

# Combinational circuit

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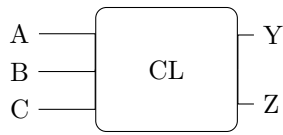
## 1 Learning objectives

1. Representing digital circuits
2. Converting between different notations: Boolean expression, logic networks and switching circuits
3. Converting between different logic network specifications: truth table, minterm, maxterms, product of sums canonical form and sum of product canonical form.

## 2 Basic Gates and notations summary

Name	C/Verilog	Boolean expr.	Truth Table	Switching circuit	(ANSI) symbol	Venn diagram															
AND Gate	L = x1 & x2	$L = x_1 \cdot x_2 = x_1x_2$	<table><tr><th><math>x_1</math></th><th><math>x_2</math></th><th><math>x_1 \cdot x_2</math></th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	$x_1$	$x_2$	$x_1 \cdot x_2$	0	0	0	0	1	0	1	0	0	1	1	1			
$x_1$	$x_2$	$x_1 \cdot x_2$																			
0	0	0																			
0	1	0																			
1	0	0																			
1	1	1																			
OR Gate	L = x1   x2	$L = x_1 + x_2$	<table><tr><th><math>x_1</math></th><th><math>x_2</math></th><th><math>x_1 + x_2</math></th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	$x_1$	$x_2$	$x_1 + x_2$	0	0	0	0	1	1	1	0	1	1	1	1			
$x_1$	$x_2$	$x_1 + x_2$																			
0	0	0																			
0	1	1																			
1	0	1																			
1	1	1																			
NOT Gate	L = ~ x1	$L = \bar{x}_1 = x'_1$	<table><tr><th><math>x_1</math></th><th><math>\bar{x}_1</math></th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	$x_1$	$\bar{x}_1$	0	1	1	0												
$x_1$	$\bar{x}_1$																				
0	1																				
1	0																				

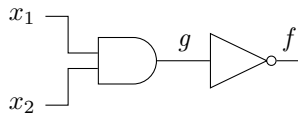
### 3 Digital circuits or networks



$$Y = F(A, B, C) \quad Z = G(A, B, C)$$

### 4 Two input networks

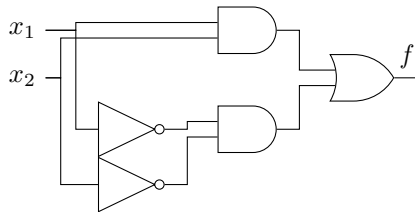
**Example 1.** Convert the following (ANSI) network into a Boolean expression, a truth table and a Venn diagram.



**Example 2.** Convert the following Boolean expression into a (ANSI) network, a truth table and a Venn diagram:

$$f = \overline{x_1 + x_2}$$

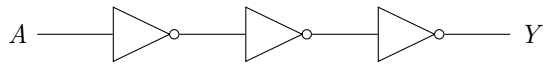
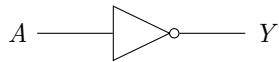
**Problem 1.** Convert the following (ANSI) network into a Boolean expression, a truth table and a Venn diagram.



**Example 3.** Convert the following Boolean expression into a network, a truth table and a Venn diagram:

$$f = x_1\bar{x}_2 + \bar{x}_1x_2$$

**Problem 2.** Can two different circuits have the same truth table? Can two different truth tables have the same circuit? Consider the following two circuits for example

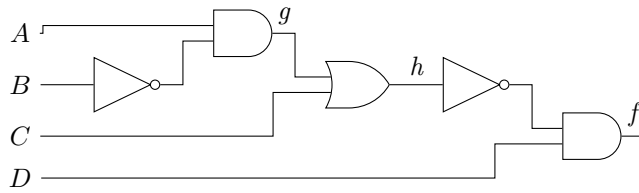


How about Venn diagrams?

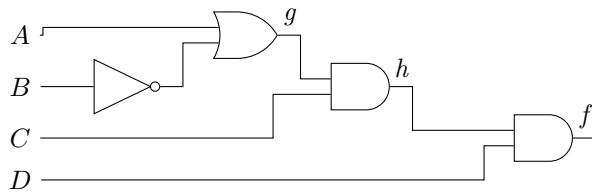
**Remark 1.** Truth tables and Venn diagrams define what the combinational circuit should do. Truth tables define output for every input. Boolean expression and networks define how to achieve the desired input output relationship.

## 5 Multi-input networks

**Example 4.** Convert the following (ANSI) network into a Boolean expression and a truth table.



**Problem 3.** Convert the following (ANSI) network into a Boolean expression and a truth table.



## 6 Minterms and Maxterms

### 6.1 Minterms

Minterm is a product involving all inputs (or complements) to a function. Every row of a truth table has a corresponding minterm. Minterm is true if and only if the corresponding row in the table is active.

Minterms defined as follows for each row of a two input truth table:

A	B	minterm	minterm name
0	0	$\bar{A}\bar{B}$	$m_0$
0	1	$\bar{A}B$	$m_1$
1	0	$A\bar{B}$	$m_2$
1	1	$AB$	$m_3$

Consider a two input circuit whose output  $Y$  is given by the truth table:

A	B	Y	minterm	minterm name
0	0	0	$\bar{A}\bar{B}$	$m_0$
0	1	1	$\bar{A}B$	$m_1$
1	0	0	$A\bar{B}$	$m_2$
1	1	1	$AB$	$m_3$

then  $Y = \bar{A}B + AB = m_1 + m_3 = \sum(1, 3)$ .

This also gives the *sum of products canonical form*.

**Example 5.** What is the minterm  $m_{13}$  for a 4-input circuit with inputs  $x, y, z, w$  (ordered from MSB to LSB).

**Problem 4.** What is the minterm  $m_{23}$  for a 5-input circuit with inputs  $a, b, c, d, e$  (ordered from MSB to LSB).

**Example 6.** Convert the following 4-input truth table into sum of minterms and sum of products canonical form.

<i>minterm name</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>f</i>
$m_0$	0	0	0	0	0
$m_1$	0	0	0	1	1
$m_2$	0	0	1	0	0
$m_3$	0	0	1	1	0
$m_4$	0	1	0	0	0
$m_5$	0	1	0	1	1
$m_6$	0	1	1	0	0
$m_7$	0	1	1	1	0
$m_8$	1	0	0	0	0
$m_9$	1	0	0	1	0
$m_{10}$	1	0	1	0	0
$m_{11}$	1	0	1	1	0
$m_{12}$	1	1	0	0	0
$m_{13}$	1	1	0	1	1
$m_{14}$	1	1	1	0	0
$m_{15}$	1	1	1	1	0

**Problem 5.** Convert the following 4-input truth table into sum of minterms and sum of products canonical form.

<i>minterm name</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>f</i>
$m_0$	0	0	0	0	0
$m_1$	0	0	0	1	0
$m_2$	0	0	1	0	0
$m_3$	0	0	1	1	1
$m_4$	0	1	0	0	0
$m_5$	0	1	0	1	0
$m_6$	0	1	1	0	0
$m_7$	0	1	1	1	1
$m_8$	1	0	0	0	0
$m_9$	1	0	0	1	0
$m_{10}$	1	0	1	0	0
$m_{11}$	1	0	1	1	1
$m_{12}$	1	1	0	0	0
$m_{13}$	1	1	0	1	1
$m_{14}$	1	1	1	0	1
$m_{15}$	1	1	1	1	0

## 6.2 Maxterms

Maxterm is a sum involving all inputs (or complements) to a function. Every row of a truth table has a corresponding maxterm. Minterm is false if and only if the corresponding row in the table is active.

Maxterms are defined as follows for each row of a two input truth table:

A	B	maxterm	maxterm name
0	0	$A + B$	$M_0$
0	1	$A + \bar{B}$	$M_1$
1	0	$\bar{A} + B$	$M_2$
1	1	$\bar{A} + \bar{B}$	$M_3$

Consider a two input circuit whose output  $Y$  is given by the truth table:

A	B	Y	maxterm	maxterm name
0	0	0	$A + B$	$M_0$
0	1	1	$A + \bar{B}$	$M_1$
1	0	0	$\bar{A} + B$	$M_2$
1	1	1	$\bar{A} + \bar{B}$	$M_3$

then  $Y = (A + B)(\bar{A} + B) = M_0M_2$ .

Writing a functional specification in terms of minterms is also called product of sums canonical form.

**Example 7.** Write the maxterm  $M_{11}$  for 4-input Boolean function with the ordered inputs  $A, B, C, D$ .

**Example 8.** Convert the following 4-input truth table into product of maxterms and product of sums canonical form.

maxterm name	A	B	C	D	f
$M_0$	0	0	0	0	0
$M_1$	0	0	0	1	0
$M_2$	0	0	1	0	0
$M_3$	0	0	1	1	1
$M_4$	0	1	0	0	0
$M_5$	0	1	0	1	0
$M_6$	0	1	1	0	0
$M_7$	0	1	1	1	1
$M_8$	1	0	0	0	0
$M_9$	1	0	0	1	0
$M_{10}$	1	0	1	0	0
$M_{11}$	1	0	1	1	1
$M_{12}$	1	1	0	0	0
$M_{13}$	1	1	0	1	1
$M_{14}$	1	1	1	0	1
$M_{15}$	1	1	1	1	0

**Problem 6.** Convert the following 4-input truth table into product of maxterms and products of sums canonical form.



<i>masterm name</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>f</i>
$M_0$	0	0	0	0	0
$M_1$	0	0	0	1	1
$M_2$	0	0	1	0	1
$M_3$	0	0	1	1	1
$M_4$	0	1	0	0	1
$M_5$	0	1	0	1	0
$M_6$	0	1	1	0	1
$M_7$	0	1	1	1	1
$M_8$	1	0	0	0	0
$M_9$	1	0	0	1	1
$M_{10}$	1	0	1	0	1
$M_{11}$	1	0	1	1	1
$M_{12}$	1	1	0	0	0
$M_{13}$	1	1	0	1	1
$M_{14}$	1	1	1	0	1
$M_{15}$	1	1	1	1	0

**Example 9.** Write the 3-input truth table for the function  $f = m_2 + m_3 + m_7$ .

**Problem 7.** Write the 3-input truth table for the function  $f = M_4M_5M_7$ .

**Problem 8.** Write the truth table for the function  $f = \bar{A}B\bar{C} + AB\bar{C}$ .

## 7 Karnaugh maps

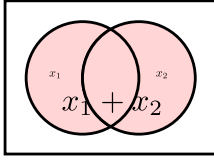
### 7.1 Two input K-maps

B	A	0	1
0		$m_0$	$m_2$
1		$m_1$	$m_3$

**Example 10.** Convert the following truth table into a K-map.

<i>A</i>	<i>B</i>	<i>f</i>
0	0	0
0	1	1
1	0	1
1	1	0

**Problem 9.** Convert the following Venn Diagram into a K-map.



## 7.2 Three input K-maps

C	AB			
	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

**Problem 10.** Draw a K-map for the function  $f = \bar{A}\bar{B}C + AB\bar{C}$ .

## 7.3 Four input K-maps

CD	AB			
	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$
01	$m_1$	$m_5$	$m_{13}$	$m_9$
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

**Problem 11.** Draw a K-map for a 4-input function  $f = m_1 + m_2 + m_7$ .

## 7.4 Five input K-maps

A = 0

DE \ BC	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$
01	$m_1$	$m_5$	$m_{13}$	$m_9$
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

A = 1

DE \ BC	00	01	11	10
00	$m_{16}$	$m_{20}$	$m_{28}$	$m_{24}$
01	$m_{17}$	$m_{21}$	$m_{29}$	$m_{25}$
11	$m_{19}$	$m_{23}$	$m_{31}$	$m_{27}$
10	$m_{18}$	$m_{22}$	$m_{30}$	$m_{26}$

**Problem 12.** Draw a K-map for a 5-input function  $f = M_1 M_2 M_7$ .