Conditional Independence test for Categorical Data towards Causal Discovery and Application in Bibliometrics

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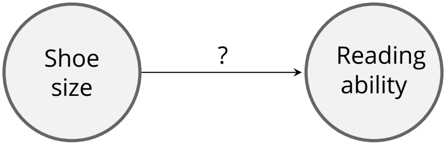
Overview

Motivation

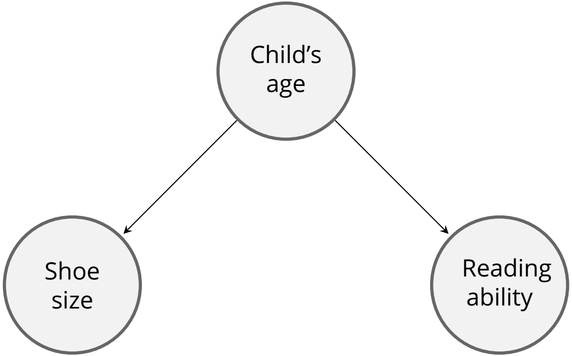
Correlation is not causation

Correlation is not causation

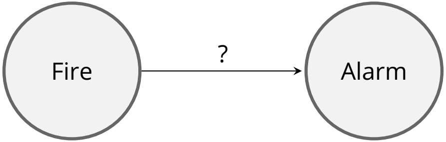
In a study on child development ...



Confounder

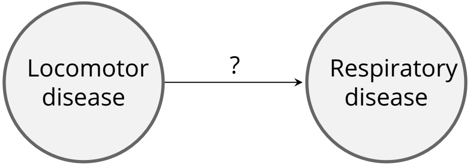


Correlation is not causation



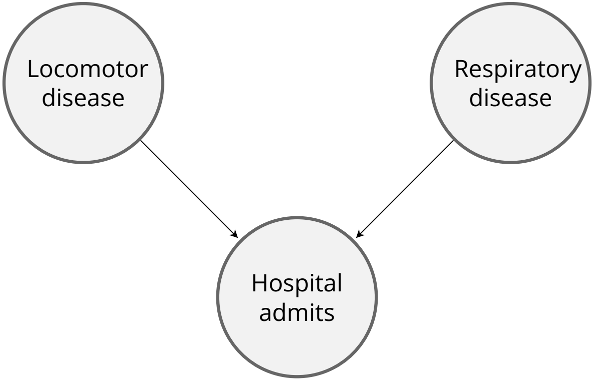
Mediator

Correlation is not causation



[Lee et al, 2019]

Selection bias (Collider bias)



PC Algorithm [Spirtes et al, 2000]

PC Algorithm

Under the assumptions of faithfulness, causal markov condition and causal sufficiency,

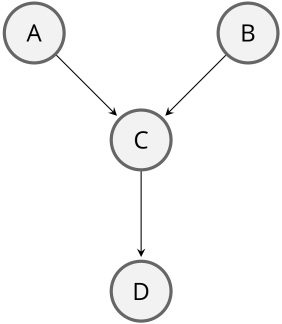


Figure: True causal graph

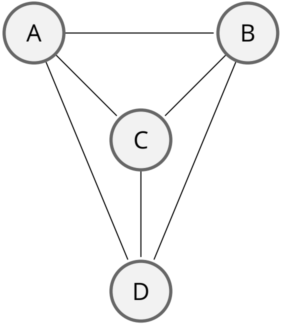


Figure: PC : Bi-partite graph

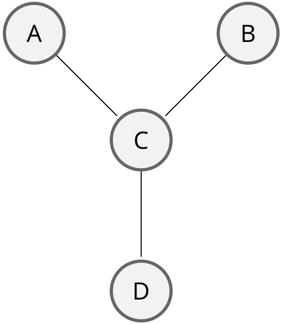


Figure: PC 1: Skeleton discovery. $X \perp\!\!\!\perp Y \mid Z$

PC Algorithm

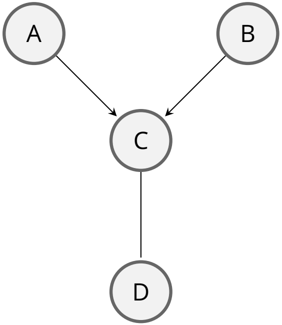


Figure: PC 2: Orienting colliders

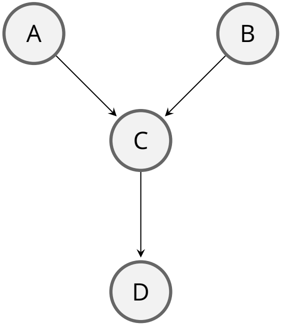


Figure: PC 3: Orientation rules

Motivation for design of CI test

- CI testing is crucial for constraint-based causal discovery algorithms.
- Focus on categorical/symbolic variables. E.g. Blood type of a person, number of sides in dice.
- Calibration of CI test (type I error <= α, maximize power).
- Evaluation using Bayesian Network (DGP).
- Comparison of exact and asymptotic test.
- Work heavily based on CMIknn, a non-parametric test for continuous variables [Runge, 2018]

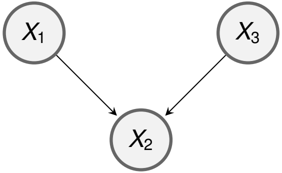
Data Generating Process

Structural Causal Model [Peters et al, 2017]

A structural causal model $\mathfrak{C} := (\mathrm{S}, \mathrm{P}_N)$ consists of a collection S of d structural assignments over random variables $X = \{X_1, ..., X_j\}$, where $X_j := f_j(PA_j, N_j), j = 1, 2, ...d$

- X_j is caused by parents $PA_j \subseteq \{X_1, ..., X_d\}$ through mechanism f_j
- Entails a joint distribution over all X
- N_j noise corresponding to X_j

A causal graph (DAG) is then $G(\mathfrak{C})$



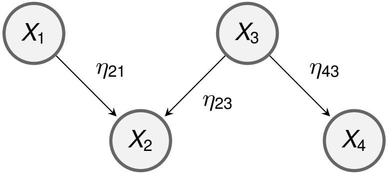
Bayesian Network

Categorical data generating process

A Bayesian Network is a DAG,

$$P(X_1, X_2, ..., X_n) = \prod_i P(X_i | PA_i)$$

- $X_1, ..., X_n$ are categorical random variables with m states each.
- Modeling dependencies using conditional probability tables (CPTs)
- Link strenth $\eta \in [0, 1]$ -> CPTs [Kokkonen et al, 2005]
- Assume that directed arrows denote causal directions.



Conditional independence testing

Exact vs Asymptotic tests

Given categorical random variables X, Y and Z,

$$H_0 : X \perp\!\!\!\perp Y \mid Z \tag{1}$$

$$H_1 : X \not\!\perp\!\!\!\perp Y \mid Z \tag{2}$$

Comparison [Tsamardinos et al, 2010] :

- Exact test (CMISymbPerm)
- Asymptotic test (G^2)
- For large sample sizes, the exact and asymptotic p-values are very similar.

Conditional mutual information

$$\begin{aligned} I_{(X;Y|Z)} &= \sum_{x,y,z} p(x,y,z)\, log\left(\frac{p(x,y|z)}{p(x|z)p(y|z)}\right) \\ &= H(X|Z) - H(X|YZ) \\ &= H(XZ) + H(YZ) - H(XYZ) - H(Z) \end{aligned} \tag{3}$$

- H denotes the Shannon entropy assuming that $p(\cdot)$ exist.
- The CMI $I_{X;Y|Z} = 0$ iff $X \perp\!\!\!\perp Y \mid Z$.

Algorithm 1 Algorithm for permutation scheme

1. Compute CMI t_{obs} of data $\{x_i, y_i, z_i\}_{i=1}^N$.
2. Compute neighbors for each sample in z.
3. Randomly permute x within neighbors. Compute CMI T.
4. Repeat B times : $T_1, T_2, \ldots T_B$.
5. The approximate p-value is

$$\frac{1}{B}\sum_{j=1}^{B} I(T_j > t_{obs})$$

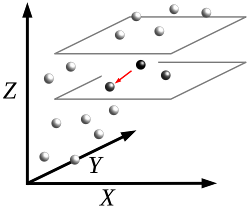


Figure: Permuting within z

G^2 test of conditional independence

$$G^2 = 2 \sum_{xyz} N_{xyz} \ln \left(\frac{N_{xyz}}{E_{xyz}} \right) \tag{4}$$

N_{xyz} = observed frequencies of $X = x, Y = y, Z = z$, where $z = (z_1, ..., z_k)$.
N_{xz} = marginal total of $X = x, Z = z$.
N_{yz} = marginal total of $Y = y, Z = z$.
N = Total sample size.
E_{xyz} = $\frac{N_{xz} N_{yz}}{N_z}$ = Expected frequencies under the assumption of independence.

Null distribution - χ^2 distributed with degrees of freedom $(|X| - 1)(|Y| - 1) \prod_{i=1}^{k} |Z_i|$.

Experimental setup

CMISymbPerm and G^2 are run over,

- Model for DGP, $c \in [0, 1]$

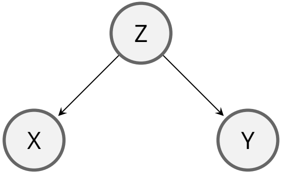


Figure: $H_0 : X \perp\!\!\!\perp Y \mid Z$

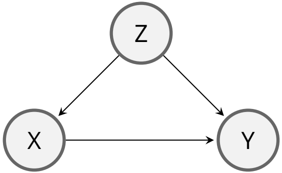


Figure: $H_1 : X \not\!\perp\!\!\!\perp Y \mid Z$

- Number of samples $N \in [50, 100, 150, 200, 250, 500, 1000, 2000]$
- Number of categories/symbols $n_{symbs} \in [2, 3, 4, 5, 6]$
- Number of dimensions of Z, $D_z = |Z| \in [1, 2, 3, 4]$

Results

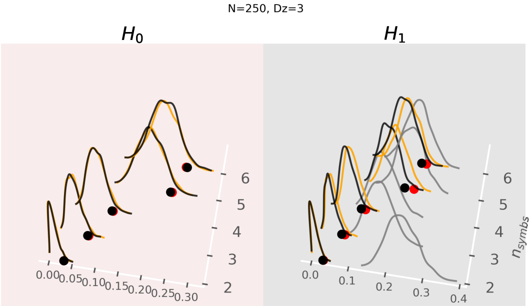


Figure: CMISymbPerm - null approximation

Results

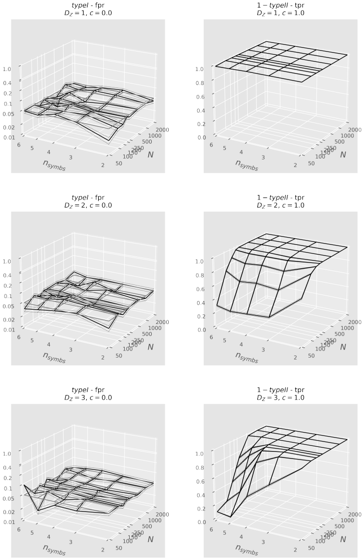


Figure: CMISymbPerm over n_{symbs} and N

Results

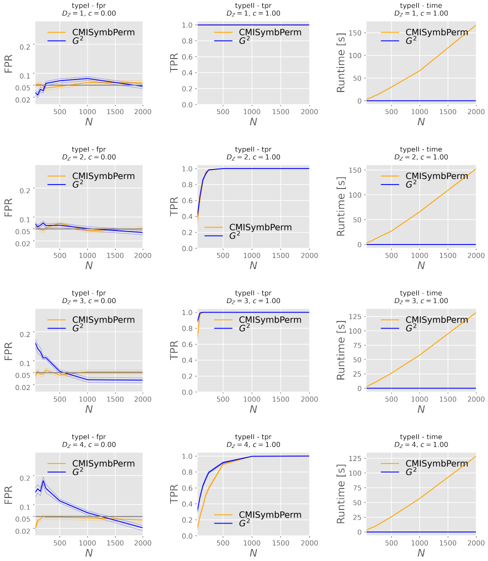


Figure: CMISymbPerm vs G^2. Over N and D_z with $n_{symbs} = 3$

Causal Discovery - Bibliometrics

Goal : Discover causal relationships amongst bibliometric features that influence a researcher's position in academia.

Dataset : Open Academic Graph 2.1 [Zhang et al, 2019]

- 240 million papers
- 243 million authors
- 53 thousand venues
- 25 thousand affiliation

Sample data :

- 10 features - continuous valued.
- Stratified sampling across 'position' (professor, associate professor, assistant professor)
- 1600 samples in each category. Total samples = 4800.

Correlation

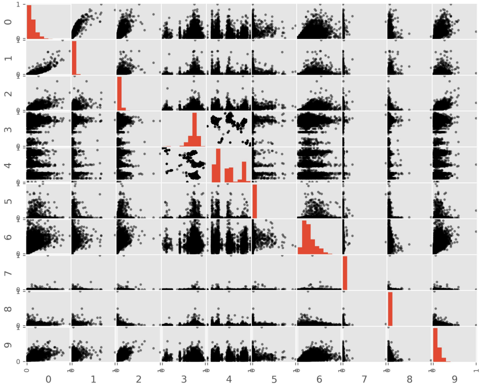


Figure: Scatter plot of features

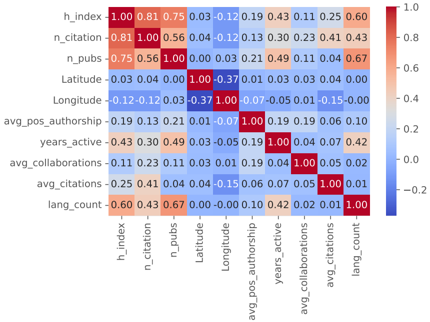


Figure: Correlation matrix

Multinomial logistic regression on *position*

k-fold cross validation
Mean accuracy = 0.582. Standard deviation = 0.02
Permutation feature importance (Dominant features)

Feature	Mean score	Standard deviation
years_active	0.132	0.006
n_pubs	0.036	0.005
h_index	0.014	0.003
n_citation	0.011	0.004
longitude	0.010	0.003
avg_citations	0.006	0.002

Causal graph

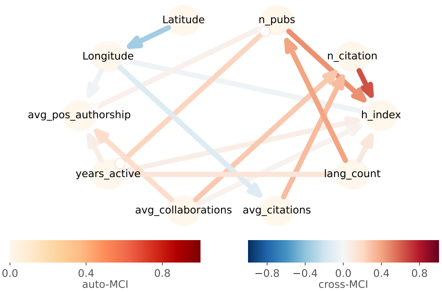


Figure: Causal graph. pc_alpha=0.005

- Confounded, no direct link
  - n_pubs and n_citations (corr = 0.56)
  - years_active and n_citations (corr = 0.3)
- lang_count and n_citations has no direct link but corr = 0.43.
- Mediated path amongst dominant features. Longitude of the affiliation –> average citations –> citations –> h-index

Conclusion

CI test :

- G^2 test and permutation test converge in type I error for larger samples.
- Time complexity - Permutation test is $\mathcal{O}(c^n)$, G^2 is $\mathcal{O}(1)$ for large samples.
- Prefer permutation test for lower sample size and larger dimensions, while G^2 for larger sample sizes.
- Prefer G^2 when time is a constraint.

Bibliometrics:

- Identified confounded and mediated paths amongst dominant features.

Future Work

- Parallelizing permutation scheme to improve time complexity.
- Evaulate using more models (chains, colliders ...)
- Selection bias in location and position
- Violation of causal sufficiency
- Location non-linearly related to other variables.
- pc_alpha=0.005 very conservative. Varied pc_alpha.
- Mixed variable CI test

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References II

Fanjin Zhang, Xiao Liu, Jie Tang, Yuxiao Dong, Peiran Yao, Jie Zhang, Xi-aotao Gu, Yan Wang, Bin Shao, Rui Li, and Kuansan Wang. Oag: Toward linking large-scale heterogeneous entity graphs. In ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD), August 2019.

Thank you

Appendix

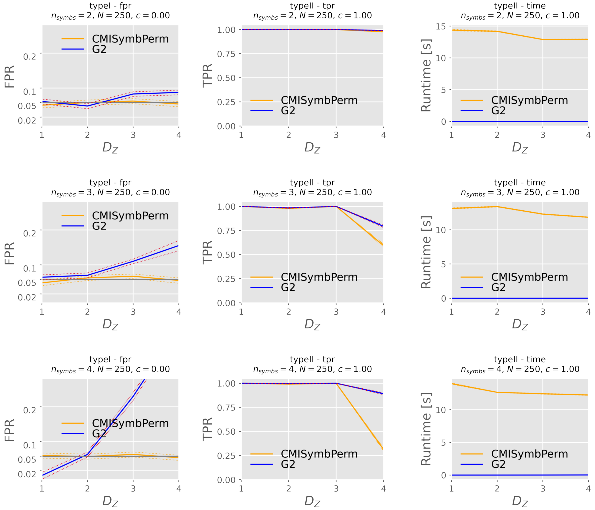

Figure: CMISymbPerm vs G^2. Over D_z with $N = 250$ and $n_{symbs} = 2, 3, 4$

Appendix

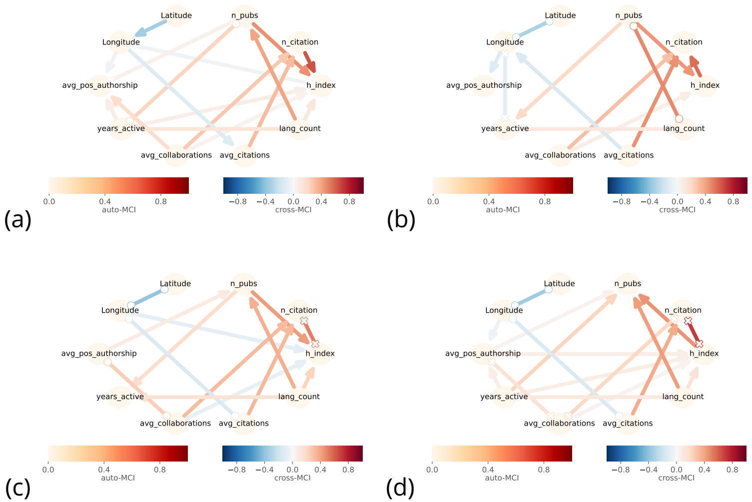

Figure: pc_alpha=0.005 (a) All authors (b) Professors (c) Associate professors (d) Assistant professors

Link strength [Kokkonen et al, 2005] $\eta = [0, 1]$ defines the strength of association between a parent node p and child x, all nodes having m states each.

1. $\eta = 0$, child is independent of the parent p.
2. $\eta_1 = \eta_2$, both parents p_1 and p_2 have equal effect on child x.
3. When all $\eta = 0$, the CPT is non-informative.

- Reduces CPT elicitation complexity from m^{p+1} to p.
- Helps elicit relative causal strengths.

Appendix

Generalized Noisy-Or model with link strength

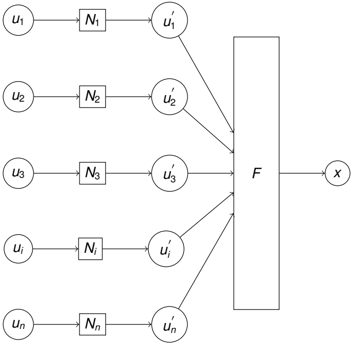

Figure: Schematic of the generalized Noisy-Or model.

$$N_i = P(u_i^{'}(r)|u_i(c)) = \begin{cases} \frac{1}{m} + \eta_i(1 - \frac{1}{m}) & \text{if } r = c \\ \frac{1}{m-1}[1 - \frac{1}{m} - \eta_i(1 - \frac{1}{m})] & \text{if } r \neq c \end{cases} \tag{5}$$

$$F(u^{'}) = x\left(ceiling\left(\frac{1}{\sum_i \eta_i)}\sum_i[\eta_i I(u_i^{'})]\right)\right) \tag{6}$$

$$P(x|u) = \sum_{u^{'}:x=F(u^{'})} P(u^{'}|u) = \sum_{u^{'}:x=F(u^{'})} \prod_{u^{'}} P(u_i^{'}|u_i) \tag{7}$$