Course 02249 Computationally Hard Problems
Fall 2013, DTU Compute

Solution to assignment PROJECT

Students' names: Andreas Hallberg Kjeldsen and Morten Chabert Eskesen

Study nr. s092638 and s133304

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| Not quite adequate | Adequate | Good |
|---|---|---|
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**Problem:** [MirrorFriendlyMinimumSpanningTree (MFMST)]
**Input:** An undirected, connected weighted graph $G = (V, E, w)$, where $V = \{1, \ldots, n\}$, $E = \{e_1, \ldots, e_m\}$ and $w : E \to \mathbb{N}_0$, and a number $B \in \mathbb{N}$.
**Output:** YES if there is a spanning tree $T \subseteq E$ for $G$ such that

$$max \left\{ \sum_{e_i \in T} w(e_i), \sum_{e_i \in T} w(e_{m+1-i}) \right\} \leq B$$

and NO otherwise.

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## a) Description of the problem in colloquial terms

A minimum spanning tree is a subgraph within a undirected, connected weighted graph that is a tree and connects all the vertices together with a weight less or equal to the weight of every other spanning tree. The main difference between a minimum spanning tree and a mirror friendly minimum spanning tree is the inequality described above. In a mirror friendly minimum spanning tree the inequality must be satisfied. It should be possible to mirror the spanning tree in such a way that the maximum of the spanning tree and the mirrored spanning tree is less than or equal to a fixed value, $B$. This also means that the mirror friendly minimum spanning tree may not be equal to the minimum spanning tree in the graph, i.e. it may have a larger weight than the minimum spanning tree.

### Solve an example problem

**Input:** $V = \{1, 2, 3\}$, $E = \{e_1 = \{1, 2\}, e_2 = \{2, 3\}, e_3 = \{1, 3\}\}$, $w(e_i) = i$ for $i \in \{1, 2, 3\}$ and $B = 4$.

| Spanning Tree | Mirrored Spanning Tree |
|---|---|
| $e_1 + e_2 = 3$ | $e_{3+1-1} + e_{3+1-2} = 5$ |
| $e_3 + e_1 = 4$ | $e_{3+1-3} + e_{3+1-1} = 4$ |

$$max \{e_3 + e_1, e_{3+1-3} + e_{3+1-1}\} \leq 4$$

$$max \{4, 4\} \leq 4$$

Output would be a spanning tree consisting of the edges: $e_3$ and $e_1$.

## b) Show that MFMST is in $NP$

**1. Design a deterministic algorithm $A$ which takes as input a problem instance $X$ and random sequence $R$**

**1a. Specify what the random sequence $R$ consists of**

**1b. Specify how $A$ interprets $R$ as a guess**

**1c. Specify how $A$ verifies the guess**

**2. Show that the two conditions are met**

**If the answer to $X$ is YES, then there is a string $R^*$ with positive probability such that $A(X, R^*) = YES$**

**If the answer to $X$ is NO, then $A(X, R) = NO$ for all $R$**

**3. Show that $A$ is $p$-bounded for some polynomial $p$**

## c) Show that MFMST is $NP$-complete

**Suitable problem $P_c$ known to be $NP$-complete**

Prove $P_c \leq_p$ MirrorFriendlyMinimumSpanningTree.
**Outline of the transformation**

**Answer to $X$ is YES then answer to $T(X)$ is YES**

**Answer to $T(X)$ is YES then answer to $X$ is YES**

**d) Find an algorithm which solves the optimizing version of the problem**

**e) Prove the worst-case running time of the algorithm**

**f) Implement the algorithm developed in d)**