Number system and conversions (section 1.4 of textbook)

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6 Signed binary numbers

Signed numbers include both negative and positive numbers. There three common signed number representations

1. Sign magnitude representation
2. One's complement
3. Two's complement

6.1 Sign-magnitude representation

The Most significant (left most) *bit* (binary digit) represents sign (0 = + and 1 = −), the rest represent the magnitude. Example, a 5-bit number $(11010)_2$ in signed magnitude representation has the value of $(-1010)_2 = -10$. Note that $+10$ has to be represented by a leading 0 at the most significant bit (MSB) $+10 = (01010)_2$. Hence, the number of bits have to be specified.

Problem 5 • *Write down all possible 4-digit binary numbers and corresponding decimal values if they are in signed magnitude format? What is the minimum and maximum value?*

• *What is the minimum and maximum value of n-digit signed binary number in sign-magnitude format?*

4-digit Binary		Decimal		4-digit	n-digit
0000	=	+0		$(0000)_2 = (1000)_2$	
0001	=	+1			
0010	=	+2			
0011	=	+3			
0100	=	+4			
0101	=	+5			
0110	=	+6			
0111	=	+7	← Max	$2^3 - 1$	$2^{n-1} - 1$
1000	=	−0			
1001	=	−1			
1010	=	−2			
1011	=	−3			
1100	=	−4			
1101	=	−5			
1110	=	−6			
1111	=	−7	← Min	$-(2^3-1)$	$-(2^{n-1}-1)$

6.2 One's complement negation

You can convert a positive number (say +10) to negative number by applying a negative sign in front of it (-(+10) = -10). It is more evident from taking negative of a negative number (-(-10) = +10). In case of sign-magnitude representation, the "negative operator" flips the sign bit. The next two signed number representations (1's complement and 2's complement) are designed around specific negative operator definitions.
Negate $13_{10} = 01101_2$ using 5-bit one's complement.

Negate -13_{10} using 5-bit one's complement.

6.3 One's complement binary numbers

In one's complement representation, the negative operation is obtained by flipping all the bits of the binary number. Example, a 5-bit one's complement of $+10 = (01010)_2$ is $(10101)_2 = -10$. Note that flipping bits is equivalent to subtracting the number from $(11111)_2$, hence the name. You can also confirm that double negative operator yields back the same number.

Problem 6

- *Write down all possible 4-digit binary numbers and corresponding decimal values if they are in sign magnitude format? What is the minimum and maximum value?*
- *What is the minimum and maximum value of n-digit signed binary number in one's complement?*

Problem 7 *Determine the decimal values of the following 1's complement 6-digit binary numbers :*

1. *01101110*
2. *10101101*

Problem 8 *Convert the decimal numbers -17 and +23 into the 6-digit one's complement binary numbers and try adding them. What adjustments will you need to make to get the right result's (23-17=6) in binary representation.*

6.4 Two's complement negation

In two's complement representation, the n-digit negative number is obtained by subtracting the positive number from 2^n. Example, two's complement of 5-digit binary number $+10 = (01010)_2$ is $2^5 - 10 = 22 = (11000)_2$. An easier algorithm to get two's complement goes via one's complement. Note that $(11111)_2 = 2^5 - 1$. We can get two's complement by adding 1 to one's complement. To get two's complement:

1. Flip all the bits. (Same as taking one's complement).
2. Add 1 to the number.

Negate $13_{10} = 01101_2$ using 5-bit two's complement.

Negate -13_{10} using 5-bit two's complement.

How to convert one's complement number representation into sign-magnitude numbers?

1. Check if the number is positive or negative. Even for one's complement representation, or two's complement representation, if the MSB (Most-significant bit) is 1, then the number is negative, otherwise positive.
2. If positive: For positive numbers, two's complement, one's complement and sign magnitude are the same. No conversion between different representation is needed. 2.b If negative: For negative numbers. Flip the bits of 1's complement. Once you flip the 1's complement bits of a negative number, you get the corresponding positive number.
3. We still want to represent the original negative number. So we set the MSB of sign-magnitude representation to 1. Since the range (min and max) for both n-bit 1's complement and sign-magnitude are the same (between $-(2^{n-1}-1)$ and $2^{n-1}-1$), you can always represent 8-bit 1's complement numbers with needing to extend the 8-bit number to 9-bits.

Example: Convert 8-bit one's complement 10101010 to 8-bit sign-magnitude Let number n = 10101010

1. Is the number +ve or -ve: It is negative because it starts with 1.
2. The number is not positive.
3. Take the 1's complement of the negative number to get the positive part. i.e. Flip the bits: -n = 01010101 or n = -(01010101)
4. We got the positive part of the number, but we want to represent the original negative number, so we set the MSB bit one. Hence, the equivalent sign-magnitude representation is:
n = 11010101

6.5 Two's complement representation

Problem 9 *Determine the decimal values of the following 2's complement 6-digit numbers :*

1. *01011110*
2. *10010111*

Problem 10 *Convert the decimal numbers -17 and +23 into the 6-digit two's complement binary numbers and try adding them. What adjustments will you need to make to get the right result's (23-17=6) in binary representation.*

Problem 11 *Convert the decimal numbers 73, 23, -17, and -163 into signed 8-bit numbers in the following representations:*

1. *Sign and magnitude*
2. *1's complement*
3. *2's complement*

6.6 Arithmetic overflow

Problem 12 *Consider addition of 4-digit two's complement binary numbers*

1. $1010_2 + 1101_2$
2. $1011_2 + 1100_2$

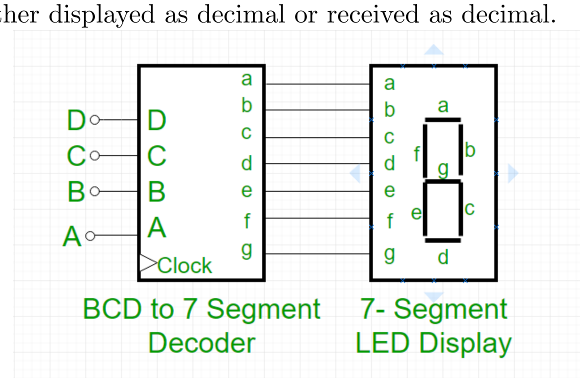
In which of the two case overflow happens? Can you come up with a rule to "easily" detect overflow?

6.6.1 Rules for detecting arithmetic overflow:

1. Adding numbers of different signs never produces an overflow.
2. Adding numbers of the same sign may produce an overflow
   (a) Wrong approach: Adding two negative 2's complement numbers always produces an additional carry-over 1, but that in itself isn't an overflow. An example, the range of 4-bit 2's complement numbers is between -8 to +7. Adding -3 to -4 in 2's complement is 1101 + 1100 produces an additional carry over 1. You can ignore the additional carry-over 1 to get the correct answer 1001 = -7 which is within range -8 to 7.
   (b) Approach 1: The easiest way for now to detect overflow is if adding two -ve numbers results in a +ve number, or adding +ve numbers results in a -ve number.
   (c) Approach 2: You can also do a range test in decimal based range test. The range of n-bit 2's complement numbers is between -2^{n-1} and $2^{n-1}-1$. For 5-bit 2's complement numbers, it is between -16 and 15. For 6-bit 2's complement numbers, it is between -32 and 31.
   (d) Approach 3: You can also check the carry-overs of the most significant two bits. If they match, i.e. 0 and 0, or 1 and 1, then there is no overflow. If they do not match, i.e. 0 and 1 or 1 and 0, then there is an overflow.

7 Binary coded decimal

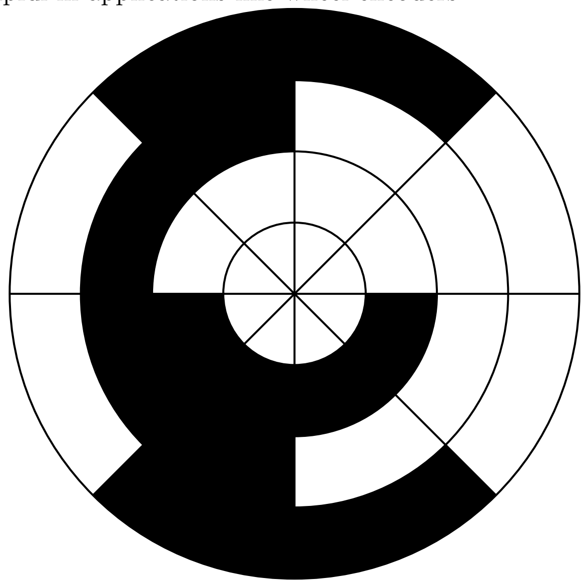
In Binary coded decimal (BCD), each decimal digit is represented by 4 bits. For example, 1047 = $(0001\_0000\_0100\_0111)_{\mathrm{BCD}}$. It is useful in input-output applications where the number has to be either displayed as decimal or received as decimal.



Problem 13 *Convert 11, 23, 35, 57 and 103897 to BCD?*

8 Gray code

A sequence of binary numbers where only one bit changes when the number increases by 1. It is helpful in applications like wheel encoders



Problem 14 *Write all possible 3-bit binary numbers in gray-code*