

Combinational circuit

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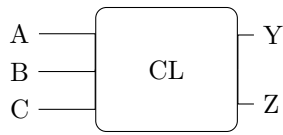
1 Learning objectives

1. Representing digital circuits
2. Converting between different notations: Boolean expression, logic networks and switching circuits
3. Converting between different logic network specifications: truth table, minterm, maxterms, product of sums canonical form and sum of product canonical form.

2 Basic Gates and notations summary

Name	C/Verilog	Boolean expr.	Truth Table	Switching circuit	(ANSI) symbol	Venn diagram															
AND Gate	L = x1 & x2	$L = x_1 \cdot x_2 = x_1x_2$	<table><tr><th>x_1</th><th>x_2</th><th>$x_1 \cdot x_2$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	x_1	x_2	$x_1 \cdot x_2$	0	0	0	0	1	0	1	0	0	1	1	1			
x_1	x_2	$x_1 \cdot x_2$																			
0	0	0																			
0	1	0																			
1	0	0																			
1	1	1																			
OR Gate	L = x1 x2	$L = x_1 + x_2$	<table><tr><th>x_1</th><th>x_2</th><th>$x_1 + x_2$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	x_1	x_2	$x_1 + x_2$	0	0	0	0	1	1	1	0	1	1	1	1			
x_1	x_2	$x_1 + x_2$																			
0	0	0																			
0	1	1																			
1	0	1																			
1	1	1																			
NOT Gate	L = ~ x1	$L = \bar{x}_1 = x'_1$	<table><tr><th>x_1</th><th>\bar{x}_1</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	x_1	\bar{x}_1	0	1	1	0												
x_1	\bar{x}_1																				
0	1																				
1	0																				

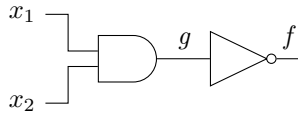
3 Digital circuits or networks



$$Y = F(A, B, C) \quad Z = G(A, B, C)$$

4 Two input networks

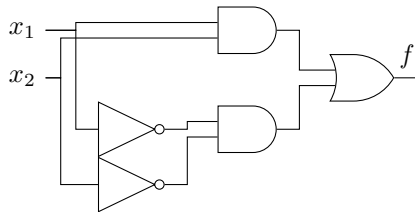
Example 1. Convert the following (ANSI) network into a Boolean expression, a truth table and a Venn diagram.



Example 2. Convert the following Boolean expression into a (ANSI) network, a truth table and a Venn diagram:

$$f = \overline{x_1 + x_2}$$

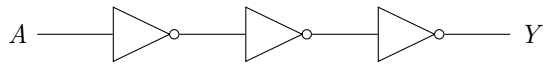
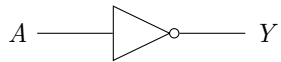
Problem 1. Convert the following (ANSI) network into a Boolean expression, a truth table and a Venn diagram.



Example 3. Convert the following Boolean expression into a network, a truth table and a Venn diagram:

$$f = x_1\bar{x}_2 + \bar{x}_1x_2$$

Problem 2. Can two different circuits have the same truth table? Can two different truth tables have the same circuit? Consider the following two circuits for example

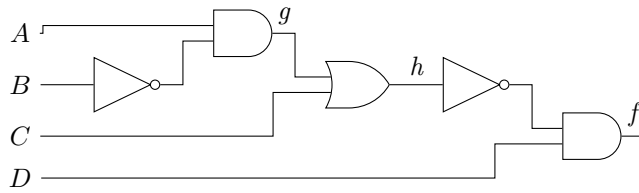


How about Venn diagrams?

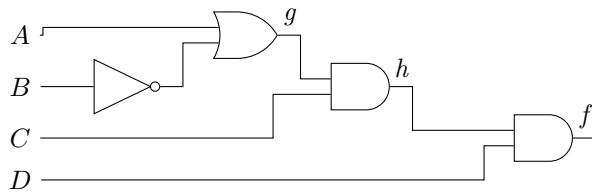
Remark 1. Truth tables and Venn diagrams define what the combinational circuit should do. Truth tables define output for every input. Boolean expression and networks define how to achieve the desired input output relationship.

5 Multi-input networks

Example 4. Convert the following (ANSI) network into a Boolean expression and a truth table.



Problem 3. Convert the following (ANSI) network into a Boolean expression and a truth table.



6 Minterms and Maxterms

6.1 Minterms

Minterm is a product involving all inputs (or complements) to a function. Every row of a truth table has a corresponding minterm. Minterm is true if and only if the corresponding row in the table is active.

Minterms defined as follows for each row of a two input truth table:

A	B	minterm	minterm name
0	0	$\bar{A}\bar{B}$	m_0
0	1	$\bar{A}B$	m_1
1	0	$A\bar{B}$	m_2
1	1	AB	m_3

Consider a two input circuit whose output Y is given by the truth table:

A	B	Y	minterm	minterm name
0	0	0	$\bar{A}\bar{B}$	m_0
0	1	1	$\bar{A}B$	m_1
1	0	0	$A\bar{B}$	m_2
1	1	1	AB	m_3

then $Y = \bar{A}B + AB = m_1 + m_3 = \sum(1, 3)$.

This also gives the *sum of products canonical form*.

Example 5. What is the minterm m_{13} for a 4-input circuit with inputs x, y, z, w (ordered from MSB to LSB).

Problem 4. What is the minterm m_{23} for a 5-input circuit with inputs a, b, c, d, e (ordered from MSB to LSB).

Example 6. Convert the following 4-input truth table into sum of minterms and sum of products canonical form.

<i>minterm name</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>f</i>
m_0	0	0	0	0	0
m_1	0	0	0	1	1
m_2	0	0	1	0	0
m_3	0	0	1	1	0
m_4	0	1	0	0	0
m_5	0	1	0	1	1
m_6	0	1	1	0	0
m_7	0	1	1	1	0
m_8	1	0	0	0	0
m_9	1	0	0	1	0
m_{10}	1	0	1	0	0
m_{11}	1	0	1	1	0
m_{12}	1	1	0	0	0
m_{13}	1	1	0	1	1
m_{14}	1	1	1	0	0
m_{15}	1	1	1	1	0

Problem 5. Convert the following 4-input truth table into sum of minterms and sum of products canonical form.

<i>minterm name</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>f</i>
m_0	0	0	0	0	0
m_1	0	0	0	1	0
m_2	0	0	1	0	0
m_3	0	0	1	1	1
m_4	0	1	0	0	0
m_5	0	1	0	1	0
m_6	0	1	1	0	0
m_7	0	1	1	1	1
m_8	1	0	0	0	0
m_9	1	0	0	1	0
m_{10}	1	0	1	0	0
m_{11}	1	0	1	1	1
m_{12}	1	1	0	0	0
m_{13}	1	1	0	1	1
m_{14}	1	1	1	0	1
m_{15}	1	1	1	1	0

6.2 Maxterms

Maxterm is a sum involving all inputs (or complements) to a function. Every row of a truth table has a corresponding maxterm. Minterm is false if and only if the corresponding row in the table is active.

Maxterms are defined as follows for each row of a two input truth table:

A	B	maxterm	maxterm name
0	0	$A + B$	M_0
0	1	$A + \bar{B}$	M_1
1	0	$\bar{A} + B$	M_2
1	1	$\bar{A} + \bar{B}$	M_3

Consider a two input circuit whose output Y is given by the truth table:

A	B	Y	maxterm	maxterm name
0	0	0	$A + B$	M_0
0	1	1	$A + \bar{B}$	M_1
1	0	0	$\bar{A} + B$	M_2
1	1	1	$\bar{A} + \bar{B}$	M_3

then $Y = (A + B)(\bar{A} + B) = M_0M_2$.

Writing a functional specification in terms of minterms is also called product of sums canonical form.

Example 7. Write the maxterm M_{11} for 4-input Boolean function with the ordered inputs A, B, C, D .

Example 8. Convert the following 4-input truth table into product of maxterms and product of sums canonical form.

maxterm name	A	B	C	D	f
M_0	0	0	0	0	0
M_1	0	0	0	1	0
M_2	0	0	1	0	0
M_3	0	0	1	1	1
M_4	0	1	0	0	0
M_5	0	1	0	1	0
M_6	0	1	1	0	0
M_7	0	1	1	1	1
M_8	1	0	0	0	0
M_9	1	0	0	1	0
M_{10}	1	0	1	0	0
M_{11}	1	0	1	1	1
M_{12}	1	1	0	0	0
M_{13}	1	1	0	1	1
M_{14}	1	1	1	0	1
M_{15}	1	1	1	1	0

Problem 6. Convert the following 4-input truth table into product of maxterms and products of sums canonical form.

<i>masterm name</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>f</i>
M_0	0	0	0	0	0
M_1	0	0	0	1	1
M_2	0	0	1	0	1
M_3	0	0	1	1	1
M_4	0	1	0	0	1
M_5	0	1	0	1	0
M_6	0	1	1	0	1
M_7	0	1	1	1	1
M_8	1	0	0	0	0
M_9	1	0	0	1	1
M_{10}	1	0	1	0	1
M_{11}	1	0	1	1	1
M_{12}	1	1	0	0	0
M_{13}	1	1	0	1	1
M_{14}	1	1	1	0	1
M_{15}	1	1	1	1	0

Example 9. Write the 3-input truth table for the function $f = m_2 + m_3 + m_7$.

Problem 7. Write the 3-input truth table for the function $f = M_4M_5M_7$.

Problem 8. Write the truth table for the function $f = \bar{A}B\bar{C} + AB\bar{C}$.

7 Karnaugh maps

7.1 Two input K-maps

		A	
		0	1
B	0	m_0	m_2
	1	m_1	m_3

7.2 Three input K-maps

		AB			
		00	01	11	10
C	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

7.3 Four input K-maps

		AB			
		00	01	11	10
CD	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

7.4 Five input K-maps


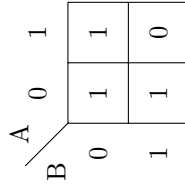

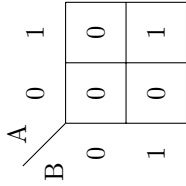

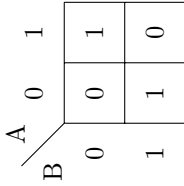
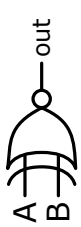
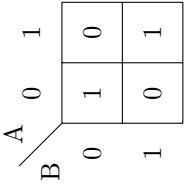
A = 0

		BC			
		00	01	11	10
DE	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

A = 1

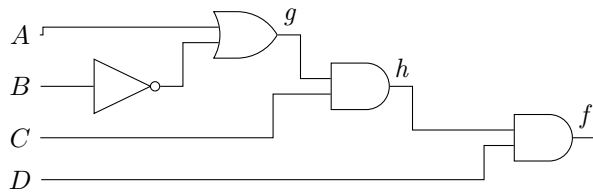
		BC			
		00	01	11	10
DE	00	m_{16}	m_{20}	m_{28}	m_{24}
	01	m_{17}	m_{21}	m_{29}	m_{25}
	11	m_{19}	m_{23}	m_{31}	m_{27}
	10	m_{18}	m_{22}	m_{30}	m_{26}

8 More Gates and notations summary

Name	C/Verilog	Boolean expr.	Truth Table	(ANSI) symbol	K-map															
NAND Gate	$Q = \sim(x1 \ \& \ x2)$	$Q = \overline{x_1 \cdot x_2} = \overline{x_1}x_2 + x_1\overline{x_2}$	<table><tr><th>x_1</th><th>x_2</th><th>$\overline{x_1 \cdot x_2}$</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	x_1	x_2	$\overline{x_1 \cdot x_2}$	0	0	1	0	1	1	1	0	1	1	1	0		
x_1	x_2	$\overline{x_1 \cdot x_2}$																		
0	0	1																		
0	1	1																		
1	0	1																		
1	1	0																		
NOR Gate	$Q = \sim(x1 \ \ x2)$	$Q = \overline{x_1 + x_2} = \overline{x_1}\overline{x_2}$	<table><tr><th>x_1</th><th>x_2</th><th>$\overline{x_1 + x_2}$</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	x_1	x_2	$\overline{x_1 + x_2}$	0	0	1	0	1	0	1	0	0	1	1	0		
x_1	x_2	$\overline{x_1 + x_2}$																		
0	0	1																		
0	1	0																		
1	0	0																		
1	1	0																		
XOR Gate	$Q = x1 \ \sim \ x2$	$Q = x_1 \oplus x_2$	<table><tr><th>x_1</th><th>x_2</th><th>$x_1 \oplus x_2$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	x_1	x_2	$x_1 \oplus x_2$	0	0	0	0	1	1	1	0	1	1	1	0		
x_1	x_2	$x_1 \oplus x_2$																		
0	0	0																		
0	1	1																		
1	0	1																		
1	1	0																		
XNOR Gate	$Q = \sim(x1 \ \sim \ x2)$	$Q = \overline{x_1 \oplus x_2}$	<table><tr><th>x_1</th><th>x_2</th><th>$\overline{x_1 \oplus x_2}$</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	x_1	x_2	$\overline{x_1 \oplus x_2}$	0	0	1	0	1	0	1	0	0	1	1	1		
x_1	x_2	$\overline{x_1 \oplus x_2}$																		
0	0	1																		
0	1	0																		
1	0	0																		
1	1	1																		

Example 10. Convert the following Boolean expression into a K-map. $f = \overline{A\overline{B}} + CD$

Problem 9. Convert the following logic circuit into a K-map.



9 Boolean Algebra

9.1 Axioms of Boolean algebra

1. $0 \cdot 0 = 0$
2. $1 + 1 = 1$

3. $1 \cdot 1 = 1$

4. $0 + 0 = 0$

5. $0 \cdot 1 = 1 \cdot 0 = 0$

6. $\bar{0} = 1$

7. $\bar{1} = 0$

8. $x = 0$ if $x \neq 1$

9. $x = 1$ if $x \neq 0$

9.2 Single variable theorems (Prove by drawing K-maps)

1. $x \cdot 0 = 0$

2. $x + 1 = 1$

3. $x \cdot 1 = x$

4. $x + 0 = x$

5. $x \cdot x = x$

6. $x + x = x$

7. $x \cdot \bar{x} = 0$

8. $x + \bar{x} = 1$

9. $\bar{\bar{x}} = x$

Remark 2 (Duality). *Swap $+$ with \cdot and 0 with 1 to get another theorem*

9.3 Two and three variable properties (Prove by K-maps)

1. Commutative: $x \cdot y = y \cdot x$, $x + y = y + x$

2. Associative: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$, $x + (y + z) = (x + y) + z$

3. Distributive: $x \cdot (y + z) = x \cdot y + x \cdot z$, $x + y \cdot z = (x + y) \cdot (y + z)$

4. Absorption: $x + x \cdot y = x$, $x \cdot (x + y) = x$

5. Combining: $x \cdot y + x \cdot \bar{y}, (x + y) \cdot (x + \bar{y}) = x$

6. DeMorgan's theorem: $\overline{x \cdot y} = \bar{x} + \bar{y}, \overline{x + y} = \bar{x} \cdot \bar{y}.$

7. Concensus:

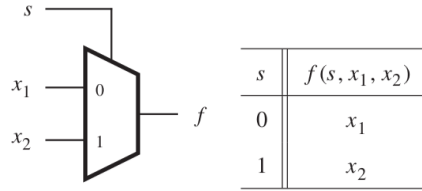
(a) $x + \bar{x} \cdot y = x + y$

(b) $x \cdot (\bar{x} + y) = x \cdot y$

(c) $x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$

(d) $(x + y) \cdot (y + z) \cdot (\bar{x} + z) = (x + y) \cdot (\bar{x} + z)$

Example 11 (Multiplexer). *Multiplexer is a circuit used to select one of the input lines x_1 and x_2 based only select input s . When $s = 0$, x_1 is selected, x_2 is selected otherwise. Find a boolean expression and a circuit for multiplexer*



Example 12. *Simplify $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$ using boolean algebra.*

Example 13. *Simplify $f = \bar{A}\bar{A}\bar{C} + \bar{A}\bar{B}C$ using K-maps.*