Fuzzy connected object definition in images with respect to co-objects

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ABSTRACT

Tangible solutions to practical image segmentation are vital to ensure progress in many applications of medical imaging. Toward this goal, we previously proposed a theory and algorithms for fuzzy connected object definition in n-dimensional images. Their effectiveness has been demonstrated in several applications including multiple sclerosis lesion detection/delineation, MR Angiography, and craniofacial imaging. The purpose of this work is to extend the earlier theory and algorithms to fuzzy connected object definition that considers all relevant objects in the image simultaneously. In the previous theory, delineation of the final object from the fuzzy connectivity scene required the selection of a threshold that specifies the weakest "hanging-togetherness" of image elements relative to each other in the object. Selection of such a threshold was not trivial and has been an active research area. In the proposed method of relative fuzzy connectivity, instead of defining an object on its own based on the strength of connectedness, all co-objects of importance that are present in the image are also considered and the objects are let to compete among themselves in having image elements as their members. In this competition, every pair of elements in the image will have a strength of connectedness in each object. The object in which this strength is highest will claim membership of the elements. This approach to fuzzy object definition using a relative strength of connectedness eliminates the need for a threshold of strength of connectedness that was part of the previous definition. It seems to be more natural since it relies on the fact that an object gets defined in an image by the presence of other objects that coexist in the image. All specified objects are defined simultaneously in this approach. The concept of iterative relative fuzzy connectivity has also been introduced. Robustness of relative fuzzy objects with respect to selection of reference image elements has been established. The effectiveness of the proposed method has been demonstrated using a patient's 3D contrast enhanced MR angiogram and a 2D phantom scene.

Keywords: Image segmentation, fuzzy connectivity, object definition, object delineation

1. INTRODUCTION

Two- and higher-dimensional images are currently available through sensing devices that operate on a wide range of frequency in the electromagnetic spectrum — from ultrasound to visible light to X- and γ-rays.[1] The activity of defining meaningful objects in these images, generally referred to as image segmentation, spans over three decades.[2] The present paper falls in this category and deals with an extension of a previous work[3] which was also motivated by the problem of defining objects in multidimensional medical images. Defining objects in these image data is fundamental to most image-related applications. It is obvious that defining objects is essential prior to their visualization, manipulation, and analysis. Even operations such as image interpolation and filtering, seemingly unrelated to object definition, can be made more effective with object knowledge.

Object definition in images may be considered to consist of mainly two related tasks – recognition and delineation. *Recognition* is the process of determining roughly the whereabouts of the object in the image. *Delineation*, on the other hand, is a process that defines the precise spatial extent and composition of the object in the image. A variety of approaches have been taken in biomedical imaging applications, wherein the degree of automation for recognition and delineation ranges from completely manual to completely automatic.

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Among automatic approaches to recognition, two classes may be identified: knowledge-based and model-based. Knowledge-based methods[4,5] form hypotheses relating to objects and test them for recognizing object parts. Usually, some preliminary delineation is needed for forming and testing hypotheses relating to object components. Model-based methods[6–8] utilize predefined object models to optimally match image information to models for recognizing object components.

Approaches to delineation may be broadly classified into two groups: boundary-based and region-based. Boundary-based methods[9–11] produce a delineation of the object boundaries in the image whereas region-based methods[3,12–14] generate delineations in the form of the region occupied by the object in the image. Each of these groups may be further divided into subgroups — hard and fuzzy — depending on whether the defined regions/boundaries are described by hard or fuzzy sets.

The subject matter of this paper is related to delineation. In a previous paper,[3] we described a theory and algorithms for fuzzy connected object definition, treating a given image as a fuzzy subset of the set of spatial elements (spels) comprising the image. The method is currently utilized in several medical imaging applications including multiple sclerosis lesion segmentation and quantification,[15–17] MR angiography,[18] and hard and soft tissue 3D imaging for craniofacial surgery.[19]

In the present paper, an extension to the above definition of fuzzy objects is proposed. In the proposed method of relative fuzzy connectivity, instead of defining an object on its own based on the strength of connectedness, all co-objects of importance that are present in the image are also considered and the objects are let to compete among themselves in having spels as their members. In this competition, every pair of spels in the image will have a strength of connectedness in each object. The object in which this strength is highest will claim membership of the spels. This approach to fuzzy object definition using a relative strength of connectedness eliminates the need for a threshold of strength of connectedness that was part of the previous definition. It seems to be more natural since it relies on the fact that an object gets defined in an image by the presence of other objects that coexist in the image. All specified objects are defined simultaneously in this approach. Its theory and algorithms are presented in Sections 2 and 3, respectively. In section 4, we introduce the key ideas behind the extension of the relative connectivity model to an iterative framework. In Section 5, we illustrate the results of application of these methods on a contrast enhanced MR angiography data set. In the same section, using a 2D phantom, we compare the method to a method of optimum thresholding of the fuzzy connectivity scenes.[3,20] Finally, concluding remarks are drawn in Section 6.

2. THEORY

Terminologies and notations of our previous paper[3] are followed in this paper. For completeness, some of the key concepts that are required in this paper are briefly described here. Prior to this, an intuitive description of the key ideas is given using a two dimensional example.

2.1. An Outline of the Key Ideas

Consider a 2D image composed of three regions corresponding to three objects O_1, O_2, O_3 as illustrated in Figure 1, O_3 being the background. Suppose we determine an affinity relation[3] for each object that assigns to every pair of nearby spels in the image a value based on the nearness of spels in space and in intensity (or in features derived from intensities). Affinity represents local "hanging togetherness" of spels. To every "path" connecting every pair of spels, such as the solid curve π_1 connecting c and d in Figure 1, a "strength of connectedness" in every object is assigned which is simply the smallest pairwise affinity (associated with the corresponding object) of spels along the path. If the affinities are derived properly, then π_1 is likely to have a higher strength in O_1 than in O_2 or in O_3. Further, a path such as π_2 is likely to have a lower strength in O_1 than the strength of π_1 in O_1. This relative strength of connectedness in different objects offers a natural mechanism for partitioning spels into regions based on the strongest paths between every pair of spels in every object. A spel such as a in the fuzzy boundary between O_1 and O_3 will be claimed by O_1 or O_3 depending on which pool of spels it hangs together more strongly.

2.2. Notations and Definitions

Let X be any reference set. A *fuzzy subset* $\mathcal{A}$ of X is a set of ordered pairs $\{(x, \mu_{\mathcal{A}}(x)) | x \in X\}$ where $\mu_{\mathcal{A}} : X \to [0,1]$ is the *membership function* of $\mathcal{A}$ in X. A *fuzzy relation ρ in X* is a fuzzy subset, $\{((x,y), \mu_\rho(x,y)) | x, y \in X\}$, of $X \times X$. ρ will be called a *similitude relation* in X if it is *reflexive* (i.e., $\forall x \in X, \mu_\rho(x,x) = 1$), *symmetric* (i.e.,

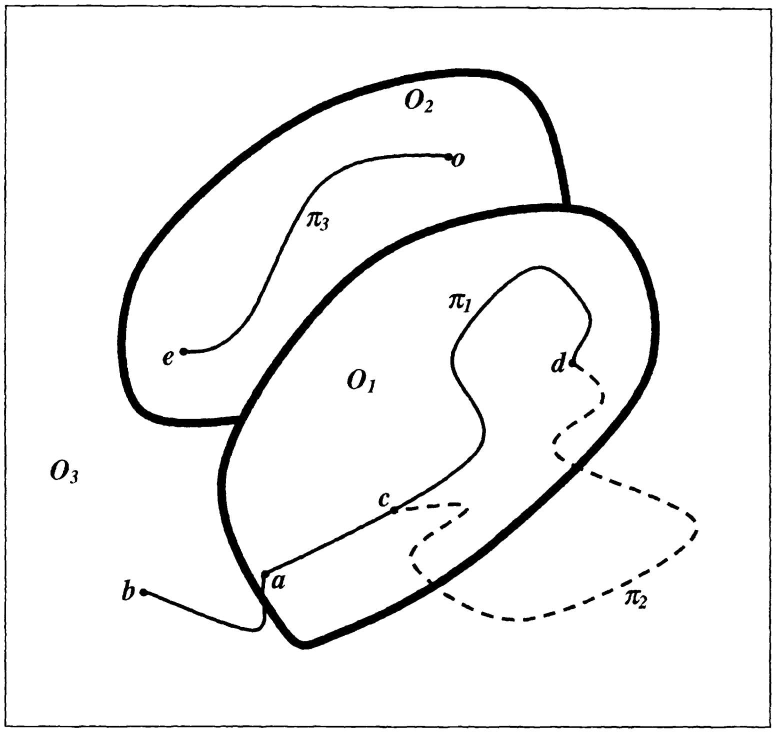

Figure 1. Illustration of the basic ideas of relative connectivity.

$\forall x, y \in X, \mu_\rho(x,y) = \mu_\rho(y,x)$) and *transitive* (i.e., $\forall x, z \in X, \mu_\rho(x,z) = \max_{y \in X}[\min[\mu_\rho(x,y), \mu_\rho(y,z)]]$). The analogous concept for a hard binary relation is an *equivalence relation.*

The pair (Z^n, α), where Z^n is the set of n-tuples of integers and α is a *fuzzy spel adjacency relation* (i.e., any fuzzy relation that is reflexive and symmetric) on Z^n, will be referred to as a *fuzzy digital space.* Elements of Z^n will be called a *spel* (short for spatial element). A *scene over* a fuzzy digital space (Z^n, α) is a pair $\mathcal{C} = (C, f)$ where $C = \{c| - b_j \leq c_j \leq b_j$ for some $b \in Z_+^n\}$, Z_+^n is the set of n-tuples of positive integers, f is a function whose domain is C, called the *scene domain,* and whose range is a set of numbers $[L, H]$.

Any fuzzy relation κ in C is said to be a *fuzzy spel affinity relation in* $\mathcal{C}$ if it is reflexive and symmetric. In practice, κ should be such that $\mu_\kappa(c,d)$ is a function of the fuzzy adjacency between c and d, the homogeneity of their intensities (or other features) and their agreement to some expected value of object intensity (or features).[20] Further, $\mu_\kappa(c,d)$ may also depend on the scale of the object at c and d.[20] Throughout this paper, κ with an appropriate subscript and/or a superscript will be used to denote fuzzy spel affinity. A *path* π *in* $\mathcal{C}$ *from* a spel c *to* a spel d is a sequence $\langle c_1, c_2, \cdots, c_m \rangle$ of $m \geq 2$ spels in C, such that $c_1 = c$ and $c_m = d$. The *strength assigned to a path* is defined as the weakest affinity between successive pairs of elements along the path. Thus, the strength of π is $\min_{1 \leq i < m}[\mu_\kappa(c_i, c_{i+1})]$. For any $S \subseteq C$, we say that the path π is *contained in* S if all all spels in π belong to S. The *strength of fuzzy* κ*-connectedness* from c to d, denoted $\mu_K(c,d)$, is the maximum of the strengths of all paths between c and d. We have shown earlier[3] that fuzzy κ-connectedness is a similitude relation. Throughout this paper, the upper case form of the symbol used to represent a fuzzy spel affinity will be used for the corresponding fuzzy connectedness relation.

For any scene $\mathcal{C} = (C, f)$ over (Z^n, α), for any fuzzy spel affinity κ in $\mathcal{C}$, and for any spel $o \in C$, the κ*-connectivity scene of* o *in* $\mathcal{C}$ is the scene $\mathcal{C}_{Ko} = (C, f_{Ko})$ such that, for any $c \in C$, $f_{Ko}(c) = \mu_K(o, c)$.

2.3. Relative Fuzzy κ-Objects

For any spels o, b in C, define

$$P_{ob_\kappa} = \{c \mid c \in C \text{ and } \mu_K(o,c) > \mu_K(b,c)\}. \tag{1}$$

The idea here is that o and b are spels specified in "object" and "background", respectively. Note that $P_{ob_\kappa} = \phi$ if $b = o$.

A *relative fuzzy κ-object* $\mathcal{O}$ of a scene $\mathcal{C} = (C, f)$ containing a spel o relative to a background containing a spel b is the fuzzy subset of C defined by the membership function

$$\mu_{\mathcal{O}}(c) = \begin{cases} \eta(f(c)), & \text{if } c \in P_{ob_\kappa} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

where η is an "objectness"function whose domain is $[L, H]$ and whose range is $[0, 1]$. The range of $f(c)$ is usually not [0,1] and $f(c)$ itself may not directly represent the degree of objectness. For example, in a CT scene of the lungs, the spels consisting of the interior of the bronchial tree have low $f(c)$ values. The proper choice of η to give accurate values of the measurements (such as volume) that are sought from the segmented fuzzy object is a non trivial issue. Since, it is out of scope of the present work, we will not delve into this here. For short, we will refer to $\mathcal{O}$ as simply a *relative fuzzy κ-object* of $\mathcal{C}$. Some particular cases are instructive to study. Suppose $\mu_K(b, o) = 1$. Then by (1), $P_{ob_\kappa} = \phi$ and the relative κ-object is empty. This makes sense since both o and b are inside the "object" in this case, and there is no meaningful separation between sets of spels "hanging together" with o and with b. Note also that, when $\mathcal{C}$ is a binary scene and $f(o) \neq f(b)$, i.e., $\mu_K(o, b) \neq 1$, then P_{ob_κ} is essentially a connected component of spels whose type is that of o that contains o. To ensure that this is a reasonable definition, we will state (but not prove) several properties of relative fuzzy κ-objects. The most important among these is that, for any spel p in P_{ob_κ} and most spels q not in P_{ob_κ}, we get the same relative fuzzy κ-object.

The following theorem states the tightness of relative fuzzy κ-objects.

Theorem 1. For any scene $\mathcal{C} = (C, f)$ over (Z^n, α), for any fuzzy spel affinity κ in $\mathcal{C}$, and for any spels o, b, p and c in C such that $p \in P_{ob_\kappa}$,

$$\mu_K(p, c) > \mu_K(b, c) \tag{3}$$

if, and only if, $c \in P_{ob_\kappa}$.

The following theorem asserts the robustness of relative fuzzy κ-objects with respect to reference spels specified in the object and in the background.

Theorem 2. For any scene $\mathcal{C} = (C, f)$ over (Z^n, α), for any fuzzy spel affinity κ in $\mathcal{C}$, and for any spels o, b, p and q in C such that $p \in P_{ob_\kappa}$,

$$P_{ob_\kappa} = P_{pq_\kappa} \text{ if } q \in P_{bo_\kappa}. \tag{4}$$

Note that the condition in (4) is sufficient for $P_{ob_\kappa} = P_{pq_\kappa}$ but not necessary. The necessary and sufficient condition is expressed in the following theorem.

Theorem 3. For any scene $\mathcal{C} = (C, f)$ over (Z^n, α), for any fuzzy spel affinity κ in $\mathcal{C}$, and for any spels o, b, p and q in C such that $p \in P_{ob_\kappa}$,

$$P_{ob_\kappa} = P_{pq_\kappa} \text{ if, and only if, } \mu_K(b, o) = \mu_K(q, o). \tag{5}$$

The above two theorems have different implications in the practical computation of relative fuzzy κ-objects in a given scene in a repeatable, consistent manner. Although less specific, and therefore more restrictive, Theorem 2 offers practically a more relevant guidance than Theorem 3 for selecting spels in the object and background so that the relative fuzzy κ-object defined is independent of the reference spels.

It follows from Theorems 2 and 3, by setting $p = o$, that $P_{oq_\kappa} = P_{ob_\kappa}$. However, the constancy of the relative fuzzy κ-object, even in this situation where the reference spel for the object is fixed but changeable only for the background, requires constraints expressed in (4) and (5).

The following theorem states an important property of relative fuzzy κ-objects, namely that their domain, that is the set P_{ob_κ}, is connected in the sense that for any two spels $p, c \in P_{ob_\kappa}$, the best path between them is contained by P_{ob_κ}.

Theorem 4. For any scene $\mathcal{C} = (C, f)$ over (Z^n, α), for any fuzzy spel affinity κ in $\mathcal{C}$, and for any spels o, b, p and c in C such that $p, c \in P_{ob_\kappa}$, the best path connecting p and c is contained in P_{ob_κ}.

3. ALGORITHMS

In this section, we present an algorithm, named $\kappa RFOE$, for extracting the relative fuzzy κ-object $\mathcal{O}$ of a scene $\mathcal{C} = (C, f)$ containing a spel, say o, relative to a background containing a spel, say b. Prior to this, we present another algorithm, named κFOE, for creating the κ-connectivity scene of o in $\mathcal{C}$. Algorithm κFOE[3] is based on dynamic programming and is called by algorithm $\kappa RFOE$.

Algorithm $\kappa FOE(o)$
Input: $\mathcal{C} = (C, f)$, and κ as defined in Section 2.
Output: κ-connectivity scene $\mathcal{C}_{Ko} = (C, f_{Ko})$.
Auxiliary Data Structures: An n-D array representing $\mathcal{C}_{Ko} = (C, f_{Ko})$ and a queue Q of spels. We refer to the array itself by $\mathcal{C}_{Ko}$ for the purpose of the algorithm.

begin
0. set all elements of $\mathcal{C}_{Ko}$ to 0 except the spel o which is set to 1;
1. push all spels $c \in C$ such that $\mu_\kappa(o, c) > 0$ to Q;
2. *while* Q is not empty do
3. remove a spel c from Q;
4. find $f_{\max} = \max_{d \in C}[\min[f_{Ko}(d), \mu_\kappa(c, d)]]$;
5. *if* $f_{\max} > f_{Ko}(c)$ then
6. set $f_{Ko}(c) = f_{\max}$;
7. push all spels e such that $\mu_\kappa(c, e) > f_{Ko}(e)$ to Q;
endif;
endwhile;
8. output the κ-connectivity scene $\mathcal{C}_{Ko}$;
end

Algorithm $\kappa RFOE(o, b)$
Input: $\mathcal{C} = (C, f)$, and κ as defined in Section 2.
Output: The relative fuzzy κ-object $\mathcal{O} = (C, \mu_{\mathcal{O}})$ of $\mathcal{C}$.
Auxiliary Data Structures: Three n-D arrays to store the following: (1) the κ-connectivity scene $\mathcal{C}_{Ko} = (C, f_{Ko})$, (2) the κ-connectivity scene $\mathcal{C}_{Kb} = (C, f_{Kb})$, and (3) $\mathcal{O}$.

begin
0. set $\mathcal{C}_{Ko} = \kappa FOE(o)$;
1. set $\mathcal{C}_{Kb} = \kappa FOE(b)$;
2. *for* all $c \in C$ do
3. *if* $f_{Ko}(c) > f_{Kb}(c)$ then
4. set $\mu_{\mathcal{O}}(c) = \eta(f(c))$;
5. *else*
6. set $\mu_{\mathcal{O}}(c) = 0$;
endif
endfor
7. output the relative fuzzy κ-object $\mathcal{O}$;
end

4. ITERATIVE RELATIVE FUZZY κ-OBJECTS

In this section, we introduce the key ideas behind an extension of relative connectedness to an iterative framework, while still satisfying the key ideas behind relative connectedness developed in Section 2.

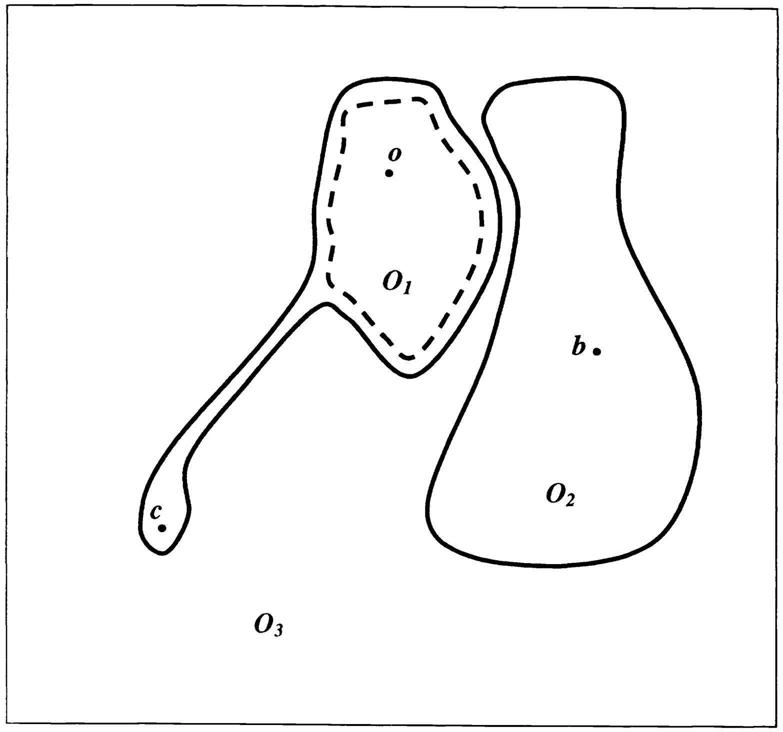

Figure 2. Illustration of the basic ideas of iterative relative connectivity.

Consider the situation illustrated in Figure 2 which demonstrates three objects O_1, O_2 and O_3. It is very likely that, for a spel such as c, $\mu_K(o,c) \approx \mu_K(b,c)$ because of the blurring that takes place in those parts where O_1 and O_2 come close together. In this case, the stongest path from b to c is likely to pass through the "core" of O_1 indicated by the dotted curve in the figure. This core which is roughly P_{ob_κ}, can be detected first and then excluded from consideration in a subsequent iteration for any path from b to c to pass through. Then, we can substantially weaken the strongest path from b to c compared to the strongest path from o to c which is still allowed to pass through the core. This leads us to an iterative strategy to grow from o (and so complementarily from b) to more accurately capture O_1 (and O_2) than if a single shot relative connectedness strategy is used. An outline of this formulation is given below.

For any fuzzy affinity κ and any two spels $c, d \in C$, define

$$\mu_{\kappa^0_{ob}}(c,d) = \mu_\kappa(c,d) \tag{6}$$

$$P^0_{ob_\kappa} = \{c \mid c \in C \text{ and } \mu_K(o,c) > \mu_{K^0_{ob}}(b,c)\}. \tag{7}$$

Note that $P^0_{ob_\kappa}$ is exactly the same as P_{ob_κ}, defined in (1). Assuming that $P^{i-1}_{ob_\kappa}$ and κ^{i-1}_{ob} for any integer i, $P^i_{ob_\kappa}$ and κ^i_{ob} are defined as follows. For all $c, d \in C$

$$\mu_{\kappa^i_{ob}}(c,d) = \begin{cases} 0, & \text{if } c \text{ or } d \in P^{i-1}_{ob_\kappa} \\ \mu_\kappa(c,d), & \text{otherwise,} \end{cases} \tag{8}$$

$$P^i_{ob_\kappa} = \{c \mid c \in C \text{ and } \mu_K(o,c) > \mu_{K^i_{ob}}(b,c)\}. \tag{9}$$

An *iterative relative fuzzy κ^i-object O^i* of a scene $\mathcal{C} = (C, f)$ containing a spel o relative to a background containing a spel b is the fuzzy subset of C defined by the membership function

$$\mu_{O^i}(c) = \begin{cases} \eta(f(c)), & \text{if } c \in P^i_{ob_\kappa}, \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

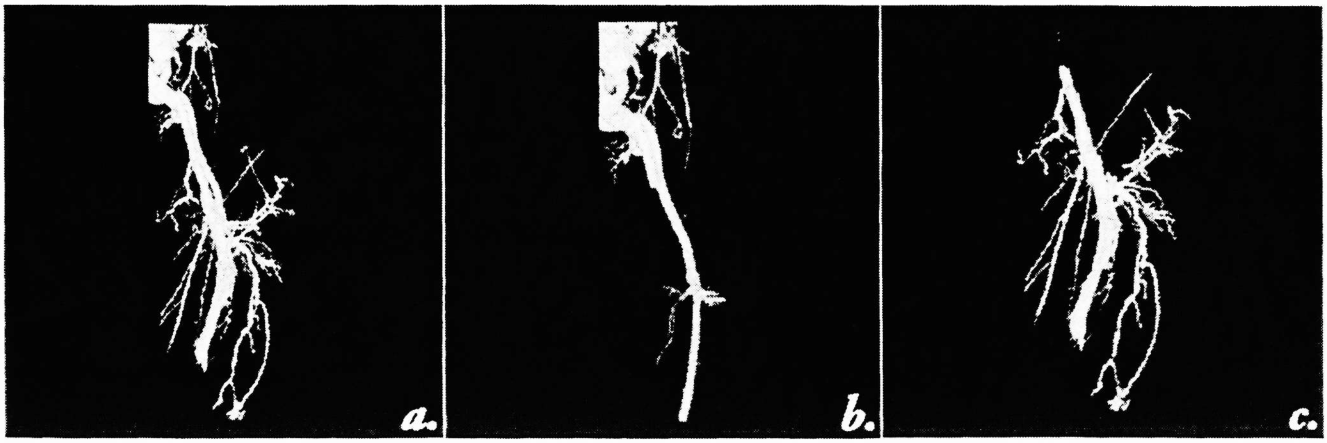

Figure 3. Results of application on a contrast enhanced 3D MR angiogram of a patient's left thigh. (a) Whole vessel tree structure automatically segmented using scale-based fuzzy connectedness. (b) Artery tree structure segmented via relative fuzzy connectivity. (c) Same as (b) for vein.

We are currently pursuing a rigorous mathematical and experimental study of the properties, robustness and effectiveness of this iterative relative fuzzy connectedness strategy for image segmentation.

5. RESULTS AND DISCUSSION

In this section, we demonstrate the effectiveness of the proposed relative fuzzy connectedness method both qualitatively and quantitatively. Figure 3 demonstrates the results of application of the method on a 3D contrast enhanced MR angiogram of a patient's left thigh. Figure 3(a) shows the whole vessel structure which was automatically segmented from the MR angiogram using scale-based fuzzy connectedness[20] and was rendered using shell rendering.[21] Figure 3(b) shows the segmented artery structure obtained using relative fuzzy connectedness while Figure 3(c) shows the segmented vein structure. Here, reference voxels were manually selected inside artery and inside vein. The same fuzzy affinity relation that was used for the whole vessel segmentation was adopted for artery-vein separation. The angiography scene has a domain of $512 \times 512 \times 60$ and a voxel size of $0.94\,\text{mm} \times 0.94\,\text{mm} \times 1.80\,\text{mm}$.

Figure 4 quantitatively demonstrates the effectiveness of the method. Figure 4(a) shows the white matter region manually segmented from a 2D slice of a 3D MR scene of a multiple sclerosis patient's head. The segmentation of the white matter region was carefully performed using 3DVIEWNIX[22] supported Live-wire[11] tool. A new 2D scene was then obtained by assigning to every pixel in the segmented white matter region a constant intensity equal to the average intensity within the segmented white matter region in the original slice, and by assigning to each of the rest of the pixels a constant intensity equal to the average intensity within the segmented gray matter region in the original slice. From the simulated scene, we created the actual test phantom by adding (a) a blurring using a 2D Gaussian kernel, (b) a 0-mean Gaussian correlated noise, and (c) a slowly varying (ramp) background component from 0 to 100 across the columns. Figure 4(b) shows the final test phantom scene. Figure 4(c) shows the scale-based fuzzy connectivity scene for the phantom scene of Figure 4(b). From the fuzzy connectivity scene, the best possible hard segmentation of white matter region was obtained as follows. A distance between two binary scenes is defined as the percentage of normalized counts of pixel mismatches between the two binary scenes. The fuzzy connectivity scene was then segmented at a threshold at which the distance between the segmented binary scene and the white matter scene (i.e., the initial truth) of Figure 4(a) is minimum. Figure 4(d) shows the best possible hard segmentation for white matter region from the fuzzy connectivity scene of Figure 4(c). (Note that the best threshold was selected by exhaustive search.) The computed distance for the segmented scene of Figure 4(d) is 2.81574. Figure 4(e) shows the segmented white matter region using relative fuzzy connectivity; here, the computed distance for the segmented scene is 2.60409. In both cases, the same parameters for fuzzy affinity relations and the same set of reference pixels (for white matter region) were used. One background reference pixel was selected for relative fuzzy connectivity. The phantom scene has a domain of 189×230 and a pixel size of $0.86\,\text{mm} \times 0.86\,\text{mm}$. Although, the final segmentations in

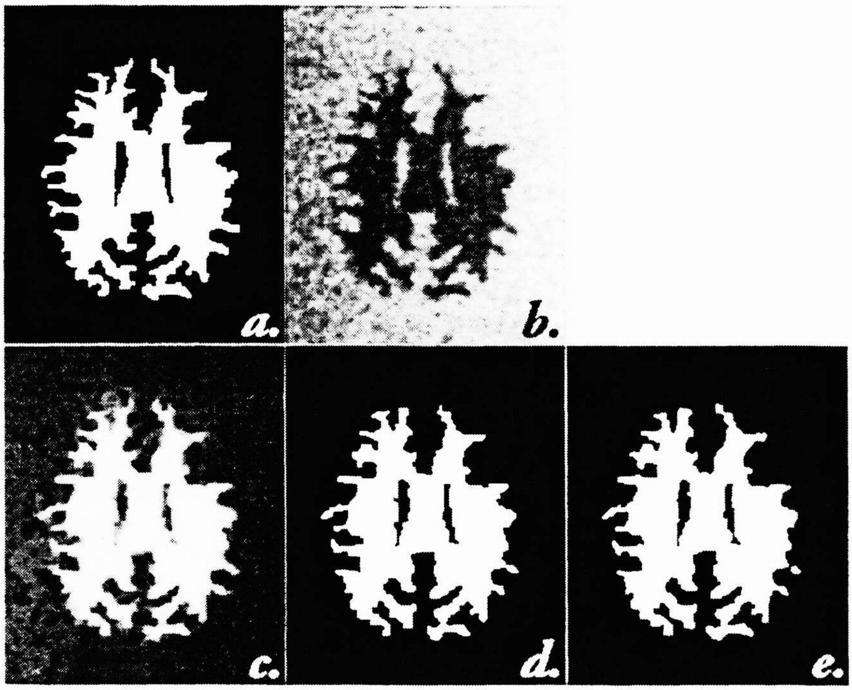

Figure 4. Results of application of relative fuzzy connectivity on a 2D phantom. (a) Binary white matter region manually segmented out from a 2D slice of a 3D MR scene of a multiple sclerosis patient's head. (b) Test phantom scene generated from (a) after adding noise, blurring and background variation. (c) Scale-based fuzzy connectivity scene for white matter region. (d) Hard segmented white matter region obtained from (c) at the best possible threshold. (e) Hard segmented white matter region obtained via relative fuzzy connectivity.

Figures 4(d) and (e) are visually very close, the segmentation using relative connectivity is closer to the initial truth. This observation may be argued by the fact that in absolute fuzzy connectivity, a fixed global threshold is chosen while in relative connectivity the competition between objects (foreground and background) is spatially variant. Effectively, relative connectedness allows a variable threshold in the strength of connectivity in different parts of the scene. Most importantly, to achieve the best threshold in absolute fuzzy connectivity, the initial phantom truth is used which is not available in any real application. Relative fuzzy connectivity eliminates this need.

6. CONCLUSION

Based on our previously developed framework of fuzzy connectedness and object definition,[3,20] we have proposed an extension of this framework that considers all relevant objects simultaneously. The fundamental premise on which this is developed is that an object gets defined in an image by the existence of other co-objects (including the background). We consider certain regions in the image as part of the object because these regions hang-together more strongly with object elements than with background elements.

One drawback of the previous fuzzy connectivity theory is having to select a threshold for the fuzzy connectivity scene to delineate the object region. Relative fuzzy connectivity provides an effective and robust solution to this problem. The robustness of relative fuzzy objects with respect to reference pixel selection has been stated. An algorithm for computing relative fuzzy connected objects using dynamic programming has also been presented. The concept of iterative relative fuzzy connectivity has been introduced.

We have demonstrated the effectiveness of the proposed method in artery-vein separation in a contrast enhanced 3D MR angiogram. Based on a 2D phantom scene, we have shown that the segmentation by relative fuzzy connectivity is better than that obtainable via the best thresholding of the absolute fuzzy-connectivity scene. More extensive experiments in several ongoing applications are currently undergoing, as well as the development of the theory for multiple objects.

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