Homework 2 solution

Max marks: 110

Due on September 17, 2021, 9 AM, before class.

Row	x_1	x_2	x_3	f
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

Table 1: Truth table for a 3-way light switch

1 Sept 10th Lecture

Problem 1 *If the SOP form for $\bar{f} = A\bar{B}\bar{C} + \bar{A}\bar{B}$, then give the POS form for f. [10 marks]*

Solution

Take inversion on both sides

$$\begin{aligned} \bar{\bar{f}} &= \overline{A\bar{B}\bar{C} + \bar{A}\bar{B}} \\ f &= \overline{A\bar{B}\bar{C}} \cdot \overline{\bar{A}\bar{B}} && \text{by DeMorgan's} \\ &= (\bar{A} + B + C)(A + B) && \text{by DeMorgan's} \end{aligned}$$

Problem 2 *Use DeMorgan's Theorem to find f if $\bar{f} = (A + BC)D + EF$. [10 marks]*

Solution

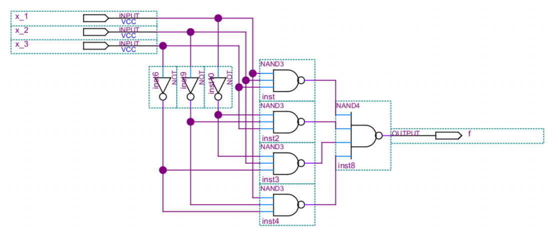
Take inversion on both sides

$$\begin{aligned} \bar{\bar{f}} &= \overline{(A + BC)D + EF} \\ f &= \overline{((A + BC)D)} \cdot \overline{EF} && \text{by DeMorgan's} \\ &= (\overline{(A + BC)} + \bar{D})(\bar{E} + \bar{F}) && \text{by DeMorgan's} \\ &= (\bar{A}\overline{(BC)} + \bar{D})(\bar{E} + \bar{F}) && \text{by DeMorgan's} \\ &= (\bar{A}(\bar{B} + \bar{C}) + \bar{D})(\bar{E} + \bar{F}) && \text{by DeMorgan's} \end{aligned}$$

Problem 3 *Implement the function in Table 1 using only NAND gates. [10 marks]*

Solution

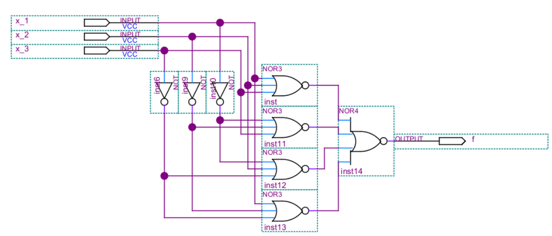
$$\begin{aligned} f &= \bar{x}_1\bar{x}_2x_3 + \bar{x}_1x_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + x_1x_2x_3 \\ &= \overline{\overline{\bar{x}_1\bar{x}_2x_3}} + \overline{\overline{\bar{x}_1x_2\bar{x}_3}} + \overline{\overline{x_1\bar{x}_2\bar{x}_3}} + \overline{\overline{x_1x_2x_3}} \\ &= \overline{\overline{\bar{x}_1\bar{x}_2x_3} \cdot \overline{\bar{x}_1x_2\bar{x}_3} \cdot \overline{x_1\bar{x}_2\bar{x}_3} \cdot \overline{x_1x_2x_3}} \end{aligned}$$



Problem 4 *Implement the function in Table 1 using only NOR gates. [10 marks]*

Solution

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + x_3) \\ &= \overline{\overline{(x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + x_3)}} \\ &= \overline{\overline{(x_1 + x_2 + x_3)} + \overline{(x_1 + \bar{x}_2 + \bar{x}_3)} + \overline{(\bar{x}_1 + x_2 + \bar{x}_3)} + \overline{(\bar{x}_1 + \bar{x}_2 + x_3)}} \end{aligned}$$



2 Sept 13th Lecture

Problem 5 *Find the minimum-cost SOP and POS forms for the function $f(x_1, x_2, x_3) = m(1, 2, 3, 5)$. [1, Prob 2.37] [10 marks]*

Solution

Minimum cost SOP

	$\bar{x}_1 \bar{x}_2$	$\bar{x}_1 x_2$	$x_1 x_2$	$x_1 \bar{x}_2$
$\bar{x}_3$	0	1	0	0
x_3	1	1	0	1

$$f = \bar{x}_1 x_2 + \bar{x}_2 x_3 \tag{1}$$

Cost = 2 AND + 1 OR + (2*2 + 2) inputs = 9

Minimum cost POS

	$\bar{x}_1 \bar{x}_2$	$\bar{x}_1 x_2$	$x_1 x_2$	$x_1 \bar{x}_2$
$\bar{x}_3$	0	1	0	0
x_3	1	1	0	1

$$\bar{f} = \bar{x}_2 \bar{x}_3 + x_1 x_2 \tag{2}$$

$$\implies f = (x_2 + x_3)(\bar{x}_1 + \bar{x}_2) \tag{3}$$

Cost = 2 OR + 1 AND + (2*2+2) inputs = 9

Problem 6 *Find the minimum-cost SOP and POS forms for the function* $f(x_1, x_2, x_3) = \sum m(1, 4, 7) + D(2, 5)$. *[1, Prob 2.38] [10 marks]*

Solution

Minimum cost SOP

	$\bar{x}_1 \bar{x}_2$	$\bar{x}_1 x_2$	$x_1 x_2$	$x_1 \bar{x}_2$
$\bar{x}_3$	0	d	0	1
x_3	1	0	1	d + d + d

$$f = x_1 \bar{x}_2 + x_1 x_3 + \bar{x}_2 x_3 \tag{4}$$

Cost = 3 AND + 1 OR + (3*2 + 3) inputs = 13

Minimum cost POS

	$\bar{x}_1 \bar{x}_2$	$\bar{x}_1 x_2$	$x_1 x_2$	$x_1 \bar{x}_2$
$\bar{x}_3$	0	1	0	0
x_3	1	1	0	1

$$\bar{f} = \bar{x}_2 \bar{x}_3 + x_1 x_2 \tag{5}$$

$$\implies f = (x_2 + x_3)(\bar{x}_1 + \bar{x}_2) \tag{6}$$

Cost = 2 OR + 1 AND + (2*2+2) inputs = 9

Problem 7 *Find the minimum-cost SOP and POS forms for the function* $f(x_1, x_2, x_3, x_4) = \prod M(0, 1, 2, 4, 5, 7, 8, 9, 10, 12, 14, 15)$. *[1, Prob 2.39] [10 marks]*

Problem 8 *Find the minimum-cost SOP and POS forms for the function* $f(x_1, x_2, x_3, x_4) = \sum m(0, 2, 8, 9, 12, 15) + D(1, 3, 6, 7)$. *[1, Prob 2.40] [10 marks]*

Problem 9 *Derive a minimum-cost realization of the four-variable function that is equal to 1 if exactly two or exactly three of its variables are equal to 1; otherwise it is equal to 0. [1, Prob 2.46] [10 marks]*

Problem 10 *Find the minimum-cost SOP and POS forms for the function* $f(x_1, \ldots, x_5) = \sum m(0, 1, 3, 4, 6, 8, 9, 11, 13, 14, 16, 19, 20, 21, 22, 24, 25) + D(5, 7, 12, 15, 17, 23)$. *[1, Prob 2.42] [10 marks]*

References

[1] S. Brown and Z. Vranesic. *Fundamentals of Digital Logic with Verilog Design: Third Edition.* McGraw-Hill Higher Education, 2013.