Basic gates
1. AND
2. OR
3. NOT

Other gates
1. NAND → Not AND $f(A,B) = \overline{A \cdot B}$
2. NOR → Not OR $f(A,B) = \overline{A+B}$
3. XOR → Exclusive OR
4. XNOR → Not Exclusive OR

XOR

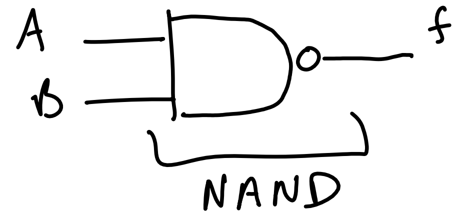
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

XNOR

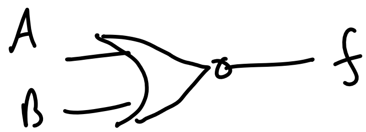
A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

ANSI network symbols

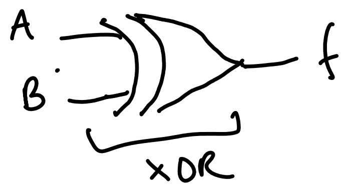
NAND $\equiv$ Not of AND



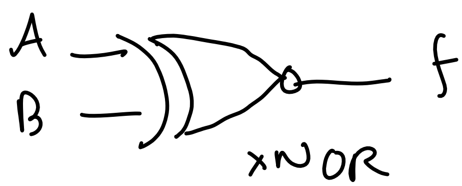
NOR $\equiv$ Not of OR



XOR $\equiv$



XNOR $\equiv$



$$\text{XOR} = A \oplus B = m_1 + m_2 = \bar{A}B + A\bar{B}$$

$$\text{XNOR} = \overline{A \oplus B} = m_0 + m_3 = \bar{A}\bar{B} + AB$$

XOR:

B \ A	0	1
0	0	1
1	1	0

XNOR:

B \ A	0	1
0	1	0
1	0	1

NAND/NOR gates + Petricks

Vikas Dhiman for ECE275

September 25, 2023

1 Circuit design using NAND/NOR gates

Example 1. *Implement the function $f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$ using (1) NAND gates only and (2) NOR gates only.*

NAND gates only ≡ Sum of products (SOP)

$$f = A\bar{B} + \bar{A}B = \overline{\underset{x}{\boxed{\overline{A\bar{B}}}}} + \overline{\underset{y}{\boxed{\overline{\bar{A}B}}}}$$

$x = \overline{A\bar{B}}$

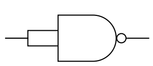
$y = \overline{\bar{A}B}$

$= \bar{x} + \bar{y}$

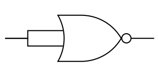
$= \overline{x \cdot y}$ | By Demorgan's Theorem

Remark 1. *NAND-NAND logic is generated from SOP form. NOR-NOR logic is generated from POS form.*

Remark 2. *NOT gate can also be created from a NAND gate $\bar{x} = \overline{x \cdot x}$.*

x 

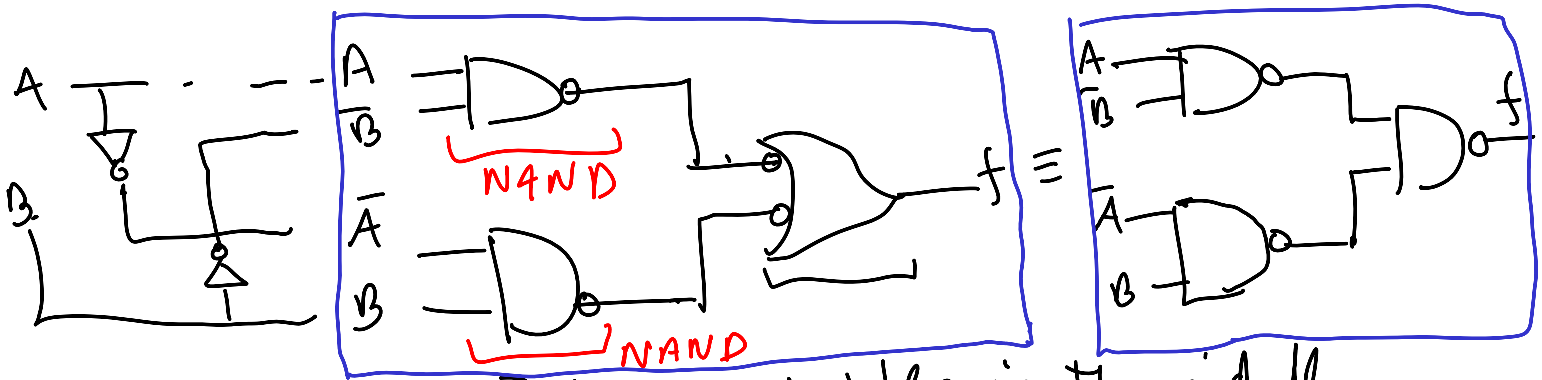
Remark 3. *NOT gate can also be created from a NOR gate $\bar{x} = \overline{x + x}$.*

x 

Problem 1. *Design the simplest circuit that implements the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7)$ using (1) NAND gates only (2) NOR gates only.*

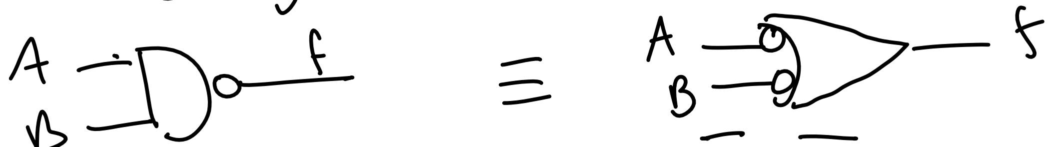
$f = A\bar{B} + \bar{A}B$

Bubble pushing

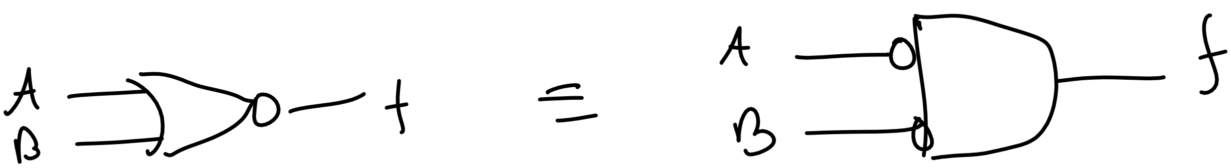


(1) Introduce bubbles in the middle

DeMorgan's theorem



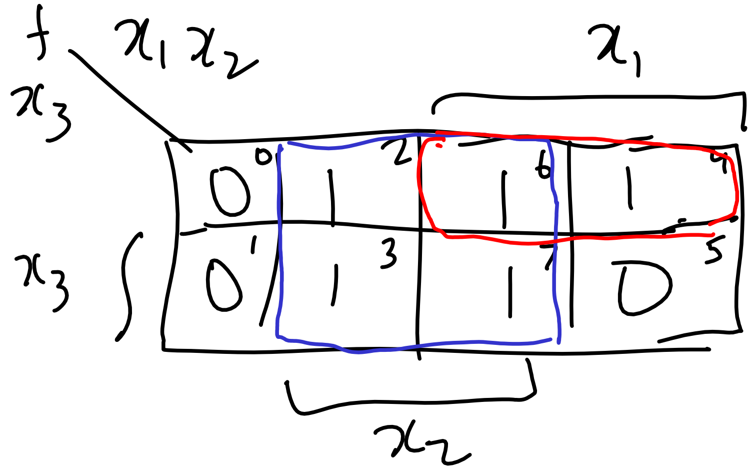
$f = \overline{A \cdot B} = \bar{A} + \bar{B}$



Ex.1) Find the min cost NAND implementation

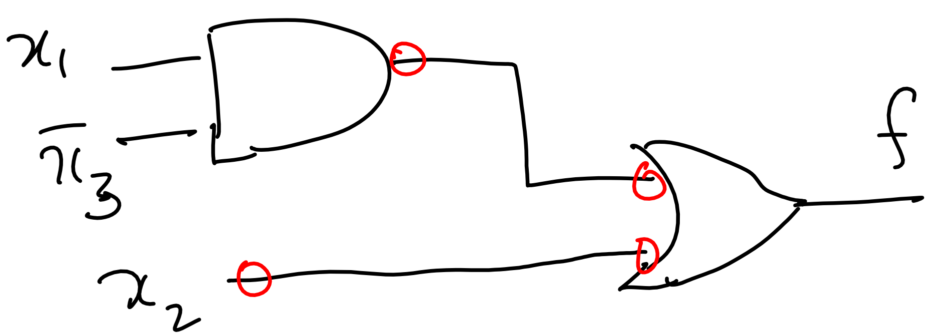
$f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$

for NAND, we design SOP implementation

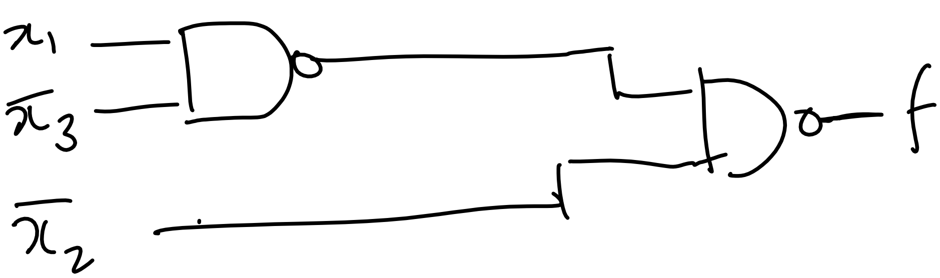


$PI = \{x_2, x_1 \bar{x}_3\} = EPI$

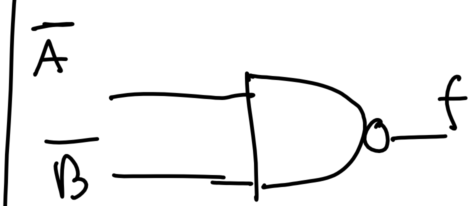
$f = x_2 + x_1 \bar{x}_3$



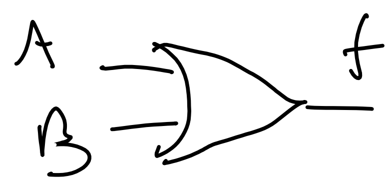
$\equiv$



Q



$\equiv$

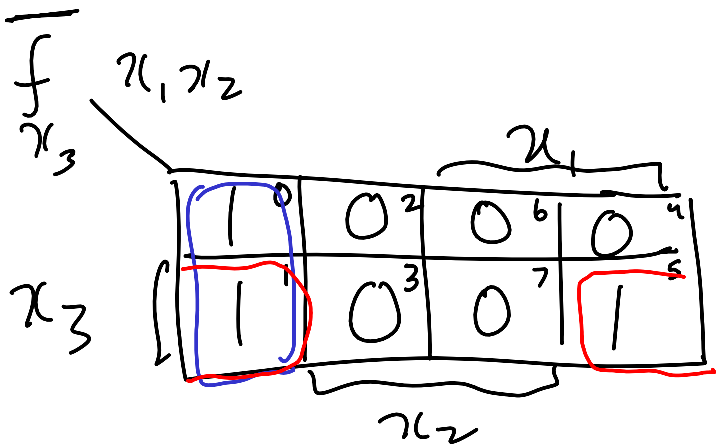


$f = \overline{\bar{A} \cdot \bar{B}}$

$= A + B$

$$f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$$

for NOR implementation find POS

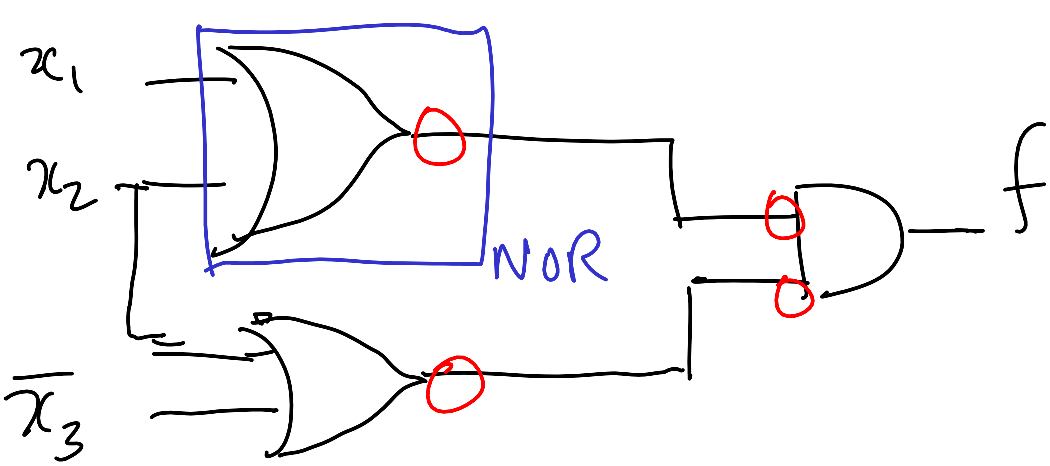


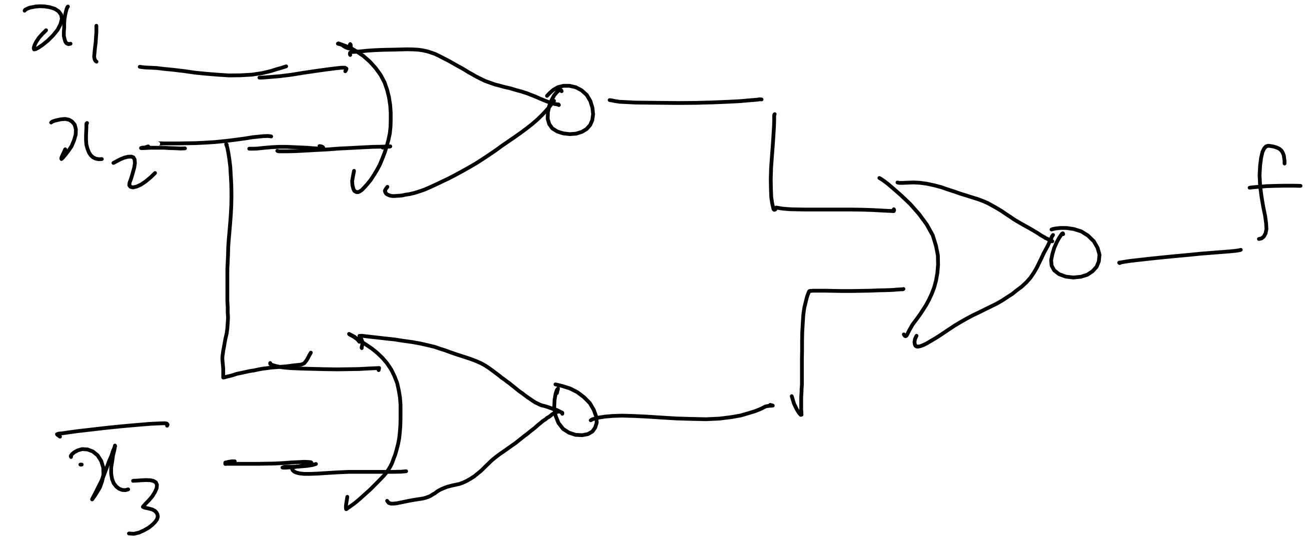
$$PI = \{\bar{x}_1 \bar{x}_2 , \bar{x}_2 x_3\} = EPI$$

$$\bar{f} = \bar{x}_1 \bar{x}_2 + \bar{x}_2 x_3$$

$$f = \overline{\bar{x}_1 \bar{x}_2 + \bar{x}_2 x_3}$$

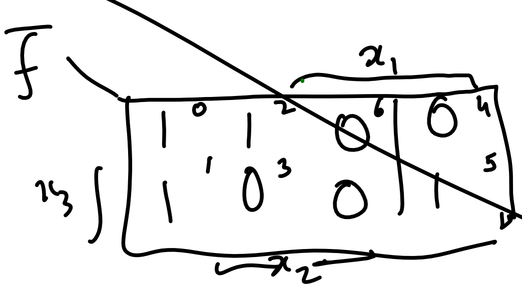
$$= (x_1 + x_2) \cdot (x_2 + \bar{x}_3)$$





Prob 1 $f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7)$

for NOR find POS

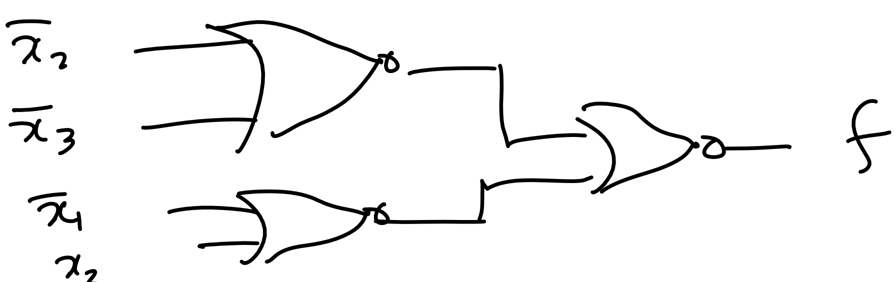


PI = { x_2x_3 , x_1x_2 , $x_1\overline{x_3}$ }

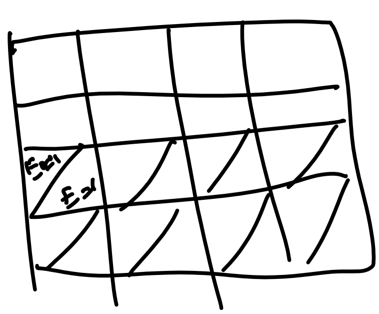
EPI = { x_2x_3 , $x_1\overline{x_3}$ }

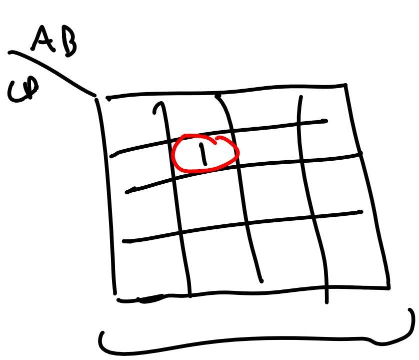
$\overline{f} = x_2x_3 + x_1\overline{x_3}$

$f = (\overline{x_2} + \overline{x_3})(\overline{x_1} + x_3)$



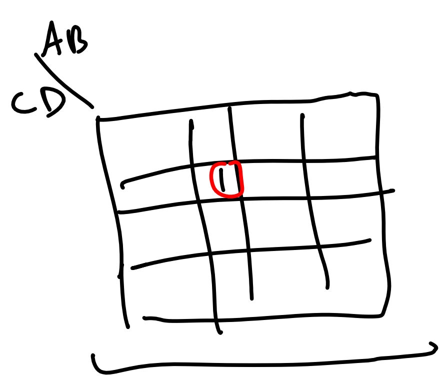
5-var K-map





$\overline{E} = 1$

E=



E=1

① Finding all the PI

Quine McCluskey method ② Finding minimum cost cover from the PIs

2a 2b

2 PI Table reduction and Petrick's method

This is not in the text-book. For additional reading, please refer to the linked resources on the website.

Definition 1 (Implicant)**.** *Given a function f of n variables, a product term P is an implicant of f if and only if for every combination of values of the n variables for which $P = 1$, f is also equal to 1.*

Definition 2 (Prime Implicant)**.** *A prime implicant of a function f is an implicant which is no longer an implicant if any literal is removed from it.*

There are 4 main steps in the Quine-McCluskey algorithm/PI Table reduction and Petrick's method:

1. Generate Prime Implicants
2. Construct Prime Implicant Table. PIs as columns, and minterms as rows (don't cares are excluded).
3. Reduce Prime Implicant Table by repeating following steps until they it cannot be reduced further
   (a) Remove Essential Prime Implicants
   (b) Row Dominance: Remove *dominating* rows. (i.e. unnecessary minterms)
   (c) Column Dominance: Remove *dominated* columns. (i.e. remove unnecessary PIs)
4. Solve Prime Implicant Table by Petrick's method

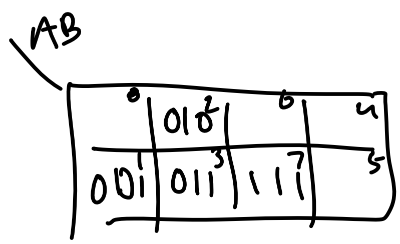
2.1 Generate Prime Implicants

Example 2. *Generate prime implicants of the function $F(A, B, C, D) = \sum m(0, 2, 5, 6, 7, 8, 10, 12, 13, 14, 15)$ using Quine-McCluskey method*

Group	Minterms	ABCD
0	m_0	0000
1	m_2	0010
	m_8	1000
2	m_5	0101
	m_6	0110
	m_{10}	1010
	m_{12}	1100
3	m_{13}	1101
	m_{14}	1110
	m_7	0111
4	m_{15}	1111

$\Rightarrow$ Binary number

G		A	B	C	D
0	m(0,2)	0	0	*	0
	m(0,8)	*	0	0	0
1	m(2,6)	0	*	1	0
	m(2,10)	*	0	1	0
	m(8,10)	1	0	*	0
	m(8,12)	1	*	0	0
2	m(5,13)	*	1	0	1
	m(5,7)	0	1	*	1
	m(6,14)	*	1	1	0
	m(6,7)	0	1	1	*
	m(10,14)	1	*	1	0
	m(12,13)	1	1	0	*
	m(12,14)	1	1	*	0



$m_0 + m_2$:

$m_0 + m_8$

$= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D}$

$= \bar{A}\bar{B}(\bar{C}+C)\bar{D}$

$= \bar{A}\bar{B}\bar{D}$

Steps:

1. Start with writing minterms in binary format (include don't cares as minterms).

3	m(13,15)	1	1	*	1	
	m(14,15)	1	1	1	*	
	m(7,15)	*	1	1	1	

Binary number

G		A	B	C	D	
0	m(0,2)	0	0	*	0	✓
	m(0,8)	*	0	0	0	✓
1	m(2,6)	0	*	1	0	✓
	m(2,10)	*	0	1	0	✓
	m(8,10)	1	0	*	0	✓
	m(8,12)	1	*	0	0	
2	m(5,13)	*	1	0	1	✓
	m(5,7)	0	1	*	1	✓
	m(6,14)	*	1	1	0	✓
	m(6,7)	0	1	1	*	✓
	m(10,14)	1	*	1	0	✓
	m(12,13)	1	1	0	*	✓
	m(12,14)	1	1	*	0	✓
3	m(13,15)	1	1	*	1	✓
	m(14,15)	1	1	1	*	✓
	m(7,15)	*	1	1	1	✓

G	A	B	C	D
0	*	0	*	0
	*	0	*	0
1	*	*	1	0
	*	*	1	0
	1	*	*	0
	1	*	*	0
2	*	1	*	1
	1	1	*	*
	*	1	1	*
	1	1	*	*
	*	1	*	1

G	A	B	C	D
0				
1				

$PI = \{\bar{B}\bar{D}, C\bar{D}, A\bar{D}, BD, AB, BC, BD\}$

PI table reduction

2. Create potential groups of minterms that can be combined (merged). The only minterms that can be combined differ only be single 1. Create a new list of combined minterms as n-1 literal implicants.
3. Check off the minterms that could be combined. Unchecked minterms are prime implicants (PIs).
4. Repeat the grouping process with n-1 literal implicants.

Problem 2. *Generate PIs for the function* $F(A,B,C,D) = \sum m(0,2,3,4,5,6,7,8,9,10,11,12,13)$.

2.2 Prime Implicants table and reduction

Example 3. *Reduce the prime implicants* $\underbrace{\{\bar{B}\bar{D}, C\bar{D}, BD, BC, A\bar{D}, AB\}}$ *using prime implicants table.*

PI table   Draw all the minterms as row and PIs as columns

minterms ↓ / PI →	$\bar{B}\bar{D}$	$C\bar{D}$	BD	BC	$A\bar{D}$	AB
0						
2						
5						
6						
7						
8						
10						
12						
13						
14						
15						

Example 4.

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

$*0*0 \quad **10 \quad *1*1 \quad *11* \quad 1**0 \quad 11**$

PI table: Draw all the minterms as row and PIs as columns

minterms		$\bar{B}\bar{D}$	$C\bar{D}$	BD	BC	$A\bar{D}$	AB
0000	0	✓					
0010	2	✓	✓				
0101	5			✓			
0110	6		✓		✓		
0111	7			✓	✓		
1000	8	✓				✓	
1010	10	✓	✓			✓	
1100	12					✓	✓
1101	13			✓			✓
1110	14		✓		✓	✓	✓
1111	15			✓	✓		✓

(1) Find EPIs

EPI = $\{\bar{B}\bar{D}, BD\}$

(2) Row dominance: Remove $\boxed{\text{dominating}}$ row

minterm 14 dominates minterm 12

(3) Column dominance: Remove $\boxed{\text{dominated}}$ cols

(3a) PI $A\bar{D}$ and AB dominate each other so pick the lowest cost PI

I pick AB

(3b) PI BC and $C\bar{D}$ dominate each other so pick the lowes cost PI among them. I pick BC.

PI cover = EPI + $\{AB, BC\}$

$= \{\bar{B}\bar{D}, BD, AB, BC\}$

$f(A,B,C,D) = \bar{B}\bar{D} + BD + AB + BC$

Example 5.

$CD \backslash AB$	00	01	11	10
00	d	0	0	0
01	1	1	d	d
11	1	1	0	0
10	1	d	0	0

$f = \bar{A}\bar{D} + \bar{B}C$

$p_1 = p_3 = p_4 = \dots = 0$

$p_2 = 1$

Example 6. *Reduce the following PI table*

	~~$\bar{A}\bar{D}$~~	~~$\bar{B}\bar{D}$~~	~~$\bar{C}\bar{D}$~~	$\bar{A}C$ (p_1)	$\bar{B}C$ (p_2)	$\bar{A}B$ (p_3)	$B\bar{C}$ (p_4)	$A\bar{B}$ (p_5)	$A\bar{C}$ (p_6)
0	X	X	X						
~~2~~	~~X~~	~~X~~		~~X~~	~~X~~				
3				X	X				
~~4~~	~~X~~		~~X~~			~~X~~	~~X~~		
5						X	X		
~~6~~	~~X~~			~~X~~		~~X~~			
7				X		X			
~~8~~		~~X~~	~~X~~					~~X~~	~~X~~
9								X	X
~~10~~		~~X~~			~~X~~			~~X~~	
11					X			X	
~~12~~			~~X~~		~~X~~		~~X~~		~~X~~
13							X		X

All minterms are covered iff

$$= \underbrace{(P_1 + P_2)}_{m_3 \text{ is covered}} \underbrace{(P_3 + P_4)}_{m_5} \underbrace{(P_1 + P_3)}_{m_7} (P_5 + P_6)(P_2 + P_5)(P_4 + P_6)$$

① Find EPIs = { } No EPIs

② Row dominance (2a) minterm 2 dominates minterm 3
Remove 2.

2(b) m(4) dominates m(5)
Remove m(4)

2(c) m(6) > m(7) ⇒ remove m(6)

2(d) m(8) > m(9) ⇒ remove m(8)

m(10) > m(11) ⇒ " m(10)

m(12) > m(13) ⇒ " m(12)

③ Column dominance (3a) $\bar{A}\bar{D} \equiv \bar{B}\bar{D} \equiv \bar{C}\bar{D}$, so pick min cost.
I pick $\bar{A}\bar{D}$

Second Round

① Find EPIs

② No row dominance

③ No col dominance

Petrick's method

P_i := true if PI p_i is included in the final cover

All minterms are covered iff

$x \cdot x = x$

$= (P_1 + P_2)(P_3 + P_4)(P_1 + P_3)(P_5 + P_6)(P_2 + P_5)(P_4 + P_6)$

$= \underbrace{P_1 P_3 P_5 P_2 P_4}_{\text{PIs are in the cover}} + P_1 P_3 P_5 \quad \ldots \quad - \quad +$

↑ OR

$p_1 = \bar{A} C$

$p_3 = \bar{A} B$

$p_5 = A \bar{B}$

$f = \bar{A}\bar{D} + \bar{A}C + \bar{A}B + A\bar{B}$

2.3 Petrick's method

Example 7. *Solve the Prime Implicant table using Petrick's method*

	$p_1 = \bar{A}C$	$p_2 = \bar{B}C$	$p_3 = \bar{A}B$	$p_4 = B\bar{C}$	$p_5 = A\bar{B}$	$p_6 = A\bar{C}$
3	X	X				
5			X	X		
7	X		X			
9					X	X
11		X			X	
13				X		X

Example 8. *Find the minimum SOP expression for the function* $F(A, B, C, D) = \sum m(2, 3, 7, 9, 11, 13) + \sum d(1, 10, 15)$ *using Quine-McCluskey method.*