Example: Suppose $F(A,B,C) = \sum(2,3,5,7)$. Give the truth table and the canonical SOP expression.

Row	A B C	TRUE Minterm	F
0	0 0 0		0
1	0 0 1		0
2	0 1 0	$\bar{A}B\bar{C}$	1
3	0 1 1	$\bar{A}BC$	1
4	1 0 0		0
5	1 0 1	$A\bar{B}C$	1
6	1 1 0		0
7	1 1 1	ABC	1

$$F(A,B,C) = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

Products of sums form

A product of sums (POS) form contains the ANDing of one or more sum terms.

E.g., $A + B$ (a single sum term)

$(A+C)(\bar{B}+D)$

$A(\bar{B}+\bar{C})$

not a POS expression: $\overline{(A+B)(B+C)}(\bar{C}+\bar{D})$ ← disqualifies from being a true POS form

Canonical POS forms

A special case where each sum is a maxterm.

E.g., for a 3-variable system

$F = (A+B+\bar{C})(\bar{A}+\bar{B}+C)(A+\bar{B}+C)$ ← all are maxterms

$F = (A+B)\bar{C}$ ← neither sum term is a maxterm

Maxterms and the truth table. Back to our XOR.

Row	A	B	FALSE Maxterm	F
0	0	0	$A+B$	0
1	0	1	$A+\bar{B}$	1
2	1	0	$\bar{A}+B$	1
3	1	1	$\bar{A}+\bar{B}$	0

list the maxterm that evaluates to FALSE on each row. Designated M_0, M_1, M_2, M_3.

The canonical POS expression is by gathering maxterms on each row where $F = 0$.

$$F = (A+B)(\bar{A}+\bar{B})$$

which we abbreviate as $F = M_0 M_3$

or simply $F = \prod(0,3)$

"product of"

Axioms of theorems of Boolean algebra

Boolean algebra is based on 5 simple axioms:

	Axiom		Dual	Name
A1	$B=0$ if $B \neq 1$	A1'	$B=1$ if $B \neq 0$	Binary field
A2	$\bar{0}=1$	A2'	$\bar{1}=0$	NOT
A3	$0 \cdot 0 = 0$	A3'	$1+1=1$	AND/OR
A4	$1 \cdot 1 = 1$	A4'	$0+0=0$	AND/OR
A5	$1 \cdot 0 = 0 \cdot 1 = 0$	A5'	$1+0=0+1=1$	AND/OR

Axioms and theorems obey duality. Interchange 0's and 1's; interchange '·' and '+'

Theorems of one variable

	Theorem		Dual	Name
T1	$B \cdot 1 = B$	T1'	$B + 0 = B$	Identity
T2	$B \cdot 0 = 0$	T2'	$B + 1 = 1$	Null element
T3	$B \cdot B = B$	T3'	$B + B = B$	Idempotency
T4	$\bar{\bar{B}} = B$			Involution
T5	$B \cdot \bar{B} = 0$	T5'	$B + \bar{B} = 1$	Complements

Multivariable theorems

	Theorem		Dual	Name
T6	$B \cdot C = C \cdot B$	T6'	$B + C = C + B$	Commutativity
T7	$(B \cdot C) \cdot D = B \cdot (C \cdot D)$	T7'	$(B + C) + D = B + (C + D)$	Associativity

Boolean algebra is also distributive

T8 $\quad B \cdot (C + D) = BC + BD$ "MULTIPLYING OUT"

However, we also get ...

T8' $\quad B + (C \cdot D) = (B + C) \cdot (B + D)$ "ADDING OUT"

More ...

	Theorem		Dual	Name
T9	$B \cdot (B + C) = B$	T9'	$B + (B \cdot C) = B$	Covering
T10	$(B \cdot C) + (B \cdot \bar{C}) = B$	T10'	$(B + C) \cdot (B + \bar{C}) = B$	Combining

T9 and T10 are very important for simplifying expressions.
Two ways to prove T10

1. Perfect induction

(i.e., proof by using a truth table)

BC	$(B \cdot C)$	$(B \cdot \bar{C})$	$(B \cdot C) + (B \cdot \bar{C})$
00	0	0	0
01	0	0	0
10	0	1	1
11	1	0	1

B