal: $P(+) = P(-) = 0.5$.

Assumptions:

- The topics are independent of each other.
- The differences are drawn from the same distribution.
- The individual scores for each topic can be meaningfully compared.

Training and Testing

Statistical Hypothesis Testing: Sign Test

	Average Precision						Mean
	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6	
System 1	0.78	0.44	0.54	0.62	0.45	0.22	0.51
System 2	0.52	0.44	0.55	0.32	0.12	0.13	0.35
Difference	$+0.26$	± 0.00	-0.01	$+0.30$	$+0.33$	$+0.09$	$+0.16$
Sign	$+$	$=$	$-$	$+$	$+$	$+$	n/a

If the null hypothesis were true, what is the probability of observing at least $m = 4$ times $+$ out of $n = 5$ experiments?

If $P(+) = P(-) = 0.5$ holds, the test statistic is binomially distributed:

$$p \;=\; P(+ \geq m) \;=\; \sum_{k=m}^{n} \frac{n!}{k!(n-k)!} \cdot P(+)^n \;=\; \frac{5+1}{32} \;= 0.1875$$

Conclusions:

- The differences of Systems 1 and 2 are not statistically significant as $p > 0.05$.
- We cannot reject the null hypothesis.
- Under the sign test, Systems 1 and 2 must be presumed equally effective.

Training and Testing

Statistical Hypothesis Testing: Student's t-test

	Average Precision						Mean	s
	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6		
System 1	0.78	0.44	0.54	0.62	0.45	0.22	0.51	0.19
System 2	0.52	0.44	0.55	0.32	0.12	0.13	0.35	0.19
Difference	$+0.26$	± 0.00	-0.01	$+0.30$	$+0.33$	$+0.09$	$+0.16$	0.15

Procedure:

- Compute the score differences of the scores of Systems 1 and 2.
- Test statistic: $t = (\bar{d} - \mu_0)/(s_d/\sqrt{n})$ for n topics, where $\bar{d}$ denotes the average difference between Systems 1 and 2, μ_0 the expected difference, and s_d the observed standard deviation.

Null hypothesis:

- The average difference $\bar{d}$ is at most μ_0.

Assumptions:

- The topics are independent of each other.
- The differences are approximately normally distributed.

Training and Testing

Statistical Hypothesis Testing: Student's t-test

	Average Precision						Mean	s
	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6		
System 1	0.78	0.44	0.54	0.62	0.45	0.22	0.51	0.19
System 2	0.52	0.44	0.55	0.32	0.12	0.13	0.35	0.19
Difference	$+0.26$	±0.00	-0.01	$+0.30$	$+0.33$	$+0.09$	$+0.16$	0.15

If the null hypothesis were true, what is the probability of observing $\bar{d} = 0.16$ and $s_d = 0.15$ for $n = 6$ at an expected $\mu_0 = 0$?

The test statistic is t-distributed with $n - 1$ degrees of freedom:

$$t = \frac{0.16 - 0}{0.15/\sqrt{6}} = 2.613 \quad \leadsto \quad t(0.975; n-1) \; < \; 1 - p \; < \; t(0.99; n-1)$$

t-distribution table [Wikipedia]:

n	0.75	0.80	0.85	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
⋮											
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
⋮											

Training and Testing

Statistical Hypothesis Testing: Student's t-test

	Average Precision						Mean	s
	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6		
System 1	0.78	0.44	0.54	0.62	0.45	0.22	0.51	0.19
System 2	0.52	0.44	0.55	0.32	0.12	0.13	0.35	0.19
Difference	$+0.26$	± 0.00	-0.01	$+0.30$	$+0.33$	$+0.09$	$+0.16$	0.15

If the null hypothesis were true, what is the probability of observing $\bar{d} = 0.16$ and $s_d = 0.15$ for $n = 6$ at an expected $\mu_0 = 0$?

The test statistic is t-distributed:

$$t = \frac{0.16 - 0}{0.15/\sqrt{6}} = 2.613 \quad \rightsquigarrow \quad p = 0.025\,,$$

where p has been computed precisely using an implementation of the t-distribution.

Conclusions:

- The differences of Systems 1 and 2 are statistically significant as $p < 0.05$.
- We can reject the null hypothesis.
- Under the Student's t-test, System 1 may be better than System 2.

Training and Testing

Statistical Hypothesis Testing: Power Analysis and Effect Size

Power Analysis [Wikipedia] [G*Power]

- Estimation of the probability of rejecting the null hypothesis of a binary hypothesis test.
- Applied before conducting an experiment to determine the sample size (number of topics).
- Hypothesis tests with "more power" have a higher likelihood of rejecting the null hypothesis given the alternative hypothesis is true.
- The sign test has less power than the t-test.

Effect Size Estimation [Wikipedia]

- Quantification of the magnitude of a phenomenon (e.g., an observed significance)
- Effect size does not directly determine significance, nor vice versa.
- Sufficiently large sample sizes will always yield statistical significance unless the population effect size is exactly zero.
- An effect size score shows how "substantive" a statistically significant result is.
- About 50 to 100 different measures of effect size are known: For the Student's t-test, Cohen's d is a well-known effect size estimator.

Remarks:

- For the example above, Cohen's $d = 0.84$.

  Common interpretation:

Effect size	d
Very small	0.01
Small	0.20
Medium	0.50
Large	0.80
Very large	1.20
Huge	2.00

Training and Testing

Hyperparameter Optimization

Retrieval systems possess many parameters, many of which affect retrieval effectivenss. Examples: Algorithm parameters, alternative algorithms for a subtask, weights of document fields.

In IR, hyperparameter optimization often boils down to trial and error:

- Grid Search.
  Systematic trials of all parameter combinations from pre-specified value ranges and steps for each parameter.

- Random Search.
  Selection of a random subset of all parameter combinations of pre-specified value ranges and steps for each parameter.

Ideally, parameters are optimized based on a 3-way split of the available data into subsets used for training, validation, and test.

Training data are used to fine-tune learning algorithms. Validation data are used to repeatedly check a retrieval system's performance trajectory during optimization. Test data are used once at the end as a final check.