2. Algebraically

$$BC + B\bar{C} = B(C+\bar{C}) \quad \text{by T8}$$
$$= B \cdot 1 \quad \text{by T5}'$$

Similarly, for T9

$$B + BC = B \cdot 1 + BC \quad \text{by T1}$$
$$= B(1+C) \quad \text{by T8}'$$
$$= B \cdot 1 \quad \text{by T2}'$$

don't need to remember labels and names!

The final two: T11 (T11'), T12 (T12')

	Theorem		Dual	Name
T11	$BC + \bar{B}D + CD = BC + \bar{B}D$	T11'	$(B+C)(\bar{B}+D)(C+D) = (B+C)(\bar{B}+D)$	Consensus
T12	$\overline{B_0 B_1 B_2 \ldots} = \bar{B}_0 + \bar{B}_1 + \bar{B}_2 + \ldots$	T12'	$\overline{B_0 + B_1 + B_2 + \ldots} = \bar{B}_0 \bar{B}_1 \bar{B}_2 \ldots$	DeMorgan's theorem

The consensus theorem is difficult to apply algebraically, but easy to spot on Karnaugh maps!

DeMorgan's theorem

An extremely useful and important theorem. Gives a way of expressing the complements of functions.

E.g., consider $\overline{(AB)} = \bar{A} + \bar{B}$

let $F = AB$, so $\bar{F} = \bar{A} + \bar{B}$

and let A = "rich"
B = "loving"

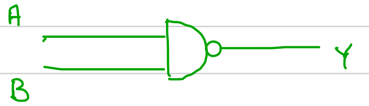
$F =$ "You may date my daughter"

Then $F = AB$ $\longrightarrow$ "You may date my daughter if you are rich AND loving"

And $\bar{F} = \bar{A} + \bar{B}$ $\longrightarrow$ "You may NOT date my daughter if you are NOT rich OR NOT loving"

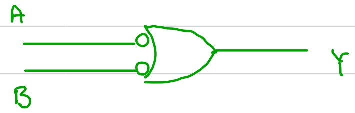
DeMorgan - equivalent gates ("pushing bubbles"!)

NAND



$Y = \overline{AB}$

$\Rightarrow$ $Y = \overline{AB}$
$= \bar{A} + \bar{B}$

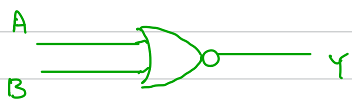


$Y = \bar{A} + \bar{B}$

AND - INVERT SYMBOL

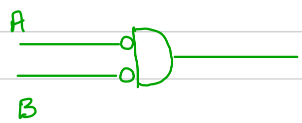
INVERT - OR SYMBOL

NOR



$Y = \overline{A + B}$

$\Rightarrow$ $Y = \overline{A + B}$
$= \bar{A} \cdot \bar{B}$



$Y = \bar{A} \cdot \bar{B}$

OR - INVERT FORM

INVERT - AND FORM

DeMorgan's theorem to switch SOP and POS forms

$$F = \bar{A}BC + A\bar{B}C$$

Apply DeMorgan's theorem

$$\bar{F} = \overline{\bar{A}BC + A\bar{B}C} \longrightarrow \text{then, } \bar{F} = (\overline{\bar{A}BC}) \cdot (\overline{A\bar{B}C})$$

And again $\bar{F} = (A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})$

Simplifying logic expressions using theorems

Simple is good (lower power consumption, fewer gates, faster)

Simplify: $F = A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + ABC$

The most-common simplification is <u>combining</u> $BC + B\bar{C} = B$

Look for pairs that can be combined in this way.

$$F = \underbrace{A\bar{B}C + A\bar{B}\bar{C}}_{A\bar{B}(\underbrace{C+\bar{C}}_{1})} + \underbrace{\bar{A}BC + ABC}_{(\underbrace{\bar{A}+A}_{1})BC}$$

$$= A\bar{B} + BC$$

Simplify: $C_{out} = AB\overline{C_{in}} + A\bar{B}C_{in} + \bar{A}BC_{in} + ABC_{in}$

can duplicate terms!

$$C_{out} = \underbrace{AB\overline{C_{in}} + ABC_{in}}_{AB(\cancel{\overline{C_{in}} + C_{in}})\,1} + \underbrace{A\bar{B}C_{in} + ABC_{in}}_{AC_{in}(\cancel{\bar{B}+B})\,1} + \underbrace{\bar{A}BC_{in} + ABC_{in}}_{(\cancel{\bar{A}+A})\,1\,BC_{in}}$$

so $C_{out} = AB + AC_{in} + BC_{in}$

This is called a "minimal SOP form".