Homework 4

Max marks: 40

Due on Oct 1st, 2021, 9 AM, before class.

Problem 1 *Hazard problem: Design a hazard free SOP for* $f(A,B,C,D) = \sum m(0,1,4,5,6,7,9,11,14,15)$

Solution

The K-map for f is

		$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}$	$\bar{D}$	1	1	0	0
$\bar{C}$	D	1	1	0	1
C	D	0	1	1	1
C	$\bar{D}$	0	1	1	0

$$f = \bar{A}B + BC + ACD + \bar{A}\bar{C} + A\bar{B}D + \bar{B}\bar{C}D$$

Cost(f) = 6 AND gates + 2*3 + 3*3 inputs to AND gates + 1 OR gate + 6 inputs = 28.

Problem 2 *Find the simplest realization of the function* $f(x_1,\ldots,x_4) = \sum m(0,3,4,7,9,10,13,14)$, *assuming that the logic gates have a maximum fan-in of two.*

Solution

The K-map for f is

		$\bar{x}_1\bar{x}_2$	$\bar{x}_1 x_2$	$x_1 x_2$	$x_1\bar{x}_2$	Row pattern
$\bar{x}_3$	$\bar{x}_4$	1	1	0	0	$\bar{g} = \bar{x}_1$
$\bar{x}_3$	x_4	0	0	1	1	$g = x_1$
x_3	x_4	1	1	0	0	$\bar{g}$
x_3	$\bar{x}_4$	0	0	1	1	g
Col pattern		$\bar{h}$	$\bar{h}$	h	h	

Writing f in terms of column pattern $h = \bar{x}_3 x_4 + x_3\bar{x}_4 = x_3 \oplus x_4$.

$$\begin{aligned} f &= h x_1 + \bar{h}\bar{x}_1 && (1)\\ &= \bar{x}_1 \oplus h && (2)\\ &= \bar{x}_1 \oplus (x_3 \oplus x_4) && (3) \end{aligned}$$

Cost(f) = 2 XOR gates + (2+2) inputs each = 6. Max fan-in = 2.

Problem 3 *Find the minimum-cost circuit for the function* $f(x_1,\ldots,x_4) = \sum m(0,4,8,13,14,15)$. *Assume that the input variables are available in uncomplemented form only. (Hint: Use functional decomposition.)*

Solution

The K-map for f is

		$\bar{x}_1\bar{x}_2$	$\bar{x}_1 x_2$	$x_1 x_2$	$x_1\bar{x}_2$	Row pattern
$\bar{x}_3$	$\bar{x}_4$	1	1	0	1	$\bar{g} = x_1 x_2$
$\bar{x}_3$	x_4	0	0	1	0	$g = x_1 x_2$
x_3	x_4	0	0	1	0	g
x_3	$\bar{x}_4$	0	0	1	0	g
Col pattern		$h = \bar{x}_3\bar{x}_4$	h	$\bar{h}$	h	

Write f in terms of g,

$$\begin{aligned} f &= \bar{x}_3\bar{x}_4\bar{g} + \overline{\bar{x}_3\bar{x}_4}\, g && (4)\\ &= \overline{x_3 + x_4}\,\bar{g} + \underbrace{(x_3 + x_4)}_{h_2} g && (5)\\ &= \bar{h}_2\bar{g} + h_2 g && (6)\\ &= \overline{h_2 \oplus g} && (7)\\ &= \overline{(x_3 + x_4) \oplus (x_1 x_2)} && (8) \end{aligned}$$

Cost(f) = 1 OR gate + 2 inputs for OR gate + 1 AND gate + 2 inputs for AND gate + 1 XOR gate + 2 inputs for XOR gate + 1 NOT gate + 1 input to NOT gate = 11.

Problem 4 *Use functional decomposition to find the best implementation of the function* $f(x_1, \ldots, x_5) = \sum m(1, 2, 7, 9, 10, 18, 19, 25, 31) + D(0, 15, 20, 26)$. *How does your implementation compare with the lowest-cost SOP implementation? Give the costs.*

Solution

The K-map for f is given in Table 1.

We try writing $\bar{x}_1 = 1$ half of f in terms of Row patterns $g = x_3$, and $x_1 = 1$ half of f in terms normal K-map grouped terms,

$$\begin{aligned} f &= \bar{x}_1(\underbrace{(\bar{x}_4 x_5 + x_4 \bar{x}_5)}_{h}\bar{x}_3 + \underbrace{x_4 x_5}_{h_2} x_3) \\ &\quad + x_1\bar{x}_2\bar{x}_3 x_4 + x_2 x_3 h_2 + x_2 \bar{x}_3 h \\ &= \bar{x}_1 h \bar{x}_3 + \bar{x}_1 h_2 x_3 + x_1\bar{x}_2\bar{x}_3 x_4 + x_2 x_3 h_2 \\ &\quad + x_2 \bar{x}_3 h \end{aligned}$$

Cost(h) = 1+2=3. Cost(h_2) = 1+2=3. Total Cost of f = Cost(h) + Cost(h_2) + 5 AND gates + 4*3 input per gate +4 inputs to AND gates + 1 OR gate + 5 inputs = 3+3+5 + 12+4+1+5=33.

Normal grouping is shown in Table 2.

$$\begin{aligned} f = {} & x_1\bar{x}_3 x_4 \bar{x}_5 + x_1 \bar{x}_3 \bar{x}_4 \bar{x}_5 + x_1 x_3 x_4 x_5 \\ & + x_1\bar{x}_2\bar{x}_3 x_4 + x_2 x_3 x_4 x_5 + x_2 \bar{x}_3 \bar{x}_4 x_5 \end{aligned} \tag{9}$$

Cost(f) = 6+6*4+1+6=37.

		$\bar{x}_1$				Row pattern	x_1				Row pattern
		$\bar{x}_2$		x_2			$\bar{x}_2$		x_2		
		$\bar{x}_3$	x_3		$\bar{x}_3$		$\bar{x}_3$	x_3		$\bar{x}_3$	
$\bar{x}_4$	$\bar{x}_5$	d	0	0	0	**0**	0	d	0	0	**0**
	x_5	1	0	0	1	$\bar{g} = \bar{x}_3$	0	0	0	1	$g_2 = x_2\bar{x}_3$
x_4		0	1	d	0	$g = x_3$	1	0	1	0	$g_3 = \bar{x}_2 \oplus x_3$
	$\bar{x}_5$	1	0	0	1	$\bar{g} = \bar{x}_3$	1	0	0	d	$g = \bar{x}_3$
Col pattern		$h = x_4 \oplus x_5$	$h_2 = x_4x_5$	**0**	h		$h_3 = x_4$	**0**	$h_2 = x_2x_4$	h	

Table 1: K-map for Problem 4.

		$\bar{x}_1$				x_1			
		$\bar{x}_2$		x_2		$\bar{x}_2$		x_2	
		$\bar{x}_3$	x_3		$\bar{x}_3$	$\bar{x}_3$	x_3		$\bar{x}_3$
$\bar{x}_4$	$\bar{x}_5$	d	0	0	0	0	d	0	0
	x_5	1	0	0	1	0	0	0	1
x_4		0	1	d	0	1	0	1	0
	$\bar{x}_5$	1	0	0	1	1	0	0	d

Table 2: K-map for Problem 4.