8 Karnaugh maps

8.1 Two input K-maps

B \ A	0	1
0	m_0	m_2
1	m_1	m_3

8.2 Three input K-maps

C \ AB	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

8.3 Four input K-maps

CD \ AB	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

8.4 Five input K-maps

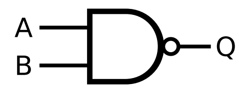
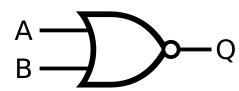
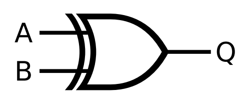
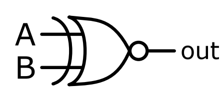
A = 0

DE \ BC	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

A = 1

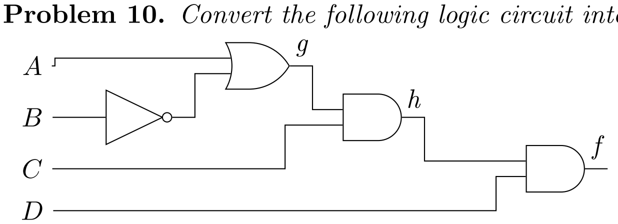
DE \ BC	00	01	11	10
00	m_{16}	m_{20}	m_{28}	m_{24}
01	m_{17}	m_{21}	m_{29}	m_{25}
11	m_{19}	m_{23}	m_{31}	m_{27}
10	m_{18}	m_{22}	m_{30}	m_{26}

9 More Gates and notations summary

<table>
<tr><th>Name</th><th>C/Verilog</th><th>Boolean expr.</th><th>Truth Table</th><th>(ANSI) symbol</th><th>K-map</th></tr>
<tr><td>NAND Gate</td><td><code>Q = ~(x1 & x2)</code></td><td>$Q = \overline{x_1 \cdot x_2} = \overline{x_1 x_2}$</td><td>$\begin{array}{cc|c} x_1 & x_2 & \overline{x_1 \cdot x_2} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$</td><td>
</td><td>$\begin{array}{c|cc} B \backslash A & 0 & 1 \\ \hline 0 & 1 & 1 \\ 1 & 1 & 0 \end{array}$</td></tr>
<tr><td>NOR Gate</td><td><code>Q = ~(x1 | x2)</code></td><td>$Q = \overline{x_1 + x_2}$</td><td>$\begin{array}{cc|c} x_1 & x_2 & \overline{x_1 + x_2} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{array}$</td><td>
</td><td>$\begin{array}{c|cc} B \backslash A & 0 & 1 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 1 \end{array}$</td></tr>
<tr><td>XOR Gate</td><td><code>Q = x1 ^ x2</code></td><td>$Q = x_1 \oplus x_2$</td><td>$\begin{array}{cc|c} x_1 & x_2 & x_1 \oplus x_2 \\ \hline 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$</td><td>
</td><td>$\begin{array}{c|cc} B \backslash A & 0 & 1 \\ \hline 0 & 0 & 1 \\ 1 & 1 & 0 \end{array}$</td></tr>
<tr><td>XNOR Gate</td><td><code>Q = ~(x1 ^ x2)</code></td><td>$Q = \overline{x_1 \oplus x_2}$</td><td>$\begin{array}{cc|c} x_1 & x_2 & \overline{x_1 \oplus x_2} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$</td><td>
</td><td>$\begin{array}{c|cc} B \backslash A & 0 & 1 \\ \hline 0 & 1 & 0 \\ 1 & 0 & 1 \end{array}$</td></tr>
</table>

Example 10. *Convert the following Boolean expression into a K-map.* $f = \overline{A\bar{B} + CD}$

Problem 10. *Convert the following logic circuit into a K-map.*



10 Boolean Algebra

10.1 Axioms of Boolean algebra

1. $0 \cdot 0 = 0$
2. $1 + 1 = 1$

3. $1 \cdot 1 = 1$
4. $0 + 0 = 0$
5. $0 \cdot 1 = 1 \cdot 0 = 0$
6. $\bar{0} = 1$
7. $\bar{1} = 0$
8. $x = 0$ if $x \neq 1$
9. $x = 1$ if $x \neq 0$

10.2 Single variable theorems (Prove by drawing K-maps)

1. $x \cdot 0 = 0$
2. $x + 1 = 1$
3. $x \cdot 1 = x$
4. $x + 0 = x$
5. $x \cdot x = x$
6. $x + x = x$
7. $x \cdot \bar{x} = 0$

8. $x + \bar{x} = 1$

9. $\bar{\bar{x}} = x$

Remark 2 (Duality)**.** *Swap + with · and 0 with 1 to get another theorem*

10.3 Two and three variable properties (Prove by K-maps)

1. Commutative: $x \cdot y = y \cdot x$, $x + y = y + x$

2. Associative: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$, $x + (y + z) = (x + y) + z$

3. Distributive: $x \cdot (y + z) = x \cdot y + x \cdot z$, $x + y \cdot z = (x + y) \cdot (y + z)$

4. Absorption: $x + x \cdot y = x$, $x \cdot (x + y) = x$

5. Combining: $x \cdot y + x \cdot \bar{y}$, $(x+y) \cdot (x+\bar{y}) = x$

6. DeMorgan's theorem: $\overline{x \cdot y} = \bar{x} + \bar{y}$, $\overline{x+y} = \bar{x} \cdot \bar{y}$.

7. Concensus:

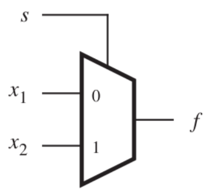
   (a) $x + \bar{x} \cdot y = x + y$

   (b) $x \cdot (\bar{x} + y) = x \cdot y$

   (c) $x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$

   (d) $(x+y) \cdot (y+z) \cdot (\bar{x}+z) = (x+y) \cdot (\bar{x}+z)$

Example 11 (Multiplexer). *Multiplexer is a circuit used to select one of the input lines x_1 and x_2 based only select input s. When $s = 0$, x_1 is selected, x_2 is selected otherwise. Find a boolean expression and a circuit for multiplexer*



s	$f(s, x_1, x_2)$
0	x_1
1	x_2

Example 12. *Simplify $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C}$ using boolean algebra.*

Example 13. *Simplify $f = \bar{A}\bar{A}\bar{C} + \bar{A}\bar{B}C$ using K-maps.*