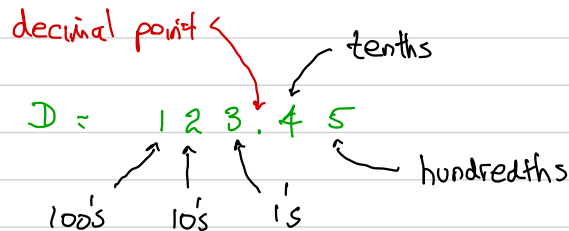


NUMBER SYSTEMS

We have adapted the decimal number system as the default number system.

- seems only natural ~ 10 fingers, 10 toes

Consider:



$$\text{So } D = 1 \times 100 + 2 \times 10 + 3 \times 1 + 4 \times \frac{1}{10} + 5 \times \frac{1}{100}$$

In general, we may express this as

$$\underbrace{d_n d_{n-1} d_{n-2} \dots d_1 d_0}_{\text{integer part}} . \underbrace{d_{-1} d_{-2} \dots d_{-p}}_{\text{fractional part}}$$

which is evaluated as a power series

$$D = d_n 10^n + d_{n-1} 10^{n-1} + \dots + d_0 + d_{-1} 10^{-1} + \dots + d_{-p} 10^{-p}$$

This is a base-10 number; it has a radix of $r = 10$

Properties:

- each digit d_j multiplied by 10^j
- $0 \leq d_j < 10$

Can be expressed this way for any radix r .

$$D = d_n r^n + d_{n-1} r^{n-1} + \dots + d_0 + d_{-1} r^{-1} + d_{-2} r^{-2} + \dots$$

or more compactly as

$$D = \sum_{j=-p}^n d_j r^j$$

Power series representation of a positional number system

where $0 \leq d_j < r$

The radix of particular importance to us is $r=2$, corresponding to the binary number system

The binary digit is called a bit

Example: Determine the decimal value of 1011.01_2 means $r=2$.

$$D = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2}$$

(note all digits d_j obey $0 \leq d_j < 2$)

$$= 11.25_{10}$$

\uparrow $r=10$

Number system conversion

Radix- r to radix-10 conversion is easy

$$D = \sum_{j=-p}^n d_j r^j$$

Radix-10 to radix- r is a little more work. In the most general case, this is done in two parts:

- integer part: successive division by r (important)
- fractional part: successive multiplication by r
(not discussed in textbook)

Integer example: convert 13_{10} to binary

Need to find: $[d_3 d_2 d_1 d_0]_2$

<u>Decimal operation</u>	<u>quotient</u>	<u>remainder</u>	<u>equivalent binary operation</u>
$13 \div 2$	6	1	$d_3 d_2 d_1 \cdot \textcircled{d_0}$ d_0 is remainder so $d_0 = 1$
$6 \div 2$	3	0	$d_3 d_2 \cdot \textcircled{d_1}$ $d_1 = 0$
$3 \div 2$	1	1	$d_3 \cdot \textcircled{d_2}$ $d_2 = 1$
$1 \div 2$	0	1	$0 \cdot \textcircled{d_3}$ $d_3 = 1$

Thus, $D = 1101_2$

Fractional part (if curious!): Convert 0.625_{10} to binary

Need to find: $[0. d_{-1} d_{-2} d_{-3}]$

<u>Decimal operation</u>	<u>product</u>	<u>integer part</u>	<u>Equivalent binary operation</u>
0.625×2	1.25	1	$d_{-1} . d_{-2} d_{-3}$ $d_{-1} = 1$
0.25×2	0.5	0	$d_{-2} . d_{-1}$ $d_{-2} = 0$
0.5×2	1.0	1	$d_{-3} . 0$ $d_{-3} = 1$

Thus, $D = 0.625_{10} = 0.101_2$

Octal and hexadecimal numbers

There are two other very common radices

- $r = 8$, octal numbers ($0 \leq d_j < 8$)
- $r = 16$, hexadecimal numbers ($0 \leq d_j < 16$)