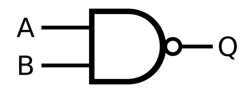
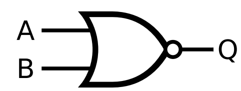
8 More Gates and notations summary

<table>
<tr><th>Name</th><th>C/Verilog</th><th>Boolean expr.</th><th>Truth Table</th><th>(ANSI) symbol</th><th>K-map</th></tr>
<tr><td>NAND Gate</td><td><code>Q = ~(x1 & x2)</code></td><td>$Q = \overline{x_1 \cdot x_2} = \overline{x_1 x_2}$</td><td>$\begin{array}{cc|c} x_1 & x_2 & \overline{x_1 \cdot x_2} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$</td><td>
</td><td>$\begin{array}{c|c|c} B \backslash A & 0 & 1 \\ \hline 0 & 1 & 1 \\ \hline 1 & 1 & 0 \end{array}$</td></tr>
<tr><td>NOR Gate</td><td><code>Q = ~(x1 | x2)</code></td><td>$Q = \overline{x_1 + x_2}$</td><td>$\begin{array}{cc|c} x_1 & x_2 & \overline{x_1 + x_2} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{array}$</td><td>
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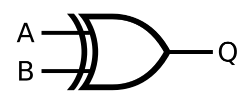
<tr><td>XOR Gate</td><td><code>Q = x1 ^ x2</code></td><td>$Q = x_1 \oplus x_2$</td><td>$\begin{array}{cc|c} x_1 & x_2 & x_1 \oplus x_2 \\ \hline 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$</td><td>
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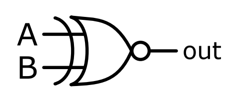
<tr><td>XNOR Gate</td><td><code>Q = ~(x1 ^ x2)</code></td><td>$Q = \overline{x_1 \oplus x_2}$</td><td>$\begin{array}{cc|c} x_1 & x_2 & \overline{x_1 \oplus x_2} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$</td><td>
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9 Boolean Algebra

9.1 Axioms of Boolean algebra

1. $0 \cdot 0 = 0$
2. $1 + 1 = 1$
3. $1 \cdot 1 = 1$
4. $0 + 0 = 0$
5. $0 \cdot 1 = 1 \cdot 0 = 0$
6. $\bar{0} = 1$
7. $\bar{1} = 0$
8. $x = 0$ if $x \neq 1$
9. $x = 1$ if $x \neq 0$

9.2 Single variable theorems

1. $x \cdot 0 = 0$

2. $x + 1 = 1$

3. $x \cdot 1 = x$

4. $x + 0 = x$

5. $x \cdot x = x$

6. $x + x = x$

7. $x \cdot \bar{x} = 0$

8. $x + \bar{x} = 1$

9. $\bar{\bar{x}} = x$

Remark 2 (Duality). *Swap + with · and 0 with 1 to get another theorem*

9.3 Two and three variable properties

1. Commutative: $x \cdot y = y \cdot x$, $x + y = y + x$

2. Associative: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$, $x + (y + z) = (x + y) + z$

3. Distributive: $x \cdot (y + z) = x \cdot y + x \cdot z$, $x + y \cdot z = (x + y) \cdot (y + z)$

4. Absorption: $x + x \cdot y = x$, $x \cdot (x + y) = x$

5. Combining: $x \cdot y + x \cdot \bar{y}$, $(x + y) \cdot (x + \bar{y}) = x$

6. DeMorgan's theorem: $\overline{x \cdot y} = \bar{x} + \bar{y}$, $\overline{x + y} = \bar{x} \cdot \bar{y}$.

7. Concensus:

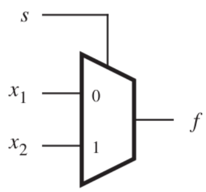
   (a) $x + \bar{x} \cdot y = x + y$

   (b) $x \cdot (\bar{x} + y) = x \cdot y$

   (c) $x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$

(d) $(x+y)\cdot(y+z)\cdot(\bar{x}+z) = (x+y)\cdot(\bar{x}+z)$

Example 10 (Multiplexer)**.** *Multiplexer is a circuit used to select one of the input lines x_1 and x_2 based only select input s. When $s=0$, x_1 is selected, x_2 is selected otherwise. Find a boolean expression and a circuit for multiplexer*



s	$f(s, x_1, x_2)$
0	x_1
1	x_2

Example 11. *Simplify $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C}$ using boolean algebra.*

Example 12. *Simplify $f = \bar{A}\bar{A}\bar{C} + \bar{A}\bar{B}C$ using K-maps.*