Divisibility

1) Which one of the following numbers is divisible by 99?
a) 3572404 b) 135792 c) 913464 d) 114345
ANS:114345
2) If n is an integer, what is the remainder when (2n + 2)2 is divided by4?
ANS:0
3) Find two nearest numbers to 19506 which are divisible by 9?
ANS:19503 and 9512
4) What is the value of M and N respectively if M39048458N is
divisible by 8 and 11, where M and N are single digit integers?
ANS:
5) How many pairs of X and Y are possible in the number
763X4Y2, if the number is divisible by 9?
ANS:1 pair x=0 y5 sum=22+x+y=22+0+5=27 which is divisible by 9 so only 1 pair (0,5)
6) When the integer n is divided by 8, the remainder is 3. What is the remainder if 6n is divided by 8?
ANS:n ÷ 8 = q with a remainder of 3

Now, we need to find the remainder when 6n is divided by 8.

6n ÷ 8 = 6(q * 8 + 3) ÷ 8

6n ÷ 8 = 6q + 18 ÷ 8

Since the remainder when 18 is divided by 8 is 2 (18 ÷ 8 = 2 remainder 2), we have:

6n ÷ 8 = 6q + 2

Now, we can see that the remainder when 6n is divided by 8 is 2.

Therefore, the remainder if 6n is divided by 8 is 2.

7) If the product 4864 x 9P2 is divisible by 12, then what is the
value of P?
ANS:P=1 since 12 is divisble if no is divisible by 3 and 4 to make 9P2 divisible by 4 and 3 complete digit shoul be divisble hence possible value is P=1
8) If the number 7X86038 is exactly divisible by 11, then the
smallest whole number in place of X?
ANS alternate digit sum must difference must be divisible by 11 so 21-(x+9) must be divisible by 11 by placing value 0,1,2,3,4,5,6,7,8,9 in X smallerst possible value is 1 (21-(1+9))=11/11=0
9) If an integer n is divisible by 3, 5 and 12, what is the next
larger integer divisible by all these numbers?
a) n^2 b) n + 180
c) 2n d) n + 60

ANS:**The LCM of 3, 5, and 12 can be found by breaking down each number into its prime factors:**

3 = 3 5 = 5 12 = 2^2 * 3

Now, to find the LCM, we take the highest power of each prime factor:

LCM = 2^2 * 3 * 5 = 60

So, the next larger integer divisible by 3, 5, and 12 is 60.

Now, let's check the options given:

a) n^2: This is not necessarily divisible by 3, 5, and 12 for all integers n. b) n + 180: This may not be divisible by 3, 5, and 12 for all integers n. c) 2n: This may not be divisible by 3, 5, and 12 for all integers n. d) n + 60: This is divisible by 3, 5, and 12 for all integers n (since 60 is the LCM).

Therefore, the correct option is d) n + 60.

10) What is the product of the largest and the smallest possible values of M for which a number 5M83M4M1 is divisible by 9?
ANS: smallest possible value for M =2 and largest value for M=8

UNIT DIGITS CYCLICITY

1) What is the unit digit in the product (365 x 659 x 771)?
ANS: 5
2) Find unit digit of product (173)45*(152)77*(777)999
ANS:->8
3) What is the unit's digit of the number 6256–4256?
ANS:0
4) Find the unit's digit in 264102+264103
ANS:5
5) What is the unit digit of (316)3n + 1?
ANS:since it is a 6 at every unit place so +1 =6+1=7
6) What is the unit digit in (795- 358)?
ANS:7
7) What is the rightmost non-zero digit of the number 302720
ANS:2
8) What will be the last digit of the number obtained by multiplying the numbers 81*82*83*84*86*87*88*89?
ANS: 81 → last digit is 1
82 → last digit is 2
83 → last digit is 3
84 → last digit is 4
86 → last digit is 6
87 → last digit is 7

88 → last digit is 8
89 → last digit is 9
Now, let's multiply these last digits:

1 * 2 * 3 * 4 * 6 * 7 * 8 * 9 = 120

The last digit of 120 is 0.

Therefore, the last digit of the number obtained by multiplying 81 * 82 * 83 * 84 * 86 * 87 * 88 * 89 is 0.

9) Find the last three-digits of the product: 12345x54321
ANS:345+900500+000=1745 so last 3 digits are 745
10) what is unit place at last digit of 1^5+2^5+3^5+.....9^5
ANS:
1^5 = 1 2^5 = 32 3^5 = 243 4^5 = 1024 5^5 = 3125 6^5 = 7776 7^5 = 16807 8^5 = 32768 9^5 = 59049

Now, let's find the sum of these fifth powers:

1 + 32 + 243 + 1024 + 3125 + 7776 + 16807 + 32768 + 59049 = 111_111

The last digit of the sum 111_111 is 1.

Therefore, the last digit of the sum 1^5 + 2^5 + 3^5 + ... + 9^5 is 1.

REMAINDERS

1) What is the remainder when 725 is divided by 6?

ANS: 5

2) What is the remainder when 345 is divided by 8?

ANS:1

3) Find the remainder when 496 is divided by 6.

ANS:4

4) What is the remainder when 141516 is divided by 5?

ANS:1

5) Find the remainder when 6799 is divided by 7.

ANS:2

6) What is the remainder when 73x75x78x57x197x37 is divided by 34.

ANS:11

7) Let N = 1421*1423*1425. What is the remainder when N is

divided by 12?

ANS:**3**

1. **1421 ÷ 12 = 118 with a remainder of 5**
2. **1423 ÷ 12 = 118 with a remainder of 7**
3. **1425 ÷ 12 = 118 with a remainder of 9**

Now, let's find the remainder when N = 1421 * 1423 * 1425 is divided by 12:

N ≡ (5 * 7 * 9) mod 12

N ≡ 315 mod 12

N ≡ 3 with a remainder of 3

Therefore, the remainder when N = 1421 * 1423 * 1425 is divided by 12 is 3.

8) Find the remainder when 2256 is divided by 17.

ANS:12

9) The remainder of 3997! /40 is:

a) 39 b) 0 c) 1 d) None of these

10) Find the remainder on dividing 1!+2!+3!.....+100! by 7?

ANS:**1! ≡ 1 (mod 7)**

2! ≡ 2 (mod 7)

3! ≡ 6 (mod 7)

4! ≡ 3 (mod 7)

5! ≡ 1 (mod 7)

6! ≡ 6 (mod 7)

7! ≡ 0 (mod 7) (since 7! = 7 * 6! ≡ 0 * 6 ≡ 0 (mod 7))

8! ≡ 0 (mod 7)

9! ≡ 0 (mod 7)

10! ≡ 0 (mod 7)

After 7!, all factorials will be divisible by 7, so their remainders will be 0.

Now, we can calculate the sum modulo 7 by considering only the remainders:

1! + 2! + 3! + ... + 6! ≡ (1 + 2 + 6 + 6 + 1 + 6) ≡ 22 ≡ 1 (mod 7)

Since the factorials starting from 7! are all divisible by 7, their contributions to the sum will be 0 (mod 7).

Therefore, the remainder when the sum 1! + 2! + 3! + ... + 100! is divided by 7 is 1.

Factors

1) What is the number of prime factors in 6^4x8^6x10^8 x 14^10 x 22^12

ANS:prime factors are 4(2,3,5,11)

2) N= a4xb3xc7 Find the number of perfect square factors of N

where a,b,c are three distinct prime numbers.

ANS:**So, there are 30 perfect square factors of N = a^4 * b^3 * c^7, where a, b, and c are three distinct prime numbers.**

3) How many factors of 123x304x352 are even numbers?

ANS:1680

4) If N=27×34

, M= 24×32×5, then find the number of factors of N

that are common with the factors of M.

ANS:common factor are 2^4 *3^2

5) N is the smallest number that has 5 factors. How many

factors does (N - 1) have?

ANS:4

6) If both 112 and 34 are factors of the number Ax4

3

x6

2

x1311

,

then what is the smallest possible value of A?

ANS:**The prime factorizations of 11^2 and 3^4 are:**

1. **11^2 = 11 x 11**
2. **3^4 = 3 x 3 x 3 x 3**

Next, let's find the prime factorization of the given number:

Ax4^3x6^2x1311 = A x 2^12 x 3^2 x 1311

To ensure that both 11^2 and 3^4 are factors of Ax4^3x6^2x1311, we need the prime factors 11 and 3 (each with their respective powers) to be present in Ax4^3x6^2x1311.

From the prime factorization of the given number, we see that 3^2 is already present, but we don't have 11^2 as a factor yet.

Therefore, A should have at least the factor of 11^2 = 11 x 11.

To minimize A, we can take A = 11 x 11 = 121.

Now, the smallest possible value of A is 121.

7) Find the total number of factors of 10!

ANS:**2^8*3^4*5^2*7 total factor are (8+1)*(4+1)*(2+1)*(1+1)=270**

8) How many factors of 2^7 x 3^6x5^4x7^3 are even perfect squares?

ANS:**Total even perfect squares = 4 (powers of 2) * 4 (powers of 3) * 3 (powers of 5) * 2 (powers of 7) = 96.**

So, there are 96 factors of 2^7 x 3^6 x 5^4 x 7^3 that are even perfect squares.

9) In how many ways can 480 be written as a product of two

natural numbers?

ANS:**480==2^5*5^1*3^1 so total ways are (5+1)*(1+1)*(1+1)=24**

10) How many factors of 2^5x3^4 x 5^3 are not the factors of 2^3x5^4 x 7^5

ANS:180

Factorials

1) What is the highest power of 21 that divides 20!?

ANS:To find the highest power of 21 that divides 20!, we need to determine how many times 21 is a factor in the prime factorization of 20!.

The prime factorization of 20!: 20! = 1 * 2 * 3 * 4 * ... * 20

To find the highest power of 21 that divides 20!, we need to express 21 in its prime factorization form:

21 = 3 * 7

Now, we count how many times both 3 and 7 appear as factors in the prime factorization of 20!. We can do this by counting the number of multiples of 3 and 7 in the range from 1 to 20:

Multiples of 3: 3, 6, 9, 12, 15, 18 Multiples of 7: 7, 14

Count of multiples of 3 = 6 Count of multiples of 7 = 2

Since we have more multiples of 3 than multiples of 7, the highest power of 21 (which is 3 * 7) that divides 20! will be determined by the count of multiples of 7, which is 2.

Therefore, the highest power of 21 that divides 20! is 21^2, which is 441.

2) What is the highest power of 32 that divides 31!?

ANS:To find the highest power of 32 that divides 31!, we need to determine how many times 32 is a factor in the prime factorization of 31!.

The prime factorization of 31!: 31! = 1 * 2 * 3 * 4 * ... * 31

To find the highest power of 32 that divides 31!, we need to express 32 in its prime factorization form:

32 = 2^5

Now, we count how many times 2^5 (32) appears as a factor in the prime factorization of 31!. We can do this by counting the number of multiples of 32 in the range from 1 to 31:

Multiples of 32: 32

Count of multiples of 32 = 1

Therefore, the highest power of 32 that divides 31! is 32^1, which is 32.

3) Find the largest number less than 28 which divides 28!?

ANS:To find the largest number less than 28 that divides 28!, we need to determine how many times that number appears as a factor in the prime factorization of 28!.

The prime factorization of 28!: 28! = 1 * 2 * 3 * 4 * ... * 28

To find the largest number less than 28 that divides 28!, we can start from 28 and work our way down to find the first number that is a divisor of 28!.

Let's check the divisibility of 28! by 27: 28! / 27 = (1 * 2 * 3 * 4 * ... * 27 * 28) / 27

Since 27 is a factor of 28! (27 * 1 = 27), we have found the largest number less than 28 that divides 28!. Therefore, the largest number less than 28 that divides 28! is 27.

4) Find the number of zeroes at the end of 97!

ANS:To find the number of zeroes at the end of 97!, we need to determine how many times 10 (which is 2 * 5) appears as a factor in the prime factorization of 97!.

Prime factorization of 97!: 97! = 1 * 2 * 3 * 4 * ... * 97

Since the number of factors of 2 is always greater than the number of factors of 5 in the prime factorization of 97!, we only need to count the number of factors of 5.

To count the factors of 5, we can use the formula:

Number of factors of 5 in n! = $\lfloor n/5 \rfloor + \lfloor n/25 \rfloor + \lfloor n/125 \rfloor + \ldots$

where $\lfloor x \rfloor$ represents the floor function, which rounds x down to the nearest integer.

Let's calculate the number of factors of 5 in 97!:

Number of factors of 5 in 97! = $\lfloor 97/5 \rfloor + \lfloor 97/25 \rfloor + \lfloor 97/125 \rfloor + \ldots = 19 + 3 + 0 + \ldots = 22$

Therefore, the number of zeroes at the end of 97! is 22.

5) What is the highest power of 12 that divides 54!?

ANS:To find the highest power of 12 that divides 54!, we need to determine how many times 12 is a factor in the prime factorization of 54!.

The prime factorization of 54!:

54! = 1 * 2 * 3 * 4 * ... * 54

To find the highest power of 12 that divides 54!, we can express 12 in its prime factorization form:

$12 = 2^2 * 3$

Now, we count how many times $2^2 * 3$ (which is 12) appears as a factor in the prime factorization of 54!. We can do this by counting the number of multiples of 12 in the range from 1 to 54:

Multiples of 12: 12, 24, 36, 48

Count of multiples of 12 = 4

Since 12 is a factor of 54! (12 * 1 = 12), we have found the highest power of 12 that divides 54!.

Therefore, the highest power of 12 that divides 54! is 12^4, which is 20736.

6) Find the least value of x such that $60!/2^x$ is an odd number.

ANS:To find the least value of x such that $60!/2^x$ is an odd number, we need to find the highest power of 2 that divides 60!.

The prime factorization of 60!:

60! = 1 * 2 * 3 * 4 * ... * 60

To find the highest power of 2 that divides 60!, we can count the number of factors of 2 in its prime factorization.

The number of factors of 2 in n! is given by the formula:

Number of factors of 2 in n! = $\lfloor n/2 \rfloor + \lfloor n/4 \rfloor + \lfloor n/8 \rfloor + \ldots$

where $\lfloor x \rfloor$ represents the floor function, which rounds x down to the nearest integer.

Let's calculate the number of factors of 2 in 60!:

Number of factors of 2 in 60! = $\lfloor 60/2 \rfloor + \lfloor 60/4 \rfloor + \lfloor 60/8 \rfloor + \ldots = 30 + 15 + 7 + 3 + 1 = 56$

Therefore, the least value of x such that $60!/2^x$ is an odd number is x = 56.

7) Find the least value of ‘n’ if no factorial can have ‘n’ zeroes?

ANS:

8) What is the highest power of 7! dividing 50! completely.

9) How many more trailing zeroes would 625! have than 624!?

10) Find the number of zeros at the end of 1^1×2^2×3^3.....×100^100

HCF/LCM

1) The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:
a) 9000 b) 9400 c) 9600 d) 9800
ANS:9000
2) The H.C.F. of two numbers is 11 and their L.C.M. is 7700. If one of the numbers is 275, then the other is:
a) 279 b) 283 c) 308 d) 318
ANS:308
3) Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together including the toll at start?
a) 4 b) 10 c) 15 d) 16
ANS:4
4) Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. Then sum of the digits in N is:
a) 4 b) 5 c) 6 d) 8
ANS:5
5) Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.
a) 4 b) 7 c) 9 d) 13
ANS:4
6) The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:
a) 101 b) 107 c) 111 d) 185

ANS:111
7) Three number are in the ratio of 3:4:5 and their L.C.M. is 2400. Their H.C.F. is:
a) 40 b) 80 c) 120 d) 200
ANS:40
8) The G.C.D. of 1.08, 0.36 and 0.9 is:
a) 0.03 b) 0.9 c) 0.18 d) 0.108
ANS:0.03
9) The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:
a) 1 b) 2 c) 3 d) 4
ANS:6
10) The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is:
a) 74 b) 94 c) 184 d) 364
ANS:
11) The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is:
a) 3 b) 13 c) 23 d) 33
ANS:23
12) The least number which when divided by 5, 6 , 7 and 8 leaves a remainder 3, but is divisible by 9, is:
a) 1677 b) 1683 c) 2523 d) 3363
ANS:1683
13) A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds. After what time will they cross the same point from where they started?
a) 26 m 18 s b) 42 m 36 s c) 45 m d) 46 m 12 s
ANS:
14) The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. The larger of the two numbers is:
a) 276 b) 299 c) 322 d) 345
ANS:299
15) What will be the least number which when doubled will be exactly divisible by 12, 18, 21 and 30?
a) 196 b) 630 c) 1260 d) 2520
ANS:630
16) A rectangular courtyard 3.78 meters long 5.25 meters wide is to be paved with square tiles of exactly same size. What is the largest size of the tile which can be used for this purpose?
a) 14 cms b) 21 cms c) 42 cms d) None of these
ANS:21cms
17) Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is:
a) 75 b) 81 c) 85 d) 89
ANS:NONe

18) The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively is:
a) 123 b) 127 c) 235 d) 305
ANS:127
19) The L.C.M. of two numbers is 48. The numbers are in the ratio 2:3. Then sum of the number is:
a) 28 b) 32 c) 40 d) 64
ANS:40
20) The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is:
a) 15 cm b) 25 cm c) 35 cm d) 42 cm
ANS:35
21) L.C.M. of two prime numbers x and y (x>y) is 161. The value of 3y-x is :
a) -2 b) -1 c) 1 d) 2
ANS:2
22) The H.C.F and L.C.M of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is
a) 77 b) 88 c) 99 d) 110
ANS:77
23) If the sum of two numbers is 55 and the H.C.F. and L.C.M. of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to:
a) 55/601 b) 601/55 c) 11/120 d) 120/11
ANS:11/120
24) The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is:
a) 91 b) 910 c) 1001 d) 1911
ANS:91