Fuzzy Sets and Fuzzy Techniques

Lecture 13 – Fuzzy connectedness

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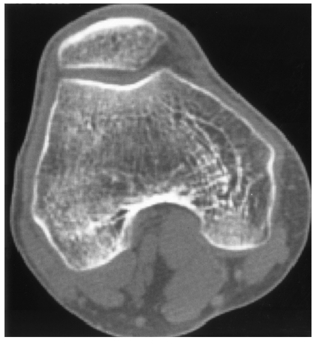
Outline

Object characteristics in images



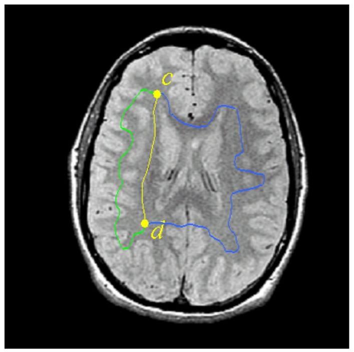
Graded composition

heterogeneity of intensity in the object region due to heterogeneity of object material and blurring caused by the imaging device

Hanging-togetherness

natural grouping of voxels constituting an object a human viewer readily sees in a display of the scene as a Gestalt in spite of intensity heterogeneity

Basic idea of fuzzy connectedness



- local hanging-togetherness (affinity) based on similarity in spatial location as well as in intensity(-derived features)
- global hanging-togetherness (connectedness)

Fuzzy set and relation

A **fuzzy subset** $\mathcal{A}$ of X is

$$\mathcal{A} = \{(x, \mu_{\mathcal{A}}(x)) \,|\, x \in X\}$$

where $\mu_{\mathcal{A}}$ is the **membership function** of $\mathcal{A}$ in X

$$\mu_{\mathcal{A}} : X \to [0, 1]$$

A **fuzzy relation** ρ in X is

$$\rho = \{((x, y), \mu_\rho(x, y)) \,|\, x, y \in X\}$$

with a membership function

$$\mu_\rho : X \times X \to [0, 1]$$

Operations on fuzzy sets

Intersection

$$\mathcal{A} \cap \mathcal{B} = \{(x, \mu_{\mathcal{A}\cup\mathcal{B}}(x)) \,|\, x \in X\} \qquad \mu_{\mathcal{A}\cap\mathcal{B}} = \min(\mu_{\mathcal{A}}, \mu_{\mathcal{B}})$$

Union

$$\mathcal{A} \cup \mathcal{B} = \{(x, \mu_{\mathcal{A}\cup\mathcal{B}}(x)) \,|\, x \in X\} \qquad \mu_{\mathcal{A}\cup\mathcal{B}} = \max(\mu_{\mathcal{A}}, \mu_{\mathcal{B}})$$

Complement

$$\bar{\mathcal{A}} = \{(x, \mu_{\bar{\mathcal{A}}}(x)) \,|\, x \in X\} \qquad \mu_{\bar{\mathcal{A}}} = 1 - \mu_{\mathcal{A}}$$

$\cap$ and $\cup$ are also called T-norm and T-conorm (S-norm).
Several (corresponding pairs) of T- and S-norms exist.
In the FC framework min and max are used.

Properties of fuzzy relations

ρ is **reflexive** if

$$\forall x \in X \quad \mu_\rho(x, x) = 1$$

ρ is **symmetric** if

$$\forall x, y \in X \quad \mu_\rho(x, y) = \mu_\rho(y, x)$$

ρ is **transitive** if

$$\forall x, z \in X \quad \mu_\rho(x, z) = \bigcup_{y \in X} \mu_\rho(x, y) \cap \mu_\rho(y, z)$$

ρ is **similitude** if it is reflexive, symmetric, and transitive

Note: this corresponds to the equivalence relation in hard sets.

Fuzzy digital space

Fuzzy spel adjacency is a reflexive and symmetric fuzzy relation α in Z^n and assigns a value to a pair of spels (c, d) based on how close they are spatially.

Example

$$\mu_\alpha(c, d) = \begin{cases} \dfrac{1}{\|c - d\|} & \text{if } \|c - d\| < \text{a small distance} \\ 0 & \text{otherwise} \end{cases}$$

Fuzzy digital space

$$(Z^n, \alpha)$$

Scene (over a fuzzy digital space)

$$\mathcal{C} = (C, f) \quad \text{where } C \subset Z^n \text{ and } f : C \to [L, H]$$

Fuzzy spel affinity

Fuzzy spel affinity is a reflexive and symmetric fuzzy relation κ in Z^n and assigns a value to a pair of spels (c, d) based on how close they are spatially and intensity-based-property-wise (local hanging-togetherness).

$$\mu_\kappa(c, d) = h(\mu_\alpha(c, d), f(c), f(d), c, d)$$

Example

$$\mu_\kappa(c, d) = \mu_\alpha(c, d)\left(w_1 G_1(f(c) + f(d)) + w_2 G_2(f(c) - f(d))\right)$$

where $G_j(x) = \exp\left(-\frac{1}{2}\frac{(x - m_j)^2}{\sigma_j^2}\right)$

Paths between spels

A **path** p_{cd} in $\mathcal{C}$ from spel $c \in C$ to spel $d \in C$ is any sequence $\langle c_1, c_2, \ldots, c_m \rangle$ of $m \geq 2$ spels in C, where $c_1 = c$ and $c_m = d$.

Let P_{cd} denote the set of all possible paths p_{cd} from c to d. Then the set of all possible paths in $\mathcal{C}$ is

$$P_{\mathcal{C}} = \bigcup_{c,d \in C} P_{cd}$$

Strength of connectedness

The **fuzzy κ-net** $\mathcal{N}_\kappa$ of $\mathcal{C}$ is a fuzzy subset of $P_{\mathcal{C}}$, where the membership (**strength of connectedness**) assigned to any path $p_{cd} \in P_{cd}$ is the smallest spel affinity along p_{cd}

$$\mu_{\mathcal{N}_\kappa}(p_{cd}) = \min_{j=1,\ldots,m-1} \mu_\kappa(c_j, c_{j+1})$$

The **fuzzy κ-connectedness** in $\mathcal{C}$ (K) is a fuzzy relation in C and assigns a value to a pair of spels (c, d) that is the maximum of the strengths of connectedness assigned to all possible paths from c to d (global hanging-togetherness).

$$\mu_K(c, d) = \max_{p_{cd} \in P_{cd}} \mu_{\mathcal{N}_\kappa}(p_{cd})$$

Fuzzy κ_θ component

Let $\theta \in [0, 1]$ be a given threshold

Let K_θ be the following binary (equivalence) relation in C

$$\mu_{K_\theta}(c, d) = \begin{cases} 1 & \text{if } \mu_\kappa(c, d) \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

Let $O_\theta(o)$ be the equivalence class of K_θ that contains $o \in C$

Let $\Omega_\theta(o)$ be defined over the fuzzy κ-connectedness K as

$$\Omega_\theta(o) = \{c \in C \mid \mu_K(o, c) \geq \theta\}$$

Practical computation of FC relies on the following equivalence

$$O_\theta(o) = \Omega_\theta(o)$$

Fuzzy connected object

The **fuzzy κ_θ object** $\mathcal{O}_\theta(o)$ of $\mathcal{C}$ containing o is

$$\mu_{\mathcal{O}_\theta(o)}(c) = \begin{cases} \eta(c) & \text{if } c \in O_\theta(o) \\ 0 & \text{otherwise} \end{cases}$$

that is

$$\mu_{\mathcal{O}_\theta(o)}(c) = \begin{cases} \eta(c) & \text{if } c \in \Omega_\theta(o) \\ 0 & \text{otherwise} \end{cases}$$

where η assigns an objectness value to each spel perhaps based on $f(c)$ and $\mu_K(o, c)$.

Fuzzy connected objects are robust to the selection of seeds.

Fuzzy connectedness as a graph search problem

- Spels $\longrightarrow$ graph nodes
- Spel faces $\longrightarrow$ graph edges
- Fuzzy spel-affinity relation $\longrightarrow$ edge costs
- Fuzzy connectedness $\longrightarrow$ all-pairs shortest-path problem
- Fuzzy connected objects $\longrightarrow$ connected components

Computing fuzzy connectedness

Dynamic programming

Algorithm

Input: $\mathcal{C}$, $o \in C$, κ
Output: A K-connectivity scene $\mathcal{C}_o = (C_o, f_o)$ of $\mathcal{C}$
Auxiliary data: a queue Q of spels

begin
  set all elements of $\mathcal{C}_o$ to 0 except o which is set to 1
  push all spels $c \in C_o$ such that $\mu_\kappa(o, c) > 0$ to Q
  while $Q \neq \emptyset$ **do**
    remove a spel c from Q
    $f_{\text{val}} \leftarrow \max_{d \in C_o}[\min(f_o(d), \mu_\kappa(c, d))]$
    if $f_{\text{val}} > f_o(c)$ **then**
      $f_o(c) \leftarrow f_{\text{val}}$
      push all spels e such that $\mu_\kappa(c, e) > 0$ $f_{\text{val}} > f_o(e)$ $f_{\text{val}} > f_o(e)$ and $\mu_\kappa(c, e) > f_o(e)$
    endif
  endwhile
end

Fuzzy connectedness variants

- Multiple seeds per object

- Scale-based fuzzy affinity
- Vectorial fuzzy affinity

- Absolute fuzzy connectedness
- Relative fuzzy connectedness
- Iterative relative fuzzy connectedness

Components of fuzzy affinity

Fuzzy spel adjacency $\mu_\alpha(c, d)$ indicates the degree of spatial adjacency of spels

The **homogeneity-based component** $\mu_\psi(c, d)$ indicates the degree of local hanging-togetherness of spels due to their similarities of intensities

The **object-feature-based component** $\mu_\phi(c, d)$ indicates the degree of local hanging-togetherness of spels with respect to some given object feature

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Defining fuzzy spel affinity

Fuzzy spel affinity

$$\mu_\kappa(c, d) = \mu_\alpha(c, d) g(\mu_\psi(c, d), \mu_\phi(c, d))$$

Expected properties of g

- range within $[0, 1]$
- monotonically non-increasing in both arguments

Examples

$$\mu_\kappa = \frac{1}{2}\mu_\alpha\left(\mu_\psi + \mu_\phi\right)$$

$$\mu_\kappa = \mu_\alpha\sqrt{\mu_\psi \mu_\phi}$$

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Homogeneity-based component

Non-scale-based

Homogeneity-based component

$$\mu_\psi(c, d) = W_\psi(|f(c) - f(d)|)$$

Expected properties of W_ψ

- range within $[0, 1]$ and $W_\psi(0) = 1$
- monotonically non-increasing
- should also be related to overall homogeneity

Examples

the right-hand-side of an appropriately scaled box, trapezoid, or Gaussian function

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Object-feature-based component

Non-scale-based

Object-feature-based component

$$\mu_\phi(c, d) = \begin{cases} 1 & \text{if } c = d \\ \frac{\mathcal{W}_o(c,d)}{\mathcal{W}_b(c,d) + \mathcal{W}_o(c,d)} & \text{otherwise} \end{cases}$$

$$\mathcal{W}_o(c, d) = \min[W_o(f(c)), W_o(f(d))]$$

$$\mathcal{W}_b(c, d) = \max[W_b(f(c)), W_b(f(d))]$$

Expected properties of W_o and W_b

- range within $[0, 1]$
- monotonically non-increasing

Examples

an appropriately scaled and shifted box, trapezoid, or Gaussian function

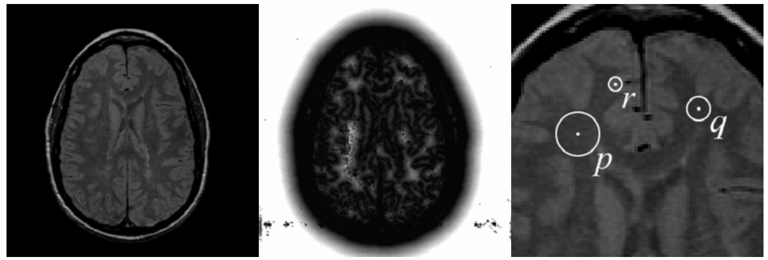
Scale-based affinity

Considers the following aspects

- spatial adjacency
- homogeneity (local and global)
- object feature (expected intensity properties)
- object scale

Object scale

Object scale in $\mathcal{C}$ at any spel $c \in C$ is the radius $r(c)$ of the largest hyperball centered at c which lies entirely within the same object region



The scale value can be simply and effectively estimated without explicit object segmentation

Computing object scale

Algorithm

Input: $\mathcal{C}$, $c \in C$, W_ψ, $\tau \in [0,1]$
Output: $r(c)$

begin
  $k \leftarrow 1$
  while $FO_k(c) \geq \tau$ **do**
    $k \leftarrow k+1$
  endwhile
  $r(c) \leftarrow k$
end

Fraction of the ball boundary homogeneous with the center spel

$$FO_k(c) = \frac{\sum\limits_{d \in B_k(c)} W_{\psi_s}(|f(c) - f(d)|)}{|B_k(c) - B_{k-1}(c)|}$$

Neighborhood selection for scale-based computations

For properties based on a single spel c use

$$B_r(c) = \{e \in C \mid \|e - c\| \leq r(c)\}$$

For properties based on a pair of spels c and d use

$$B_{cd}(c) = \{e \in C \mid \|e - c\| \leq \min[r(c), r(d)]\}$$

$$B_{cd}(d) = \{e \in C \mid \|e - d\| \leq \min[r(c), r(d)]\}$$

Intensity variations in neighborhoods

Intensity variations (inhomogeneity) surrounding c and d ($\mu_\alpha(c,d) > 0$)

- intra-object variation
  (expected to be random and to have a zero mean)
- inter-object variation
  (expected to have a direction and to be larger than the intra-object variation)

Homogeneity-based component

Scale-based

Directional inhomogeneity over the regions around c and d

$$D(c,d) = \sum_{\substack{e \in B_{cd}(c) \\ e' \in B_{cd}(d) \\ e-c=e'-d}} (1 - W_h(f(e) - f(e')))\omega_{cd}(\|e-c\|)\frac{f(e)-f(e')}{|f(e)-f(e')|}$$

Homogeneity-based component

$$\mu_{\psi_s}(c,d) = 1 - \frac{|D(c,d)|}{\sum_{e \in B_{cd}(c)} \omega_{cd}(\|e-c\|)}$$

Object-feature-based component

Scale-based

Filtered version of $f(c)$ taking into account the neighborhood

$$f_a(c) = \frac{\sum_{e \in B_r(c)} f(e)\omega_c(\|e-c\|)}{\sum_{e \in B_r(c)} \omega_c(\|e-c\|)}$$

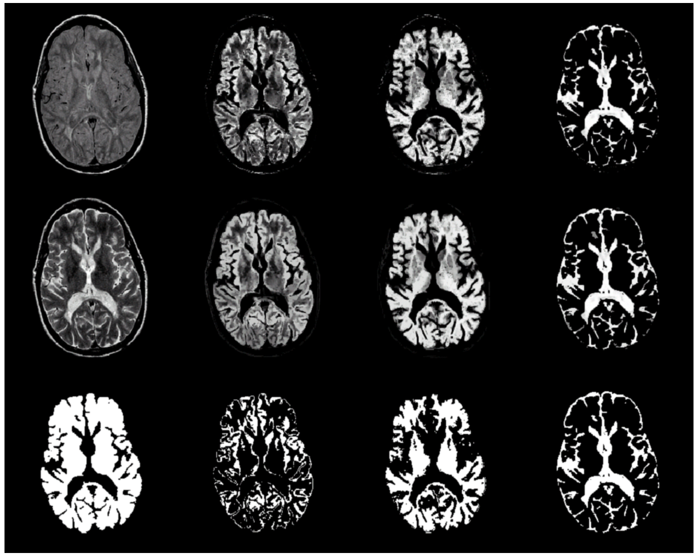
Object-feature-based component

$$\mu_{\phi_s}(c,d) = \begin{cases} 1 & \text{if } c = d \\ \dfrac{\mathcal{W}_{os}(c,d)}{\mathcal{W}_{bs}(c,d) + \mathcal{W}_{os}(c,d)} & \text{otherwise} \end{cases}$$

$$\mathcal{W}_{os}(c,d) = \min[W_{os}(f_a(c)), W_{os}(f_a(d))]$$
$$\mathcal{W}_{bs}(c,d) = \max[W_{bs}(f_a(c)), W_{bs}(f_a(d))]$$

Brain tissue segmentation

FSE



Efficiency problems with FC (1)

Problem
it takes too long to compute

Solution
use more powerful computer

- scales with CPU speed but also depends on the algorithm

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Efficiency problems with FC (2)

Problem
most time is spent with affinity computation

Solution
compute affinity only for spels that are used

- can reduce 35–55 % depending on the object

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Efficiency problems with FC (3)

Problem
many spels are unnecessarily visited

Solution
use pre-determined connectedness thresholds

- can reduce 70–90 % depending on the object

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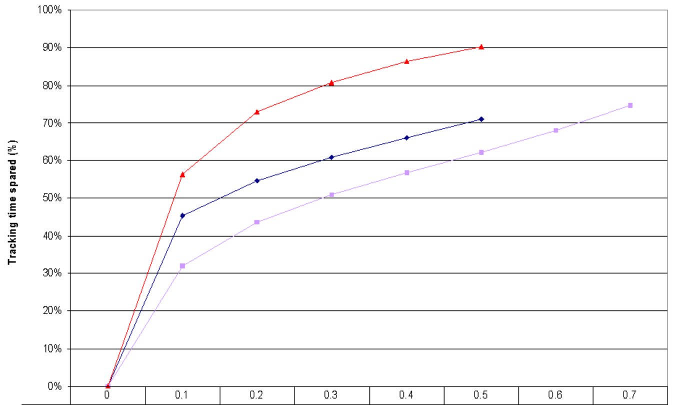
Effect of affinity thresholds



	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
WM	0.00%	45.35%	54.60%	60.99%	66.04%	71.04%		
GM	0.00%	31.86%	43.52%	50.93%	56.73%	62.26%	68.06%	74.75%
CSF	0.00%	56.36%	72.93%	80.89%	86.41%	90.37%		

Threshold

Efficiency problems with FC (4)

Problem

many spels are "revisited"

Solution

reduce the number of "revisits" by ordering

- label-correcting vs. label-setting algorithms
- binary, d-ary, and Fibonacci heaps
- LIFO and FIFO lists
- hash tables of various sizes with various hash functions
- additional pointer arrays

Computing fuzzy connectedness

Dynamic programming

Algorithm

Input: $\mathcal{C}$, $o \in C$, κ
Output: A K-connectivity scene $\mathcal{C}_o = (C_o, f_o)$ of $\mathcal{C}$
Auxiliary data: a queue Q of spels

begin
set all elements of $\mathcal{C}_o$ to 0 except o which is set to 1
push all spels $c \in C_o$ such that $\mu_\kappa(o, c) > 0$ to Q
while $Q \neq \emptyset$ **do**
remove a spel c from Q
$f_{\text{val}} \leftarrow \max_{d \in C_o}[\min(f_o(d), \mu_\kappa(c, d))]$
if $f_{\text{val}} > f_o(c)$ **then**
$f_o(c) \leftarrow f_{\text{val}}$
push all spels e such that $\mu_\kappa(c, e) > 0$ $f_{\text{val}} > f_o(e)$ $f_{\text{val}} > f_o(e)$ and $\mu_\kappa(c, e) > f_o($
endif
endwhile
end

Computing fuzzy connectedness

Dijkstra's-like

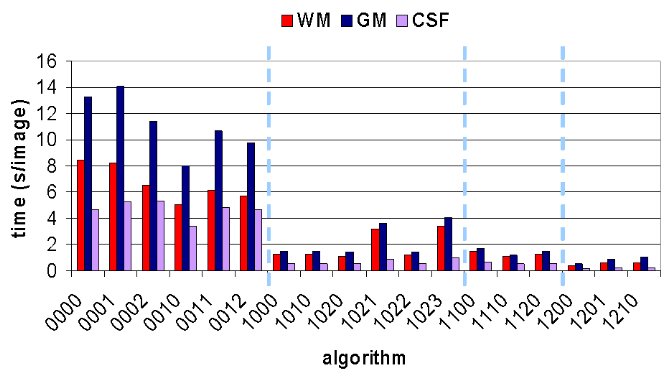
Algorithm

Input: $\mathcal{C}$, $o \in C$, κ
Output: A K-connectivity scene $\mathcal{C}_o = (C_o, f_o)$ of $\mathcal{C}$
Auxiliary data: a priority queue Q of spels

begin
set all elements of $\mathcal{C}_o$ to 0 except o which is set to 1
push o to Q
while $Q \neq \emptyset$ **do**
remove a spel c from Q for which $f_o(c)$ is maximal
for each spel e such that $\mu_\kappa(c, e) > 0$ **do**
$f_{\text{val}} \leftarrow \min(f_o(c), \mu_\kappa(c, e))$
if $f_{\text{val}} > f_o(e)$ **then**
$f_o(e) \leftarrow f_{\text{val}}$
update e in Q (or push if not yet in)
endif
endfor
endwhile
end

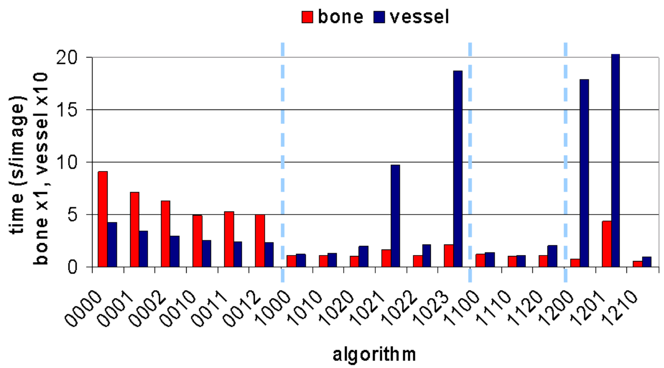
FC with threshold

MRI



FC with threshold

CT and MRA



Vector-valued scenes

Scene

$$\mathcal{C} = (C, \mathbf{f}) \quad \text{where } C \subset Z^n \text{ and } \mathbf{f} = (f_1, f_2, \ldots, f_l)^{\mathsf{T}}$$

$$\mathbf{f} : C \to [L_1, H_1] \times [L_2, H_2] \times \cdots \times [L_l, H_l]$$

Object scale

$$FO_k(c) = \frac{\displaystyle\sum_{d \in B_k(c)} W_h(\mathbf{f}(c) - \mathbf{f}(d))}{|B_k(c) - B_{k-1}(c)|}$$

$$W_h(\mathbf{x}) = \exp\left(-\frac{1}{2}\mathbf{x}^{\mathsf{T}}\mathbf{\Sigma}_h^{-1}\mathbf{x}\right)$$

Homogeneity-based component

Vectorial scale-based

Total unidirectional inhomogeneity over the scale regions around c and d

$$\mathbf{D}(c, d) = \sum_{\substack{e \in B_{cd}(c) \\ e' \in B_{cd}(d) \\ e - c = e' - d}} (1 - W_h(\mathbf{f}(e) - \mathbf{f}(e')))\omega_{cd}(\|e - c\|)\frac{\mathbf{f}(e) - \mathbf{f}(e')}{|\mathbf{f}(e) - \mathbf{f}(e')|}$$

$$\mu_{\psi_{vs}}(c, d) = 1 - \frac{|\mathbf{D}(c, d)|}{\displaystyle\sum_{e \in B_{cd}(c)} \omega_{cd}(\|e - c\|)}$$

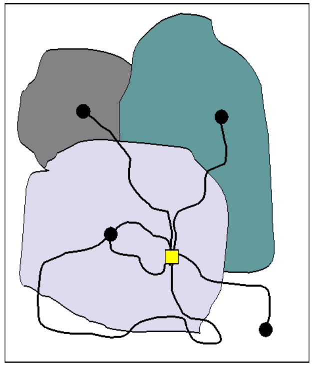
Object-feature-based component

Vectorial scale-based

$$W_o(\mathbf{f}_a(c)) = \exp\left(-\frac{1}{2}(\mathbf{f}_a(c) - \mathbf{M}_o)^{\mathsf{T}}\mathbf{\Sigma}_o^{-1}(\mathbf{f}_a(c) - \mathbf{M}_o)\right)$$

$$\mu_{\phi_{vs}}(c, d) = \begin{cases} 1 & \text{if } c = d \\ \min[W_o(\mathbf{f}_a(c)), W_o(\mathbf{f}_a(d))] & \text{otherwise} \end{cases}$$

Relative fuzzy connectedness



- always at least two objects
- automatic/adaptive thresholds on the object boundaries
- objects (object seeds) "compete" for spels and the one with stronger connectedness wins

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Relative fuzzy connectedness

Let $O_1, O_2, \ldots, O_m$, a given set of objects ($m \geq 2$), $S = \{o_1, o_2 \ldots, o_m\}$ a set of corresponding seeds, and let $b(o_j) = S \setminus \{o_j\}$ denote the 'background' seeds w.r.t. seed o_j.

1. define affinity for each object $\Rightarrow \kappa_1, \kappa_2, \ldots, \kappa_m$
2. combine them into a single affinity $\Rightarrow \kappa = \bigcup_j \kappa_j$
3. compute fuzzy connectedness using $\kappa \Rightarrow K$
4. determine the fuzzy connected objects $\Rightarrow$

$$O_{ob}(o) = \{c \in C \mid \forall o' \in b(o) \quad \mu_K(o,c) > \mu_K(o',c)\}$$

$$\mu_{O_{ob}}(c) = \begin{cases} \eta(c) & \text{if } c \in O_{ob}(o) \\ 0 & \text{otherwise} \end{cases}$$

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kNN vs. VSRFC

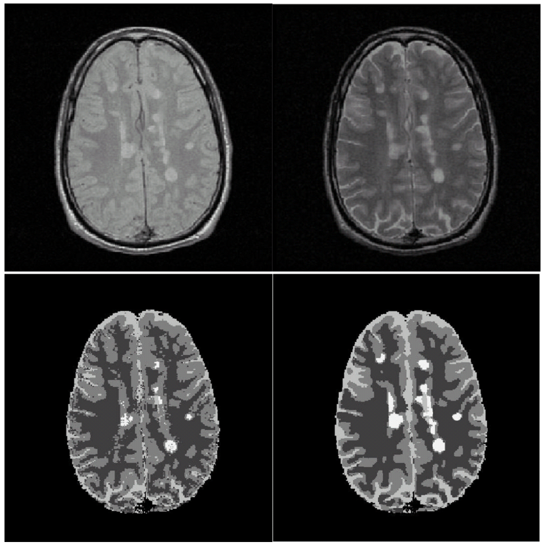

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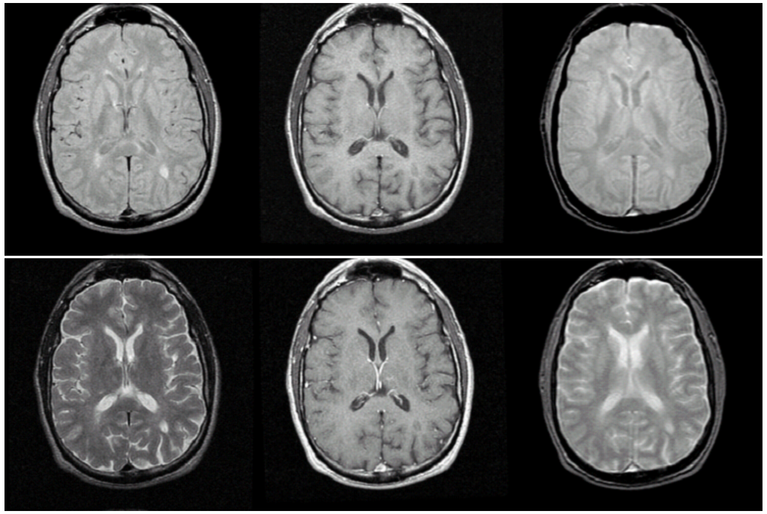
Image segmentation using FC

- MR
  - brain tissue, tumor, MS lesion segmentation
- MRA
  - vessel segmentation and artery-vein separation
- CT bone segmentation
  - kinematics studies
  - measuring bone density
  - stress-and-strain modeling
- CT soft tissue segmentation
  - cancer, cyst, polyp detection and quantification
  - stenosis and aneurism detection and quantification
- Digitized mammography
  - detecting microcalcifications
- Craniofacial 3D imaging
  - visualization and surgical planning

Protocols for brain MRI



FC segmentation of brain tissues

1. Correct for RF field inhomogeneity
2. Standardize MR image intensities
3. Compute fuzzy affinity for GM, WM, CSF
4. Specify seeds and VOI (interaction)
5. Compute relative FC for GM, WM, CSF
6. Create brain intracranial mask
7. Correct brain mask (interaction)
8. Create masks for FC objects

9. Detect potential lesion sites
10. Compute relative FC for GM, WM, CSF, LS
11. Verify the segmented lesions (interaction)

Segmentation in two phases

Phase 1: Training

performed once for each task (protocol, body region, organ)

- a few datasets are selected and used to extract the values for the parameters
- mostly requires continuous user control

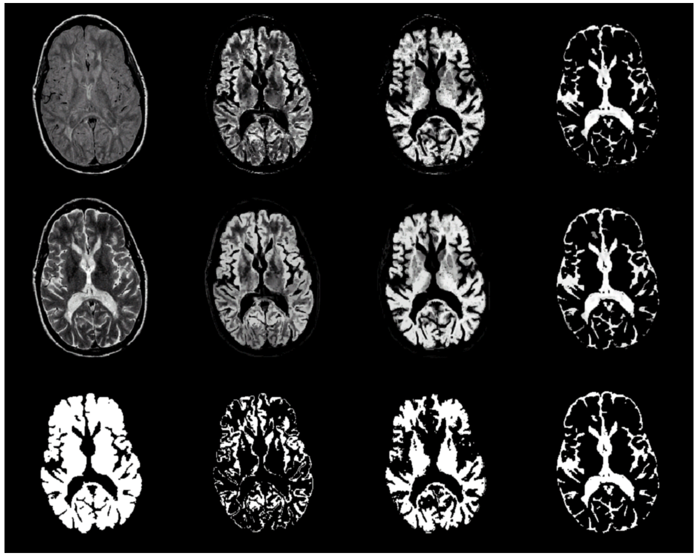
Phase 2: Segmentation

performed for each individual dataset

- most steps are automatic (parameters are fixed in Phase 1)
- interactive steps require
  - mouse clicks from the user to specify points
  - “cut” and “add” when correcting the brain mask

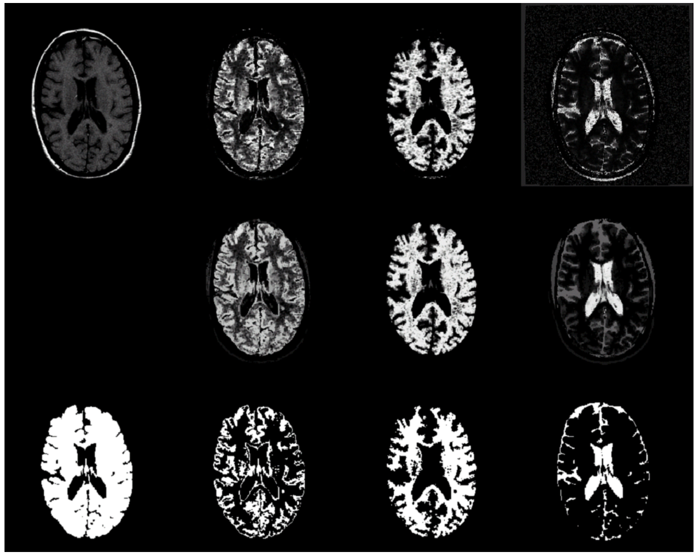
Brain tissue segmentation

FSE



Brain tissue segmentation

T1



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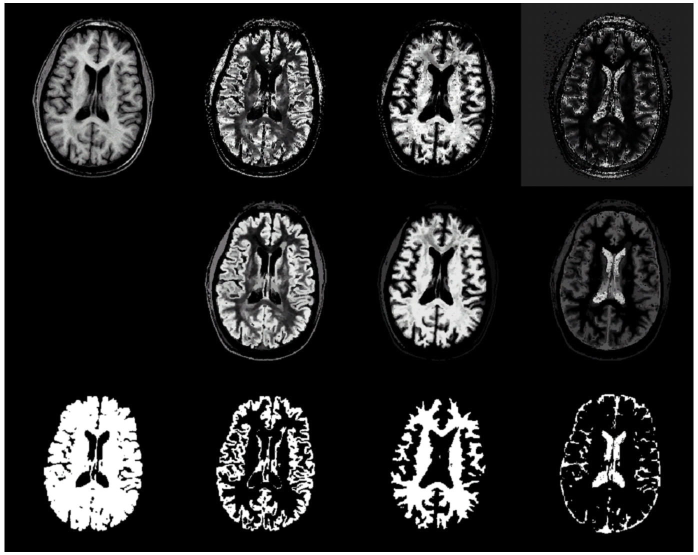
Brain tissue segmentation

SPGR



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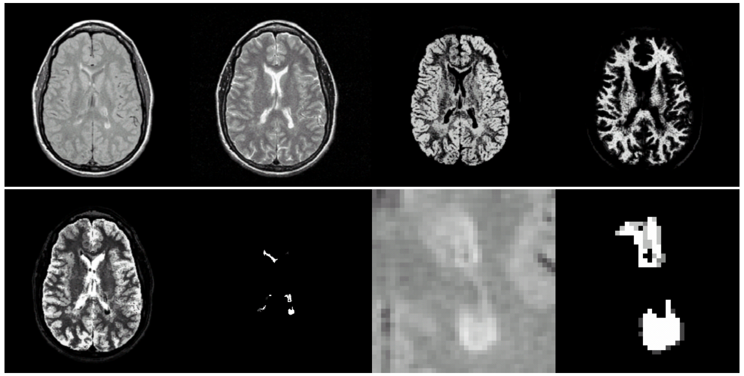
FSE



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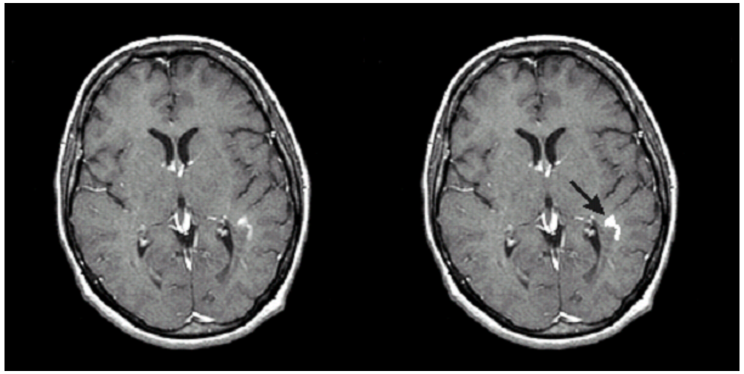
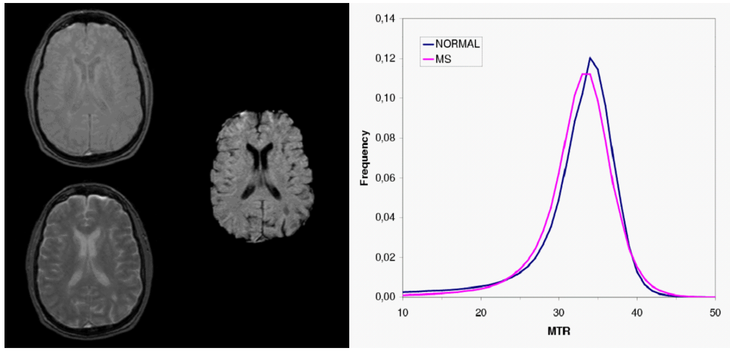
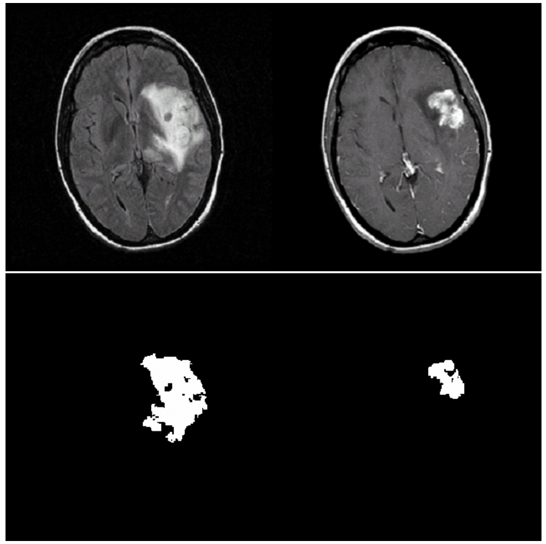
T1E



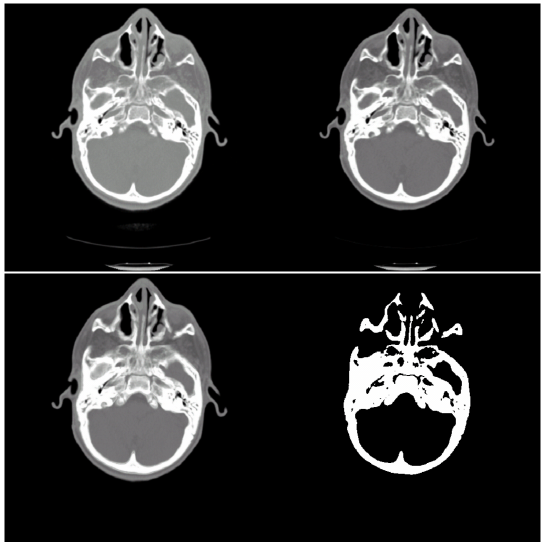
MTR analysis



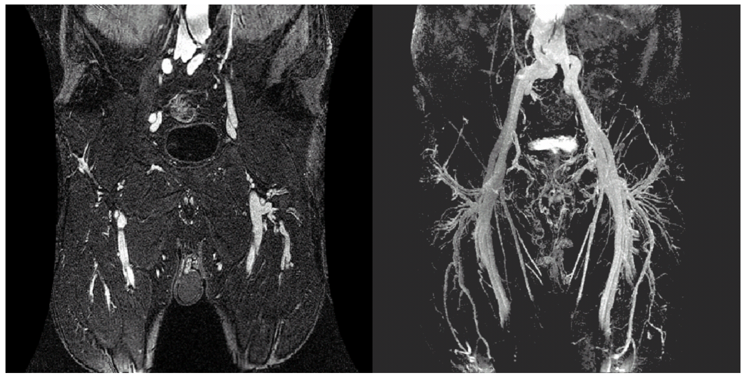
Brain tumor quantification



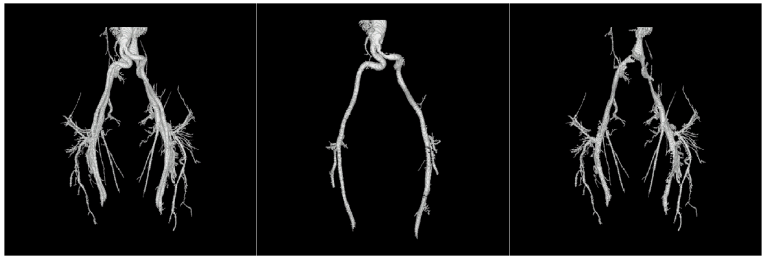
Skull object from CT



MRA slice and MIP rendering



MRA vessel segmentation and artery/vein separation



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