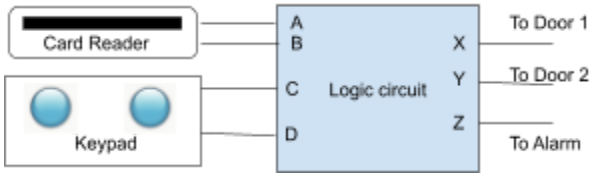
ECE 275 Midterm 1 (Fall 2021)

Max marks: 80 marks | Time: 50 minutes

Name: _____ | Oct 6, 2021

A simple security system for two doors consists of a card reader and a keypad.



A person may open a particular door if he or she has a card containing the corresponding code and enters an authorized keypad code for that card. Note that card-code and keypad-code are different. The outputs from the card reader are given in the table below.

Keypad.
CD

	A	B
No card inserted	0	0
Valid card-code for door 1	0	1
Valid card-code for door 2	1	1
Invalid card code	1	0

10
11

To unlock a door, a person must hold down the proper keys on the keypad and, then, insert the card in the reader. The authorized keypad code for door 1 is 10, and the authorized keypad code for door 2 is 11. If the card has an invalid code or if the wrong keypad code is entered, the alarm will ring when the card is inserted. If the correct keypad code is entered, the corresponding door will be unlocked when the card is inserted.

Design the logic circuit for this simple security system. Your circuit's inputs will consist of a card code AB, and a keypad code CD. The circuit will have three outputs
XYZ (if X is 1, door 1 will be opened; if Y is 1, door 2 will be opened; if Z 1, the alarm will sound).

1. Find the minimal cost two-level circuit using K-maps for X, Y, Z. Provide the minimal cost. (It can be either of SOP/POS forms) (30 marks)
2. For output Z, verify your answer using Quine McCluskey method. Verify only the form (either SOP/POS) with the lowest cost. (20 marks)
3. Try finding a lower cost implementation for Z using functional decomposition with a maximum fan-in of 2. Does functional decomposition lower the cost? (20 marks)
4. Is the circuit for Z hazard free? If not, design a Hazard free circuit. (10 marks)

For ease of grading, please use the following K-map format. The minterm numbering of the K-map is provided.

Row	A	B	C	D	Door1 X	Door2 Y	Alarm Z
0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0
2	0	0	1	0	0	0	0
3	0	0	1	1	0	0	0
4	0	1	0	0	0	0	1
5	0	1	0	1	1	0	0
6	0	1	1	0	0	0	1
7	0	1	1	1	0	0	1
8	1	0	0	0	0	0	1
9	1	0	0	1	0	0	1
10	1	0	1	0	0	0	1
11	1	0	1	1	0	0	1
12	1	1	0	0	0	0	1
13	1	1	0	1	0	0	1
14	1	1	1	0	0	0	1
15	1	1	1	1	0	1	0

X		~A		A	
		~B	B		~B
~C	~D	0	4	12	8
	D	1	5	13	9
C		3	7	15	11
	~D	2	6	14	10

Y		~A		A	
		~B	B		~B
~C	~D	0	4	12	8
	D	1	5	13	9
C		3	7	15	11
	~D	2	6	14	10

Z		~A		A	
		~B	B		~B
~C	~D	0	4	12	8
	D	1	5	13	9
C		3	7	15	11
	~D	2	6	14	10

POS

$X = \bar{A}BC\bar{D}$

SOP

$\bar{X} = A + \bar{C} + D + B$

$X = \bar{A}BC\bar{D}$

$Y = ABCD$

$\bar{Y} = \bar{B} + \bar{C} + \bar{D} + \bar{A}$

$Y = ABCD$

SOP

$Z = \bar{A}B\bar{D} + A\bar{D} + B\bar{C} + A\bar{B}$

POS 14 = 4 3 3

$\bar{Z} = ABCD + \bar{A}C\bar{D} + \bar{A}\bar{B}$

Cost(Z, POS) = 19

$Z = (\overline{A+B} + \overline{C+D})(\overline{A+C+D})$

$(A+B)$

Cost(Z) = 16

		~A		A	
		~B	B		~B
~C	~D	0	4	12	8
	D	1	5	13	9
C		3	7	15	11
	~D	2	6	14	10

12 + 4

Quine McCluskey Alg.

Z | SOP

$Z = \Sigma m(0,1,2,3,6,15)$

group	Minterms	Binary	
0	0	0000	✓
	1	0001	✓
1	2	0010	✓
	3	0011	✓
2	6	0110	✓
4	15	1111	

		~A		A	
		~B	B		~B
~C	~D	0	4	12	8
	D	1	5	13	9
C		3	7	15	11
	~D	2	6	14	10

		~A		A	
		~B	B		~B
~C	~D	0	4	12	8
	D	1	5	13	9
C		3	7	15	11
	~D	2	6	14	10

group	Minterm pair	Binary	
0	0, 1	000*	✓
	0, 2	00*0	✓
1	1, 3	00*1	✓
	2, 3	001*	✓

| 2,6 | 0*10

group	Minterm	Binary
0	0, 1, 2, 3	00**

$\bar{Z}$ = PI = {00**, 0*10, 1111}

$\bar{Z} = \bar{A}\bar{B} + \bar{A}C\bar{D} + ABCD$

$Z = (A+B)(A+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})$

		~A		A	
		~B	B		~B
~C	~D	0	4	12	8
	D	1	5	13	9
C		3	7	15	11
	~D	2	6	14	10

		~A		A	
		~B	B		~B
~C	~D	0	4	12	8
	D	1	5	13	9
C		3	7	15	11
	~D	2	6	14	10

Z		~A		A		
		~B	B		~B	
~C	~D	0 0	4 1	12 1	8 1	g_1
	D	1 0	5 1	13 1	9 1	g_1
C		3 0	7 1	15 0	11 1	g_2
	~D	2 0	6 0	14 1	10 1	g_3
		0	h_2	h_1	1	

functional decomposit

$\bar{Z} = \bar{A}\bar{B} + \bar{A}C\bar{D} + ABCD$

$= \bar{A}(\bar{B} + C\bar{D}) + ABCD$

3 3 3 5

= 12 + 5

= 17 > 16

(9) Hazards

SOP form is hazard free