Wednesday, September 25, 2019

Minterms (continued)

2-variable system A, B

not minterms: A, $\bar{B}$, etc.
(missing literals)

3-variable system A, B, C

not minterms: AC, $\bar{B}$, $B\bar{C}$, etc.
(here too)

Sum terms and maxterms

- Sum term can be a single literal
- or the ORing of 2 or more literals

E.g.,
- A
- $\bar{A} + B + \bar{D}$
- $(\bar{A} + B + \bar{D})C$ (not a sum term overall, but the expression in parentheses is!)

- Analogous to a minterm, a <u>maxterm</u> contains the sum of all system variables.

2-variable system

maxterms:
$\bar{A} + \bar{B}$
$\bar{A} + B$
$A + \bar{B}$
$A + B$

3-variable system

$\bar{A} + \bar{B} + \bar{C}$	$A + \bar{B} + \bar{C}$
$\bar{A} + \bar{B} + C$	$A + \bar{B} + C$
$\bar{A} + B + \bar{C}$	$A + B + \bar{C}$
$\bar{A} + B + C$	$A + B + C$

Boolean operator precedence

1. NOT (highest precedence)
2. AND
3. OR (lowest precedence)

- Parenthesized expressions can force precedence

e.g., $A \cdot B + C \longrightarrow (AB)$ ORed with C
$A \cdot (B + C) \longrightarrow (B+C)$ ANDed with A

Sum-of-products form

A sum-of-products (SOP) expression contains the ORing of one or more product terms

E.g.,

AB (a single product term)

$AB + \bar{B} + ACD$

$AB\bar{C} + \bar{A}BC + A\bar{B}C$

$A + B + \bar{C}$ (sum of 3 single-literal product terms)

Not SOP expressions:

$BC\underbrace{(CD + A\bar{B}\bar{C})}_{\text{but this is !}}$

$\overline{A\bar{B}C + BC\bar{D} + \bar{A}C\bar{D}}$

- overall, not SOP, but yes to what's under the bar !

The canonical SOP form

A special case of SOP expressions where all product terms are minterms

E.g., for a 3-variable system A, B, C

$F = \bar{A}BC + A\bar{B}C + AB\bar{C}$ ← all product terms are minterms

$F = AB + B + A\bar{B}\bar{C}$ ← Nope!

missing literals

Truth tables, minterms, and canonical forms

A very simple and important relationship between all these, This relationship allows us to determine a logic function from a truth table.

Consider $F = A \oplus B$ (exclusive-or function)

Truth table:

Row	A	B	TRUE minterm	F
0	0	0	$\bar{A}\bar{B}$	0
1	0	1	$\bar{A}B$	1
2	1	0	$A\bar{B}$	1
3	1	1	AB	0

number rows corresponding to binary counting order of A,B

in each row, list the minterm that evaluates to TRUE for A and B on that row

- minterms are designated m_0, m_1, m_2, m_3

Then gather the minterms on all rows in which $F = 1$.

$$F = \bar{A}B + A\bar{B}$$

A compact expression for the canonical SOP:

$$F = \sum(\underbrace{m_1}_{\text{minterm for row 1}}, \underbrace{m_2}_{\text{for row 2}})$$

or simply: $F = \sum(1, 2)$

<u>Example:</u> Write the canonical SOP expression for the C_{out} bit in a 1-bit full adder

Truth table:

Row	A	B	C_{in}	TRUE minterm	C_{out}
0	0	0	0	$\bar{A}\bar{B}\bar{C}_{in}$	0
1	0	0	1	$A B C_{in}$	0
2	0	1	0	$\bar{A}B\bar{C}_{in}$	0
3	0	1	1	$\bar{A}BC_{in}$	1
4	1	0	0	$A\bar{B}\bar{C}_{in}$	0
5	1	0	1	$A\bar{B}C_{in}$	1
6	1	1	0	$AB\bar{C}_{in}$	1
7	1	1	1	ABC_{in}	1

Pick off and sum the minterms for which $C_{out} = 1$.

$$C_{out} = \bar{A}BC_{in} + A\bar{B}C_{in} + AB\bar{C}_{in} + ABC_{in}$$

or $$C_{out} = \sum (m_3, m_5, m_6, m_7)$$

$$= \sum (3, 5, 6, 7)$$