Monday, September 9, 2019

NUMBER SYSTEMS

We have adopted the decimal number system as the default number system.

· seems only natural ~ 10 fingers, 10 toes

Consider:

decimal point, tenths

$D = 123.45$

100's, 10's, 1's, hundredths

so $D = 1 \times 100 + 2 \times 10 + 3 \times 1 + 4 \times \frac{1}{10} + 5 \times \frac{1}{100}$

In general, we may express this as

$$\underbrace{d_n\, d_{n-1}\, d_{n-2} \cdots d_1\, d_0}_{\text{integer part}} . \underbrace{d_{-1}\, d_{-2} \cdots d_{-p}}_{\text{fractional part}}$$

which is evaluated as a power series

$$D = d_n 10^n + d_{n-1} 10^{n-1} + \ldots + d_0 + d_{-1} 10^{-1} + \ldots + d_{-p} 10^{-p}$$

This is a base-10 number; it has a radix of $r = 10$

Properties:
- each digit d_j multiplied by 10^j
- $0 \le d_j < 10$

Can be expressed this way for any radix r.

$$D = d_n r^n + d_{n-1} r^{n-1} + \ldots + d_0 + d_{-1} r^{-1} + d_{-2} r^{-2} + \ldots$$

or more compactly as

$$\boxed{D = \sum_{j=-p}^{n} d_j r^j}$$

Power series representation of a positional number system

where $0 \le d_j < r$

The radix of particular importance to us is $r = 2$, corresponding to the binary number system

The binary digit is called a bit

Example: Determine the decimal value of 1011.01_2 — means $r = 2$.

$$D = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2}$$

(note all digits d_j obey $0 \le d_j < 2$)

$$= 11.25_{10}$$

$r = 10$

Number system conversion

Radix-r to radix-10 conversion is easy

$$D = \sum_{j=-p}^{n} d_j r^j$$

Radix-10 to radix-r is a little more work. In the most general case, this is done in two parts:

- integer part: successive division by r (important)
- fractional part: successive multiplication by r (not discussed in textbook)

Integer example: convert 13_{10} to binary

Need to find: $[d_3\ d_2\ d_1\ d_0]_2$

Decimal operation	quotient	remainder	equivalent binary operation
$13 \div 2$	6	1	$d_3 d_2 d_1 . d_0$ — d_0 is remainder so $d_0 = 1$
$6 \div 2$	3	0	$d_3 d_2 . d_1$ — $d_1 = 0$
$3 \div 2$	1	1	$d_3 . d_2$ — $d_2 = 1$
$1 \div 2$	0	1	$0 . d_3$ — $d_3 = 1$

Thus, $D = 1101_2$

Fractional part (if curious!): Convert 0.625_{10} to binary

Need to find: $[0.\,d_{-1}\,d_{-2}\,d_{-3}]$

Decimal operation	product	integer part	equivalent binary operation	
0.625×2	1.25	1	$d_{-1}.d_{-2}d_{-3}$	$d_{-1} = 1$
0.25×2	0.5	0	$d_{-2}.d_{-1}$	$d_{-2} = 0$
0.5×2	1.0	1	$d_{-3}.0$	$d_{-3} = 1$

Thus, $D = 0.625_{10} = 0.101_2$

Octal and hexadecimal numbers

There are two other very common radices

- $r = 8$, octal numbers $(0 \le d_j < 8)$
- $r = 16$, hexadecimal numbers $(0 \le d_j < 16)$