Wednesday, September 11, 2019

Octal and hexadecimal (continued)

For "hex" numbers, we need 16 different numerals (but only have 10!)

d_j : 0-9, A-F

first 10 — use letters

Hex	Decimal
A	10
B	11
C	12
D	13
E	14
F	15

Example: Find decimal equivalent of $3A9.C_{16}$

$$D = 3 \times 16^2 + 10 \times 16^1 + 9 \times 16^0 + 12 \times 16^{-1}$$
$$= 937.75_{10}$$

Binary, octal, hex are all closely related. Octal and hex are groups of bits

- octal: 3-bit groupings
- hex: 4-bit groupings

Example: Express $1101 0_2$ in octal and hex

pad to make 3-bit group → 0 1 1 0 1 0. ← always start here

3 2

$= 32_8$

pad → 0 0 0 1 1 0 1 0.

1 A_{16} (10_{10})

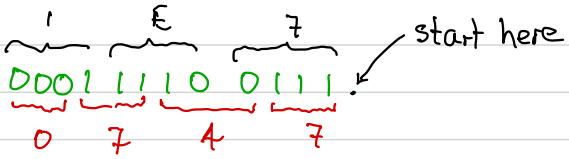
$= 1A_{16}$

Example: Decimal to hexadecimal conversion. Convert 487_{10} to hex and octal.

$487 \div 16 =$	30,	7 remainder	$d_0 = 7$
$30 \div 16 =$	1,	14 remainder	$d_1 = 14_{10} = E_{16}$
$1 \div 16 =$	0,	1 remainder	$d_2 = 1$

Therefore, $487_{10} = 1E7_{16}$

Regroup bits to form an octal number



Therefore, $487_{10} = 747_8$

Unsigned arithmetic in the binary number system

Procedures we have long used for decimal arithmetic apply for any radix.

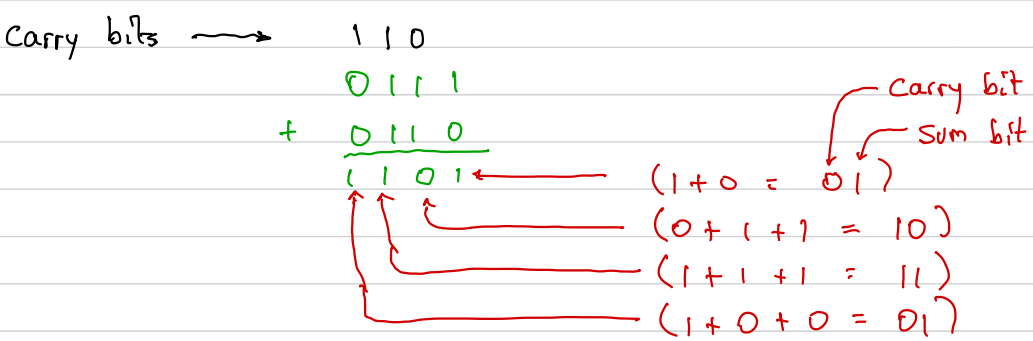
Adding decimal numbers: $5 + 3 + 6 = 14$

carry digit — sum digit

Adding bits: $1 + 1 + 0 = 2_{10} = 10_2$

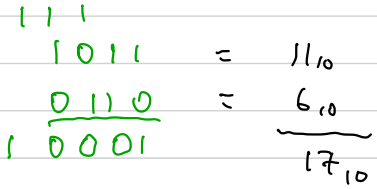
carry bit — sum bit

Example: Add the 4-bit unsigned binary numbers $0111_2 + 0110_2$



An overflow condition occurs when not enough bits to represent the correct answer.

Example: Add 4-bit numbers 1011_2 and 0110_2 to obtain a 4-bit result.



Here, the correct result needs 5 bits, but can only use 4 bits, so the answer is 0001_2

Thus, $11_{10} + 6_{10} = 1_{10}$?!

OVERFLOW

(largest 4-bit number $1111_2 = 15_{10}$)

Each column in the addition look like

C_{in} (carry-in)

$+ a$

$+ b$

C_{out} S

carry-out ← C_{out} ; S → sum bit

This can be carried out with a 1-bit full adder circuit with the following truth table

a	b	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

all 8 possible combinations