Hand Strength Evaluation in Bridge Games with the Genetic Algorithm

Jing-Ping Jan, Kuo-Si Huang and Chang-Biau Yang

***Abstract*—For the evaluation of hand strength in the contract bridge game, the common methods used by human players involve the combination of the high card point (HCP), extra strength in a long suit and a short suit method. However, the parameters (such as A=4, K=3, Q=2 and J=1) in the commonly used methods may not be optimal. The goal of this paper is to design a better hand strength evaluation method. To achieve this goal, we examine many previously used formulas, and generalize them to become more flexible by adding some parameters to these formulas. In the suit contract, the correlation coefficient is improved from 0.859 made by the Goren point count (A=4, K=3, Q=2, J=1, void=5, singleton=3 and doubleton=1) to 0.918 made by our formula.**

***Index Terms*—contract bridge, genetic algorithm, double dummy solver, high card point (HCP), correlation coefficient**

I. Introduction

In the computer programs for playing games, AlphaGo [12] is one of the most famous robots, and it defeated Sedol Lee, the world Go champion, by winning a 4:1 score in 2016. In addition, the team, DeepMind, also developed many agents to play the Atari games [6]. In the Go and Atari games, the full game information can be seen for all the players in the game, so they belong to the games of perfect information.

The *contract bridge* is a 52-card game with four players. There are two stages for playing the bridge: bidding and playing the cards. The contract bridge is a game of imperfect information because a player can see only his own hand in the bidding stage. And then, in the card playing stage, the cards of the dummy are revealed on the table. The prediction of win tricks or the double-dummy problem for contract bridge was solved successfully by using machine learning [3–5, 7–11].

The works proposed by Mańdziuk *et al.* are shown in Table I. The studies from 2004 to 2021 evaluate the hand strength with four hands and predict the win tricks by the neural network. They fed the complete information into the neural network and used it to predict the number of win tricks obtained by the NS side. Then, the predicted number of win tricks was compared with the answer provided by the *double dummy solver* (DDS).

To build a useful hand evaluation method, we designed several formulas for evaluating hand strength in this paper. We train the parameters of these formulas and then fine-tuned them manually to design a better formula that can be used in a real game of bridge. The performance of each trained formula is measured by the correlation coefficient between the hand strength and the win tricks provided by the DDS.

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We design four types of hand strength evaluation formulas for the suit contract as follows.

- **The strength of each individual card.** The point value of each individual card can be assigned because the cards are ranked. For example, the most famous *Goren point count* system assigns A = 4, K = 3, Q = 2, and J = 1.
- **The strength of a short suit**. It is usually set as void = 5, singleton = 3, and doubleton = 1.
- **The strength of a long suit**. For example, an extra one point is added for each card exceeding four cards in one suit.
- **The distribution of the suit.** With the combination of the short and long suits, a simple formula is used to represent the strength of a suit based on its length.

We apply the genetic algorithm (GA) to find the better parameter values of the formulas. The fitness is measured by the correlation coefficient between the hand strength and the number of win tricks. In the suit contract, the correlation coefficient is improved from 0.859 made by the Goren point count to 0.918 made by our formula.

The following is the organization of this paper. Section II introduces the background knowledge of this paper. Section III presents the training methods and the possible formulas we design. Section IV shows the experimental results of the training. Finally, Section V summarizes this paper.

II. Preliminaries

In the bidding stage of bridge game, the players should estimate their hand strength as precisely as possible. Three methods are commonly used in strength evaluation as follows.
(1) *Goren point count* [2], also known as *high card point* (HCP): evaluating the power of each individual card;
(2) long count: evaluating the power of the long distribution in one hand;
(3) short count: estimating the power of the short distribution in one hand.

Even if an exact formula is used to calculate the hand strength, it is still hard to estimate the number of win tricks precisely. Figure 1 shows an example of such situation, where the win tricks are obtained from the DDS.

In a double-dummy bridge game, the hands of four players are all revealed. The DDS usually uses the SCOUT algorithm

TABLE I: The prediction of the number of win tricks with neural networks.

Year	Author(s)	Input size	Note
2004[7]	Mossakowski and Mańdziuk	52	The card distribution is assumed to be unknown
2006[8]	Mossakowski and Mańdziuk	52	Figure out that different contracts strongly affect result
2007[9]	Mossakowski and Mańdziuk	52×4+84	Make the network know the card distribution of a team
2009[10]	Mossakowski and Mańdziuk	52×4+84	Compare with the bridge expert
2018[4]	Mańdziuk and Suchan	52×4	Extract the features by the auto encoder
2021[3]	Kowalik and Mańdziuk	156×4	Use the CNN to extract the features

N: 42 / AT74 / 853 / QJ95
W: 853 / K832 / 9 / 87642
E: JT7 / J / AKJ7642 / AT
S: AKQ96 / Q965 / QT / K3

	♠	♡	♢	♣	NT
N	10	9	5	8	5
E	3	3	8	5	4
S	10	9	5	8	5
W	3	3	8	5	4

(a)

N: 42 / AT74 / 853 / QJ95
W: T8753 / J3 / 72 / AT64
E: J / K82 / AKJ964 / 872
S: AKQ96 / Q965 / QT / K3

	♠	♡	♢	♣	NT
N	7	8	5	6	6
E	6	4	8	6	7
S	7	8	5	6	5
W	6	4	8	6	7

(b)

N: AK / AT74 / 853 / QJ95
W: 853 / K832 / 9 / 87642
E: JT7 / J / AKJ7642 / AT
S: Q9642 / Q965 / QT / K3

	♠	♡	♢	♣	NT
N	9	9	5	9	5
E	3	4	8	8	8
S	9	9	5	9	5
W	3	4	8	8	8

(c)

Fig. 1: Three examples for different DDS results with the same strength on the NS side. (a) A board and the DDS results. (b) The different DDS results when some cards are exchanged between E and W from (a). (c) The different DDS results when some cards are exchanged between N and S from (a).

[11], which is the enhancement of the α-β pruning algorithm. The DDS can help the bridge players to analyze the game and to figure out a better way to play the card.

Mossakowski and Mańdziuk studied the strength evaluation method, and they assigned various points to each individual card, as shown in Table II [10].

To evaluate the hand strength more precisely, in this paper, we define the following problem and try to solve it using the genetic algorithm (GA).

Definition 1. (hand strength evaluation in bridge game) *Given a hand of 13 cards, the* hand strength evaluation problem *is to estimate the strength of the hand, by giving the weights to HCP, long count and short count. The evaluation method is measured by the correlation coefficient between the hand strength and the number of win tricks, obtained by DDS.*

TABLE II: The strength evaluation methods for each individual card in the bridge game [10].

Evaluation method	A	K	Q	J	10
Goren	4	3	2	1	0
Bamberger	7	5	3	1	0
Collet	4	3	2	0.5	0.5
Four Aces	3	2	1	0.5	0
Polish	7	4	3	0	0
Reith	6	4	3	2	1
Robertson	7	5	3	2	1
Vernes	4	3.1	1.9	0.9	0
AKQ	4	3	2	0	0

III. Our Hand Strength Evaluation Method

A basic formula for simply calculating the strength of one hand in a suit contract is given as follows.

$$\text{HCP} + \text{extra for long suit} + \text{extra for short suit} \qquad (1)$$

We find that the value of an honor card in a suit contract and a no-trump contract is different. Furthermore, in a suit contract, the value of an honor card in the trump suit and non-trump suit is also different. Thus, we design several possible formulas and try to find the best parameters with GA. We involve the following steps in our method for a suit contract.

Step 1: (Design of formulas) Design the various formulas of hand strength evaluation methods.

Step 2: (Dataset generation) Build datasets with different sizes for different training stages.

Step 3: (Preliminary formula combinations). Find 25 better formula combinations with small datasets.

Step 4: (Better formula combinations) Train the 25 better formula combinations with middle datasets.

Step 5: (Manual adjustment) Make the parameters of the formula easier to be memorized so that contract bridge players can use it during the actual game.

A. The Formulas for Suit Contracts

We first provide a simple formula as an example for evaluating the hand strength, as follows.

$$H + TL + NL \qquad (2)$$

TABLE III: The formulas for individual cards, where p denotes the number of parameters.

Formula	p	Description
H	5	$\sum_{suit\in hand}\sum_{card\in suit} M^{H}[card]$
HT	11	$\sum_{suit\in hand}\sum_{card\in suit}\begin{cases} M_t^{HT}[card] & \text{, if } suit = t. \\ M_{\bar{t}}^{HT}[card] & \text{, if } suit \neq t \end{cases}$
sH	4	$\sum_{suit\in hand}\begin{cases} a_t \times (\sum_{card\in suit} M^{sH}[card])^{b_t} & \text{, if } suit = t \\ a_{\bar{t}} \times (\sum_{card\in suit} M^{sH}[card])^{b_{\bar{t}}} & \text{, if } suit \neq t \end{cases}$

TABLE IV: The formulas for long suits.

Formula	p	Description
L	4	$\sum_{suit\in hand}\begin{cases} a_t \times (L_{suit}-4)^{b_t} & \text{, if } suit = t \text{ and } L_{suit} > 4. \\ a_{\bar{t}} \times (L_{suit}-4)^{b_{\bar{t}}} & \text{, if } suit \neq t \text{ and } L_{suit} > 4. \\ 0 & \text{, otherwise.} \end{cases}$
L_4	6	$\sum_{suit\in hand}\begin{cases} a_t \times (L_{suit}-4)^{b_t} & \text{, if } suit = t \text{ and } L_{suit} > 4 \\ c_t & \text{, if } suit = t \text{ and } L_{suit} = 4; \\ a_{\bar{t}} \times (L_{suit}-4)^{b_{\bar{t}}} & \text{, if } suit \neq t \text{ and } L_{suit} > 4 \\ c_{\bar{t}} & \text{, if } suit \neq t \text{ and } L_{suit} = 4; \\ 0 & \text{, otherwise.} \end{cases}$
L^*	6	$\sum_{suit\in hand}\begin{cases} a_t \times (L_{suit}-b_t)^{c_t} & \text{, if } suit = t \text{ and } L_{suit} > b_t. \\ a_{\bar{t}} \times (L_{suit}-b_{\bar{t}})^{c_{\bar{t}}} & \text{, if } suit \neq t \text{ and } L_{suit} > b_{\bar{t}}. \\ 0 & \text{, otherwise.} \end{cases}$
TL	2	$a \times (L_t - b)$

In Equation 2, H denotes the sum of the individual card strength, TL denotes the strength of the extra length in the trump suit, and NL denotes the strength of a short non-trump suit.

Each of H, TL and NL is a primitive formula. H can be generalized as shown in Table III, involving 20 parameters. Table IV presents the generalization of TL, with 18 parameters in total. And, Table V shows the generalization of NL, with a total of 19 parameters.

In addition, we design formulas for mixing the long and short suit methods. We believe there is a base length, and extra strength can be gained whenever the length is less than or greater than the base length. The designed formulas for calculating the strength according to the length in each suit are shown in Table VI.

TABLE V: The formulas for short suits.

Formula	p	Description
S	4	$\sum_{suit\in hand}\begin{cases} 0 & \text{, if } L_{suit} \geq 3. \\ a_t \times (3-L_{suit})^{b_t} & \text{, if } suit = t \text{ and } L_{suit} < 3. \\ a_{\bar{t}} \times (3-L_{suit})^{b_{\bar{t}}} & \text{, if } suit \neq t \text{ and } L_{suit} < 3. \end{cases}$
DS	6	$\sum_{suit\in hand}\begin{cases} 0 & \text{, if } L_{suit} \geq 3. \\ M_t^{DS}[L_{suit}] & \text{, if } suit = t \text{ and } L_{suit} < 3. \\ M_{\bar{t}}^{DS}[L_{suit}] & \text{, if } suit \neq t \text{ and } L_{suit} < 3. \end{cases}$
S^*	6	$\sum_{suit\in hand}\begin{cases} a_t \times (b_t - L_{suit})^{c_t} & \text{, if } suit = t \text{ and } L_{suit} < b_t. \\ a_{\bar{t}} \times (b_{\bar{t}} - L_{suit})^{c_{\bar{t}}} & \text{, if } suit \neq t \text{ and } L_{suit} < b_{\bar{t}}. \\ 0 & \text{, otherwise.} \end{cases}$
NL	3	$\sum_{suit\in hand, suit\neq t} M^{NL}[L_{suit}].$

TABLE VI: The length formulas.

Formula	p	Description
LS	28	$\sum_{suit\in hand}\begin{cases} M_t^{LS}[L_{suit}] & \text{, if } suit = t \\ M_{\bar{t}}^{LS}[L_{suit}] & \text{, if } suit \neq t. \end{cases}$
D	6	$\sum_{suit\in hand}\begin{cases} a_t \times (\lvert L_{suit} - b_t \rvert)^{c_t} & \text{, if } suit = t. \\ a_{\bar{t}} \times (\lvert L_{suit} - b_{\bar{t}} \rvert)^{c_{\bar{t}}} & \text{, if } suit \neq t. \end{cases}$

TABLE VII: The formulas for fine-tuning the hand strength if the east makes a bid.

Formula	p	Description
cH	12	$\sum_{card\in suit}\begin{cases} M_{\text{LHO}}^{cH}[card] & \text{, if LHO bids the suit.} \\ M_{\text{RHO}}^{cH}[card] & \text{, if RHO bids the suit.} \end{cases}$
cS	3	$M^{cS}[L_{suit}]$, if $L_{suit} < 3$ and the east bids the suit
cD	3	$a \times (\lvert L_{suit} - b \rvert)^c$, if the east bids the suit
cL	2	$a \times (L_{suit} - b)$, if $L_{suit} > b$ and the east bids the suit

TABLE VIII: The formulas for no-trump contracts.

Formula	p	Description
H	5	$\sum_{suit\in hand}\sum_{card\in suit} M^{H}[card]$
S_{wh}	2	$\sum_{suit\in hand}\begin{cases} M^{S_{wh}}[L_{suit}] & \text{, if } 2 \geq L_{suit} \geq 1 \text{, with honors} \\ 0 & \text{, otherwise.} \end{cases}$
L_{wh}	2	$\sum_{suit\in hand}\begin{cases} a \times (L_{suit} - b) & \text{, if } L_{suit} \geq b \text{, with honors} \\ 0 & \text{, otherwise.} \end{cases}$

To build a final formula for estimating the hand strength, we have to consider all possible combinations of different primitive formulas. Equation 2 is a simple example for the combinations. The number of all possible combinations is $5 \times 5 \times 5 \times 3 - 1 = 374$. For example, the first number, five, means H, HT, $H + sH$, $HT + sH$, and no high card point method. Therefore, we have to train the parameter values of each combined formula.

Furthermore, we design four formulas for fine-tuning the hand strength if the east makes a bid, as shown in Table VII. When the east makes a bid, he should have more honors in the bidding suit.

B. The Formulas for No-Trump Contracts

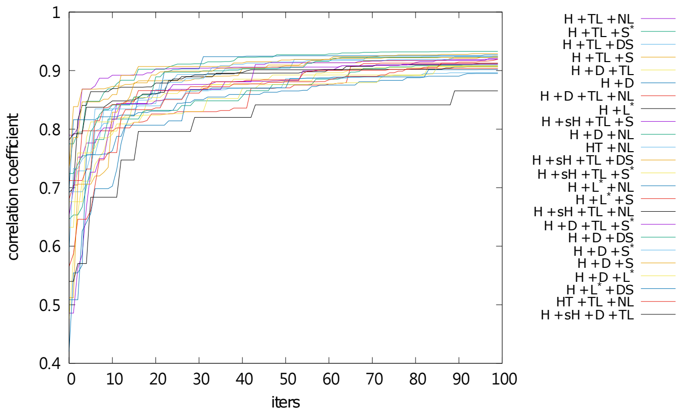
Generally, the hand distribution should be balanced in a no-trump contract. Since there is no trump suit, we cannot use the formulas TL and NL. Therefore, we cannot fine-tune the hand strength by the length of the trump suit or the non-trump suit. Table VIII shows the formulas for the no-trump contract.

IV. EXPERIMENTAL RESULTS

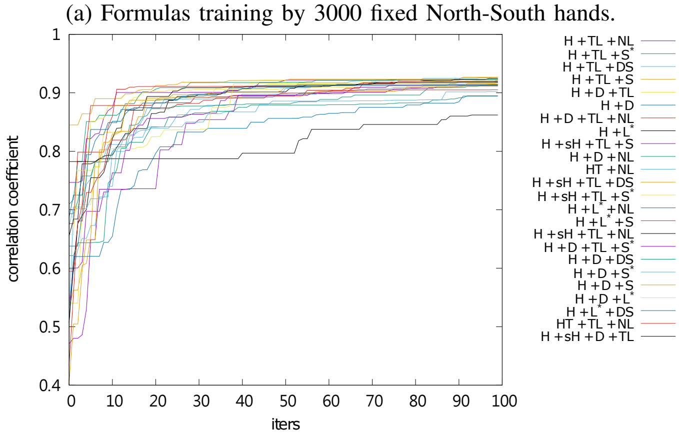
The dataset was extracted from the Vugraph project of BBO [1] that collected some famous contract bridge competitions for decades. We only use the NS hands of the Vugraph project. To obtain more suitable fitness measurement, for each fixed NS hand (26 cards), we randomly generate 100 opponent's hands. Then the number of the NS win tricks (got from DDS) is the average of these 100 boards with the same NS hand. Two kinds of data are generated, including we do not care whether opponents bid or not, and that the opponents always bid.

Among 374 possible formulas combined from the primitive formulas, we choose 25 formulas with the best correlation coefficients to fine-tune all possible parameters, including manual adjustment. Table IX shows the correlation coefficients and the accuracies of win tricks for the 25 best formulas.

To predict the number of win tricks, we have to transform the hand strength into the number of win tricks. First, we divide the dataset into seven groups, where each group corresponds to a specific number of win tricks, from 7 tricks to 13



(a) Formulas training by 3000 fixed North-South hands.



(b) Formulas training by 5000 fixed North-South hands.

Fig. 2: The convergence curves of correlation coefficients in the training, where the correlation coefficient is calculated by the fixed North-South hand and the mean of the win tricks.

tricks. Let h_i denote the *average hand strength* of the group for winning i tricks. Then, the threshold θ_i of two adjacent groups i and $i+1$ is calculated by Equation 3.

$$\theta_i = \frac{h_i + h_{i+1}}{2}. \tag{3}$$

The hand strength between θ_{i-1} and θ_i is set to win i tricks.

Dis in Table IX is the distance between our predicted tricks and the average of rounded win tricks. When $Dis = 0$, it is an exact match of our prediction and the rounded win tricks. In our prediction, the exact trick accuracies of most formulas reach about 58%. Figure 2 shows the convergence curves of the formulas during the training. Most formulas converge to a correlation coefficient of 0.9 in 100 iterations.

Our goal is to make a hand evaluation method that human bridge players can use in actual bridge games. In addition to maximizing accuracy, we aim to minimize the number of parameters and computational complexity. $H + TL + NL$ in Table IX has the least parameters, and it has no exponent calculation.Therefore, we fine-tune the parameters of $H+TL+NL$ and further reduce the resolution to make them easier to memorize. The correlation coefficients and accuracies of $H+TL+NL$ are shown in Table X. In the table, the first row is obtained from GA, while the last four rows are obtained by reducing the resolution through manual adjustment. When

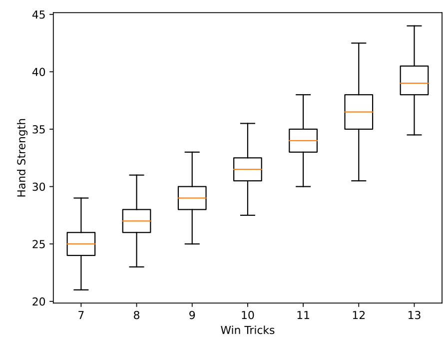


Fig. 3: The distribution of the hand strength obtained by $H + TL + NL$ with respect to the average win tricks. The parameters of $H + TL + NL$ are shown in the row of correlation coefficient 0.918 in Table X.

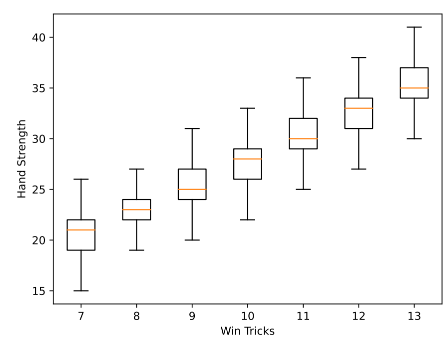


Fig. 4: The distribution of the hand strength obtained by Goren point count (A=4, K=3, Q=2, J=1) combined with the short suit method (void=5, singleton=3, doubleton=1) with respect to the average win tricks.

the resolution is 0.5, the correlation coefficient is 0.918. We recommend these parameters for $H + TL + NL$, because human players can really apply this formula in an actual bridge game.

Figure 3 shows the distribution of the hand strength obtained by $H + TL + NL$ with respect to the average win tricks. Figure 4 shows the distribution of the hand strength obtained by Goren point count (A=4, K=3, Q=2, J=1) combined with the short suit method (void=5, singleton=3, doubleton=1).

Table XI compares the correlation coefficients of our best formula $H + TL + NL$ and the previous hand strength evaluation methods. As one can see, $H + TL + NL$ has the maximal correlation coefficient.

The dataset of the no-trump contract is different from the previous one. We only play the no-trump contract when the situation is suitable for some conditions. The training result shows that the high card point has the most influence, and the fine-tuned strength of each card in the long suit formula is approximately zero for each card. Table XII shows the correlation coefficient and the accuracy.

TABLE IX: The correlation coefficients and the accuracies of win tricks for the 25 best formulas.

Formula combination	Number of parameters	Correlation coefficient	Accuracy		
			$Dis = 0$	$Dis \leq 1$	$Dis \leq 2$
H + TL + NL	10	0.923	0.585	0.975	0.999
H + TL + S^{*}	13	0.922	0.584	0.974	0.999
H + TL + DS	13	0.922	0.584	0.974	0.999
H + TL + S	11	0.922	0.585	0.974	0.999
H + D + TL	13	0.922	0.579	0.975	0.999
H + D	11	0.919	0.576	0.973	0.999
H + D + TL + NL	16	0.923	0.586	0.975	0.999
H + L^{*}	11	0.921	0.579	0.973	0.999
H + sH + TL + S	15	0.923	0.587	0.975	0.999
H + D + NL	14	0.922	0.582	0.974	0.999
HT + NL	14	0.924	0.589	0.975	0.999
H + sH + TL + DS	17	0.922	0.585	0.974	0.999
H + sH + TL + S^{*}	17	0.923	0.589	0.975	0.999
H + L^{*} + NL	14	0.921	0.585	0.973	0.999
H + L^{*} + S	15	0.923	0.586	0.975	0.999
H + sH + TL + NL	14	0.923	0.586	0.975	0.999
H + D + TL + S^{*}	19	0.921	0.585	0.973	0.999
H + D + DS	17	0.923	0.586	0.975	0.999
H + D + S^{*}	17	0.923	0.588	0.975	0.999
H + D + S	15	0.922	0.582	0.974	0.999
H + D + L^{*}	17	0.918	0.573	0.972	0.999
H + L^{*} + DS	17	0.923	0.584	0.975	0.999
H + L^{*} + S^{*}	17	0.924	0.587	0.976	0.999
HT + TL + NL	16	0.925	0.592	0.976	0.999

TABLE X: The correlation coefficients and accuracies of the formula $H + TL + NL$ with various settings (including manual adjustment) in suit contracts.

$2, 3, \cdots, 8$	9, 10	J	Q	K	A	TL		NL			Correlation coefficient	Accuracy		
						a	b	void	singleton	doubleton		$Dis = 0$	$Dis \leq 1$	$Dis \leq 2$
0	0.215	0.619	1.224	2.473	4	1.402	1.005	3.390	1.764	0.510	0.923	0.586	0.975	0.999
0	0.2	0.6	1.2	2.5	4	1.4	1.0	3.4	1.8	0.5	0.923	0.584	0.975	0.999
0	0.25	0.5	1.25	2.5	4	1.5	1	3.5	1.75	0.5	0.922	0.581	0.974	0.999
0	0	0.5	1.0	2.5	4	1.5	1	3.5	2	0.5	0.918	0.567	0.972	0.999
0	0	1	1	2	4	1	1	3	2	1	0.901	0.528	0.957	0.999

TABLE XI: The correlation coefficients of our best formula $H + TL + NL$ and the previous hand strength evaluating methods in suit contracts.

	HCP	Long	Short	Pavlicek	Loser	$H + TL + NL$	$H + TL + NL$ resolution = 0.5
Goren	0.746	0.806	0.859	0.804	0.700	0.923	0.918
NT neural	0.767	0.829	0.876	0.821			
Suit neural	0.765	0.810	0.827	0.826			
Spade neural	0.862	0.822	0.810	0.891			
Bamberger	0.758	0.804	0.852	0.794			
Collet	0.749	0.810	0.862	0.807			
Four aces	0.765	0.825	0.863	0.841			
Polish	0.750	0.797	0.845	0.787			
Reith	0.742	0.793	0.843	0.783			
Roberson	0.754	0.800	0.848	0.789			
Vernes	0.748	0.809	0.861	0.806			
AKQ	0.739	0.798	0.850	0.795			

TABLE XII: The performance of the formula H with various settings (including manual) in no-trump contracts.

$2, 3, \cdots, 8$	9, 10	J	Q	K	A	L_{wh}		Correlation	Accuracy		
						a	b	coefficient	$Dis = 0$	$Dis \leq 1$	$Dis \leq 2$
0	0.298	0.848	1.595	2.676	4	0.240	0.091	0.923	0.575	0.971	0.999
0	0.3	0.8	1.6	2.7	4	0.2	0.1	0.923	0.574	0.971	0.999
0	0.25	0.75	1.5	2.75	4	0.25	0	0.922	0.569	0.971	0.999
0	0.5	1	1.5	2.5	4	0	0	0.916	0.557	0.965	0.998
0	0	1	2	3	4	0	0	0.905	0.535	0.954	0.997

V. Conclusion

In this paper, we use the genetic algorithm to train parameters in many combined formulas in order to find a better hand evaluation method in the contract bridge. Through the experiments by training all possible combined formulas, our final hand evaluation method is $H+TL+NL$ with a resolution of 0.5 for suit contracts.

According to the experimental results, some formulas can get higher correlation coefficients. However, such formulas may not be so practical because the number of parameters in a formula is too many to memorize, or the computation complexity is too high for humans to calculate during the actual play.

Human experts of contract bridge usually fine-tune the hand strength by the bidding auctions of opponents. Therefore, there may exist some ways to fine-tune the hand strength beyond our formulas. The genetic algorithm may still not overcome this situation, or the dataset we use may not be suitable for this situation.

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