Logic minimi~~zat~~ion: Minimum-cost circuits

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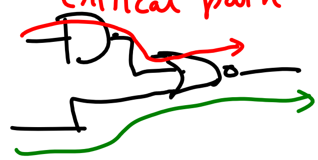
Critical path



1 Logic minimization

A general optimization criteria for multi-level logic are to Minimize some combination of:

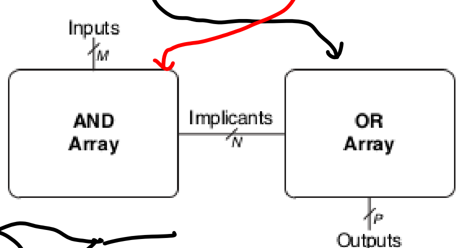
1. Area occupied by the logic gates and interconnect;
2. the Critical Path Delay of the longest path through the logic;
3. the Degree of Testability of the circuit, measured in terms of the percentage of faults covered by a specified set of test vectors, for an appropriate fault model (Eg., single stuck faults, multiple stuck faults, etc.);
4. Power consumed by the logic gates.

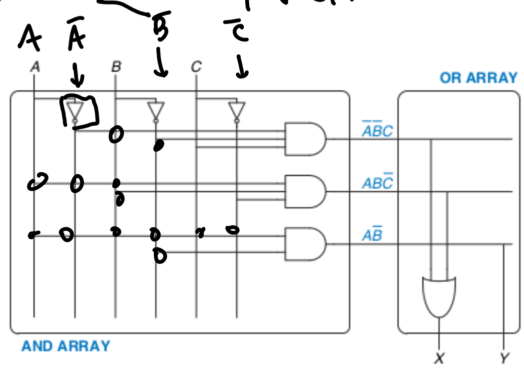
In this course, we will start with two-level multi-input circuits and a criteria based on the number of gates/transistors/diodes.

2 Programmable Logic Arrays

FPGA

Sum of Product form





3 Two-level circuits

$\overbrace{\underbrace{x + y\cdot z \cdot w}_{\text{OR}}}^{\text{AND}}$ 2 gates

OR gate has 2 inputs

AND gate has 2 inputs ~~2~~ 3

$x+y\cdot z$ has the cost $= 2+2+2 = 6$

The cost that we are going to consider in this class depend upon:

1. Number of gates. except NOT gates (Because we invert everything)
2. Number of input to the gates.

More gates need more transistors, more area on the chip. More-inputs the gate need more transistors within each gate. Number of gate inputs can be considered secondary criterion to the number of gates.

Example 1. *Find the cost of the following Boolean expression* $X = \overbrace{\underbrace{\bar{A}\bar{B}C}_{\text{AND}} + \underbrace{AB\bar{C}}_{\text{AND}} + \underbrace{A\bar{B}}_{\text{AND}}}^{\text{OR}}$.

Problem 1. *Find the cost of the following Boolean expression* $X = A\bar{B}C + \bar{A}B\bar{C} + \bar{B}C$.

3 AND gates + 1 OR gate + 3 input + 3 input + 2 input + 3 inputs to the OR gate

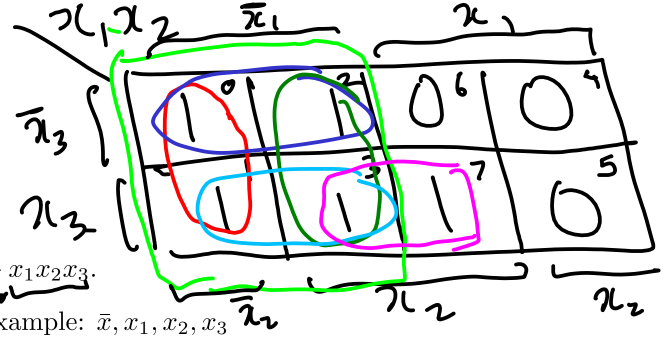
= 15

Problem 1 $A\bar{B}C + \bar{A}B\bar{C} + \bar{B}C$

3 AND gates + 1 OR gate + 3 input + 3 input + 2 input + 3 inputs to the OR gate

= 15

Implicants = $\bar{x}_1 \cdot \bar{x}_3$, $\bar{x}_1 \bar{x}_2$, $\bar{x}_1 \cdot x_3$, $\bar{x}_1 \cdot x_2$, $x_2 x_3$, $\bar{x}_1$



4 Terminology for K-maps

Running Example: $f = \sum m(0,1,2,3,7) = \bar{x}_1 + x_1x_2x_3$.

Literal A single variable or its complement. Example: $\bar{x}, x_1, x_2, x_3$

Implicant A product term which is true for a function. All minterms are implicants. Example: $x_1x_2x_3$, $\bar{x}_1$, $m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$, $\bar{x}_1x_3$, $\bar{x}_1\bar{x}_3$.

$m_0 = \overline{\bar{x}_1 \cdot \bar{x}_2 \cdot x_3}$

PI = $\bar{x}_1$, x_2x_3

Prime Implicant An implicant that cannot be combined into fewer literals. Example: $\bar{x}_1, x_2x_3$.

= EPI

Essential Prime Implicant An Prime implicant that ~~cannot be combined into fewer literals~~ that is the only one to cover some minterm. Example: x_2x_3.

Cover : List of Prime Implicants that account for all $f = 1$.

Cost : Number of gates (excluding not gate on literals) ~~and~~ + number of inputs to each gate.

Example 2. *Find minimum cost expression for the function* $f(x_1,x_2,x_3) = \prod M(4,5,6)$

x_1	x_2	x_3	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

$\bar{x}_1$

Combining : in the K-map if 1's can be grouped

in the boolean expression

$m_0 = \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3$ $\quad m_1 = \bar{x}_1 \cdot \bar{x}_2 \cdot x_3$

$f = \sum m(\ ) = m_0 + m_1 = \underbrace{\bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3 + \bar{x}_1 \cdot \bar{x}_2 \cdot x_3}_{\text{not PI}} = \bar{x}_1 \cdot \bar{x}_2 \underbrace{(\bar{x}_3 + x_3)}_{1} = \bar{x}_1 \cdot \bar{x}_2$

Problem 2. *Find minimum cost expression for the function* $f(x_1,x_2,x_3) = \prod M(2,5,6)$

4.1 Incompletely specified functions or Don't cares

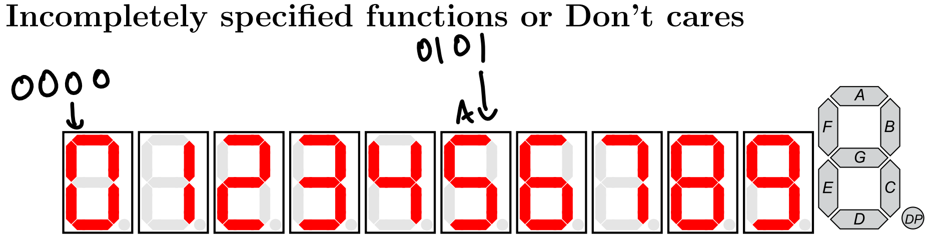
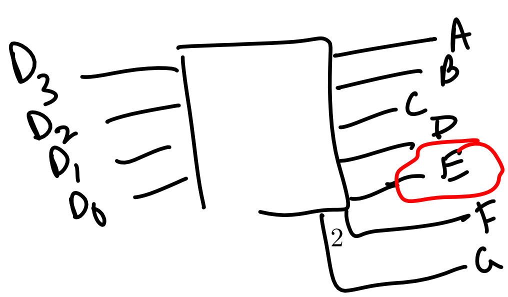
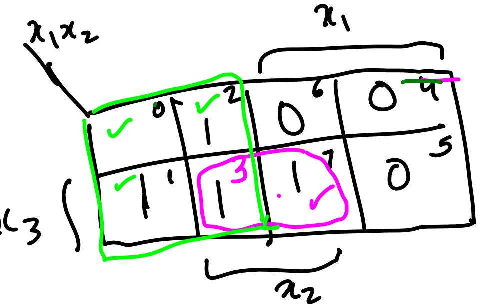


Figure 1: 7 Segment Representations of Each Integer



$f(x_1,x_2,x_3) = \prod M(4,5,6)$



① Find all PI = $\{\bar{x}_1, x_2x_3\}$

② Find all EPI = $\{\bar{x}_1, x_2x_3\}$

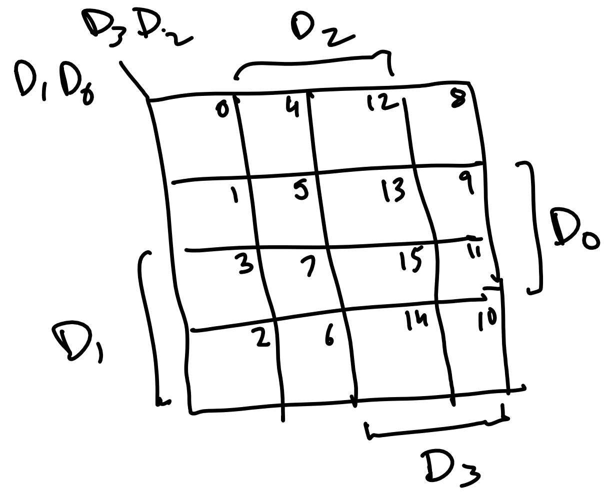
③ Remove the ones that are covered by the EPI } Take note of this EPI

↓

The list of these EPIs is the minimum cost expression/circuit

④ If some ones are still remaining the choose a subset of PIs that forms a cover

	BCD Value				LED Segment
	D_3	D_2	D_1	D_0	E
	0	0	0	0	0
	0	0	0	1	1
	0	0	1	0	0
	0	0	1	1	1
	0	1	0	0	1
	0	1	0	1	1
	0	1	1	0	0
	0	1	1	1	1
	1	0	0	0	0
9	1	0	0	1	1
	1	0	1	0	d
	1	0	1	1	d
	1	1	0	0	d
	1	1	0	1	d
	1	1	1	0	d
	1	1	1	1	d



Example 3. *Find minimum cost expression for the function*

$$f(x_1, \ldots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

Problem 3. *Find minimum cost expression for the function*

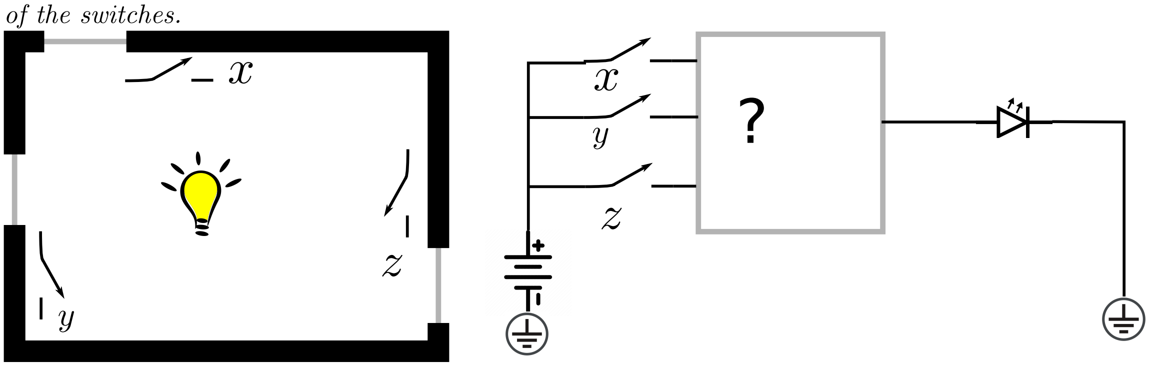
$$f(x_1, \ldots, x_4) = \sum m(0, 2, 4, 6, 7, 8, 9, 13) + D(1, 12, 15)$$

5 A few more Boolean problems

Example 4. *Simplify the following Boolean expression:*

$$f = x_1\bar{x}_3\bar{x}_4 + x_2\bar{x}_3\bar{x}_4 + x_1\bar{x}_2\bar{x}_3$$

Example 5. *Assume that a large room has three doors and that a switch near each door controls a light in the room. It has to be possible to turn the light on or off by changing the state of any one of the switches.*



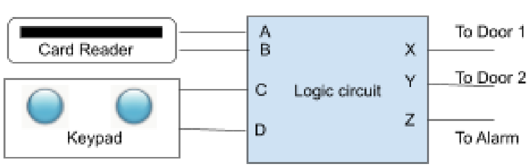
Problem 4. *A simple security system for two doors consists of a card reader and a keypad.*

A person may open a particular door if he or she has a card containing the corresponding code and enters an authorized keypad code for that card. Note that card-code and keypad-code are different. The outputs from the card reader are given in the table below.

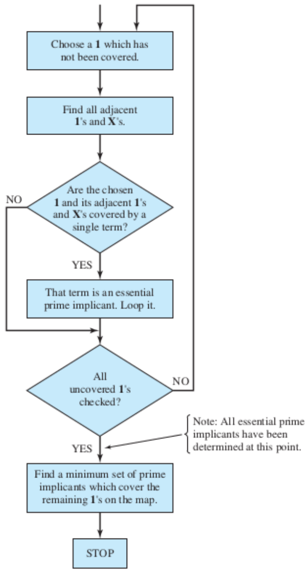
To unlock a door, a person must hold down the proper keys on the keypad and, then, insert the card in the reader. The authorized keypad code for door 1 is 10, and the authorized keypad code for door 2 is 11. If the card has an invalid code or if the wrong keypad code is entered, the alarm will ring when the card is inserted. If the correct keypad code is entered, the corresponding door will be unlocked when the card is inserted.

Design the logic circuit for this simple security system. Your circuit's inputs will consist of a card code AB, and a keypad code CD. The circuit will have three outputs XYZ (if X is 1, door 1 will be opened; if Y is 1, door 2 will be opened; if Z 1, the alarm will sound).

Find the minimal cost two-level circuit using K-maps for X, Y, Z. Provide the minimal cost. (It can be either of SOP/POS forms)



	A	B
No card inserted	0	0
Valid card-code for door 1	0	1
Valid card-code for door 2	1	1
Invalid card code	1	0



Example 6.

$CD \backslash AB$	00	01	11	10
00	X	1		1
01		1	1	1
11		X	X	
10		1		1

Problem 5. *Find the minimum SOP (sum of products) and POS (product of sum) expression for the function* $f(a,b,c,d) = \prod M(5,7,13,14,15) \cdot \prod D(1,2,3,9)$