

## **MEDICAL IMAGE COMPUTING (CAP 5937)**

**LECTURE 9:** Medical Image Segmentation (III)  
(Fuzzy Connected Image Segmentation)

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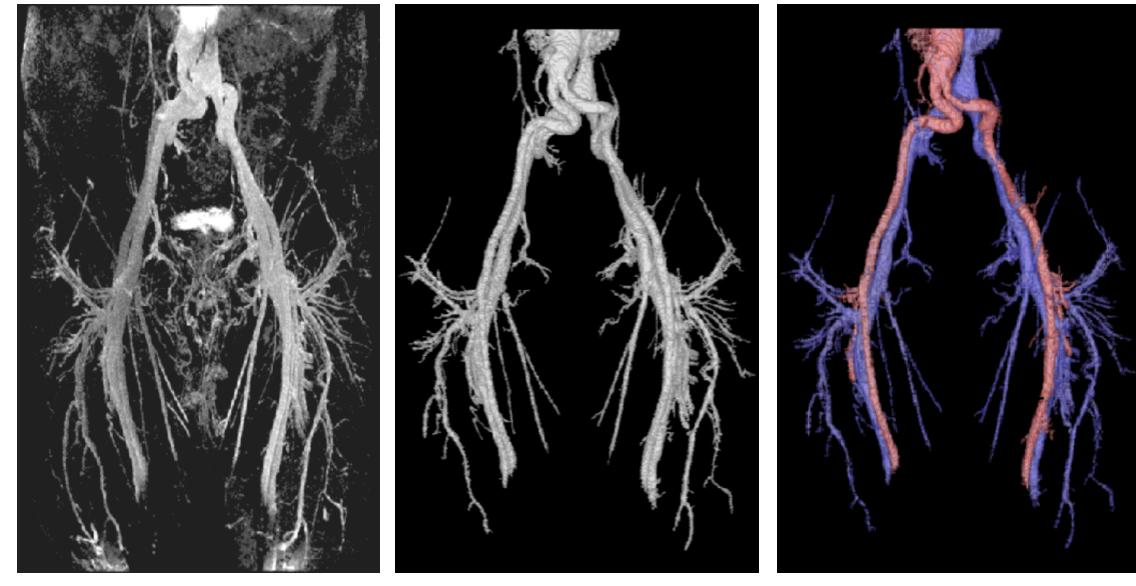
[bagci@ucf.edu](mailto:bagci@ucf.edu) or [bagci@crcv.ucf.edu](mailto:bagci@crcv.ucf.edu)

# Outline

- Fuzzy Connectivity (FC)
  - Affinity functions
- Absolute FC
- Relative FC (and Iterative Relative FC)
- Successful example applications of FC in medical imaging
- Segmentation of Airway and Airway Walls using RFC based method

# Motivation

- **Connectivity:** a popularly used tool for region growing
- **Applications:** image segmentation, object tracking, object separation
- A **fuzzy model** for connectivity analysis is essential to capture the global extent of an object using local hanging togetherness and path connectivity



CE-MRA  
Image data

Segmented  
vasculature

Separated  
arteries/veins

Separation of arteries and veins in a contrast-enhanced magnetic resonance angiographic (CE-MRA) image data using iterative relative fuzzy connectivity

*Slide credit: P. Saha*

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# Fuzzy Connected (FC) Image Segmentation

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FC segmentation is a methodology for finding M objects in a digital image based on **user-specified seed points** and **user-specified functions**, called (fuzzy) affinities, which map each pair of image points to a value in the real interval [0, 1].

# FC Family

- Absolute *FC*
- Scale-based *FC* ( $b$ -,  $t$ -,  $g$ -scale based)
- Relative *FC*
- Iterative Relative *FC*
- Vectorial *FC*
- Hierarchical *FC*
- Model-based *FC*

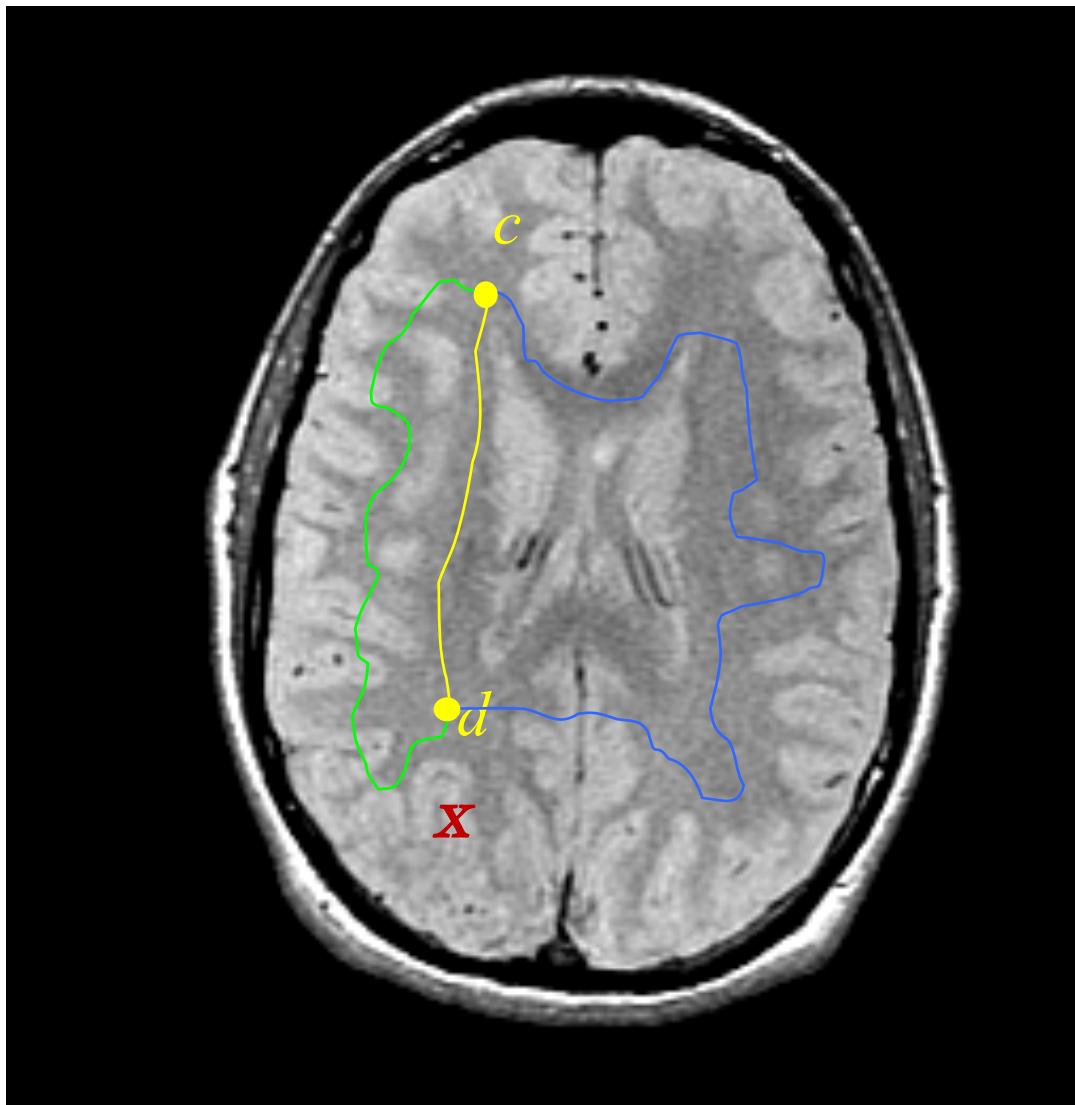


# FC Medical Image Segmentation Examples

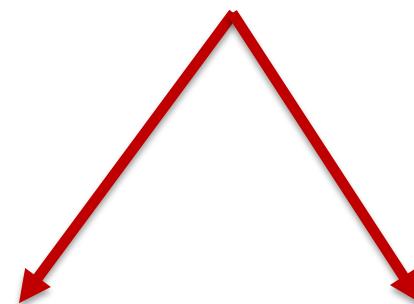
- MR
  - brain tissue, tumor, MS lesion segmentation
- MRA
  - vessel segmentation and artery-vein separation
- CT bone segmentation
  - kinematics studies
  - measuring bone density
  - stress-and-strain modeling
- CT soft tissue segmentation
  - cancer, cyst, polyp detection and quantification
  - stenosis and aneurism detection and quantification
- Digitized mammography
  - detecting microcalcifications
- Craniofacial 3D imaging
  - visualization and surgical planning



# Object Characteristics in the Images



*local hanging togetherness  
(affinity)*

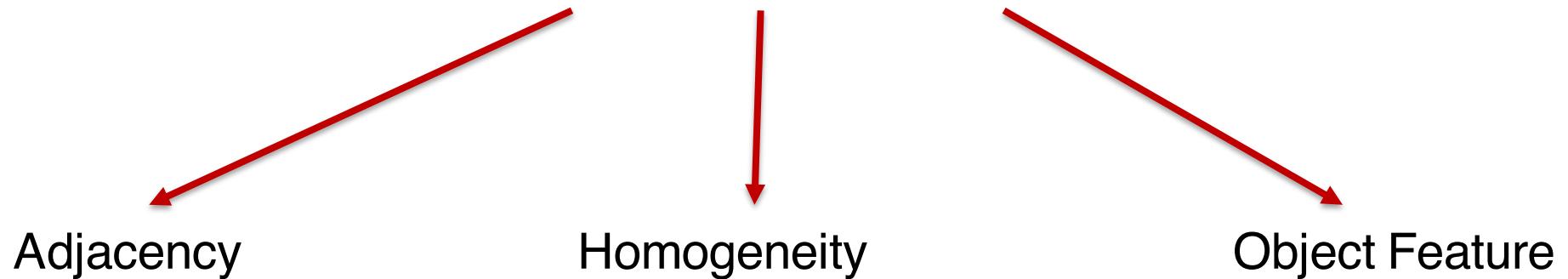


*Spatial location      intensity value  
                          (-derived)*



## FC is a global relation!

- Effectiveness of the FC algorithm is dependent on the choice of **the affinity function**, and the general setup can be divided into three components (for any voxels p and q):



**FC is a global fuzzy relation between voxels!**

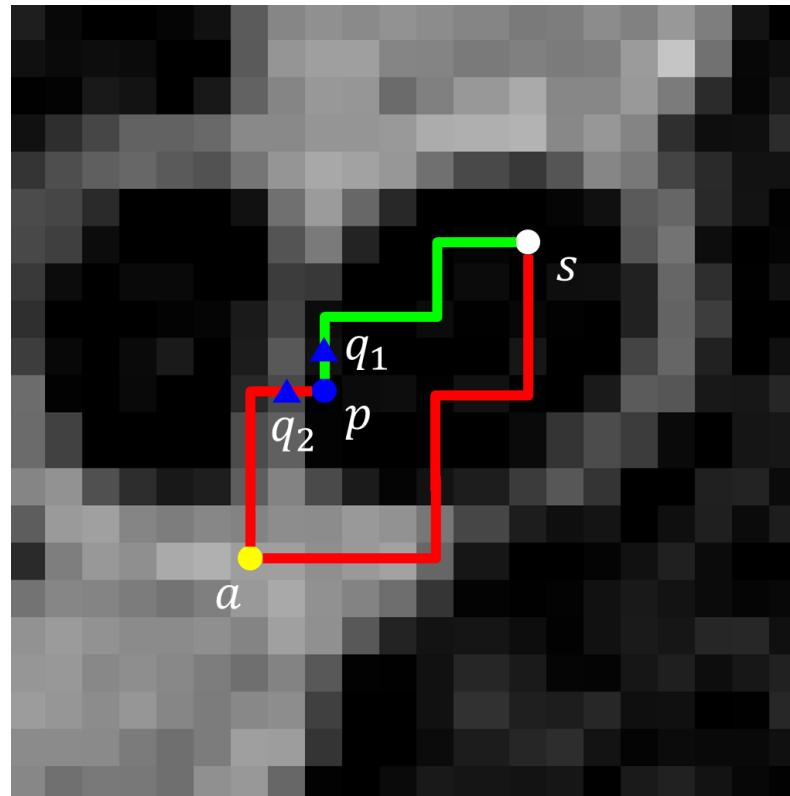
All voxels are assessed via defined affinity functions for labelling.

# Affinity

- **Definition:** local relation between every two image elements  $u$  and  $v$

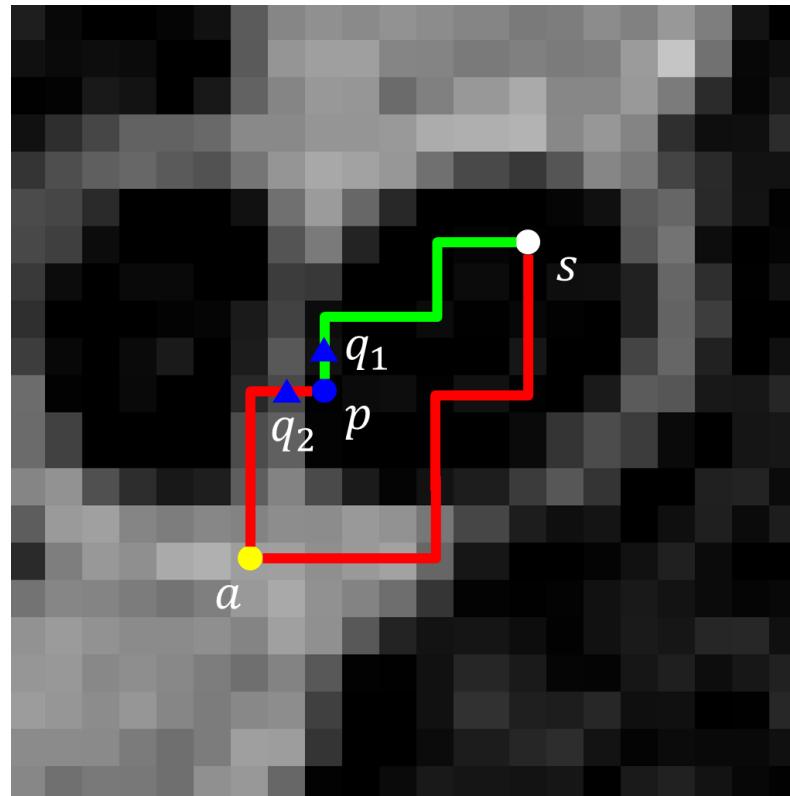
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p and q1 hang-together (than p and q2)

Green path is stronger than red path.

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- Its strength  $\alpha(a, b)$ :

$$\alpha(a, b) = \begin{cases} 1, & \text{if } a = b \\ g(\|a - b\|), & \text{if } \|a - b\| \leq D_1 \\ 0, & \text{if } \|a - b\| > D_1 \end{cases}$$

$D_1$  is a distance (known)

$g$  is a function mapping between [0,1]

# Homogeneity and Object Feature Affinities

$$\mu_\psi(p, q) = e^{-\frac{|f(p)-f(q)|^2}{2\sigma_\psi^2}},$$

$$\mu_\phi(p, q) = \min \left( e^{-\frac{|f(p)-m|^2}{2\sigma_\phi^2}}, e^{-\frac{|f(q)-m|^2}{2\sigma_\phi^2}} \right).$$



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- (1)  $\alpha(a, b)$  - Fuzzy adjacency
- (2) homogeneity  $\Psi(a, b)$  of intensity at  $a$  and  $b$ .
- (3) how close intensity features at  $a$  and  $b$  are to be expected object features -  $\Phi(a, b)$

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$$\kappa(a, b) = h[\alpha(a, b), \psi(a, b), \phi(a, b)]$$

# Different Affinity Functions can be devised!

$$\begin{aligned}\Psi(a, b) &= e^{-|f(a)-f(b)|} \\ \Phi(a, b) &= e^{-[\mu \frac{f(a)+f(b)}{2}]^2},\end{aligned}$$

f(a) and f(b): intensity values at voxel location a, b.

$\mu$ : expected object intensity

# Fuzzy Affinity and Path Strength

**Fuzzy Affinity ( $\kappa$ ):** local hanging-togetherness between two spels

- $\kappa(p, q) \in [0,1]$
- $\kappa(p, q)$  is zero if  $p, q$  are non-adjacent
- $\kappa(p, p) = 1$ , i.e., reflexive
- $\kappa(p, q) = \kappa(q, p)$ , i.e. symmetric

**Strength (  $\Pi$  ) of a path (  $\pi = \langle p_1, p_2, \dots, p_l \rangle$  )**

- $\Pi(\pi)$  = the affinity of the weakest link on the path, i.e.,

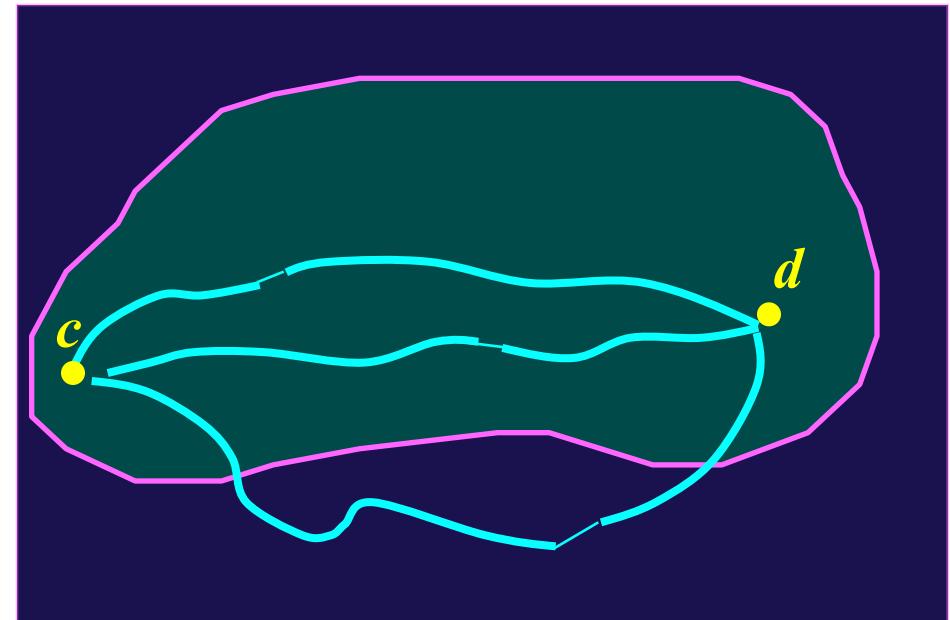
$$\Pi(\pi) = \min_{1 \leq i < l} \kappa(p_i, p_{i+1})$$

# Fuzzy Connectivity

- Fuzzy connectedness is a global fuzzy relation  $K$  among voxels. Its strength  $K(c, d)$  for any  $c, d$  is defined as:

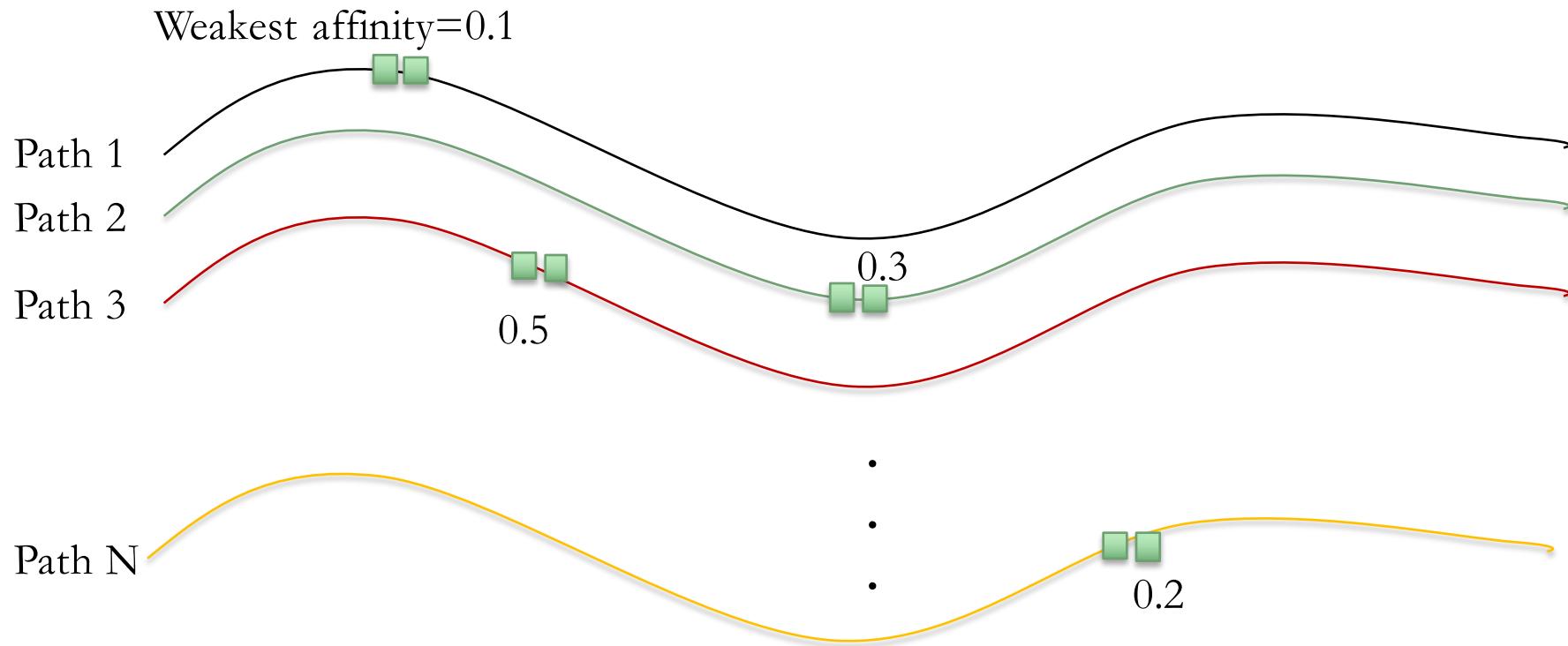
(1) Every path  $\pi$  between  $c$  and  $d$  has a strength which is the smallest affinity along  $\pi$ .

(2)  $K(c, d)$  is the strength of the strongest path.



$$K(c, d) = \max_{\pi} \left\{ \min_i [\kappa(c_i, c_{i+1})] \right\}$$

# Numerical Example



(assuming there are  $N$  paths between voxels  $c$  and  $d$ )

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6. For each voxel  $\mathbf{d}_i$ , determine its fuzzy connectedness to the seed point  $\mathbf{c}$  as the maximum strength of all possible paths  $\langle \mathbf{c}, \dots, \mathbf{d}_i \rangle$  and form connectedness map.

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7. Threshold connected map to obtain object containing  $\mathbf{c}$

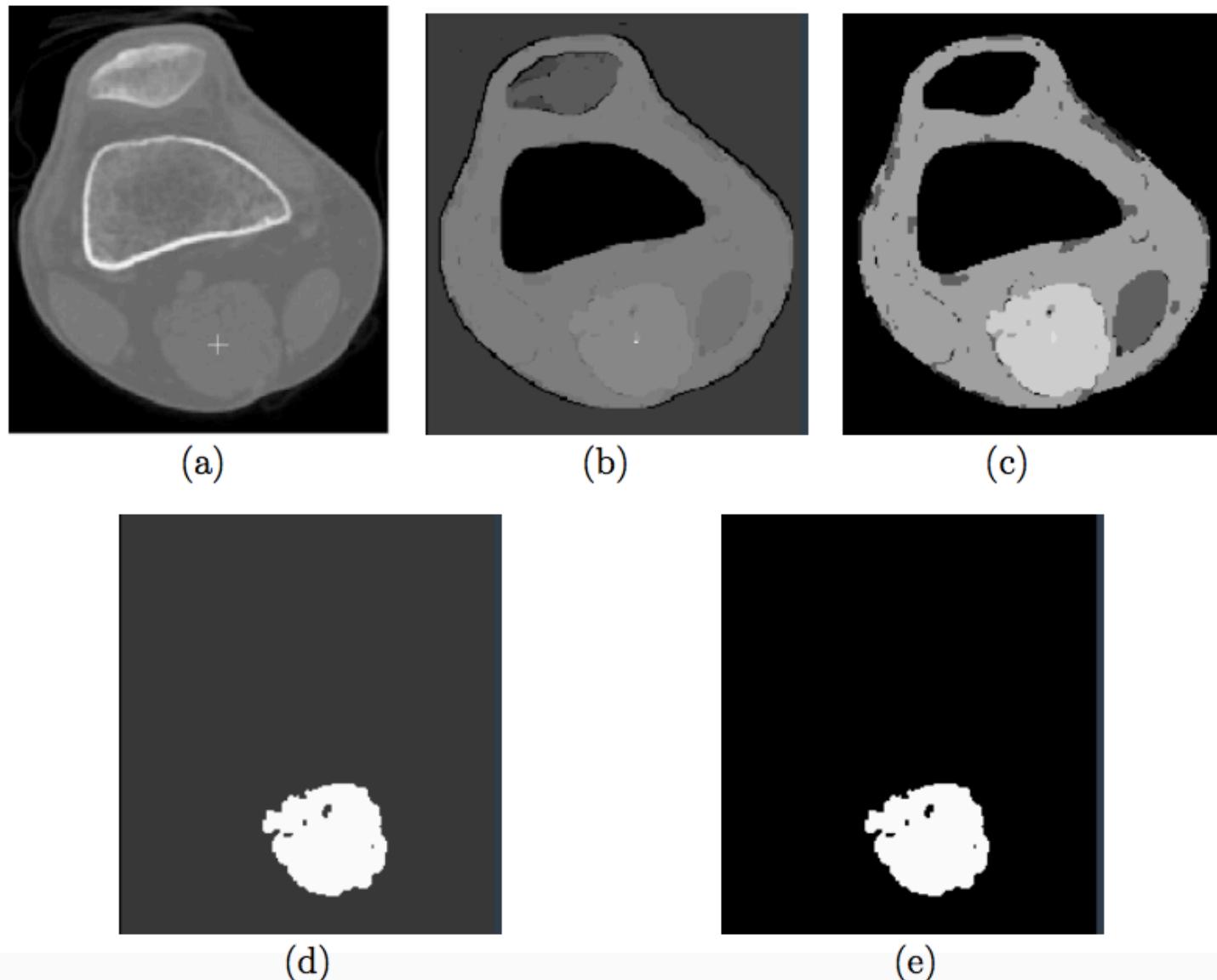
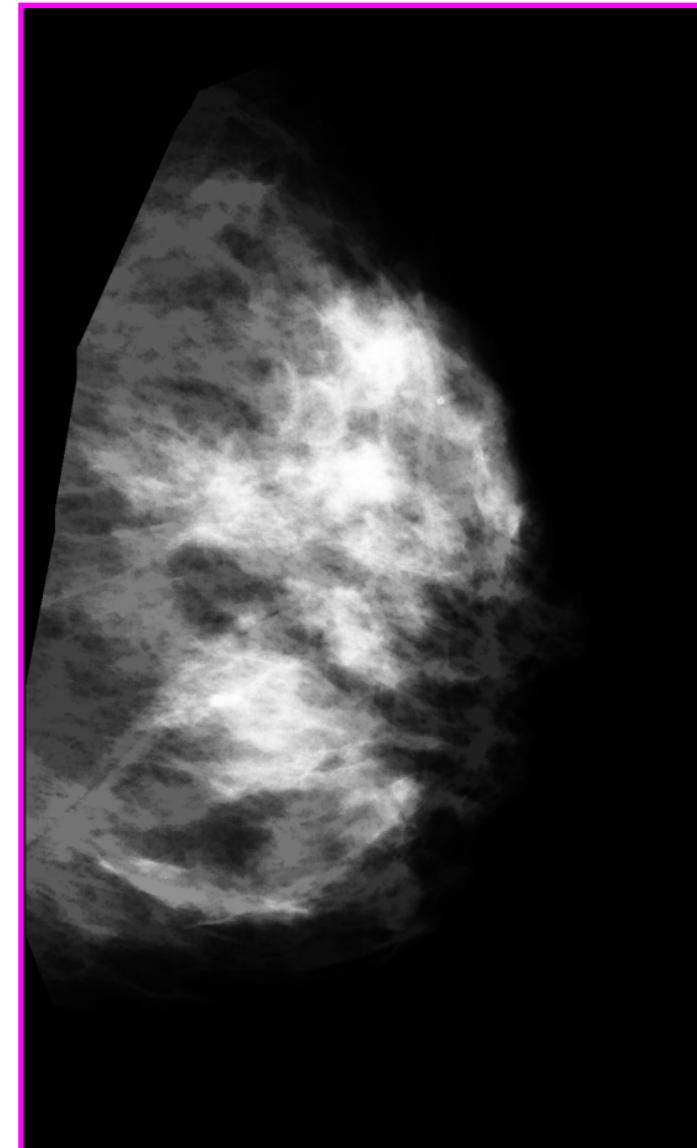
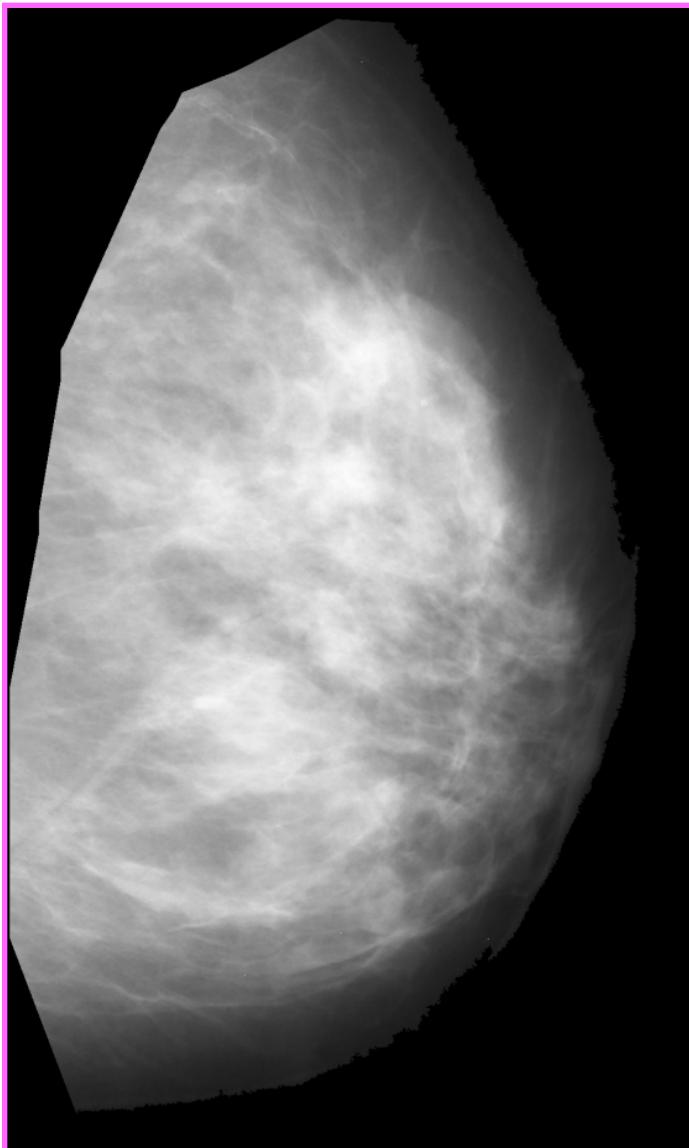


Illustration of equivalent affinities. (a) A 2D scene — a CT slice of a human knee. (b), (c) Connectivity scenes corresponding to affinities  $\psi_\sigma$  with  $\sigma = 1$  and  $\sigma = 10.8$ , respectively, and the same seed spel (indicated by + in (a)) specified in a soft tissue region of the scene in (a). (d), (e) Identical AFC objects obtained from the scenes in (b) and (c), respectively.



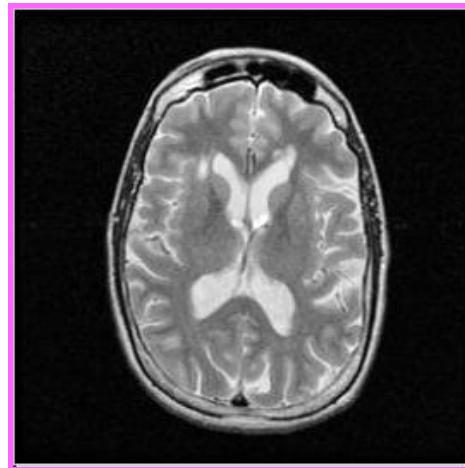
# Quantifying Breast Density



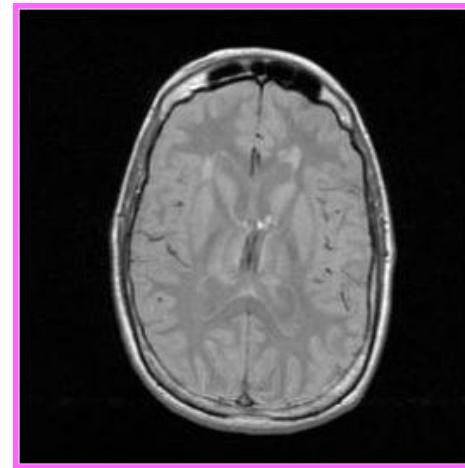


# Brain MS Lesion Quantification

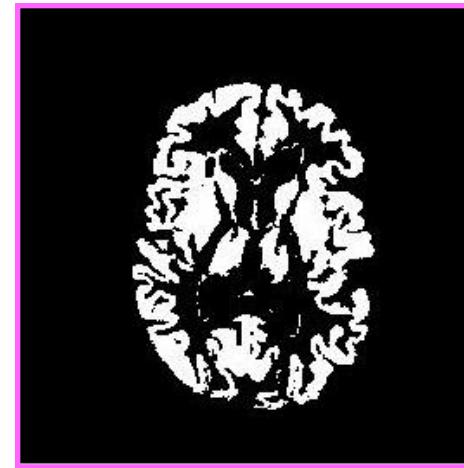
*T2*



*PD*

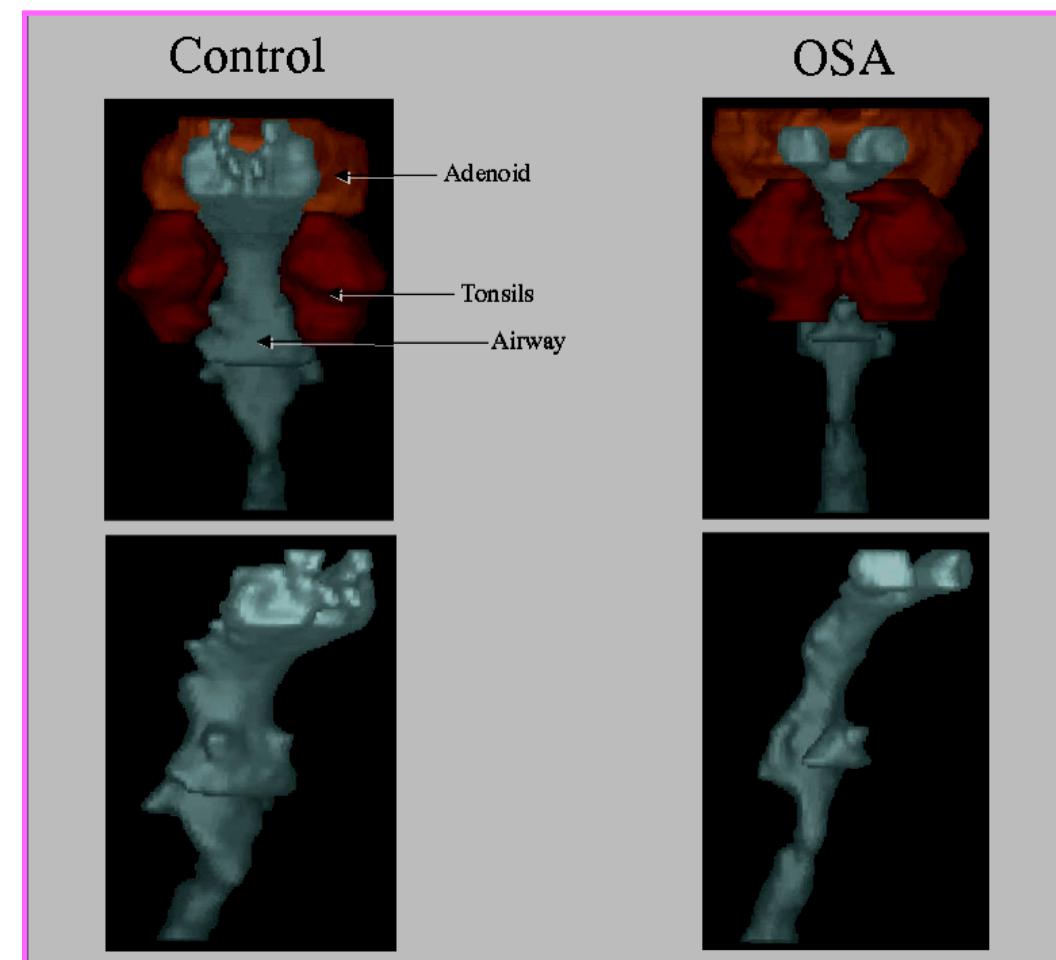
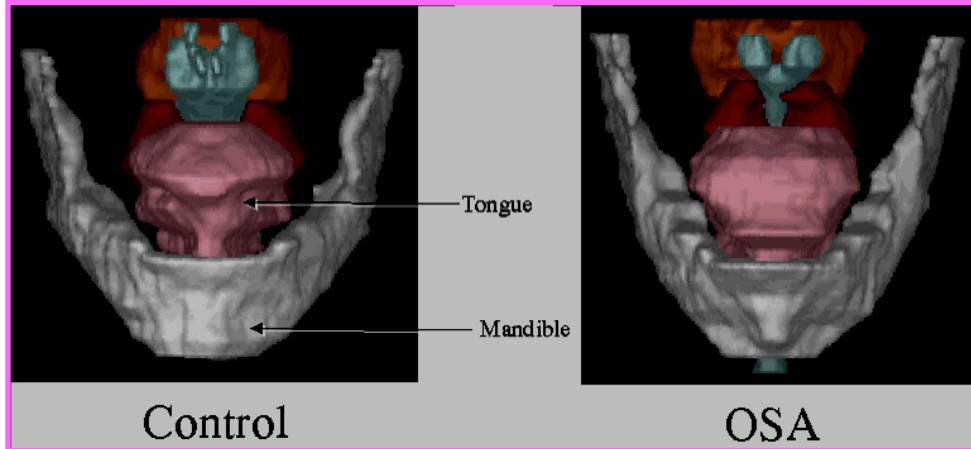


*GM*



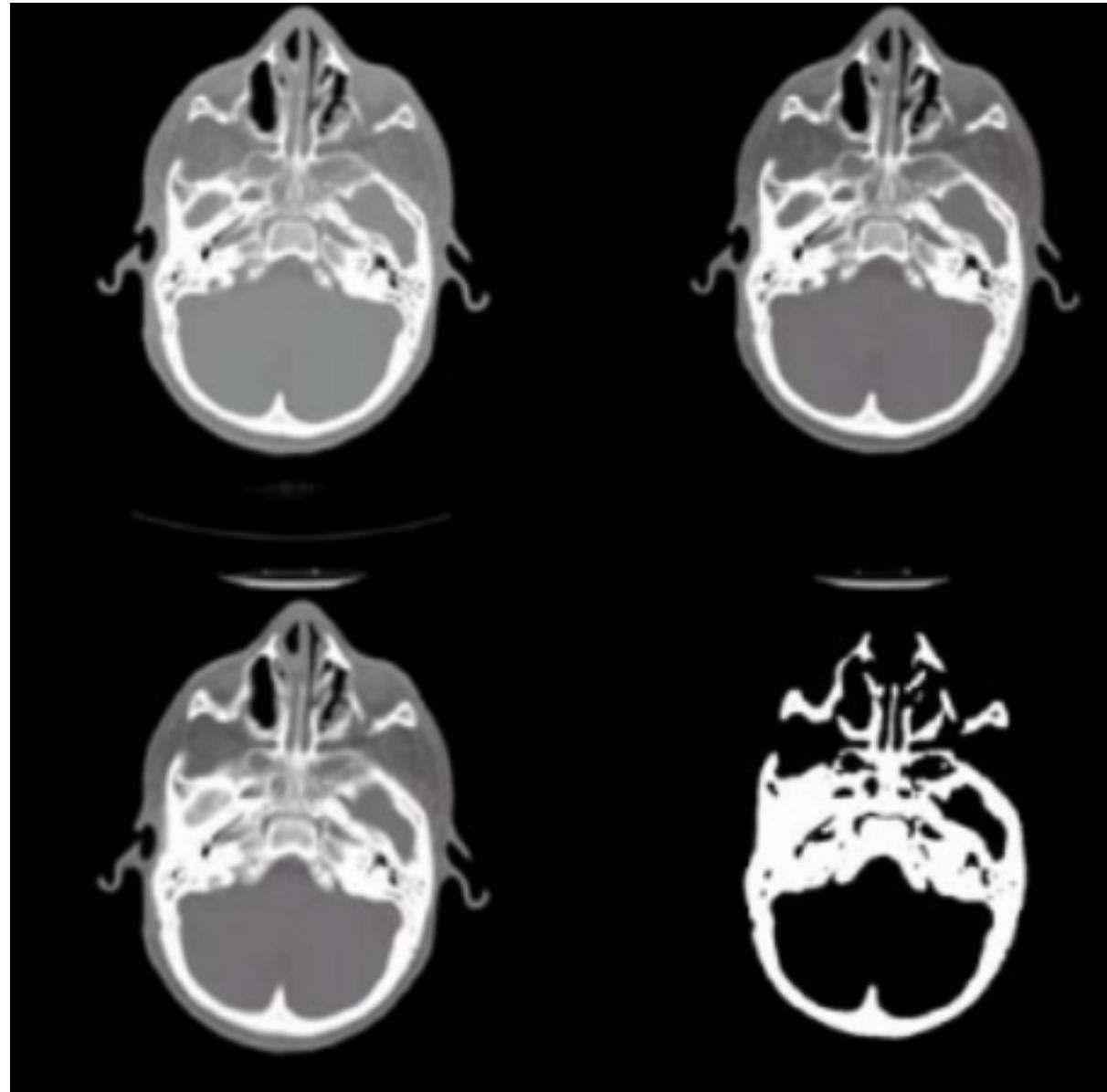


## Upper Airway Study in Children with Obstructive Sleep Apnea



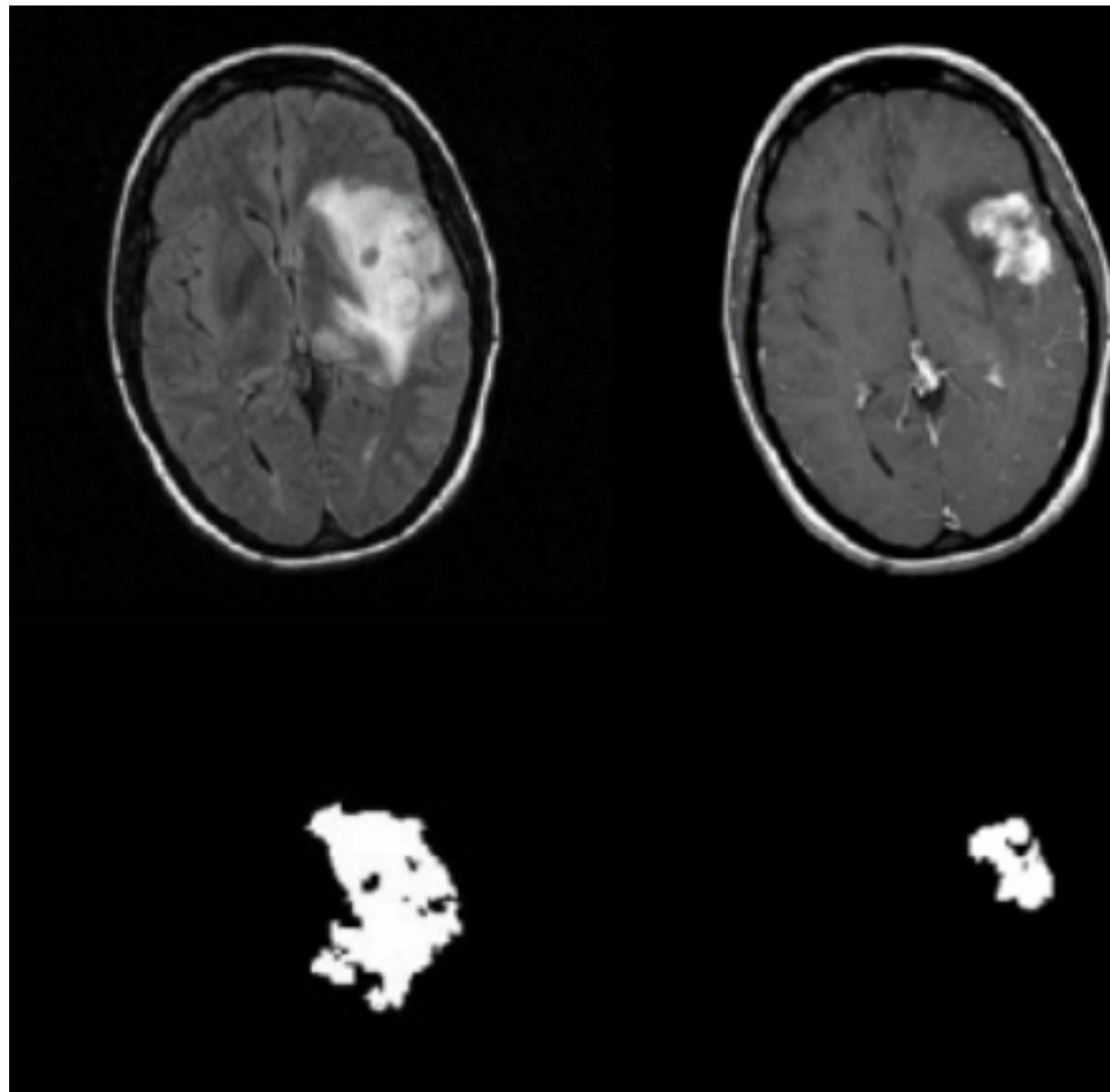


## CT Skull Extraction





# Brain Tumor Quantification - MRI



# Relative Fuzzy Connected (RFC) Image Segmentation

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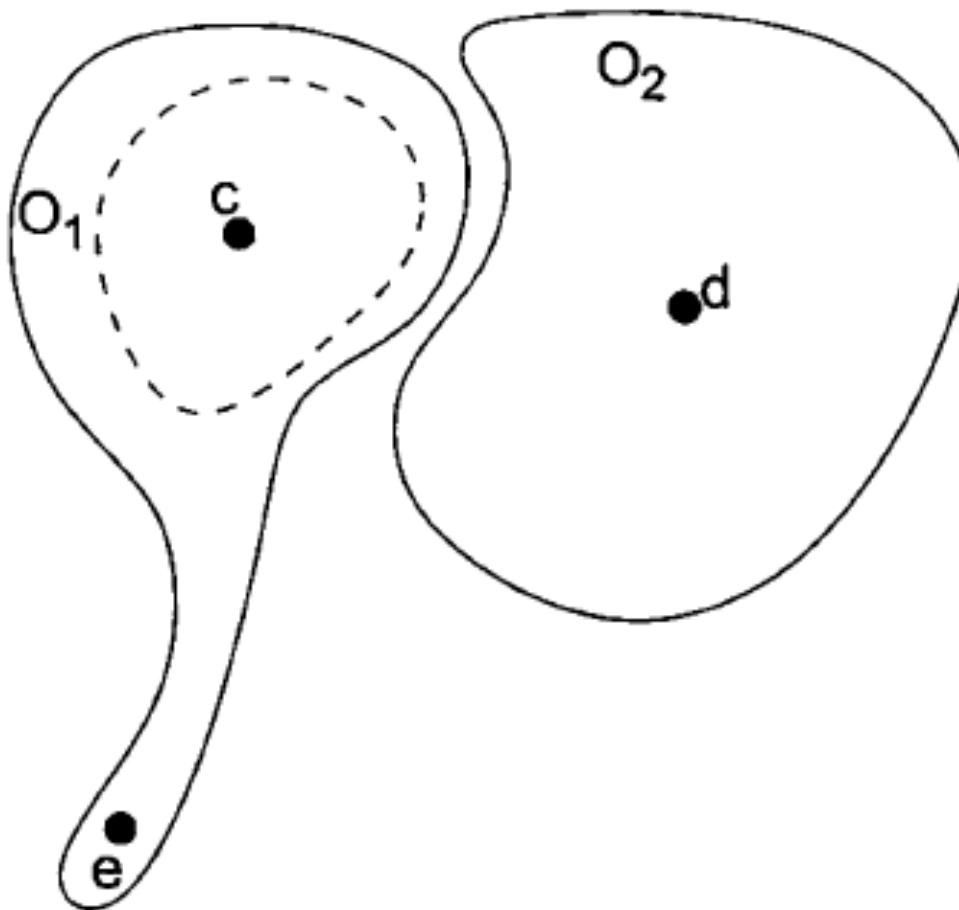
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- These 2-object RFC was extended into multiple-object RFC by the same authors



# Motivation for RFC (and IRFC)



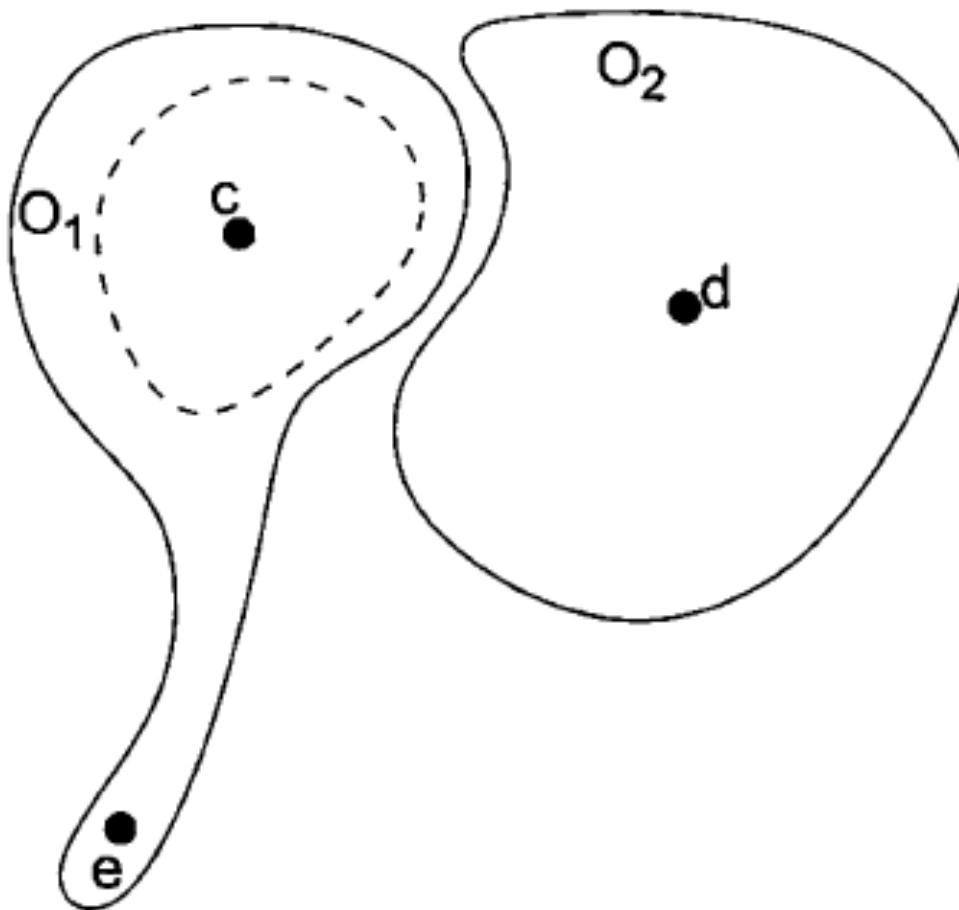
FC may fail to identify objects in this situation.

-Objects O<sub>1</sub> and O<sub>2</sub> are located very close to each other.

Due to limited resolution, border  
Between O<sub>1</sub> and O<sub>2</sub> may be weak,  
Causing homogeneity between d and e, and  
Homogeneity between c and e be similar!



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## Solution:

If O<sub>1</sub> is segmented first, paths between e and d are omitted!  
It will be iterative process, IRFC.



# Motivation for IRFC

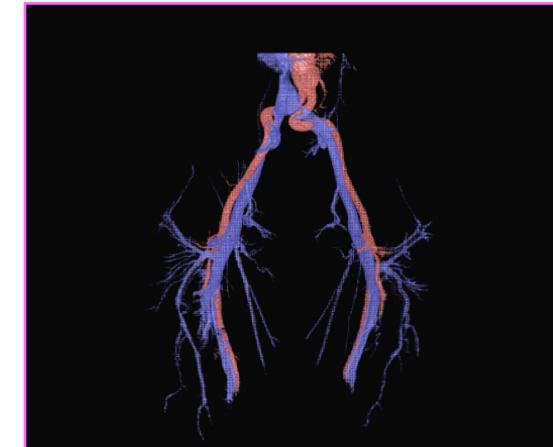
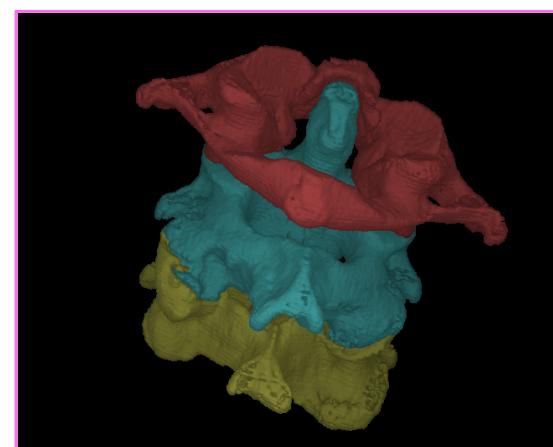
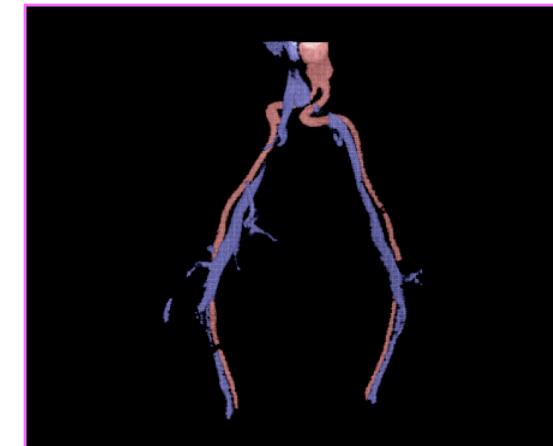
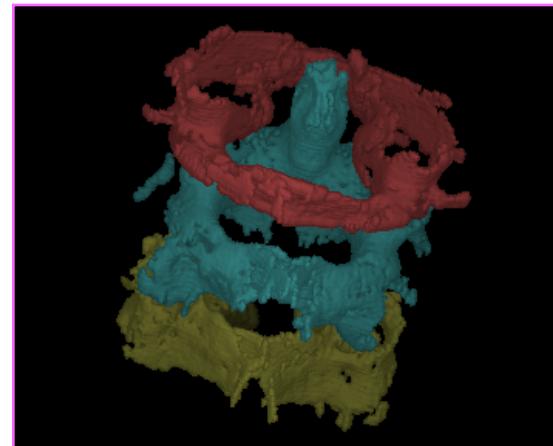
- Artery-vein separation MRA





# RFC and IRFC

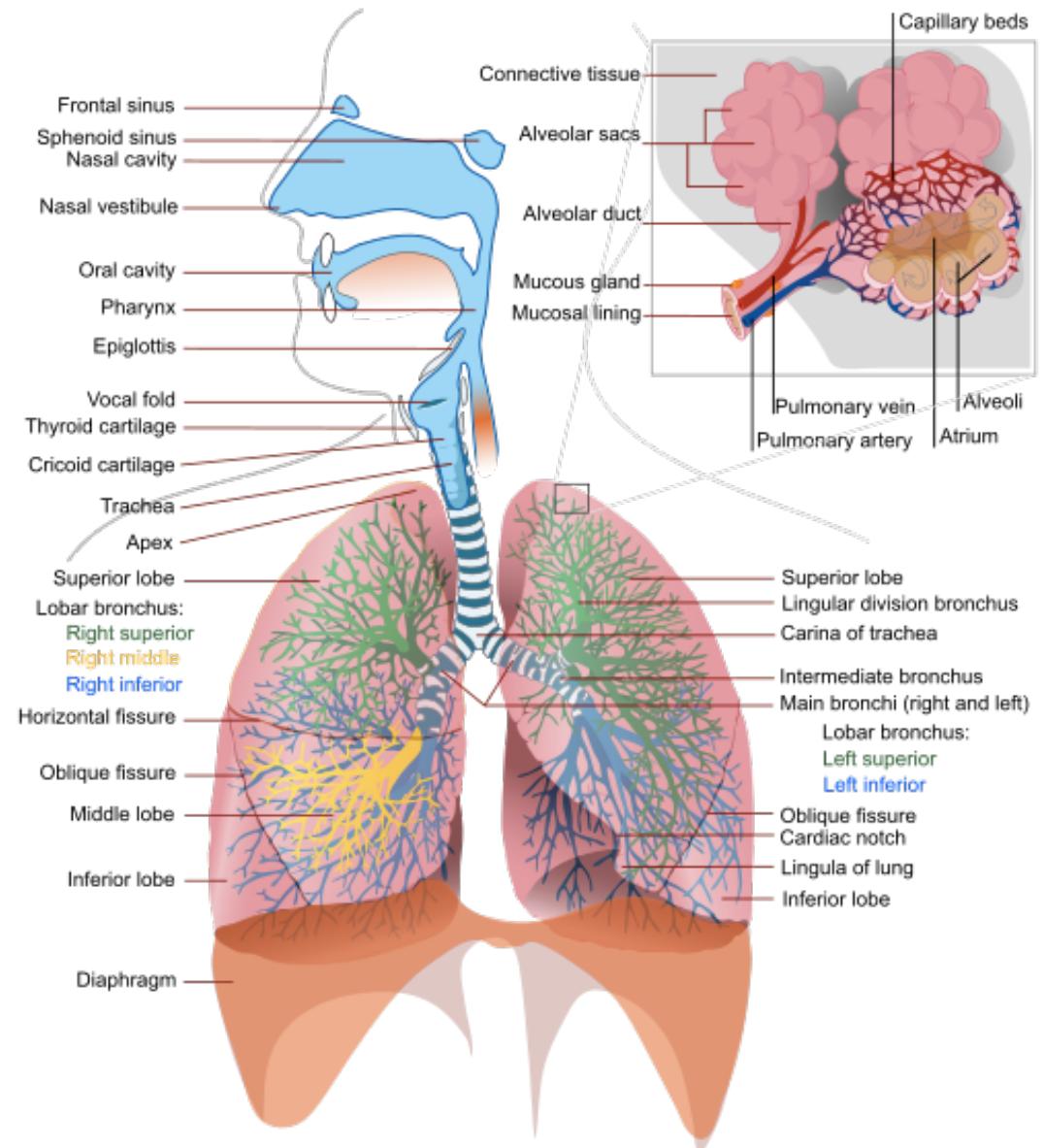
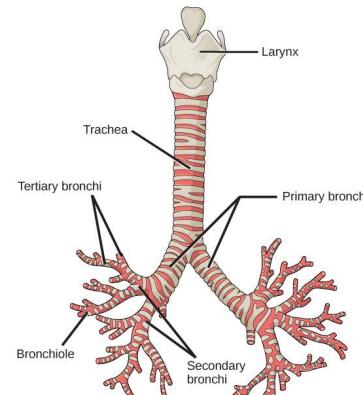
*RFC*



*IRFC*

# Airway and Airway Wall Segmentation with RFC

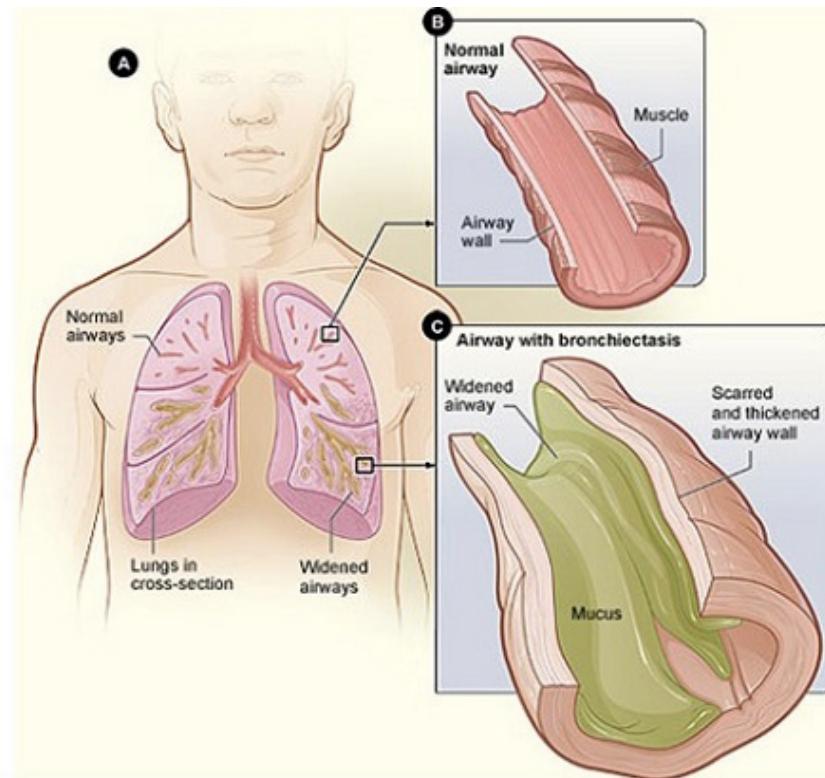
- Airways are the air-conducting structures (bronchi and bronchioles) bringing air into and out of the lungs from sites of gas exchange (alveoli).



- Credit: [healthhype.com](http://healthhype.com)

# Airway and Airway Wall Segmentation with RFC

- Airways are pathologically involved in various lung diseases. As examples, bronchiectasis is the dilation of airways (enlarged lumen), often resulting from chronic infection (Bagci et al., CMIG 2012), obstruction, and inflammation.



Credit: Corehealthclub

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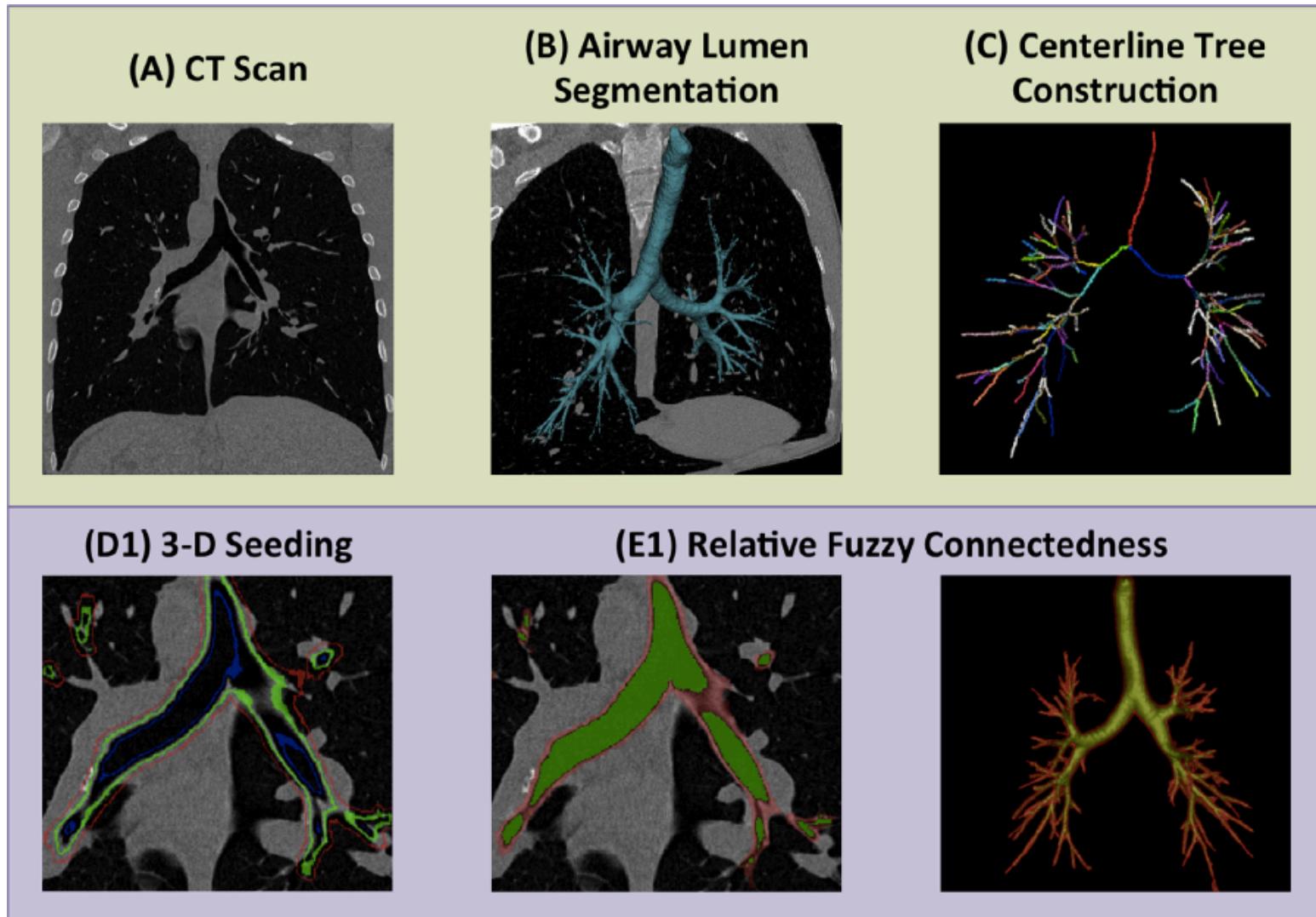
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- Due to the inherent complexity of airway structures and the resolution limitations of CT, manually tracing and analyzing airways is an extremely challenging task, taking more than 7 h of intensive work per image
- A precise method for segmentation of airways and its walls may facilitate better quantification of airway pathologies (and understanding of disease progression)



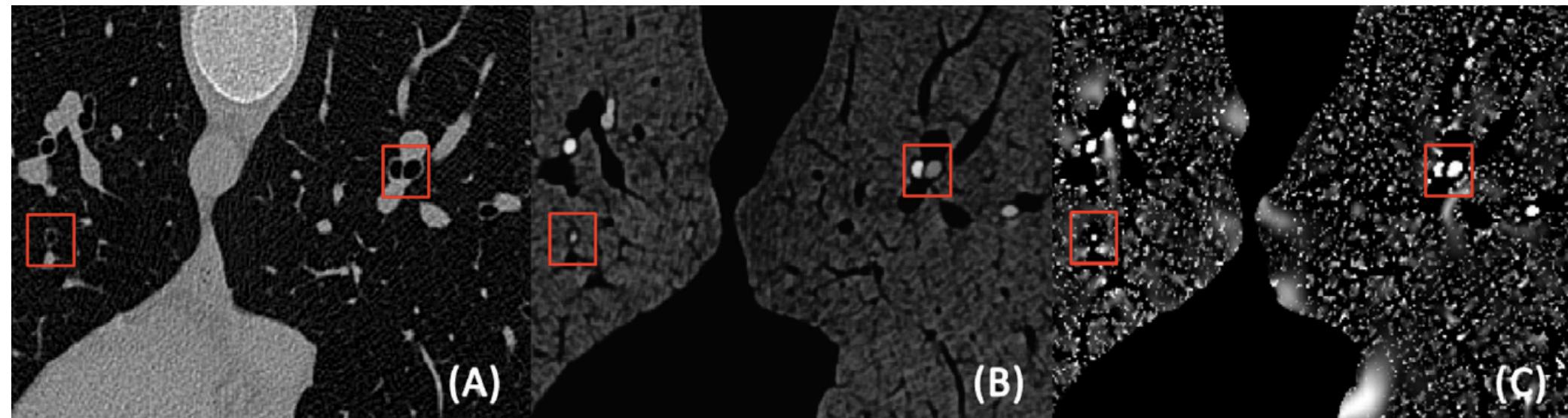
# Airway and Airway Wall Segmentation with RFC



*(Credit: Xu, Bagci, et al. Medical Image Analysis 2015. The state of the art method)*



# Airway Segmentation

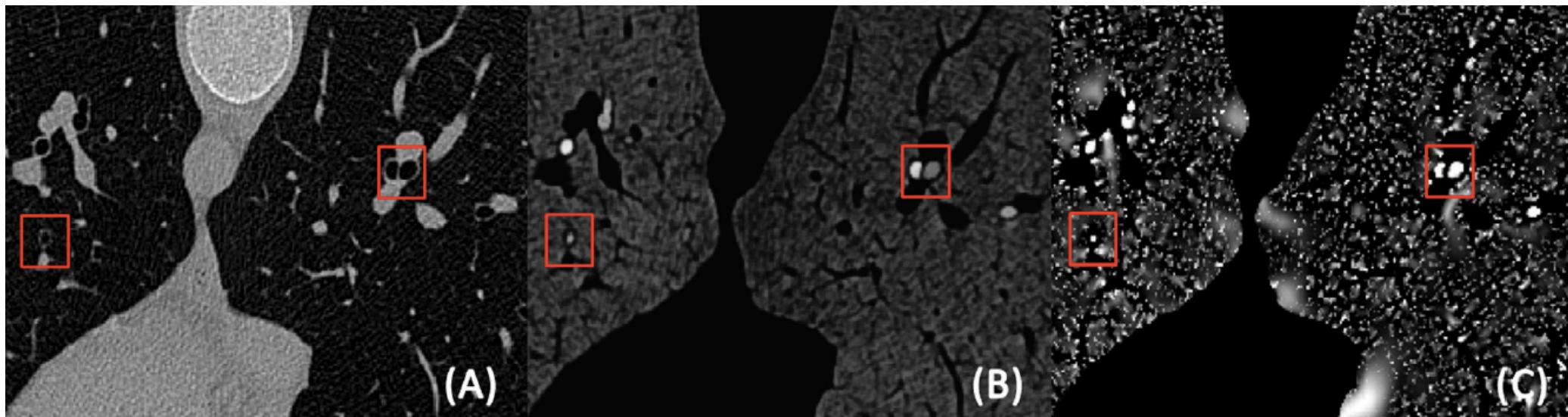


Morphological operations

Vesselness



# Airway Segmentation



(A)

(B)

(C)

Morphological operations

Good for large airways,  
Small airways can be detected  
to some extent, but limited.  
computationally expensive

Vesselness

Good for small airways,  
But numerous false positives



FC can combine these two methods within a single framework!

Large airways

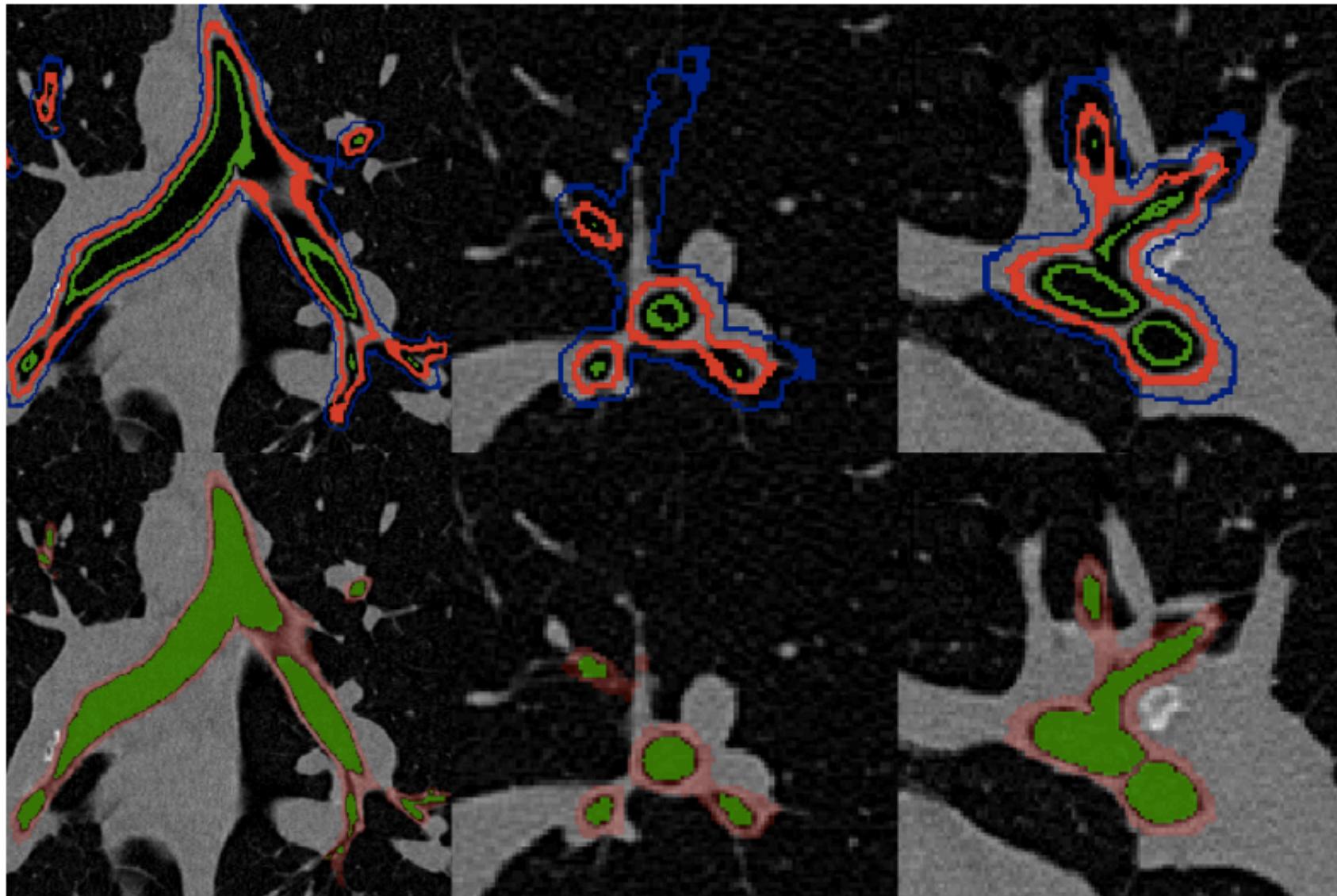
$$\mu_{\psi/\phi}^{\text{FC}} = \begin{cases} \mu_{\psi/\phi}^I, & \text{if } ls > ls^{\max}; \\ k\mu_{\psi/\phi}^I + (1 - k)\sqrt{\mu_{\psi/\phi}^D \mu_{\psi/\phi}^V}, & \text{otherwise,} \end{cases}$$

small airways

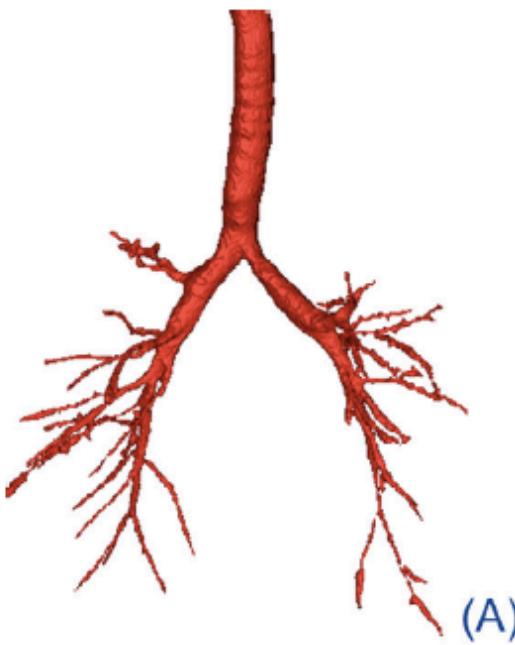
Where ls denotes local scale, k is a weight parameter, and D shows morphologically processed Image, V indicates vesselness image.



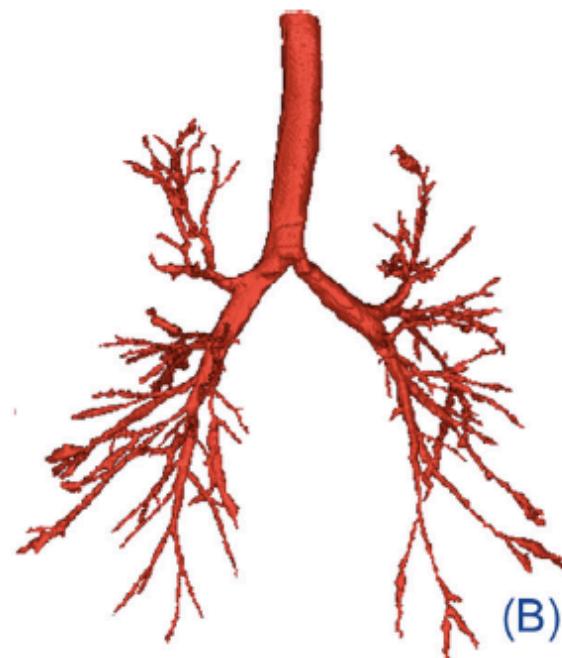
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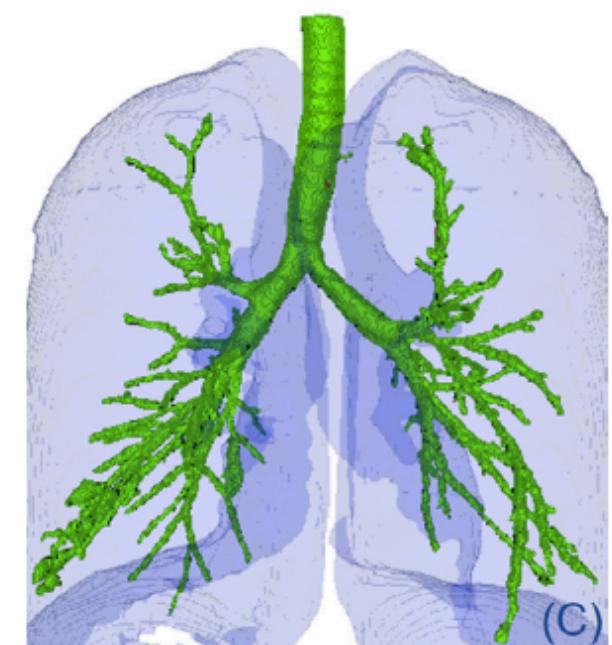
# Airway and Airway Wall Segmentation with RFC



Segmentation results  
Without fine tuning of  
parameters



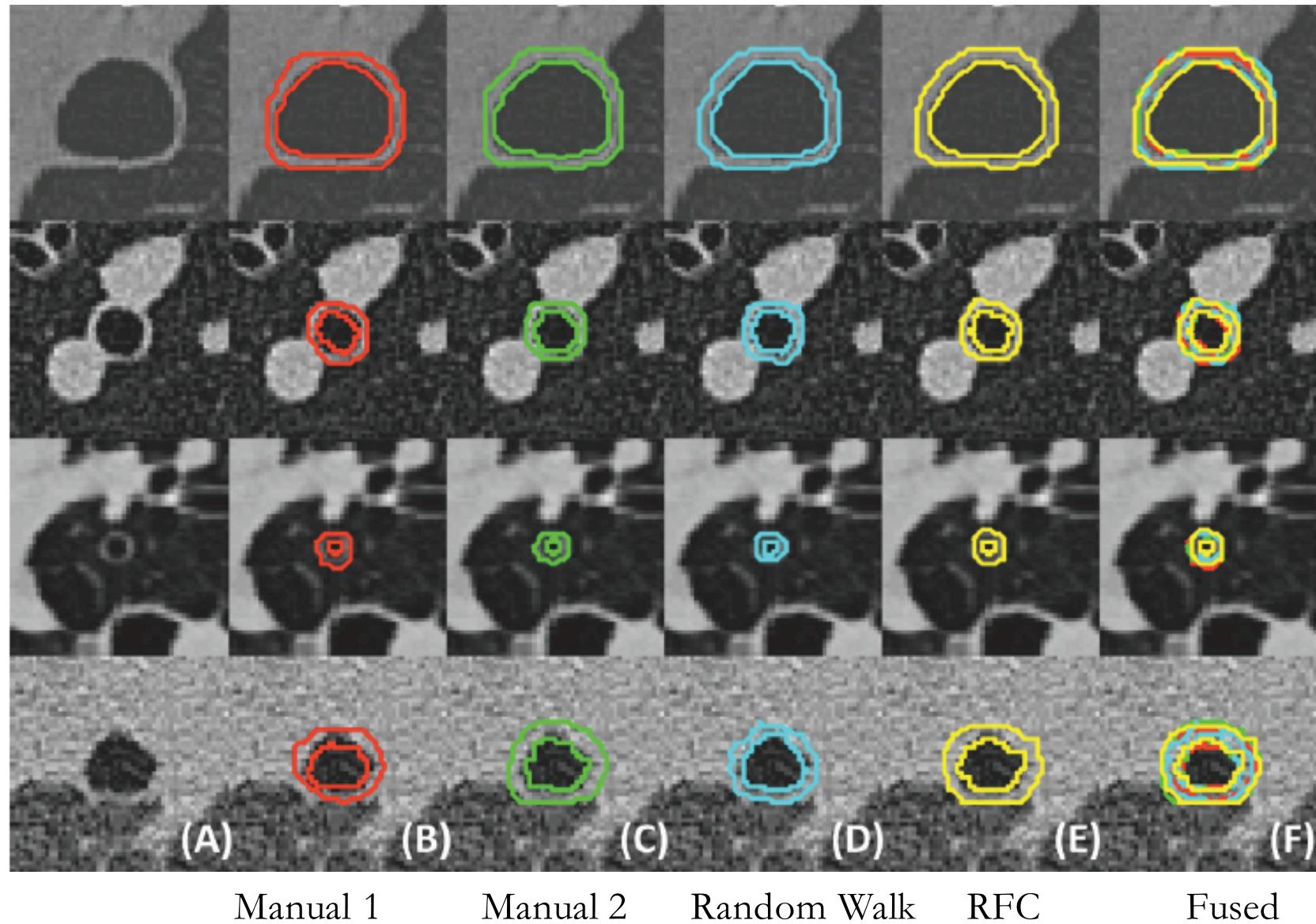
Segmentation results  
With fine tuning



Reference segmentation  
results



# Airway and Airway Wall Segmentation with RFC





# Summary

- FC is a strong segmentation tool fit for many biomedical image segmentation problems
- Affinity functions are the key stones for FC
- FC family has different version of FC, suitable for challenging tasks
- RFC and IRFC are quite successful in segmenting complex shaped objects

# Slide Credits and References

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