In [109]:

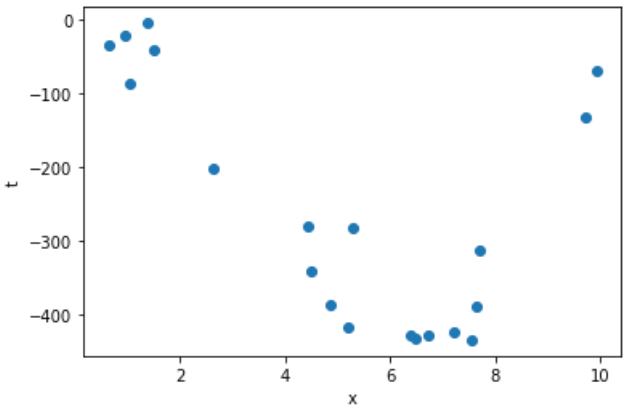
```
import numpy as np
import scipy.io as sp
import matplotlib.pyplot as plt
import pandas as pd
f = sp.loadmat('P3.mat')
x = np.array(f['x'])
t = np.array(f['t'])
M = 10
x = x.flatten()
t = t.flatten()
ds = pd.DataFrame({
    'x' : x,
    't' : t,
})
ds = ds.sort_values(by = ['x'])
x = ds['x']
t = ds['t']
```

In [110]:

```
plt.close('all')
plt.figure(1)
plt.scatter(x,t)
plt.xlabel('x')
plt.ylabel('t')
```

Out[110]:

```
Text(0, 0.5, 't')
```

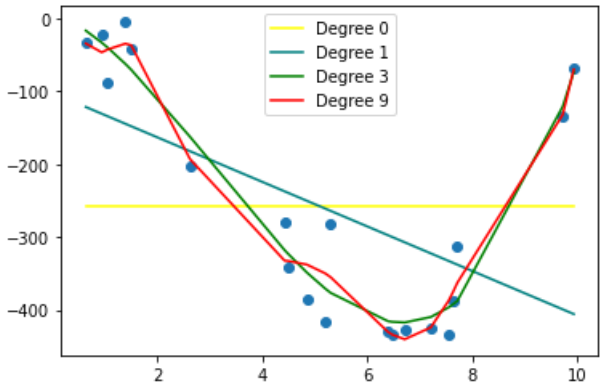


In [111]:

```
pn = [[] for _ in range(M)]
for i in range(M):
    pn[i] = np.polyfit(x,t,i)
```

In [193]:

```
y = [np.polyval(pn[i],x) for i in range(M)]
plt.figure(1)
plt.scatter(x,t)
plt.plot(x,y[0],'yellow')
plt.plot(x,y[1],'teal')
plt.plot(x,y[3],'green')
plt.plot(x,y[9],'red')
plt.legend(['Degree 0','Degree 1','Degree 3','Degree 9'],loc = 'upper center')
plt.show()
```

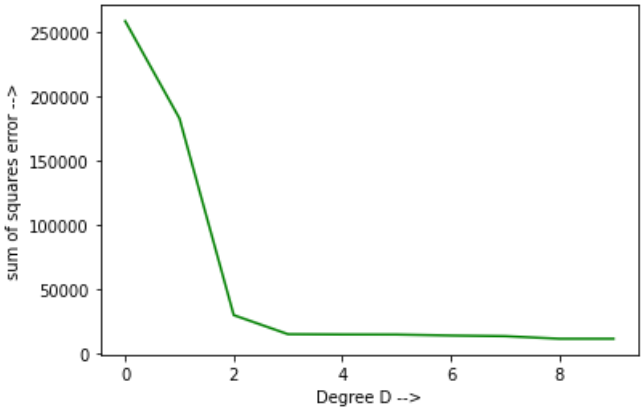


In [113]:

```
def sum_of_squares(x,t,pn):   # takes x,t,polynomial
    y = np.polyval(pn,x)
    sum = np.sum((y-t)**2)/2
    return sum
```

In [119]:

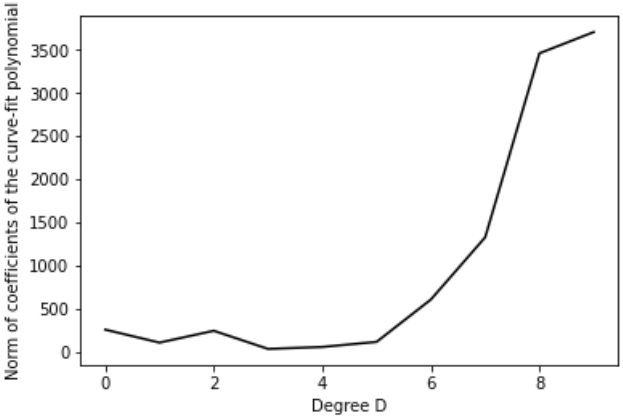
```
errs = [sum_of_squares(x,t,pn[i]) for i in range(M)]
plt.plot(range(M),errs,'g')
plt.xlabel('Degree D -->')
plt.ylabel('sum of squares error -->')
plt.show()
```



From the above figure the best choice seems to be D = 3.

In [118]:

```
norms = [np.linalg.norm(pn[i]) for i in range(M)]
plt.plot(range(M),norms,'black')
plt.xlabel('Degree D')
plt.ylabel('Norm of coefficients of the curve-fit polynomial')
plt.show()
```



From above, we can see over-fitting qunatised as the norm of the coefficient vector. It increases with degree D.

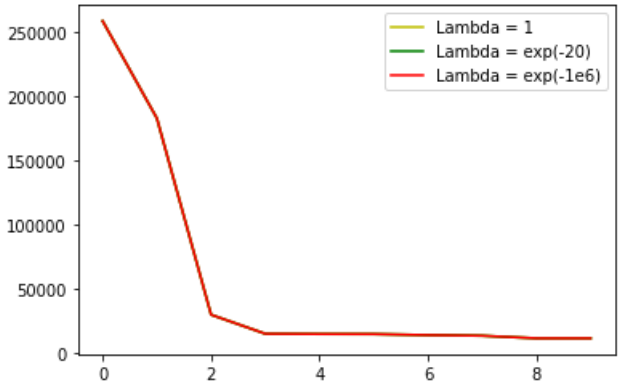
In [144]:

```
def regularised_error(x,t,pn,l):   # takes x,t,polynomial
    y = np.polyval(pn,x)
```

```
    sum = np.sum((y-t)**2)/2
    sum = sum + l*np.linalg.norm(pn)/2
    return sum
```

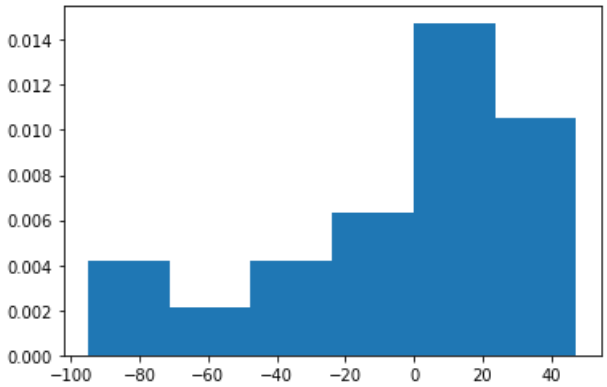
In [195]:

```
l = np.exp(-18)
rerrs = [[regularised_error(x,t,pn[i],l) for i in range(M)] for d in [1,np.exp(-20),np.exp(-1000000)]]
plt.plot(range(M),rerrs[0],'y')
plt.plot(range(M),rerrs[1],'g')
plt.plot(range(M),rerrs[2],'r')
plt.legend(['Lambda = 1','Lambda = exp(-20)','Lambda = exp(-1e6)'],loc = 'upper right')
plt.show()
```



In [188]:

```
hist = np.histogram(y[3]-t,bins = np.arange(-100,100,20))
plt.hist(y[3]-t,bins = 'auto', density=True)
plt.show()
```



The noise distribution is maximum around 0-20 and is highly present in 20-40 as well, but decreases in density from 0 to -100. There is no error after 40 or before -100.

In [ ]: