

II. Cluster Analysis

- Cluster Analysis Basics
- Hierarchical Cluster Analysis
- Iterative Cluster Analysis
- Density-Based Cluster Analysis
- Cluster Evaluation
- Constrained Cluster Analysis

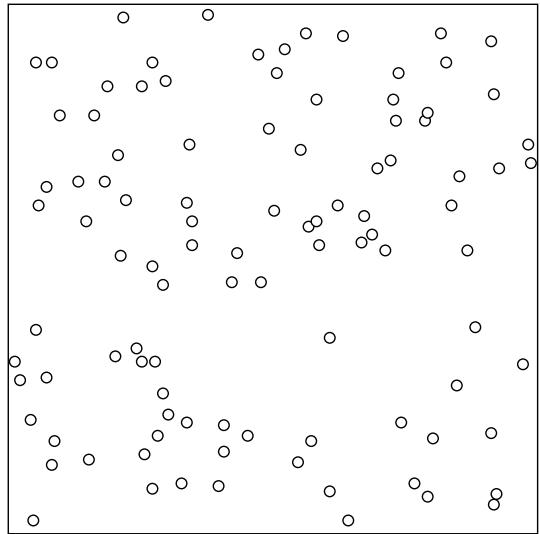
Cluster Evaluation

Overview

“The validation of clustering structures is the most difficult and frustrating part of cluster analysis. Without a strong effort in this direction, cluster analysis will remain a black art accessible only to those true believers who have experience and great courage.”

[Jain/Dubes 1990]

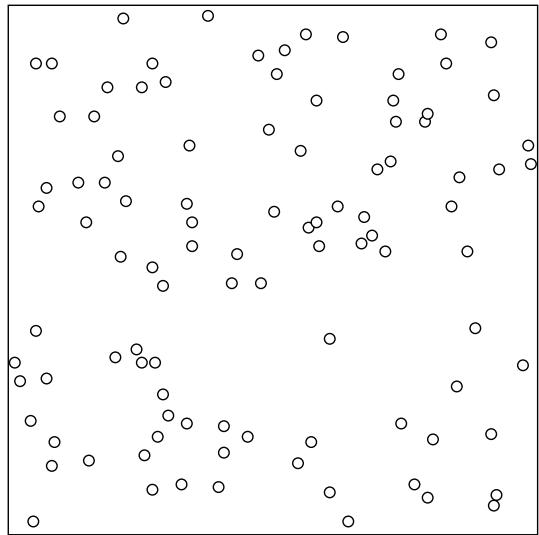
Cluster Evaluation

 [Tan/Steinbach/Kumar 2005]

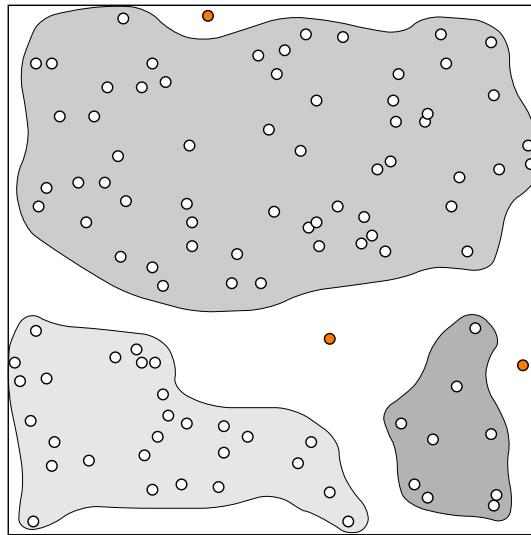
Random points

Cluster Evaluation

[Tan/Steinbach/Kumar 2005]



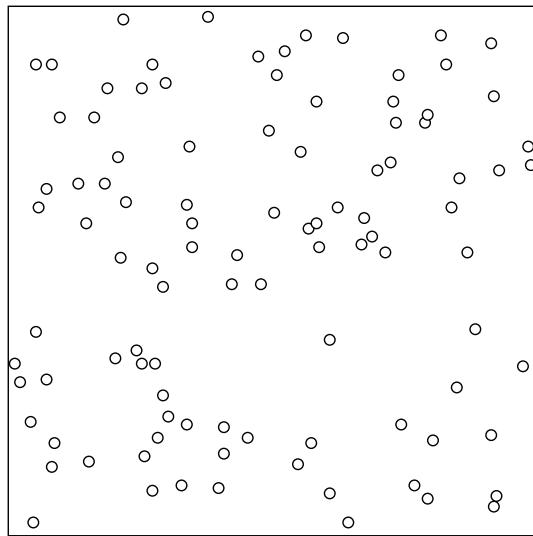
Random points



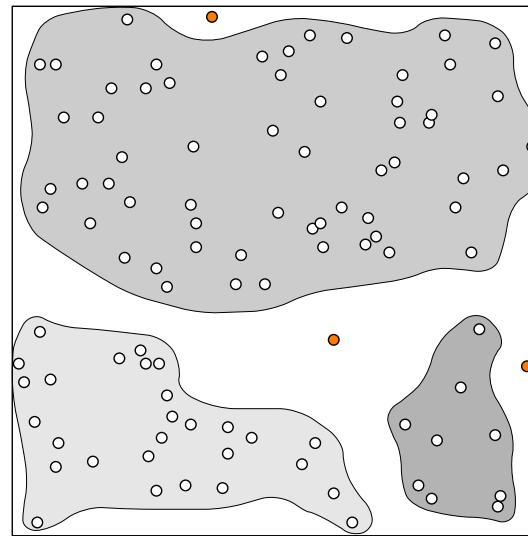
DBSCAN

Cluster Evaluation

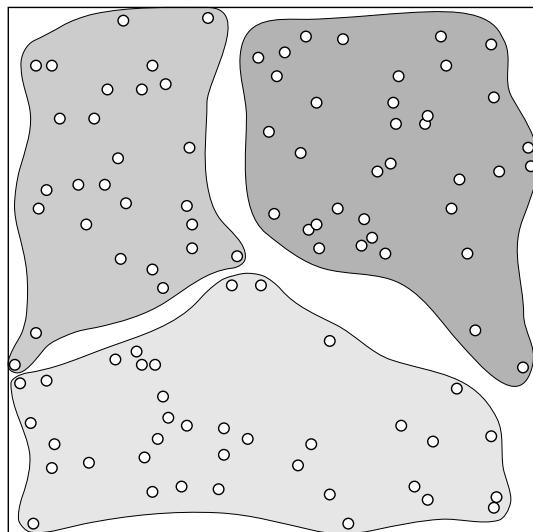
[Tan/Steinbach/Kumar 2005]



Random points



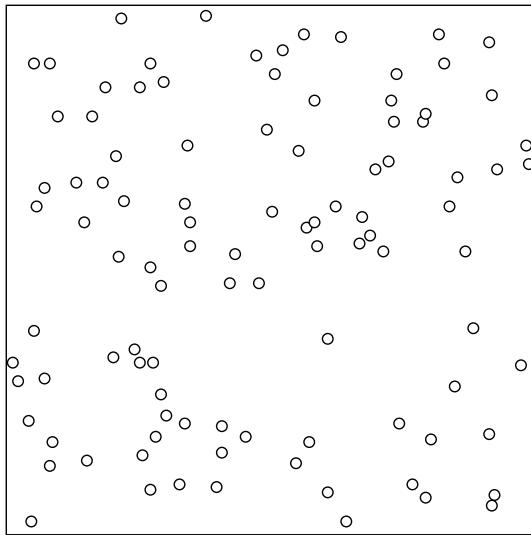
DBSCAN



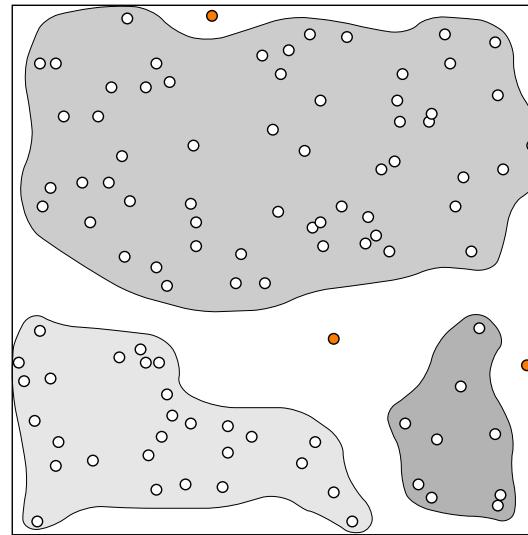
k -means

Cluster Evaluation

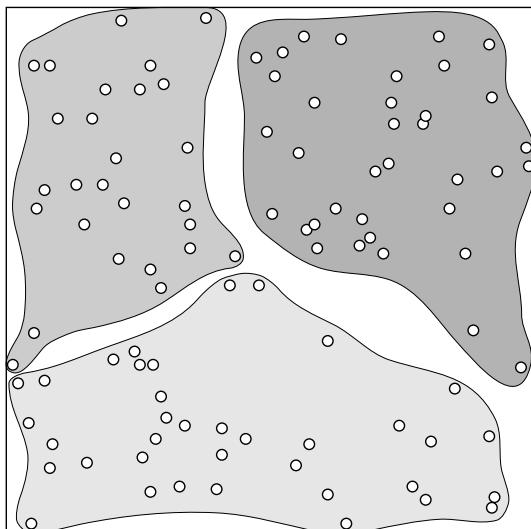
[Tan/Steinbach/Kumar 2005]



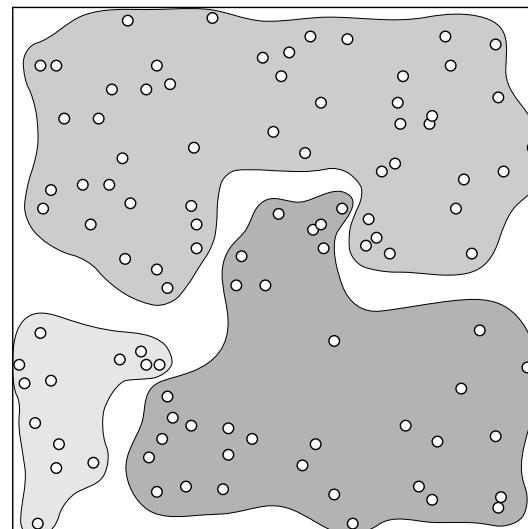
Random points



DBSCAN



k -means



Complete link

Cluster Evaluation

Overview

Cluster evaluation can address different issues:

- ❑ Provide evidence whether data contains non-random structures.
- ❑ Relate found structures in the data to externally provided class information.
- ❑ Rank alternative clusterings with regard to their quality.
- ❑ Determine the ideal number of clusters.
- ❑ Provide information to choose a suited clustering approach.

Cluster Evaluation

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(1) External validity measures:

Analyze how close is a clustering to an (external) reference.

(2) Internal validity measures:

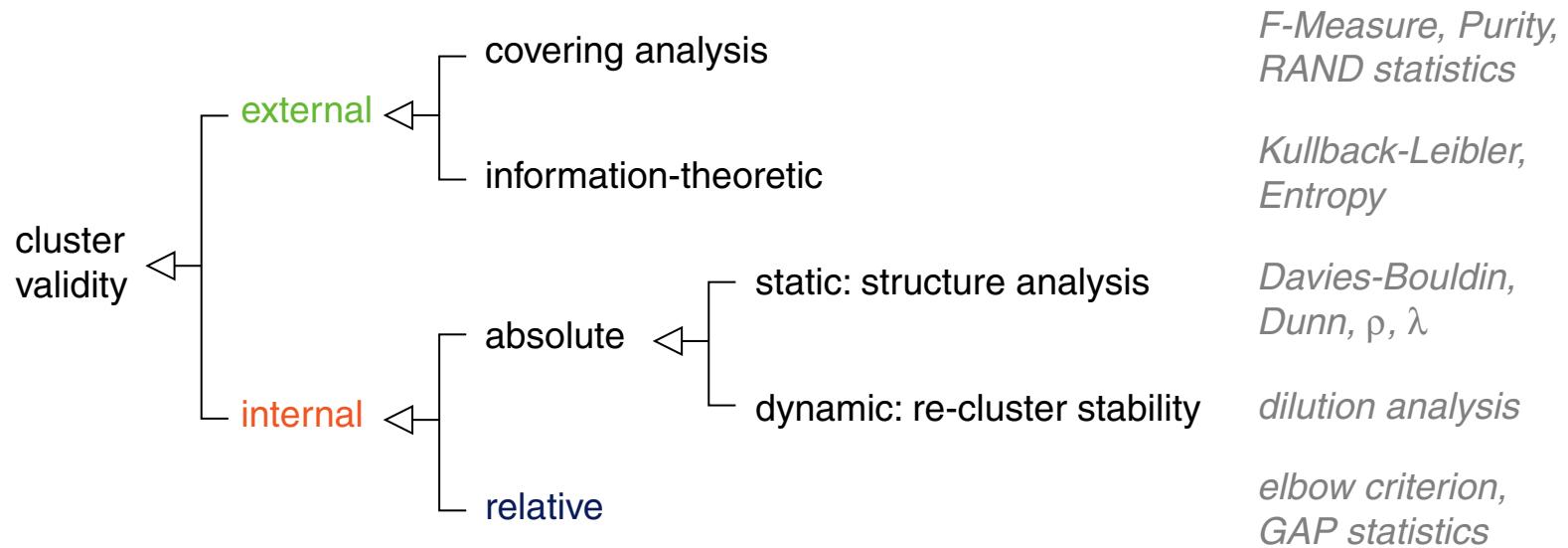
Analyze intrinsic characteristics of a clustering.

(3) Relative validity measures:

Analyze the sensitivity (of internal measures) during clustering generation.

Cluster Evaluation

Overview



(1) External validity measures:

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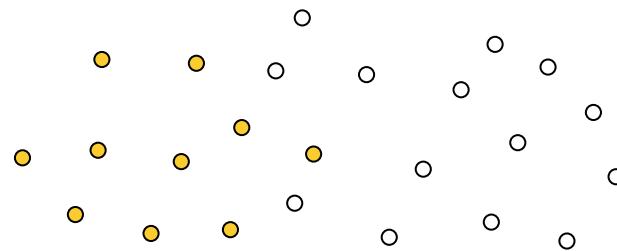
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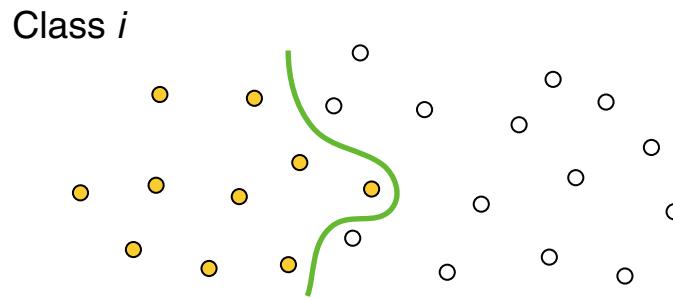
Cluster Evaluation

(1) External Validity Measures: F -Measure (for a Target Class)



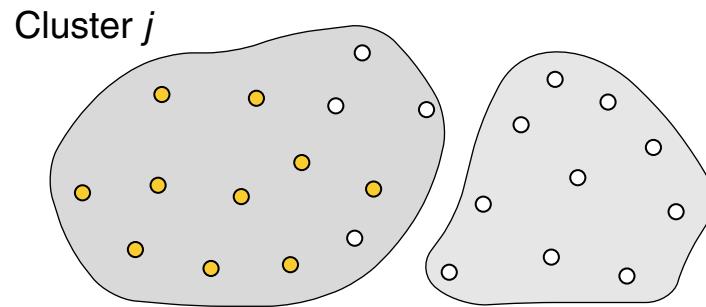
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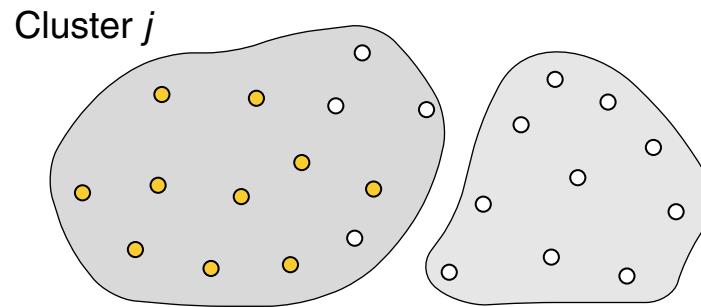
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(1) External Validity Measures: *F*-Measure (for a Target Class)



Cluster Evaluation

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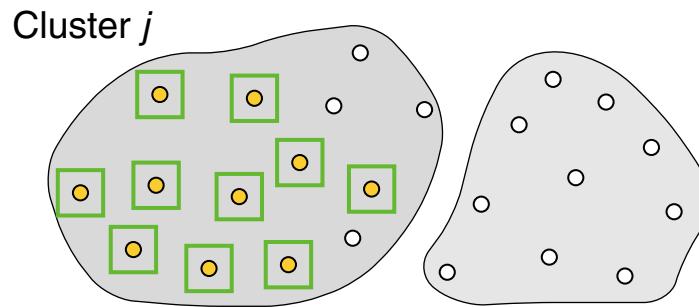


(node-based analysis)

		Truth	
		P	N
Hypothesis	P		
	N		

Cluster Evaluation

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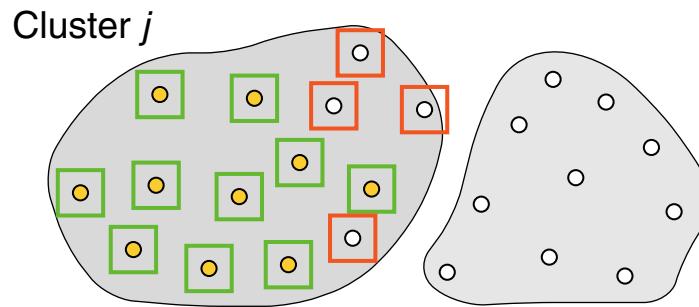


(node-based analysis)

		Truth	
		P	N
Hypothesis	P	TP (a)	
	N		

Cluster Evaluation

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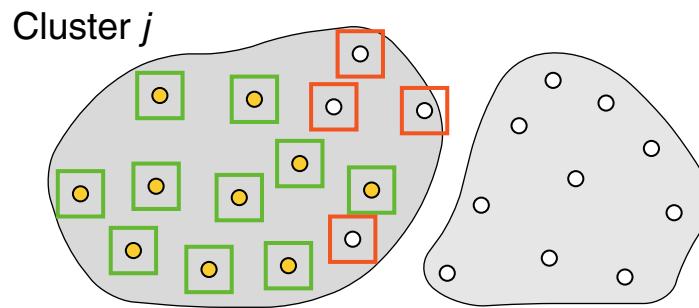


(node-based analysis)

		Truth	
		P	N
Hypothesis	P	TP (a)	FP (b)
	N		

Cluster Evaluation

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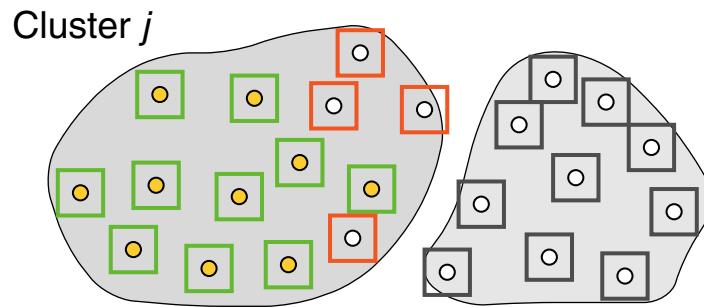


(node-based analysis)

		Truth	
		P	N
Hypothesis	P	TP (a)	FP (b)
	N	FN (c)	

Cluster Evaluation

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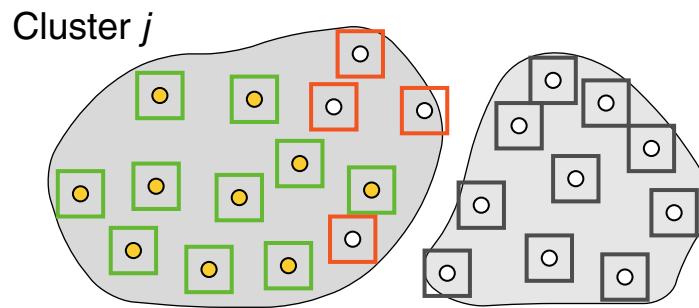


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Cluster Evaluation

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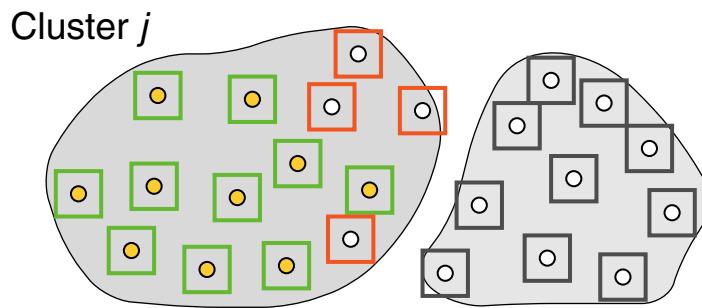
Precision: Recall:

$$\frac{a}{a + b}$$

$$\frac{a}{a + c}$$

Cluster Evaluation

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(node-based analysis)

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Hypothesis	P	TP (a)	FP (b)
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Precision:

$$\frac{a}{a + b}$$

Recall:

$$\frac{a}{a + c}$$

F-measure:

$$F_\alpha = \frac{1 + \alpha}{\frac{1}{precision} + \frac{\alpha}{recall}}$$

$$\alpha = 1$$

$$\alpha \in (0; 1)$$

$$\alpha > 1$$

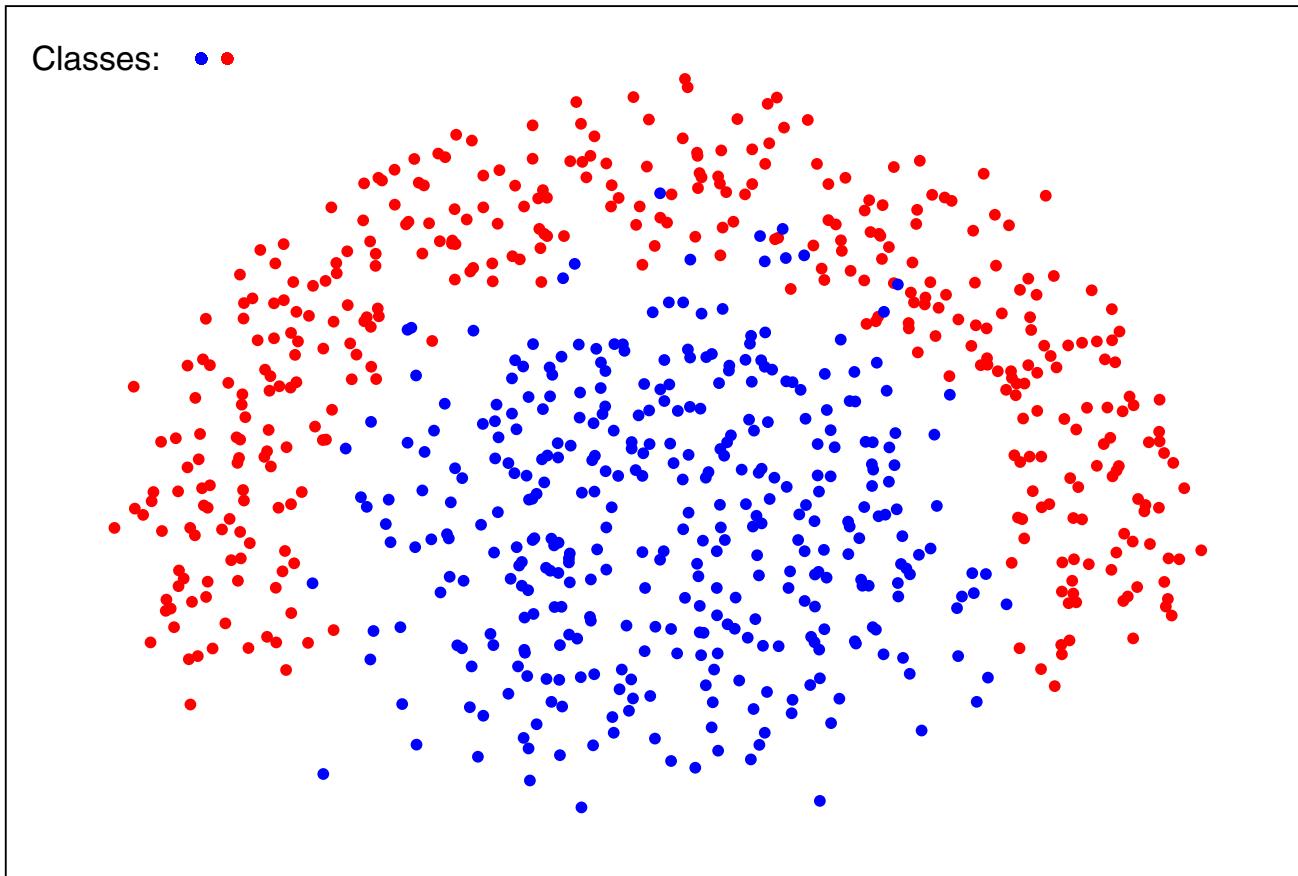
harmonic mean

favor precision over recall

favor recall over precision

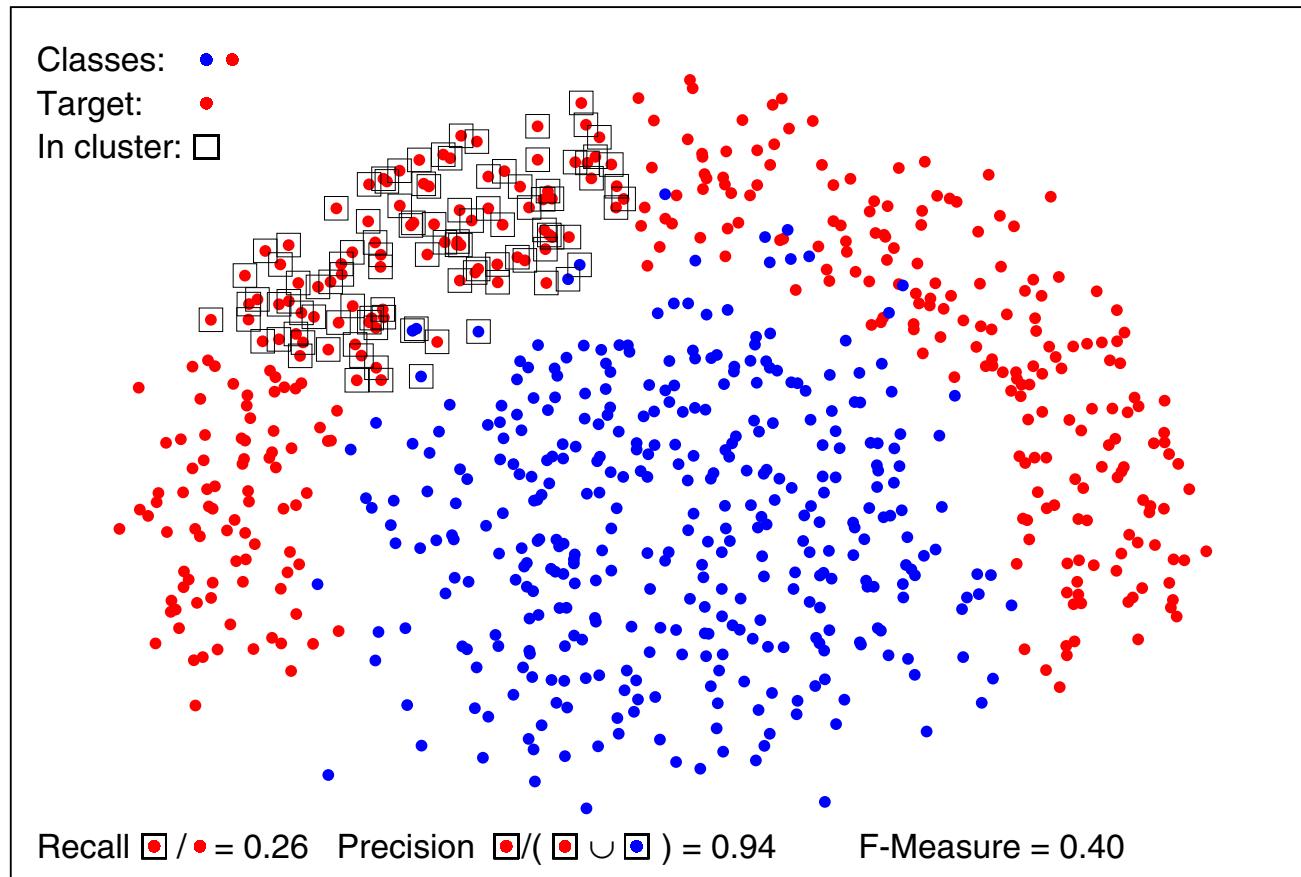
Cluster Evaluation

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Cluster Evaluation

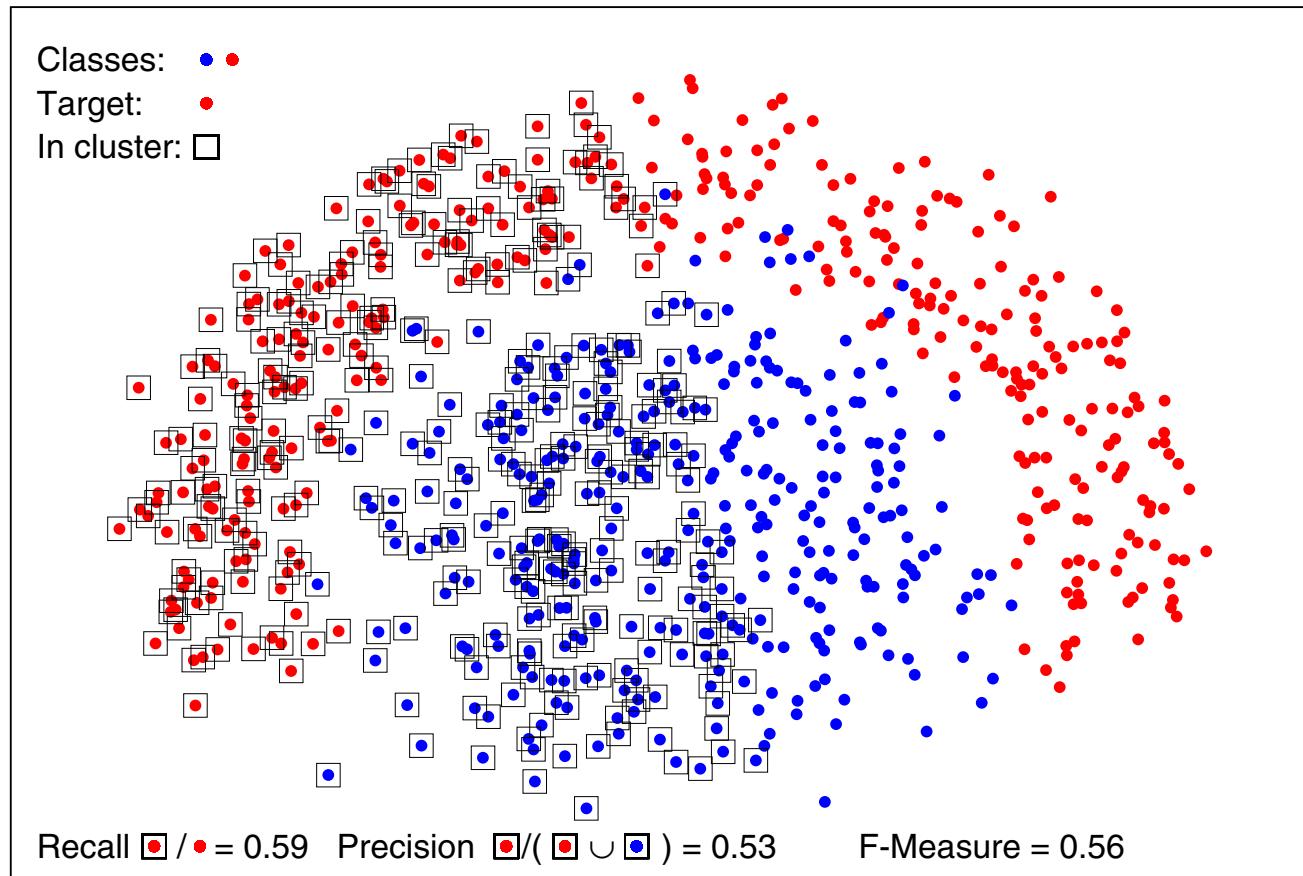
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High precision, low recall \Rightarrow low *F*-measure.

Cluster Evaluation

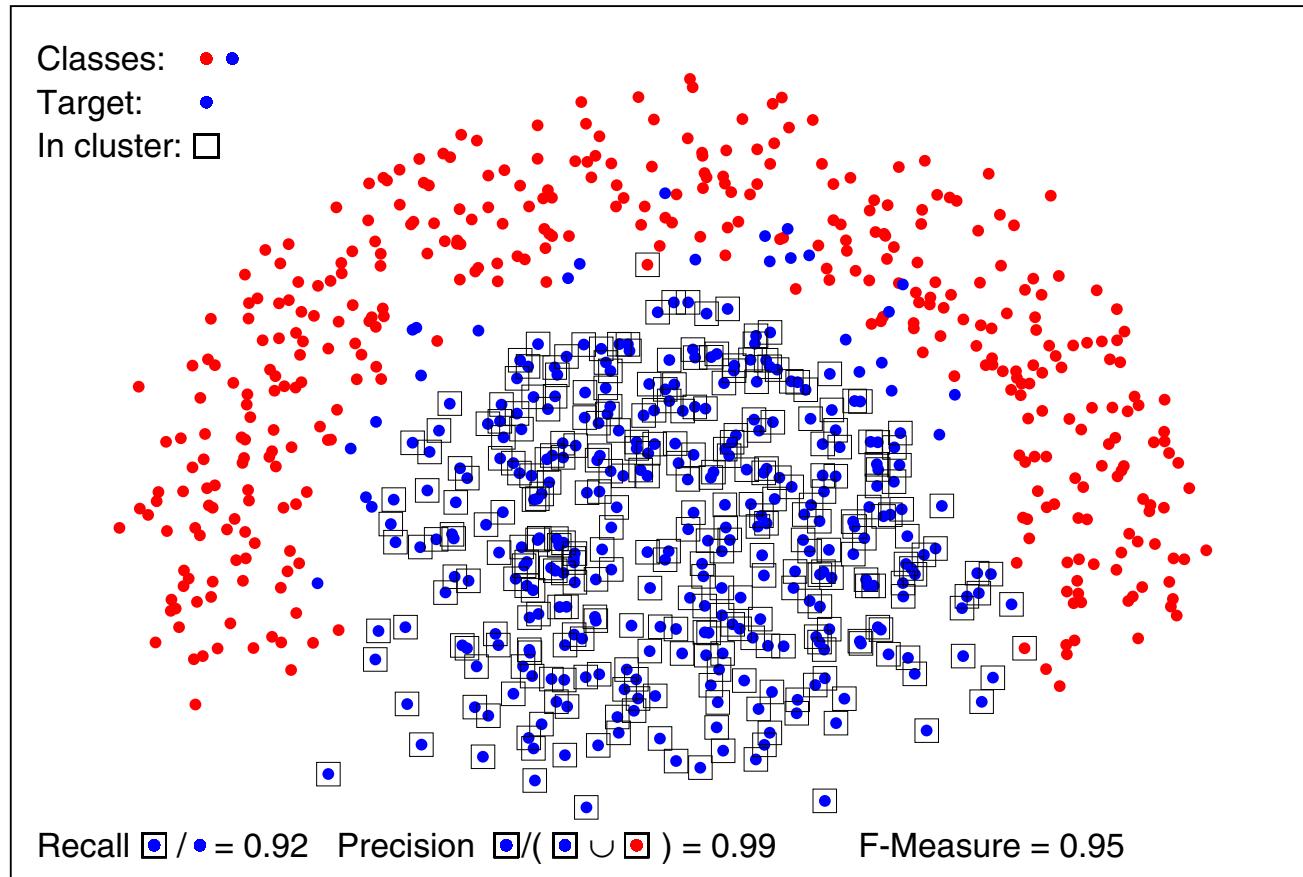
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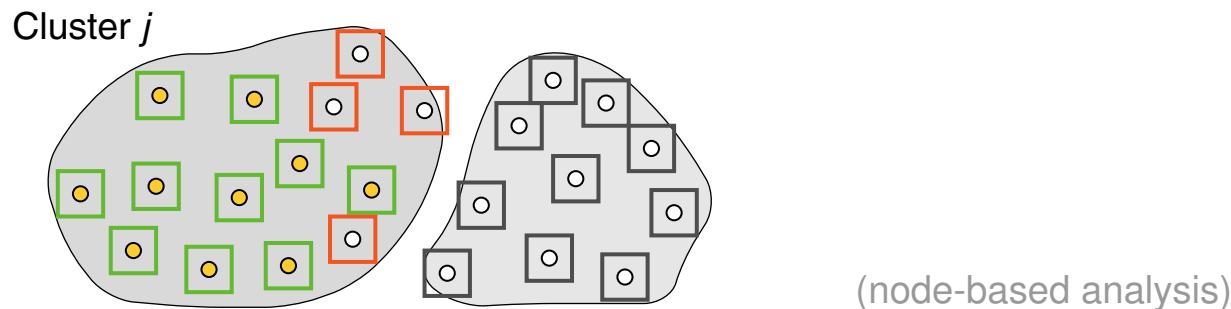
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High precision, high recall \Rightarrow high F -measure.

Cluster Evaluation

(1) External Validity Measures: F -Measure (for a Clustering)



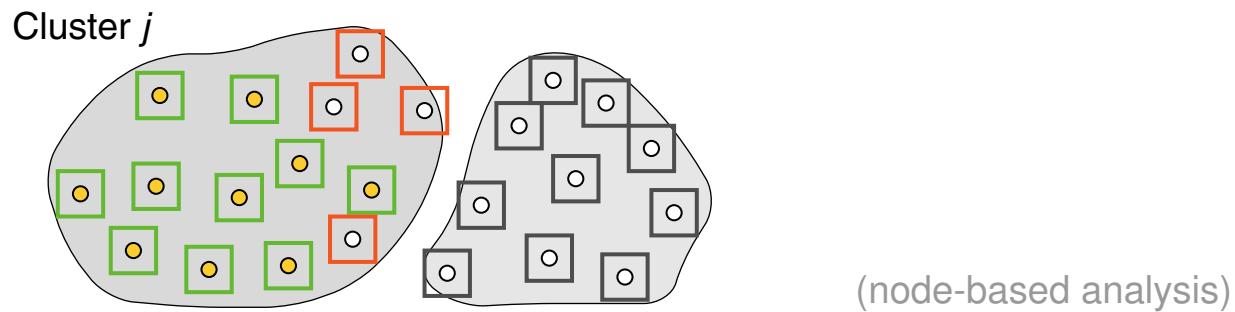
- Clustering $\mathcal{C} = \{C_1, \dots, C_k\}$ and classification $\mathcal{C}^* = \{C_1^*, \dots, C_l^*\}$ of D .
- $F_{i,j}$ is the F -measure of a cluster j computed *with respect to a class i* .

Precision of cluster j with respect to class i is $|C_j \cap C_i^*|/|C_j|$ (here: $Prec_{i,j} = 0.71$)

Recall of cluster j with respect to class i is $|C_j \cap C_i^*|/|C_i^*|$ (here: $Rec_{i,j} = 1.0$)

Cluster Evaluation

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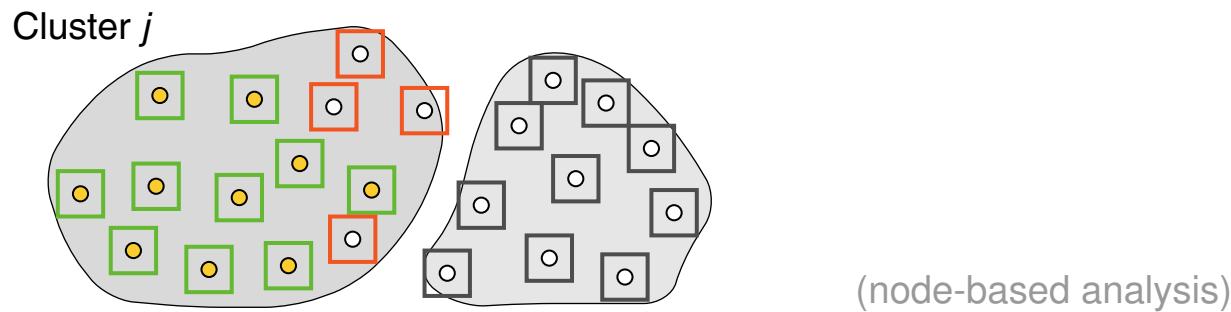
Recall of cluster j with respect to class i is $|C_j \cap C_i^*|/|C_i^*|$ (here: $Rec_{i,j} = 1.0$)

- Micro-averaged F -measure for $\langle D, \mathcal{C}, \mathcal{C}^* \rangle$:

$$F = \sum_{i=1}^l \frac{|C_i^*|}{|D|} \cdot \max_{j=1,\dots,k} \{F_{i,j}\}$$

Cluster Evaluation

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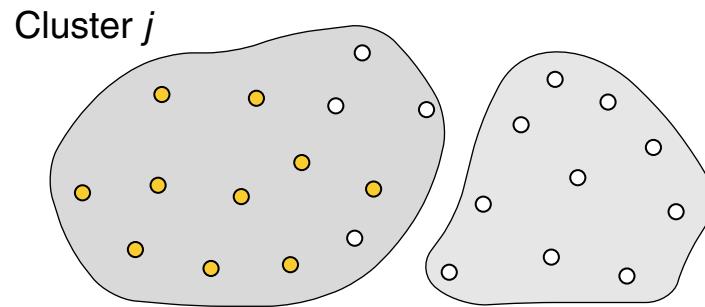
$$F = \frac{1}{l} \sum_{i=1}^l \max_{j=1,\dots,k} \{F_{i,j}\}$$

Remarks:

- Micro averaging treats objects equally, whereas macro averaging treats classes equally.

Cluster Evaluation

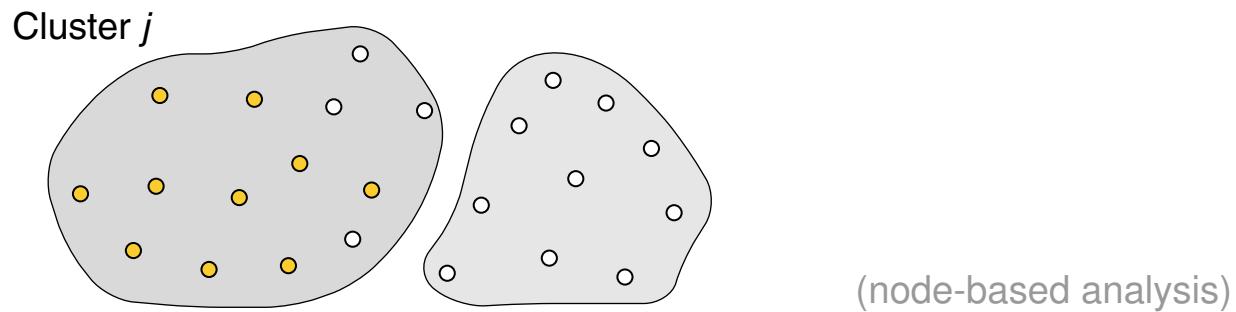
(1) External Validity Measures: Entropy



(node-based analysis)

Cluster Evaluation

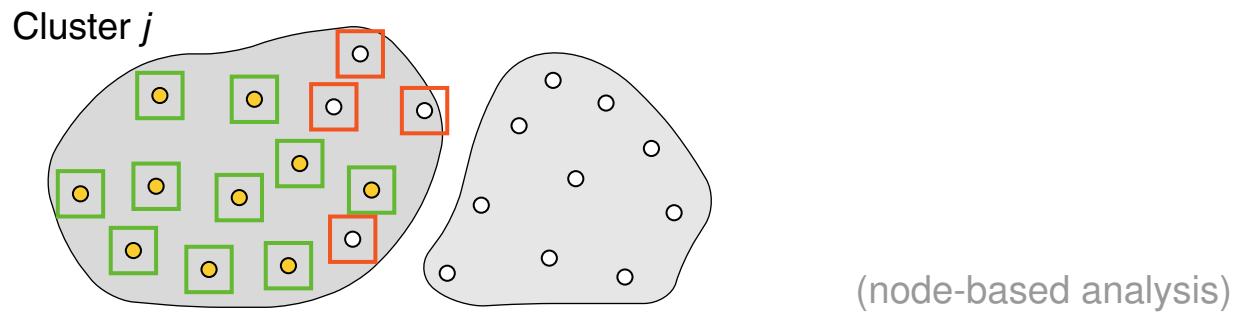
(1) External Validity Measures: Entropy



- A cluster C acts as information source \mathcal{L} .
 \mathcal{L} emits cluster labels L_1, \dots, L_l with probabilities $P(L_1), \dots, P(L_l)$.

Cluster Evaluation

(1) External Validity Measures: Entropy

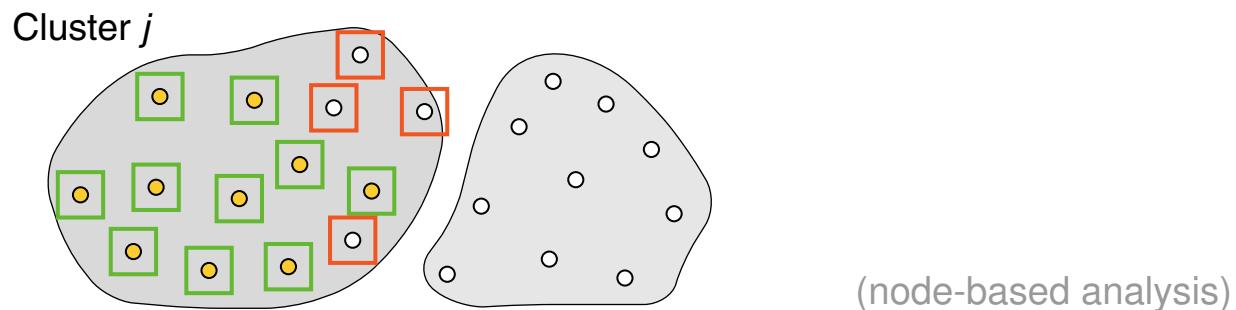


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$$L_1 = \square, \quad L_2 = \square, \quad \hat{P}(\square) = 10/14, \quad \hat{P}(\square) = 4/14$$

Cluster Evaluation

(1) External Validity Measures: Entropy



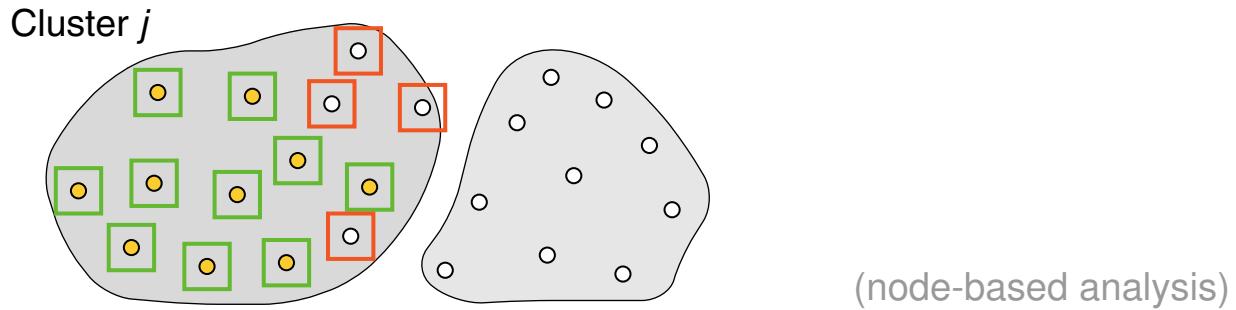
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- Entropy of \mathcal{L} :
$$H(\mathcal{L}) = -\sum_{i=1}^l P(L_i) \cdot \log_2(P(L_i))$$
- Entropy of C_j wrt. \mathcal{C}^* :
$$H(C_j) = -\sum_{C_j \cap C_i^* \neq \emptyset} |C_j \cap C_i^*| / |C_j| \cdot \log_2(|C_j \cap C_i^*| / |C_j|)$$

Cluster Evaluation

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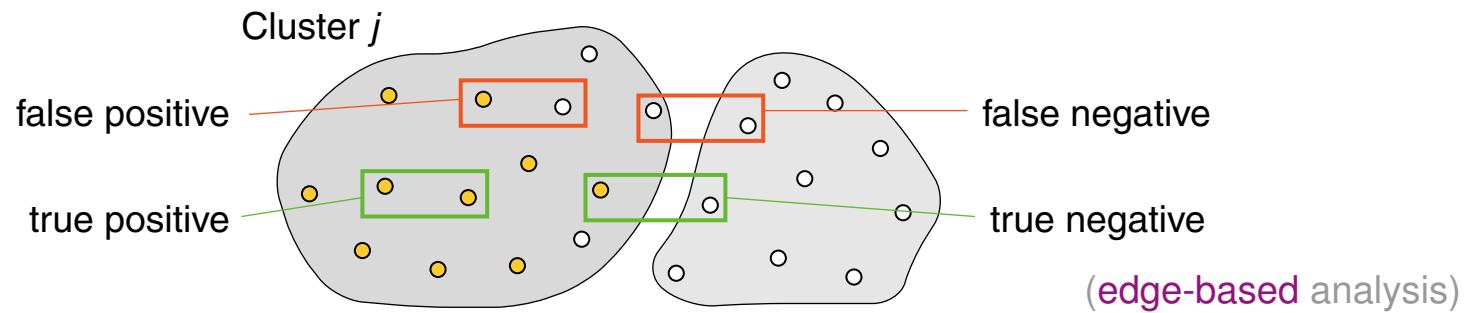
$$L_1 = \text{green square}, \quad L_2 = \text{orange square}, \quad \hat{P}(\text{green square}) = 10/14, \quad \hat{P}(\text{orange square}) = 4/14$$

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→ Entropy of \mathcal{C} wrt. \mathcal{C}^* :
$$H(\mathcal{C}) = \sum_{C_j \in \mathcal{C}} |C_j| / |D| \cdot H(C_j)$$

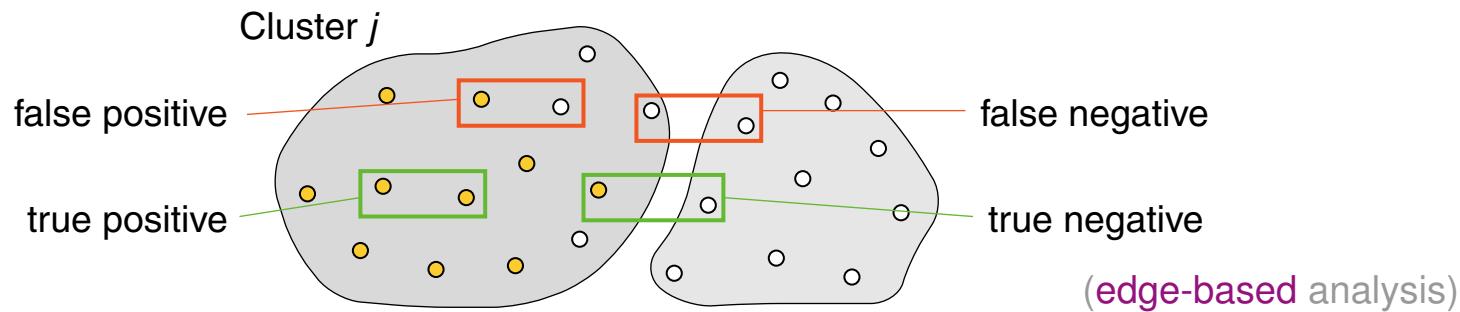
Cluster Evaluation

(1) External Validity Measures: Rand, Jaccard



Cluster Evaluation

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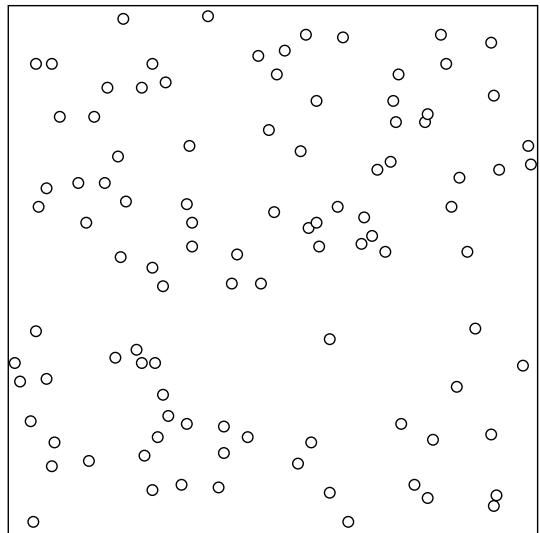


- $R(\mathcal{C}) = \frac{|TP| + |TN|}{|TP| + |TN| + |FP| + |FN|} = \frac{|TP| + |TN|}{n(n-1)/2}, \quad \text{with } n = |D|$

- $J(\mathcal{C}) = \frac{|TP|}{|TP| + |FP| + |FN|}$

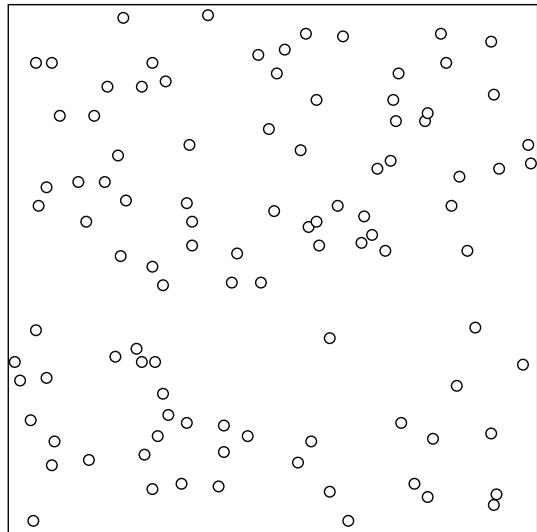
Cluster Evaluation

(2) Internal Validity Measures: Edge Correlation [Tan/Steinbach/Kumar 2005]



Cluster Evaluation

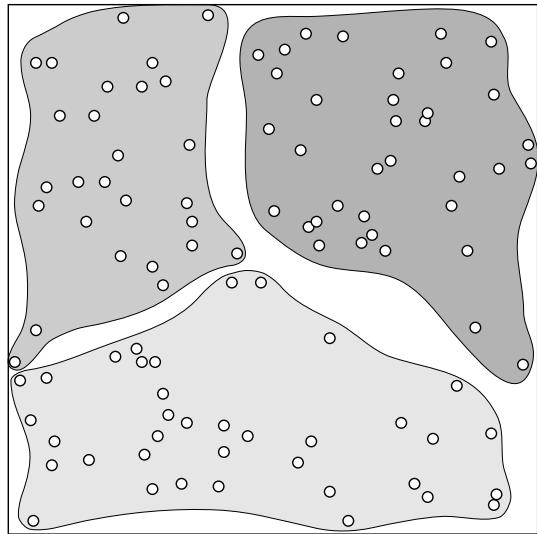
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$$\begin{pmatrix} 1.0 & 0.2 & 0.1 & 0.3 & \dots & 0.1 & 0.0 \\ - & 1.0 & 0.1 & 0.0 & \dots & 0.0 & 0.2 \\ & & & & \vdots & & \\ - & - & - & - & - & 1.0 & 0.6 \\ - & - & - & - & - & - & 1.0 \end{pmatrix}$$

Cluster Evaluation

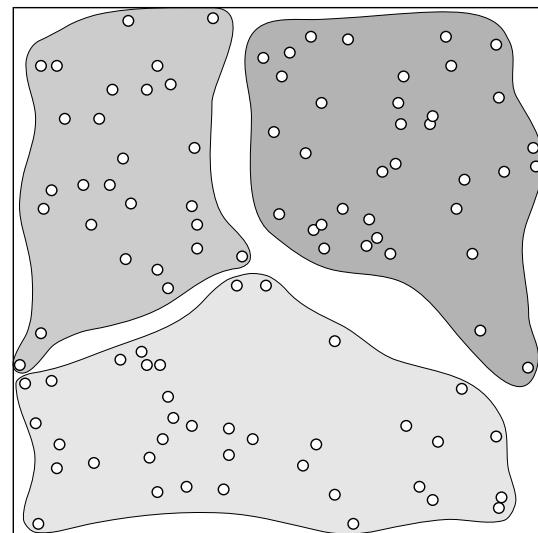
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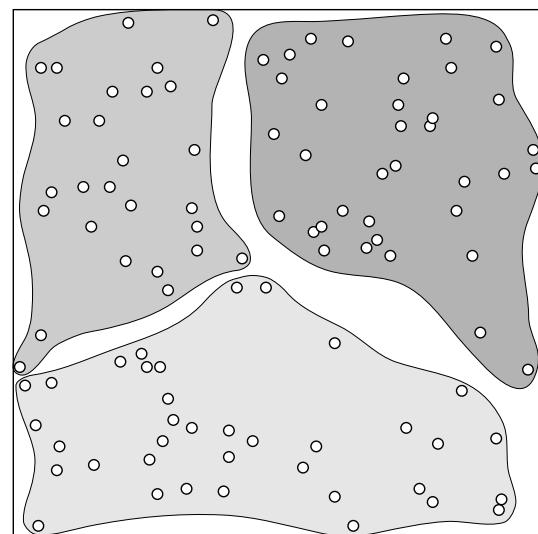


$$\begin{pmatrix} 1.0 & 0.2 & 0.1 & 0.3 & \dots & 0.1 & 0.0 \\ - & 1.0 & 0.1 & 0.0 & \dots & 0.0 & 0.2 \\ & & & & \vdots & & \\ - & - & - & - & - & 1.0 & 0.6 \\ - & - & - & - & - & - & 1.0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 & \dots & 0 & 0 \\ - & 1 & 0 & 0 & \dots & 0 & 1 \\ & & & & \vdots & & \\ - & - & - & - & - & 1 & 1 \\ - & - & - & - & - & - & 1 \end{pmatrix}$$

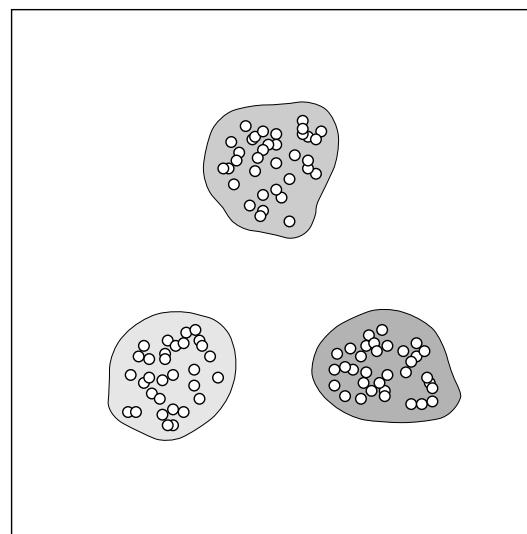
- Construct occurrence matrix based on cluster analysis.
- Compare similarity matrix to **occurrence matrix**: correlation τ

Cluster Evaluation

(2) Internal Validity Measures: Edge Correlation [Tan/Steinbach/Kumar 2005]



$k\text{-means}$
 $\tau = 0.58$



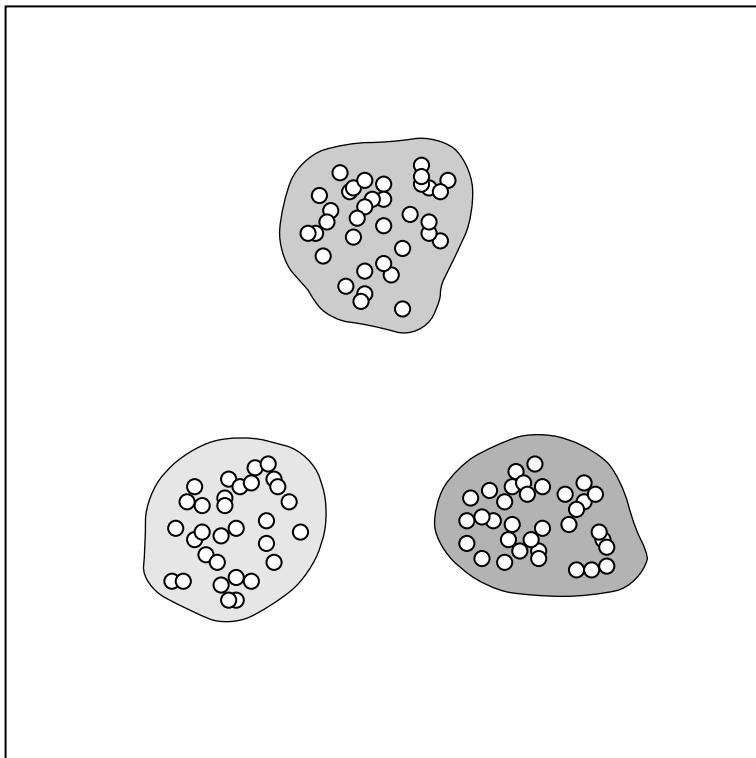
$k\text{-means}$
 $\tau = 0.92$

$$\begin{pmatrix} 1.0 & 0.2 & 0.1 & 0.3 & \dots & 0.1 & 0.0 \\ - & 1.0 & 0.1 & 0.0 & \dots & 0.0 & 0.2 \\ & & & & \vdots & & \\ - & - & - & - & - & 1.0 & 0.6 \\ - & - & - & - & - & - & 1.0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 & \dots & 0 & 0 \\ - & 1 & 0 & 0 & \dots & 0 & 1 \\ & & & & \vdots & & \\ - & - & - & - & - & 1 & 1 \\ - & - & - & - & - & - & 1 \end{pmatrix}$$

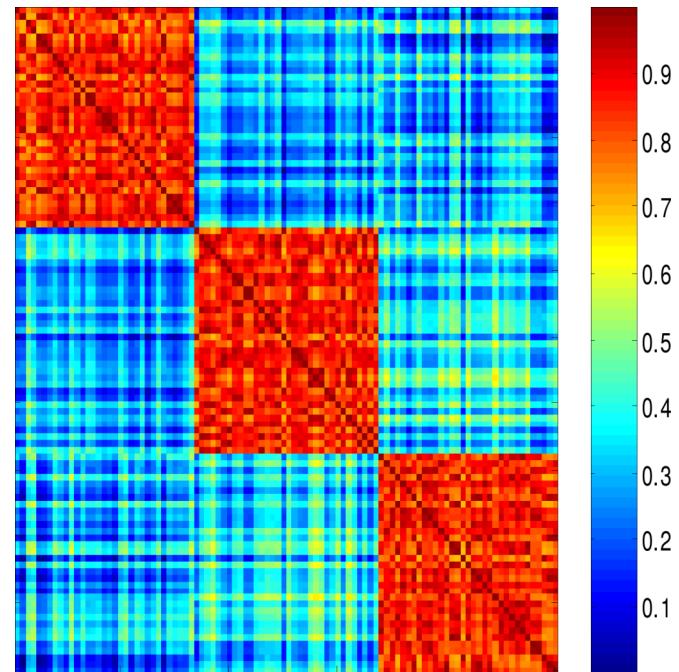
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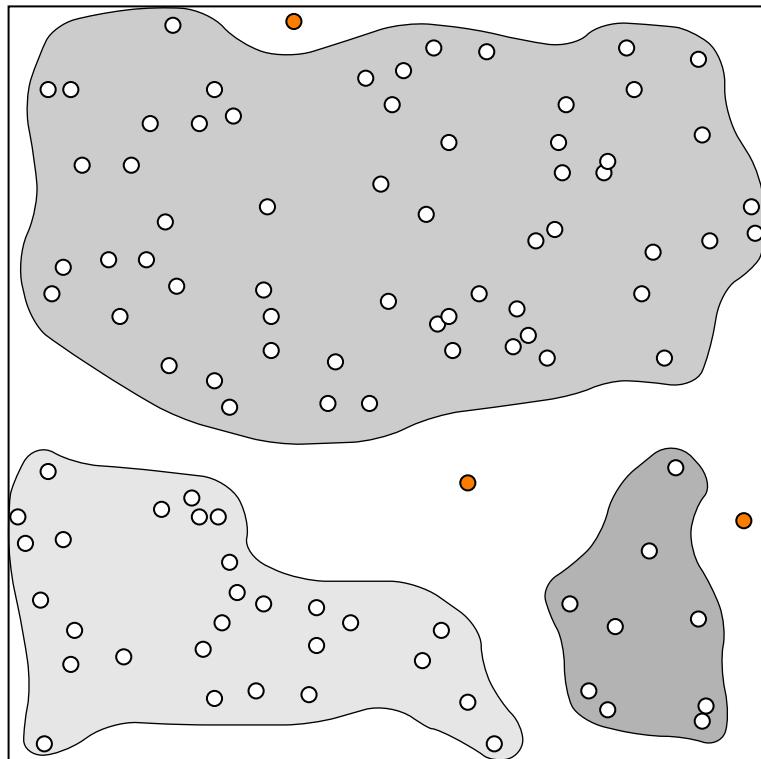
k -means at structured data.



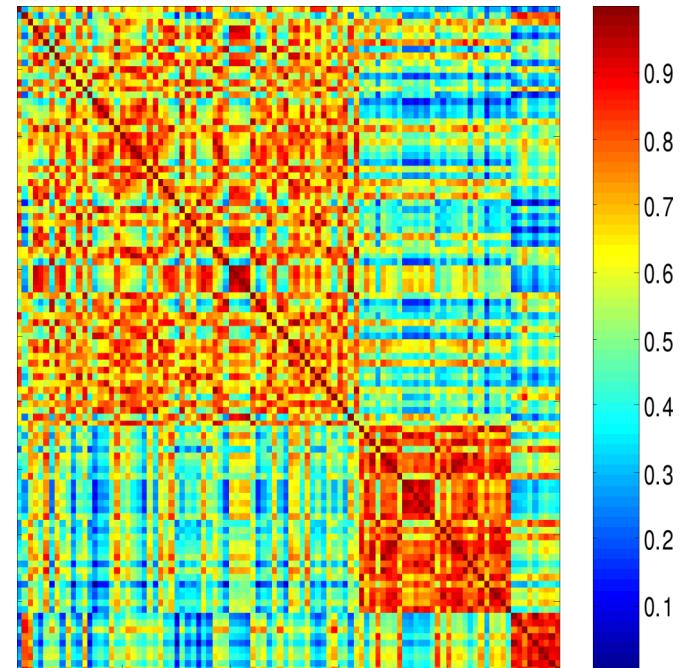
Similarity matrix sorted by cluster label.

Cluster Evaluation

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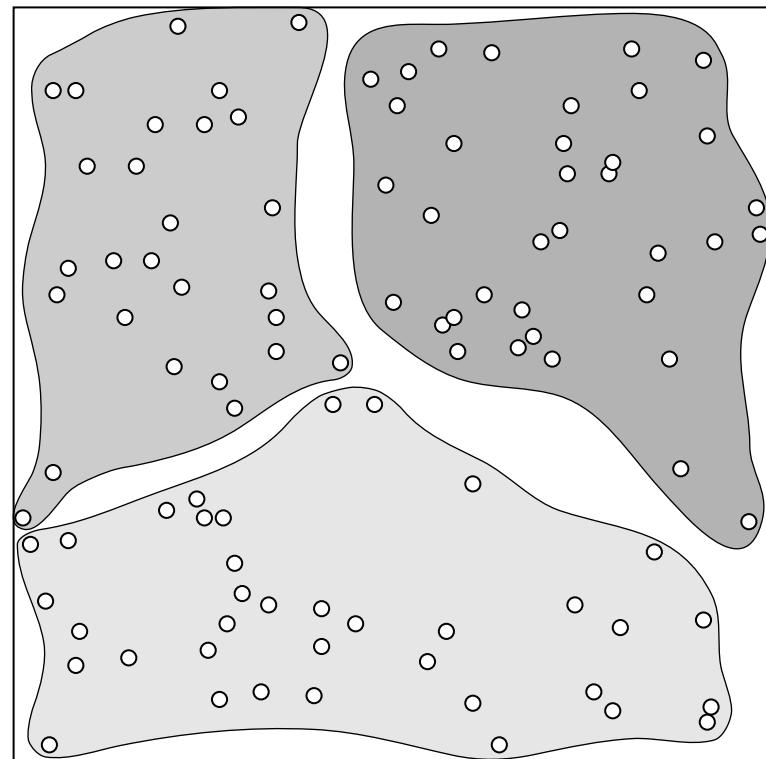
DBSCAN at random data.



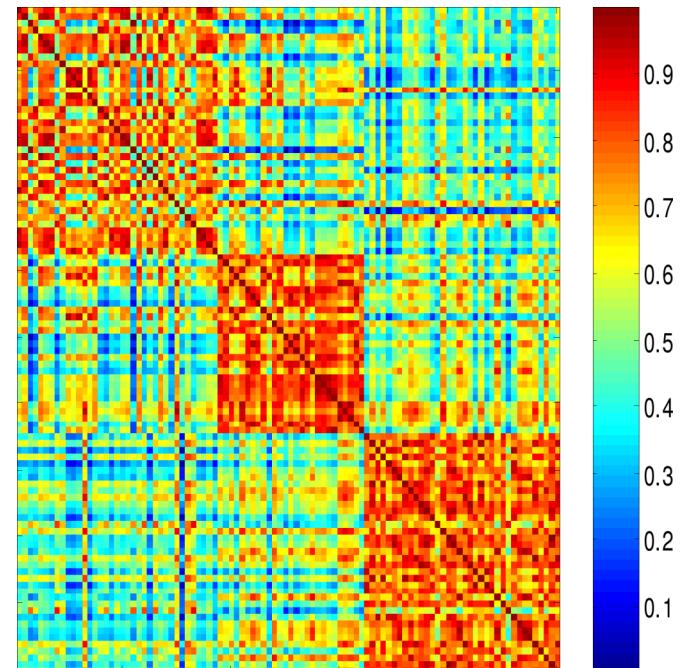
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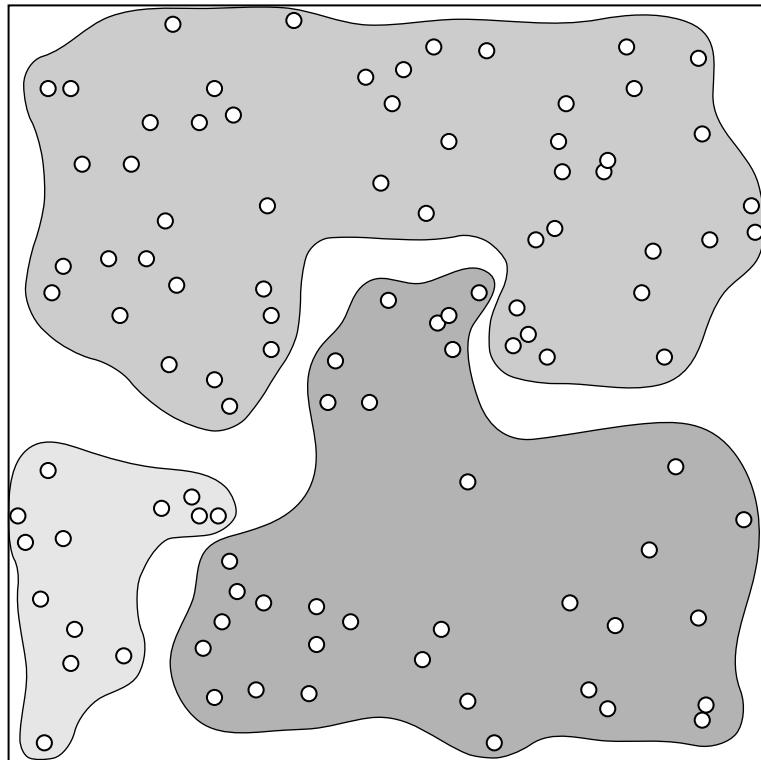
k -means at random data.



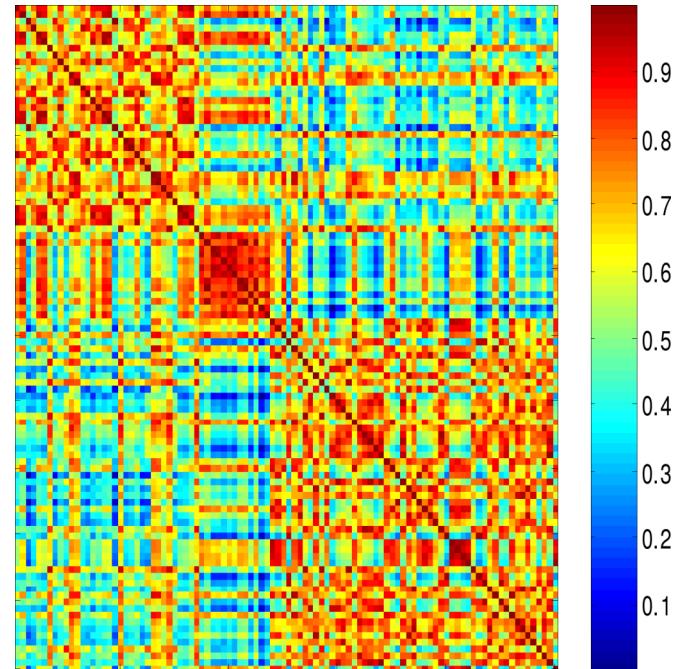
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Cluster Evaluation

(2) Internal Validity Measures: Edge Correlation [Tan/Steinbach/Kumar 2005]



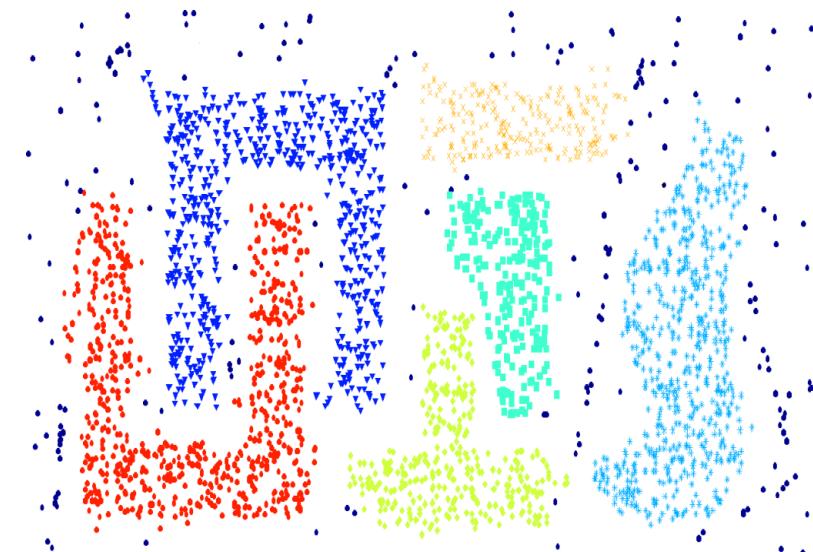
Complete link at random data.



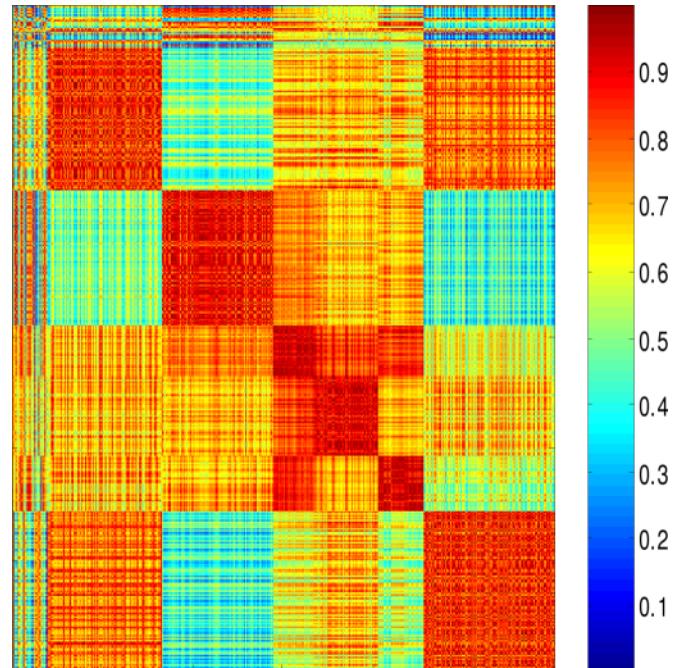
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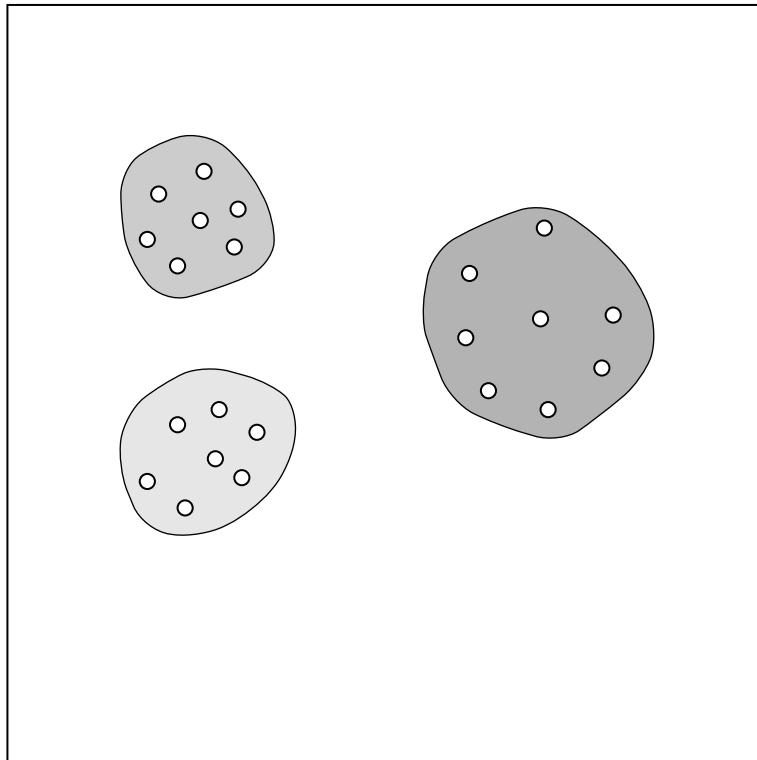
DBSCAN at structured data.



Similarity matrix sorted by cluster label.

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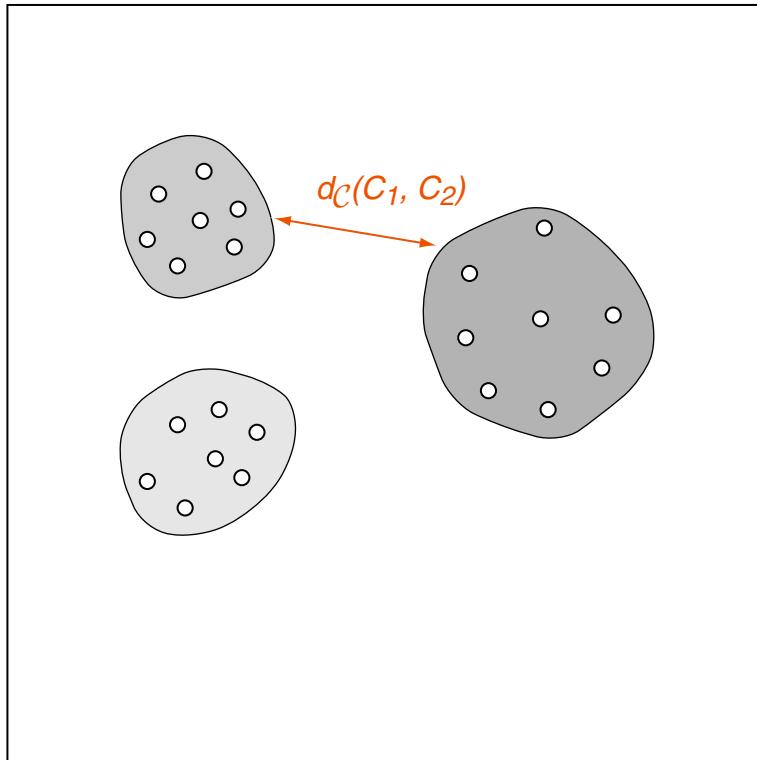
(2) Internal Validity Measures: Structural Analysis



- ❑ Distance between two clusters, $d_C(C_1, C_2)$.
- ❑ Diameter of a cluster, $\Delta(C)$.
- ❑ Scatter within a cluster, $\sigma^2(C)$, RSS.

Cluster Evaluation

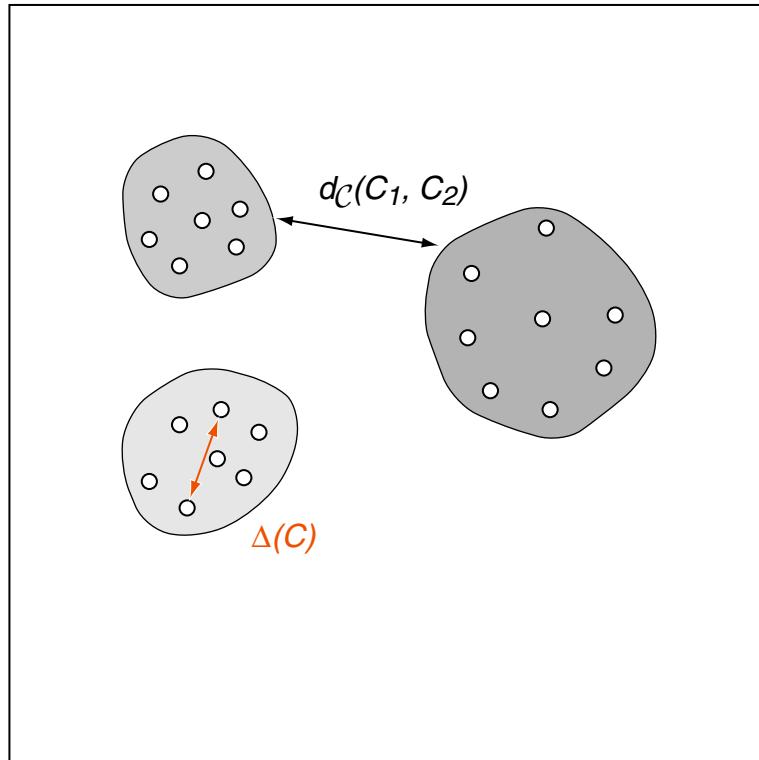
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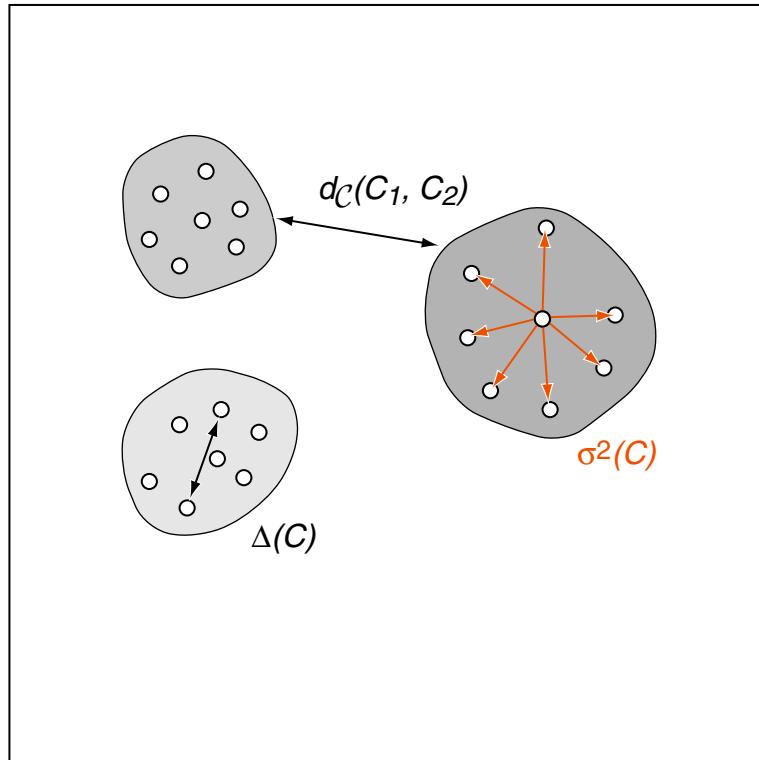
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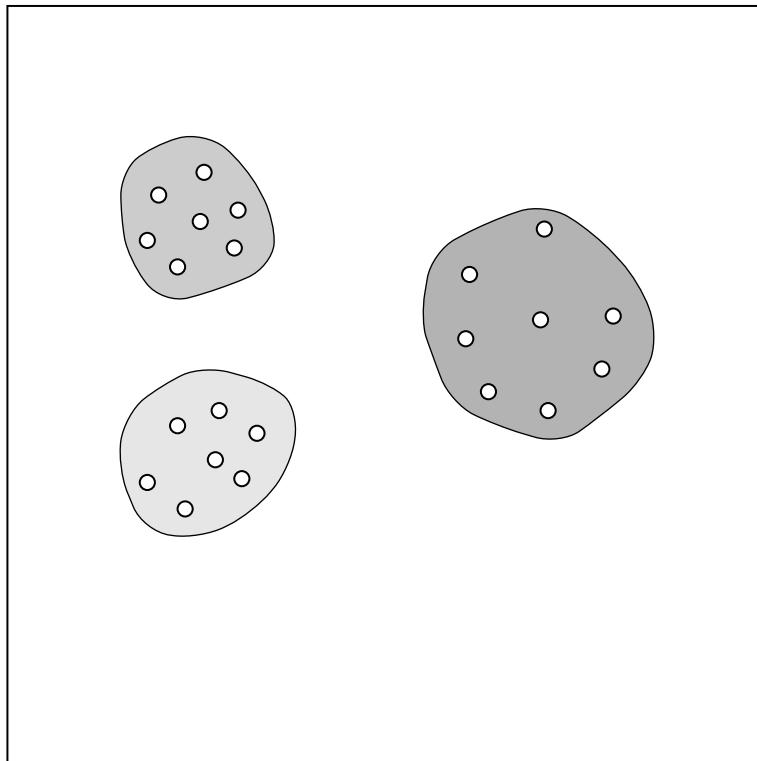
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(2) Internal Validity Measures: Dunn Index

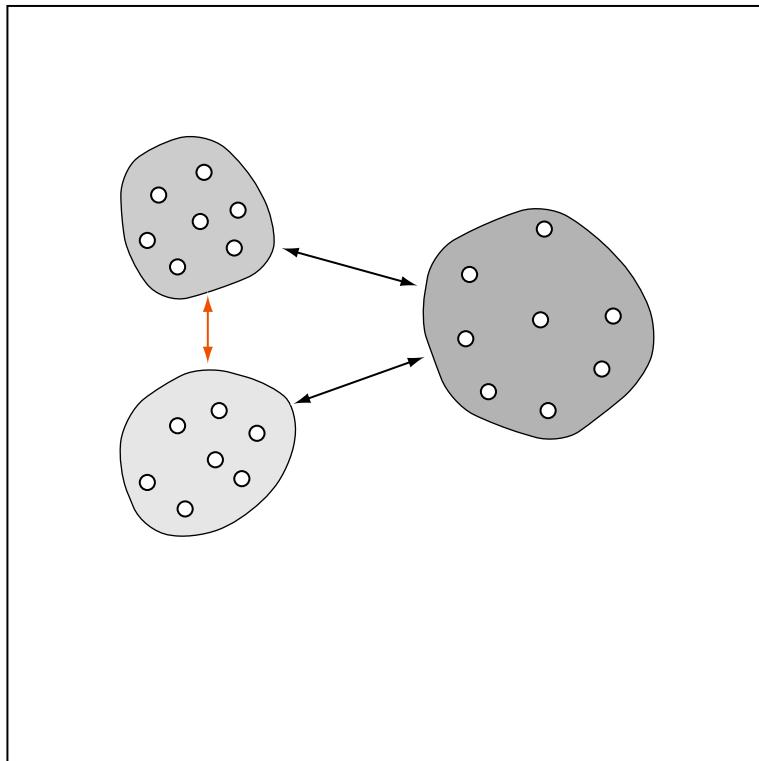


$$I(\mathcal{C}) = \frac{\min_{i \neq j} \{d_{\mathcal{C}}(C_i, C_j)\}}{\max_{1 \leq l \leq k} \{\Delta(C_l)\}},$$

$I(\mathcal{C}) \rightarrow \max$

Cluster Evaluation

(2) Internal Validity Measures: Dunn Index



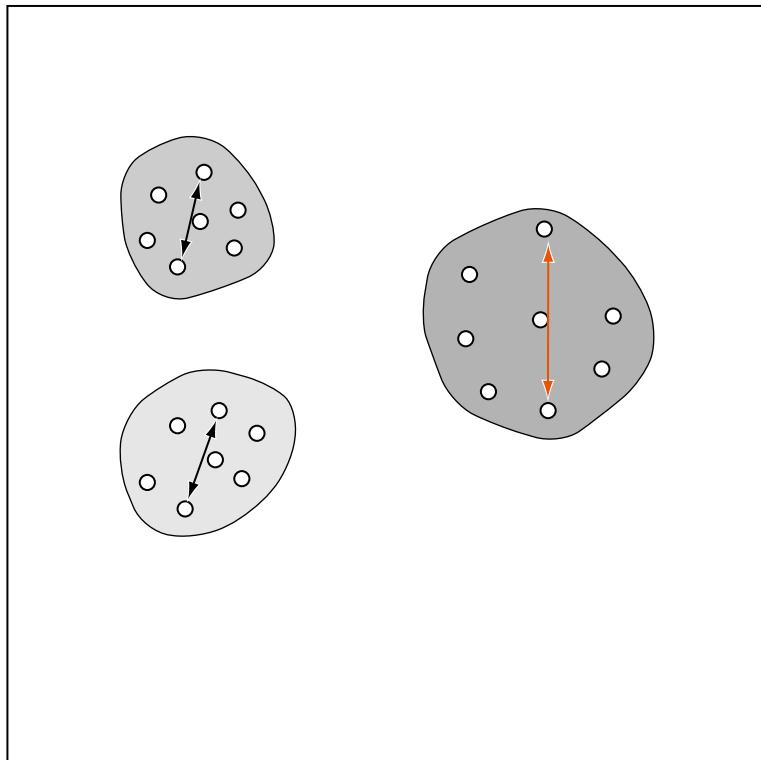
Cluster distance

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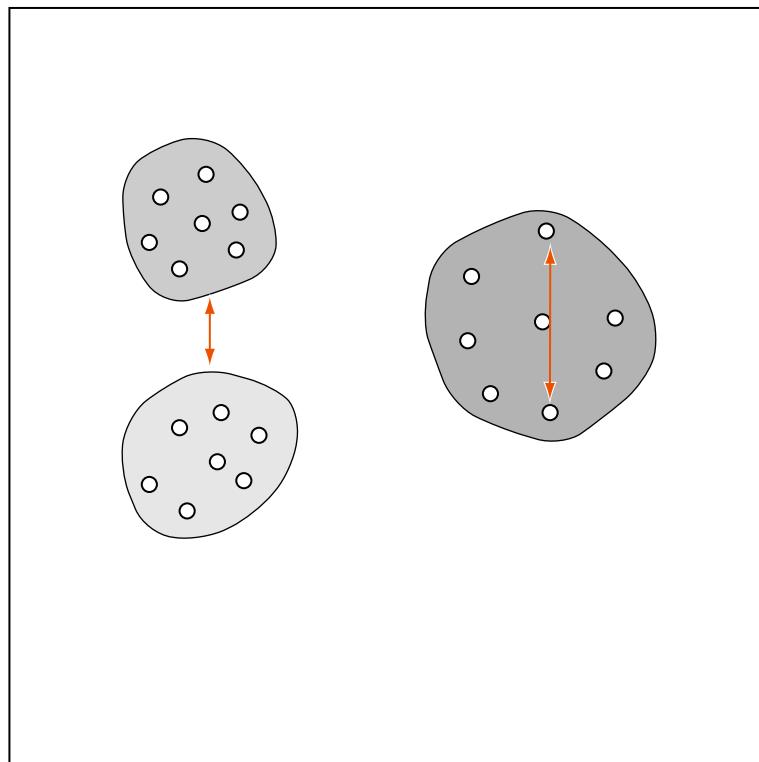
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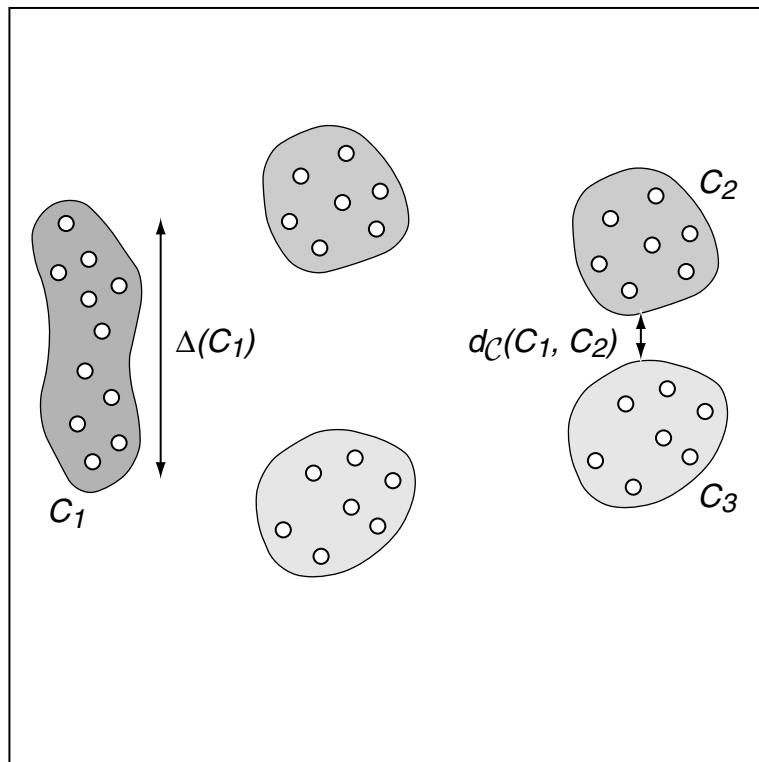
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- Dunn is susceptible to noise.
- Dunn is biased towards the worst substructure in a clustering (cf. the min).

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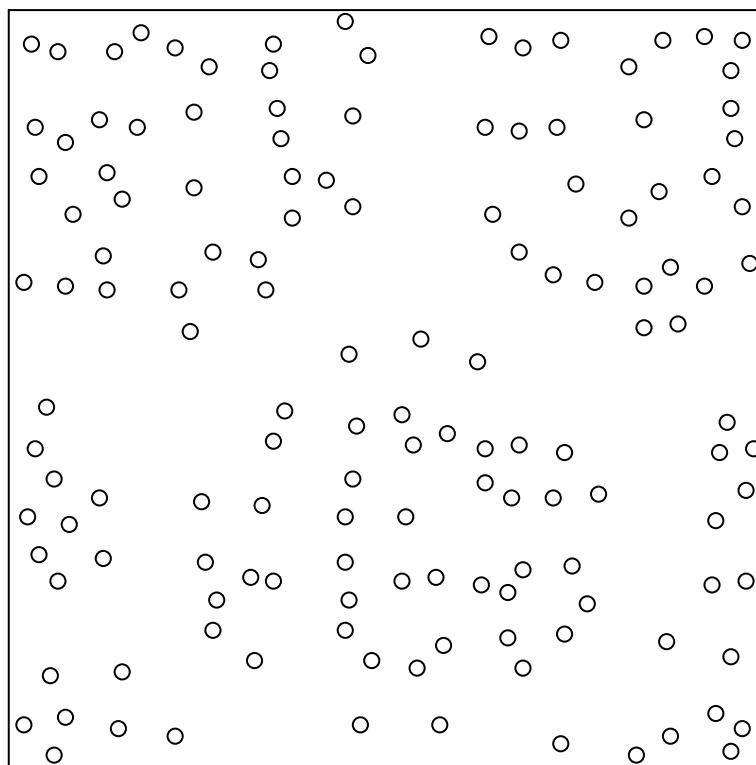
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- ❑ Dunn is susceptible to noise.
- ❑ Dunn is biased towards the worst substructure in a clustering (cf. the min).
- ❑ Dunn value too low since distances and diameters are not put into relation.

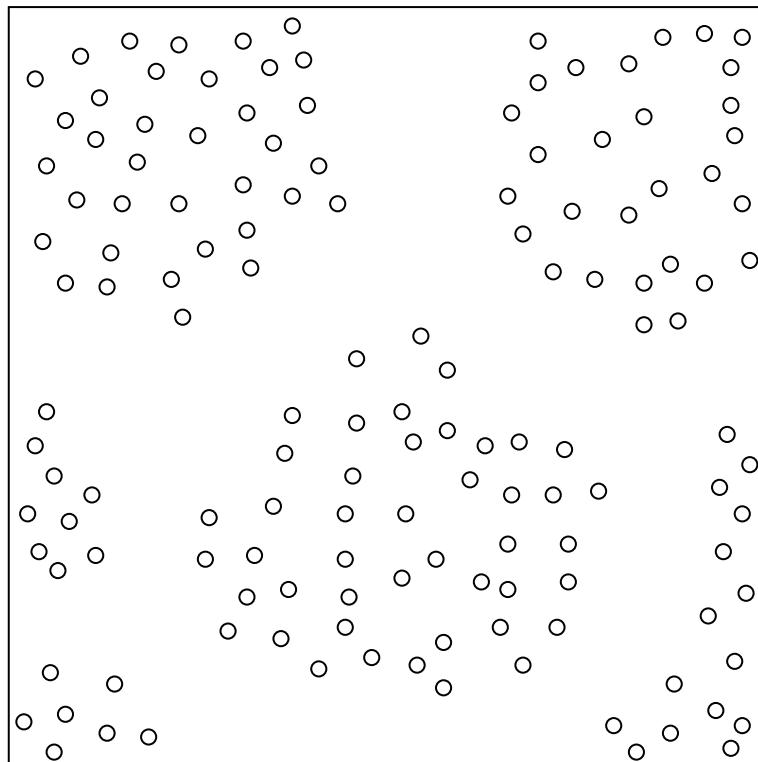
Cluster Evaluation

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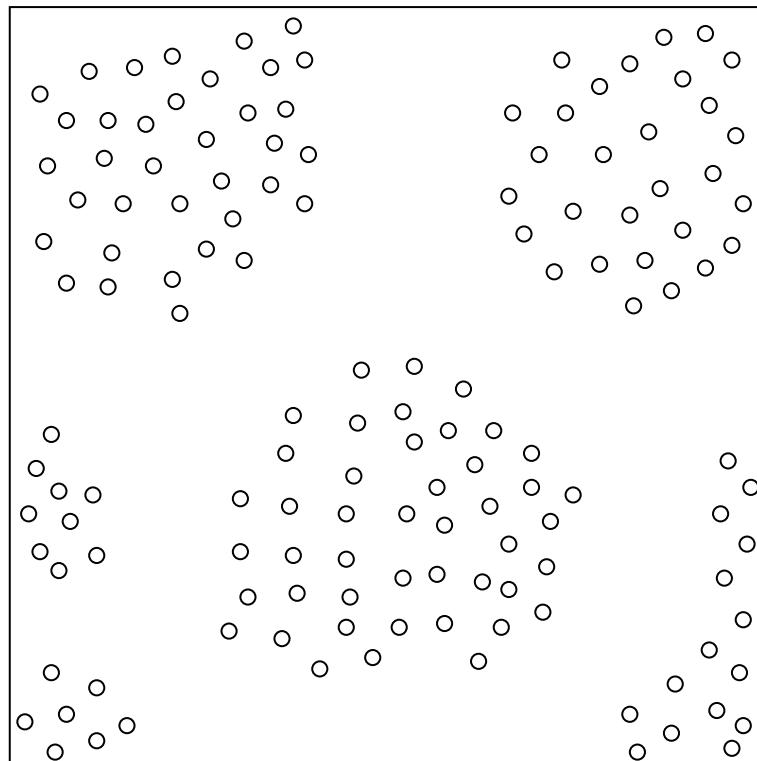
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Different models (feature sets) yield different similarity graphs.

Cluster Evaluation

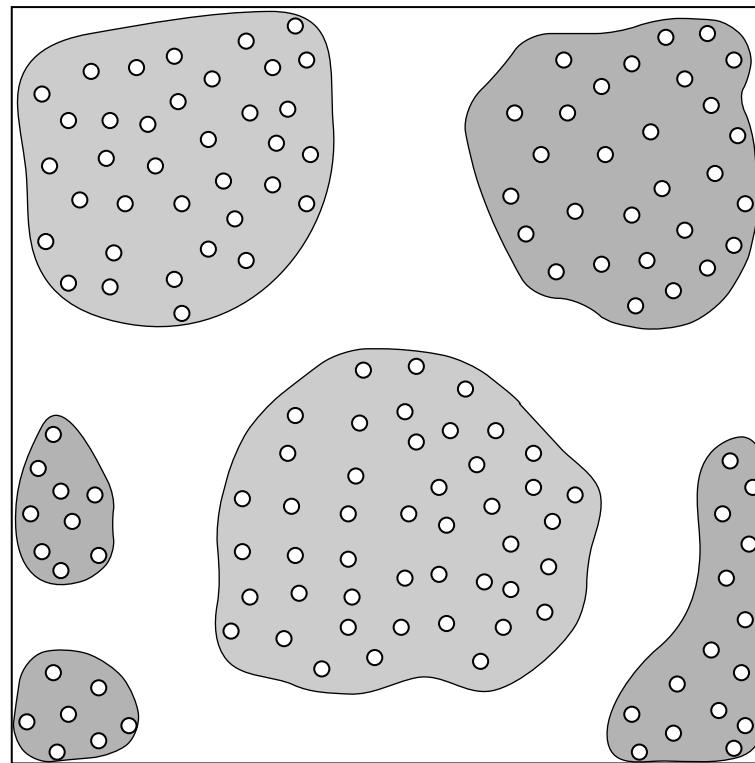
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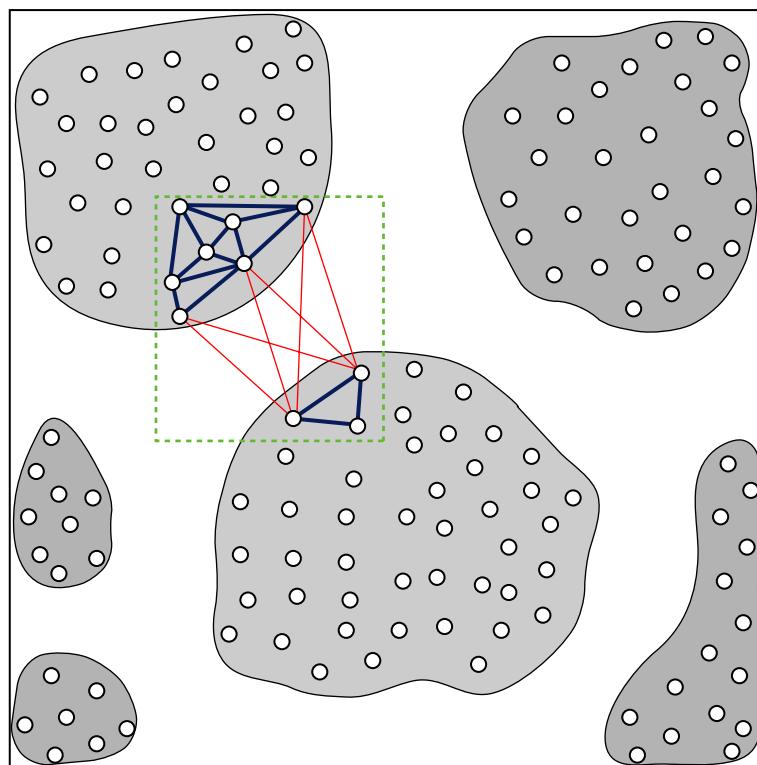
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Compare (for alternative clusterings) the similarity density within the clusters to the average similarity of the entire graph.

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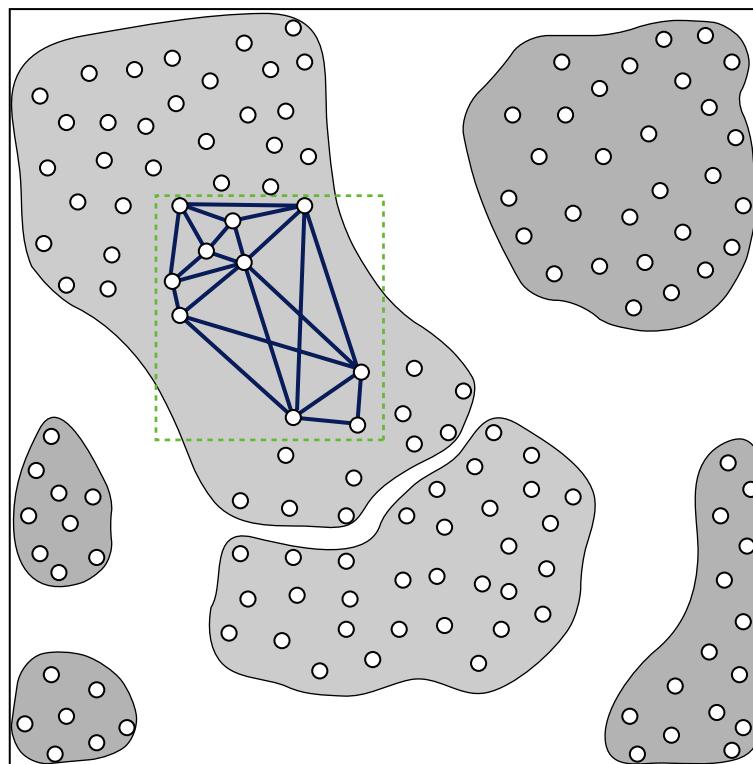
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Graph $G = \langle V, E \rangle$:

- G is called sparse if $|E| = O(|V|)$, G is called dense if $|E| = O(|V|^2)$
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Cluster C_i induces subgraph G_i :

- the **expected density ρ** relates the density of G_i to the density average in G

$$\rho(G_i) = \frac{w(G_i)}{|V_i|^\theta}$$

Cluster Evaluation

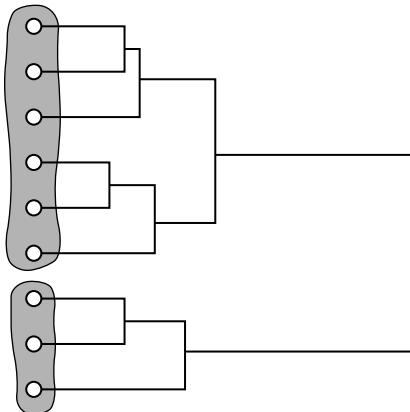
(3) Relative Validity Measures: Elbow Criterion

1. Hyperparameter alternatives of a clustering algorithm: π_1, \dots, π_m
 - number of centroids for k -means
 - stopping level for hierarchical algorithms
 - neighborhood size for DBSCAN
2. Set of clusterings $\mathcal{C} = \{\mathcal{C}_{\pi_1}, \dots, \mathcal{C}_{\pi_m}\}$ associated with π_1, \dots, π_m .
3. Points of an error curve $\{(\pi_i, e(\mathcal{C}_{\pi_i})) \mid i = 1, \dots, m\}$.

Cluster Evaluation

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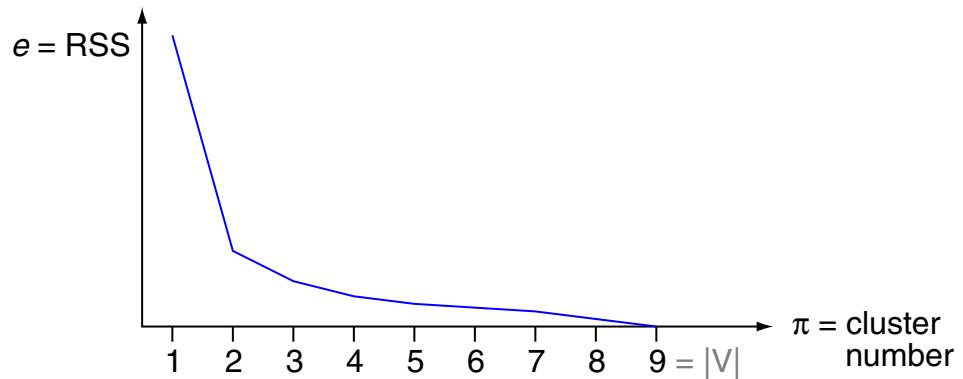
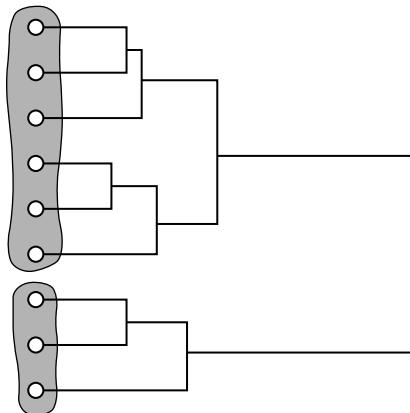
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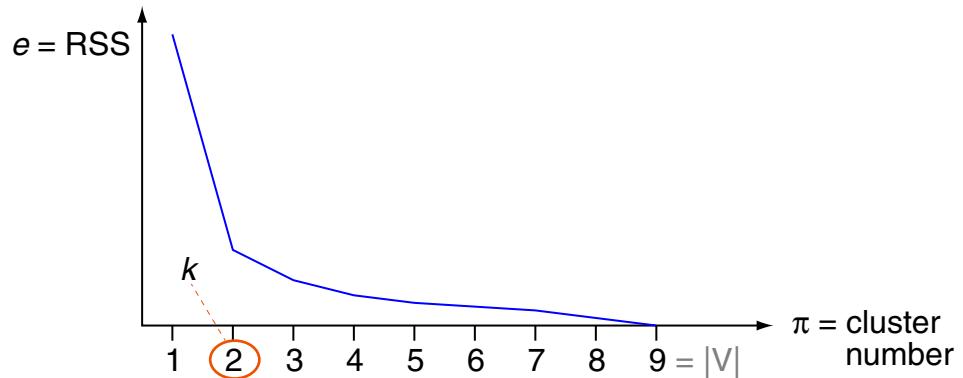
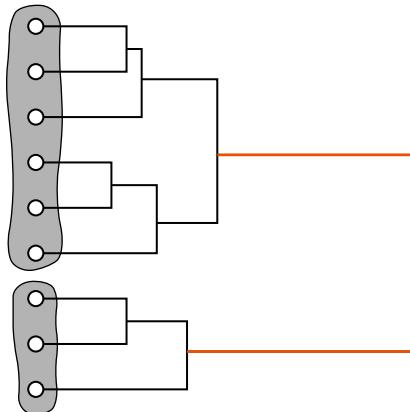
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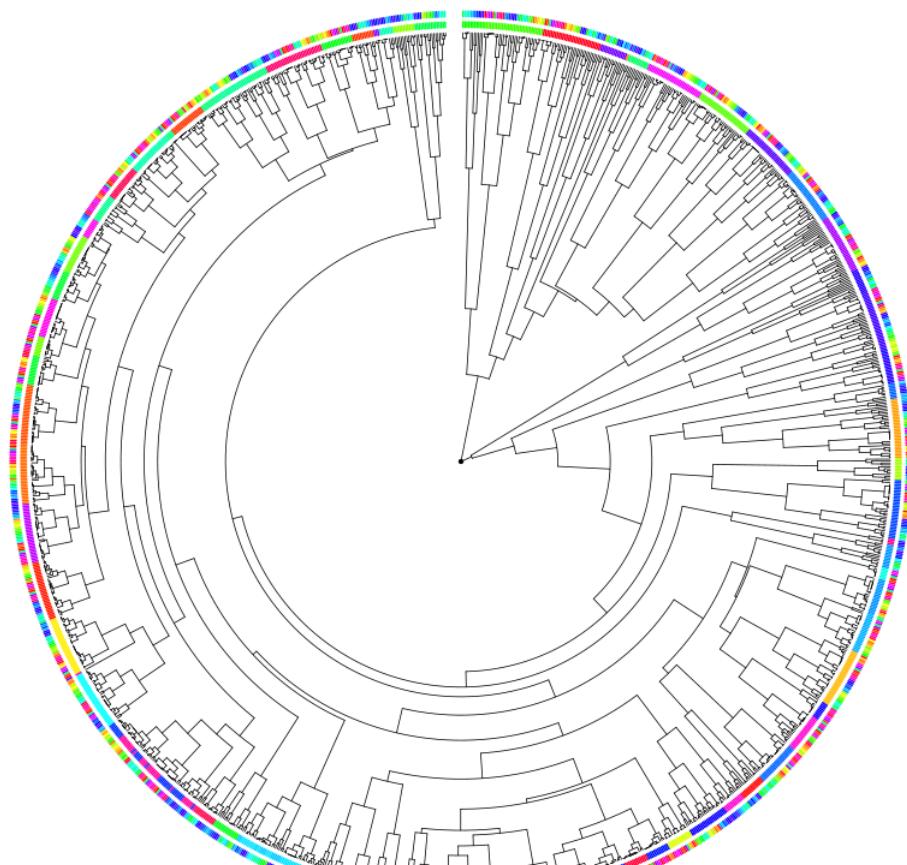
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4. Find the point that maximizes error reduction with regard to its successor.

Cluster Evaluation

(3) Relative Validity Measures: Elbow Criterion



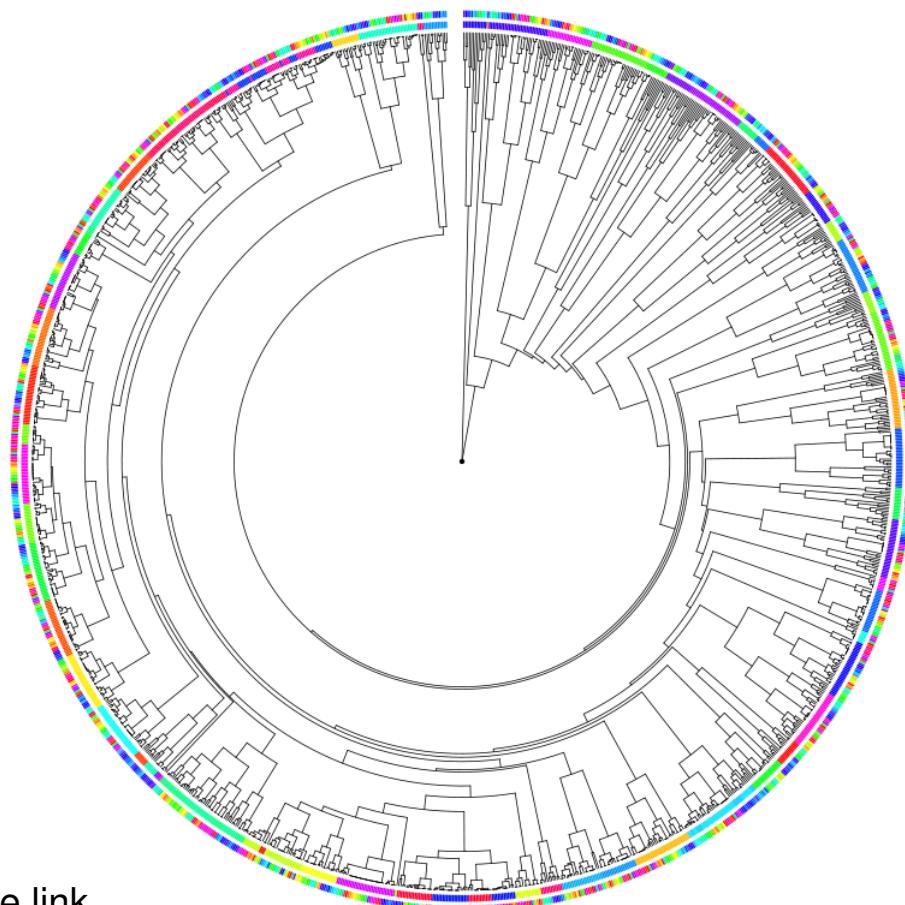
d_C : Hamming distance
Merging: complete link

<http://cs.jhu.edu/~razvanm/fs-expedition/2.6.x.html>

Relations between 1377 file systems for Linux Kernel 2.6.0. [Razvan Musaloiu 2009]

Cluster Evaluation

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d_C : Hamming distance

Merging: group average link

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Cluster Evaluation

Correlation between External and Internal Measures

In the wild, we are not given a reference classification.

- An external evaluation is not possible.

(though many papers report on such experiments)

- Resort to an internal evaluation.

(connectivity, squared error sums, distance-diameter heuristics, etc.)

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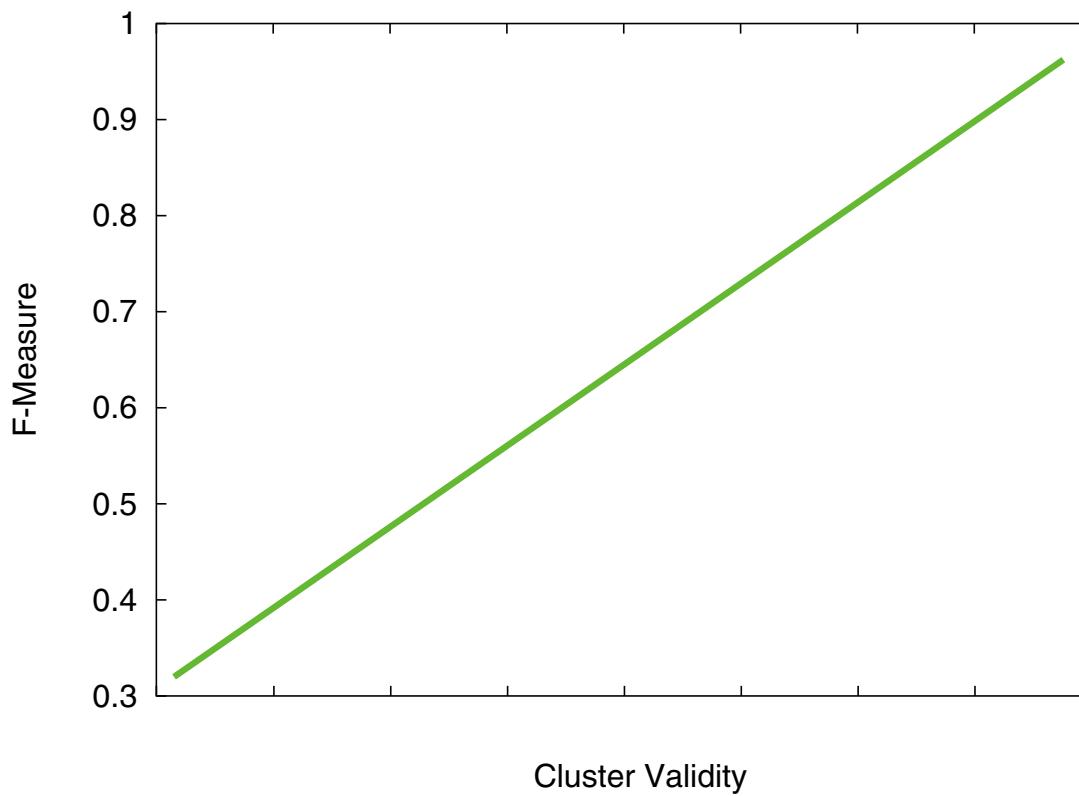
“To which extent can an internal evaluation ϕ be used to predict for a clustering its distance from the best reference classification—say, to predict the F-measure?”

$$\operatorname{argmax}_{\phi} \{\tau \langle X, Y \rangle \mid x = F(\mathcal{C}), y = \phi(\mathcal{C}), \mathcal{C} \in \mathcal{C}\}$$

[Stein/Meyer zu Eissen 2007]

Cluster Evaluation

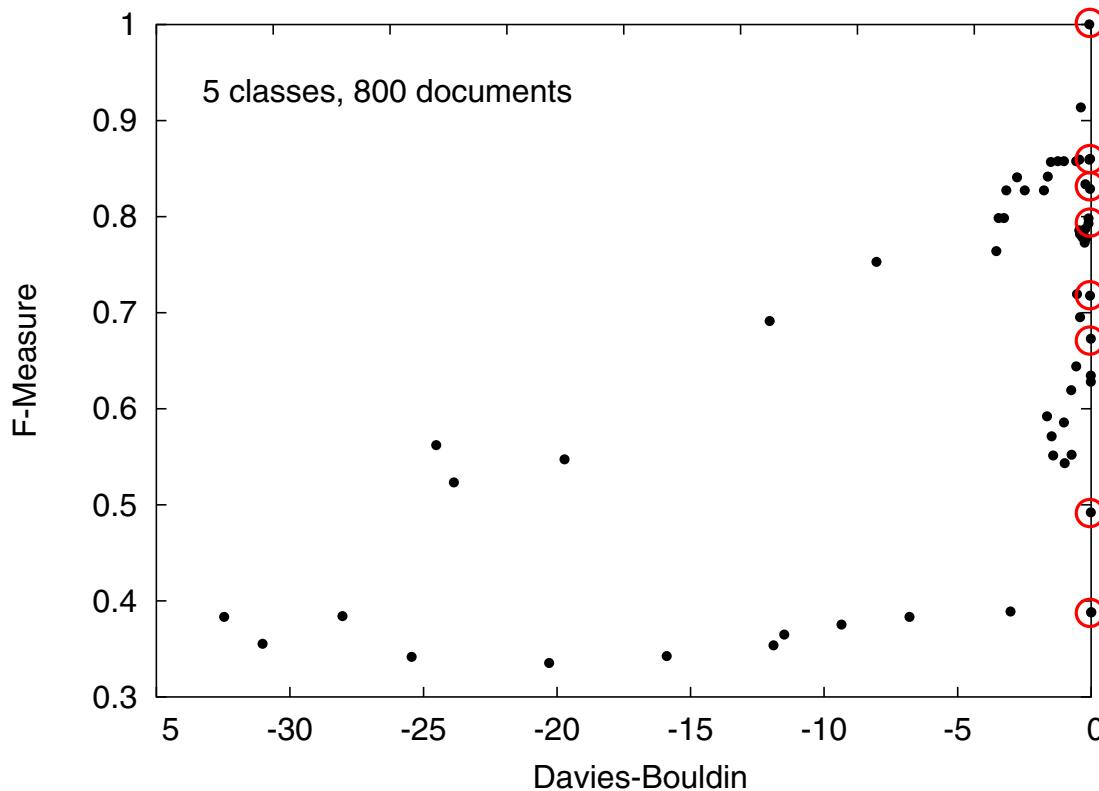
Correlation between External and Internal Measures



Perfect correlation (desired).

Cluster Evaluation

Correlation between External and Internal Measures

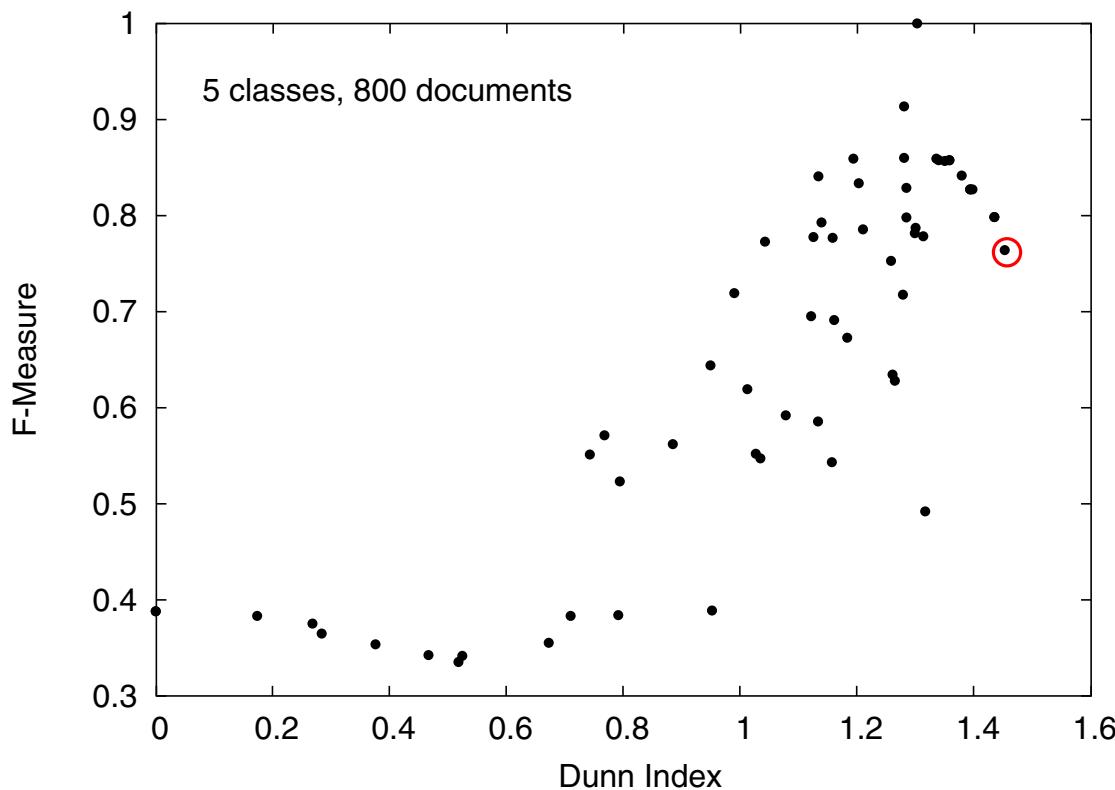


Davies-Bouldin:
$$\frac{1}{k} \cdot \sum_{i=1}^k \max_j \frac{s(C_i) + s(C_j)}{d_C(C_i, C_j)}$$

Prefers spherical clusters.

Cluster Evaluation

Correlation between External and Internal Measures



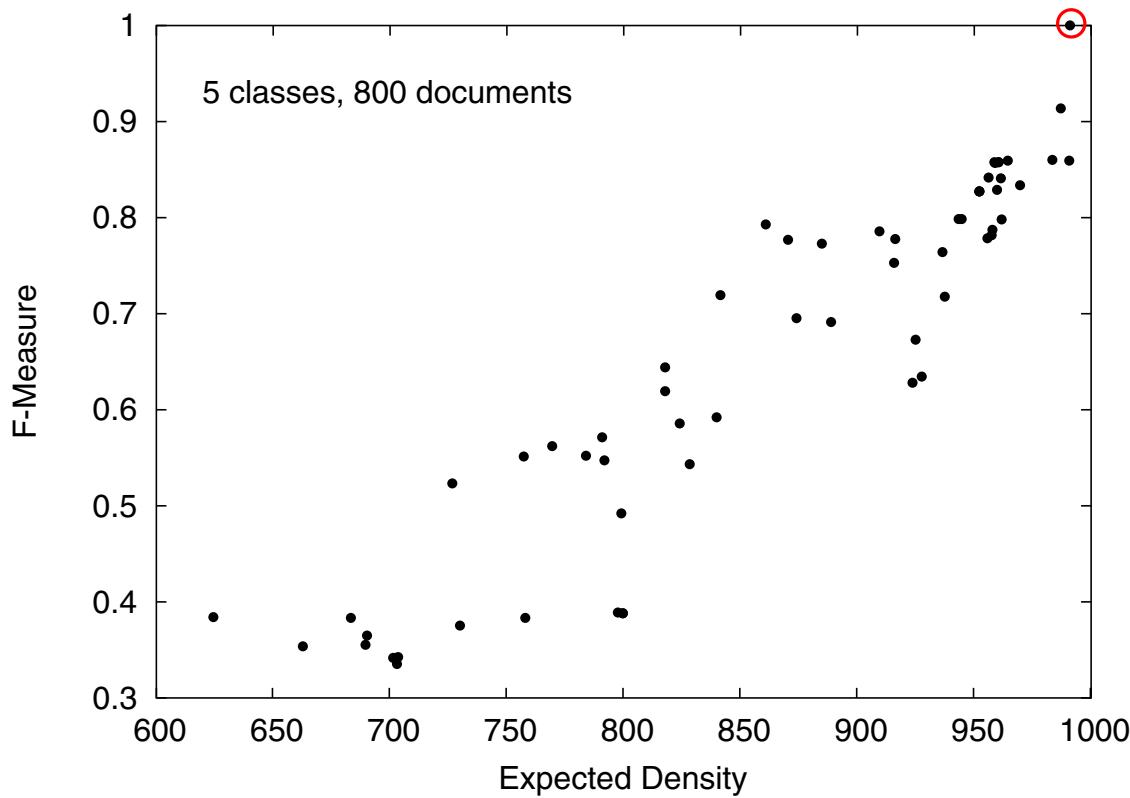
Dunn Index:

$$\frac{\min_{i \neq j} \{d_c(C_i, C_j)\}}{\max_{1 \leq l \leq k} \{\Delta(C_l)\}}$$

Maximizes dilatation = inter/intra-cluster-diameter.

Cluster Evaluation

Correlation between External and Internal Measures



Expected Density: $\bar{\rho} = \sum_{i=1}^k \frac{|V_i|}{|V|} \cdot \frac{w(G_i)}{|V_i|^\theta}$

Independent of cluster forms and sizes.