

## 8 Karnaugh maps

### 8.1 Two input K-maps

B \ A	0	1
0	$m_0$	$m_2$
1	$m_1$	$m_3$

### 8.2 Three input K-maps

C \ AB	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

①

### 8.3 Four input K-maps

bool  $x_1$ :  
bool  $x_2$ :

CD \ AB	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$
01	$m_1$	$m_5$	$m_{13}$	$m_9$
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

SystemVerilog

0 = false  
1 = true

### 8.4 Five input K-maps

A = 0

DE \ BC	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$
01	$m_1$	$m_5$	$m_{13}$	$m_9$
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

Venn Diagram

A = 1

BC

DE \ BC	00	01	11	10
00	$m_{16}$	$m_{20}$	$m_{28}$	$m_{24}$
01	$m_{17}$	$m_{21}$	$m_{29}$	$m_{25}$
11	$m_{19}$	$m_{23}$	$m_{31}$	$m_{27}$
10	$m_{18}$	$m_{22}$	$m_{30}$	$m_{26}$

$x_1$  &  $x_2$

$$L = x_1 \& x_2$$

$$L = x_1 \& x_2$$

$$L = x_1 \& x_2$$

Truth table

$\{0,1\}$

Boolean algebra

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 1$$

$+$  = OR gate

$\cdot$  = AND gate

$$0 \cdot 0 = 0$$

$$0 \cdot 1 = 0$$

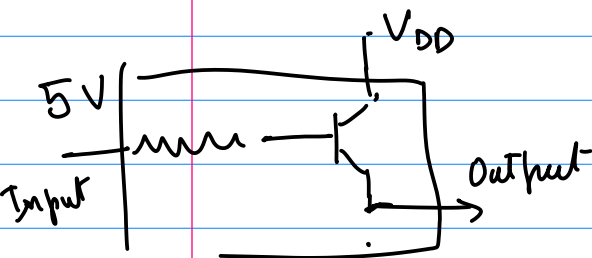
$$1 \cdot 0 = 0$$

$$1 \cdot 1 = 1$$

AND gate

$$L = x_1 \cdot x_2 = x_1 x_2$$

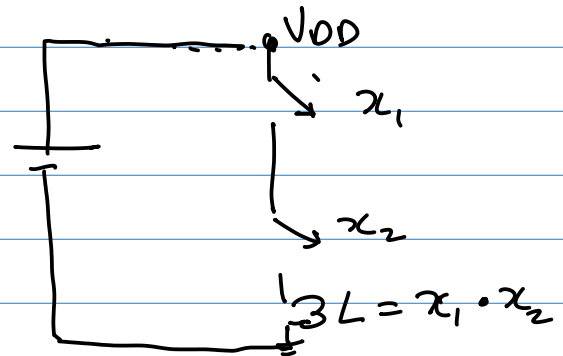
Switching circuit



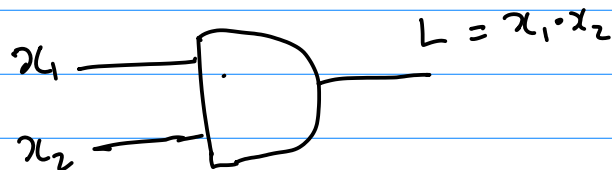
$$0 = \text{GND} \leq V_{IL}, V_{OL}$$

$$1 = V_{DD} (5V, 3.5V) \geq V_{IH}, V_{OH}$$

AND gate from switches?



ANSI symbol

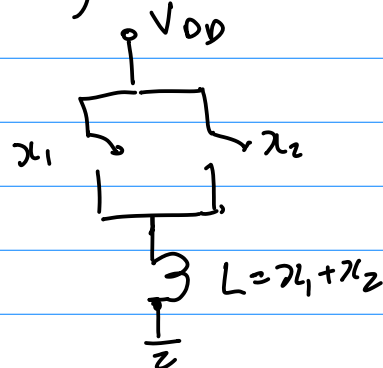


OR gate

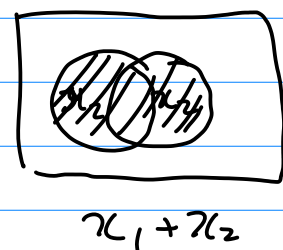
Truth table

$x_1$	$x_2$	$L = x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1

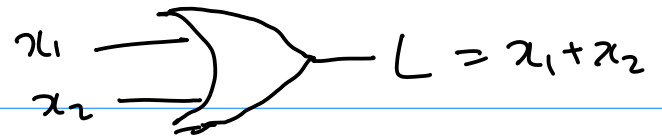
Switching circuit



Venn diagram



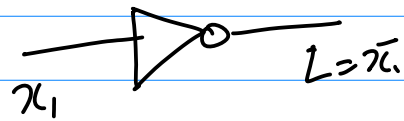
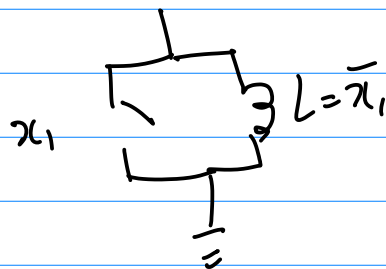
ANSI Symbol




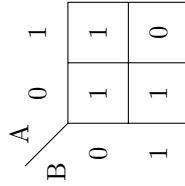

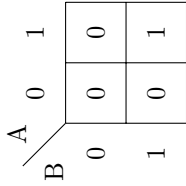

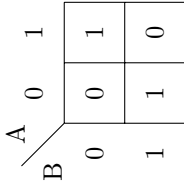
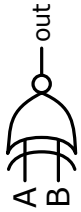
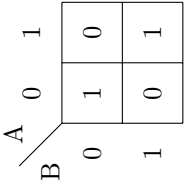
NOT gate

$x_1$	$L = \bar{x}_1 = x_1'$
0	1
1	0

Truth  
Table

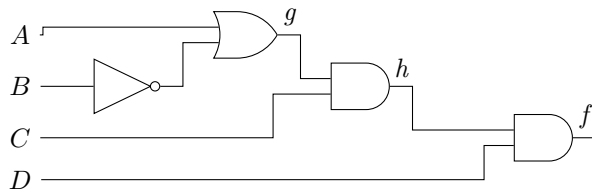


## 9 More Gates and notations summary

Name	C/Verilog	Boolean expr.	Truth Table	(ANSI) symbol	K-map															
NAND Gate	$Q = \sim(x1 \ \& \ x2)$	$Q = \overline{x_1 \cdot x_2} = \overline{x_1}x_2 + x_1\overline{x_2}$	<table><tr><th><math>x_1</math></th><th><math>x_2</math></th><th><math>\overline{x_1 \cdot x_2}</math></th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	$x_1$	$x_2$	$\overline{x_1 \cdot x_2}$	0	0	1	0	1	1	1	0	1	1	1	0		
$x_1$	$x_2$	$\overline{x_1 \cdot x_2}$																		
0	0	1																		
0	1	1																		
1	0	1																		
1	1	0																		
NOR Gate	$Q = \sim(x1 \   \ x2)$	$Q = \overline{x_1 + x_2} = \overline{x_1}\overline{x_2}$	<table><tr><th><math>x_1</math></th><th><math>x_2</math></th><th><math>\overline{x_1 + x_2}</math></th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	$x_1$	$x_2$	$\overline{x_1 + x_2}$	0	0	1	0	1	0	1	0	0	1	1	0		
$x_1$	$x_2$	$\overline{x_1 + x_2}$																		
0	0	1																		
0	1	0																		
1	0	0																		
1	1	0																		
XOR Gate	$Q = x1 \ \sim \ x2$	$Q = x_1 \oplus x_2$	<table><tr><th><math>x_1</math></th><th><math>x_2</math></th><th><math>x_1 \oplus x_2</math></th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	$x_1$	$x_2$	$x_1 \oplus x_2$	0	0	0	0	1	1	1	0	1	1	1	0		
$x_1$	$x_2$	$x_1 \oplus x_2$																		
0	0	0																		
0	1	1																		
1	0	1																		
1	1	0																		
XNOR Gate	$Q = \sim(x1 \ \sim \ x2)$	$Q = \overline{x_1 \oplus x_2}$	<table><tr><th><math>x_1</math></th><th><math>x_2</math></th><th><math>\overline{x_1 \oplus x_2}</math></th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	$x_1$	$x_2$	$\overline{x_1 \oplus x_2}$	0	0	1	0	1	0	1	0	0	1	1	1		
$x_1$	$x_2$	$\overline{x_1 \oplus x_2}$																		
0	0	1																		
0	1	0																		
1	0	0																		
1	1	1																		

**Example 10.** Convert the following Boolean expression into a K-map.  $f = \overline{A\overline{B}} + CD$

**Problem 10.** Convert the following logic circuit into a K-map.



## 10 Boolean Algebra

### 10.1 Axioms of Boolean algebra

1.  $0 \cdot 0 = 0$
2.  $1 + 1 = 1$

3.  $1 \cdot 1 = 1$

4.  $0 + 0 = 0$

5.  $0 \cdot 1 = 1 \cdot 0 = 0$

6.  $\bar{0} = 1$

7.  $\bar{1} = 0$

8.  $x = 0$  if  $x \neq 1$

9.  $x = 1$  if  $x \neq 0$

## 10.2 Single variable theorems (Prove by drawing K-maps)

1.  $x \cdot 0 = 0$

2.  $x + 1 = 1$

3.  $x \cdot 1 = x$

4.  $x + 0 = x$

5.  $x \cdot x = x$

6.  $x + x = x$

7.  $x \cdot \bar{x} = 0$

8.  $x + \bar{x} = 1$

9.  $\bar{\bar{x}} = x$

**Remark 2** (Duality). *Swap  $+$  with  $\cdot$  and  $0$  with  $1$  to get another theorem*

### 10.3 Two and three variable properties (Prove by K-maps)

1. Commutative:  $x \cdot y = y \cdot x$ ,  $x + y = y + x$

2. Associative:  $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ ,  $x + (y + z) = (x + y) + z$

3. Distributive:  $x \cdot (y + z) = x \cdot y + x \cdot z$ ,  $x + y \cdot z = (x + y) \cdot (y + z)$

4. Absorption:  $x + x \cdot y = x$ ,  $x \cdot (x + y) = x$

5. Combining:  $x \cdot y + x \cdot \bar{y}, (x + y) \cdot (x + \bar{y}) = x$

6. DeMorgan's theorem:  $\overline{x \cdot y} = \bar{x} + \bar{y}, \overline{x + y} = \bar{x} \cdot \bar{y}.$

7. Concensus:

(a)  $x + \bar{x} \cdot y = x + y$

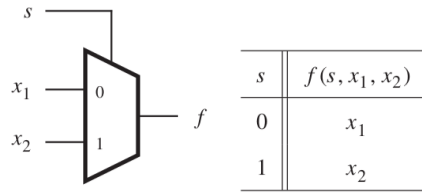
(b)  $x \cdot (\bar{x} + y) = x \cdot y$

(c)  $x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$

(d)  $(x + y) \cdot (y + z) \cdot (\bar{x} + z) = (x + y) \cdot (\bar{x} + z)$



**Example 11** (Multiplexer). *Multiplexer is a circuit used to select one of the input lines  $x_1$  and  $x_2$  based only select input  $s$ . When  $s = 0$ ,  $x_1$  is selected,  $x_2$  is selected otherwise. Find a boolean expression and a circuit for multiplexer*



**Example 12.** *Simplify  $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$  using boolean algebra.*

**Example 13.** *Simplify  $f = \bar{A}\bar{A}\bar{C} + \bar{A}\bar{B}C$  using K-maps.*



