

# ECE275 Final exam Fall 2022

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Student Name:

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## 1 Instructions

- There are seven questions. First two questions (Prob 1 and Prob 2) are mandatory. **Do any four out of the five remaining problems.** For extra credit, do all seven.
- Maximum number of marks is 120 (140 with extra-credit). This exam amounts 10% toward the final grade.
- Time allowed is 120 minutes.
- In order to minimize distraction to your fellow students, you may not leave during the last 10 minutes of the examination.
- The examination is closed-book. One  $8 \times 11$  in two-sided cheatsheet is allowed.
- Non-programmable calculators are permitted.
- Please use a pen or heavy pencil to ensure legibility. Colored pens/pencils are recommended for K-map grouping.
- Please show your work; where appropriate, marks will be awarded for proper and well-reasoned explanations.

**Problem 1.** A sequential circuit is to be used to control the operation of a vending machine which dispenses a \$0.25 product. The circuit has three inputs (N, D and Q) and two outputs (R and C). The coin detector mechanism in the vending machine is synchronized with the same clock as the sequential circuit you are to design. The coin detector outputs a single 1 to the N, D, or Q input for every nickel (5 cents), dime (10 cents), or quarter (25 cents), respectively, that the customer inserts. Only one input will be 1 at a time. When the customer has inserted at least \$0.25 in any combination of nickels, dimes, and quarters, the vending machine must give change and dispense the product. The coin return mechanism gives change by returning nickels to the customer. For every 1 output on C, the coin return mechanism will return one nickel to the customer. The product is dispensed when the circuit outputs a single 1 on output R. The circuit should reset after dispensing the product.

**Example:** The customer inserts a nickel, a dime, and a quarter. The circuit inputs and outputs could look like this:

Inputs: Money 0 0 0 5 5 5 5 15 15 15 40  
 N= 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0  
 D= 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0  
 Q= 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0  
 R= 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0  
 C= 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0

Note that any number of 0's can occur between 1 inputs. Derive a Moore state table for the sequential circuit, and for each state indicate how much money the customer has inserted or how much change is due. (30 marks)

Moore state table

Money inserted	Change due	Present State PS	Next State				Output (R,C)
			NDQ = 000	N (Nickel) 100	D (Dime) 010	Q (Quarter) 001	
0	0	S <sub>0</sub>	S <sub>0</sub>	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	0 0
5	0	S <sub>1</sub>	S <sub>1</sub>	S <sub>2</sub>	S <sub>4</sub>	S <sub>5</sub>	0 0
10	0	S <sub>2</sub>	S <sub>2</sub>	S <sub>4</sub>	S <sub>6</sub>	S <sub>7</sub>	0 0
25	0	S <sub>3</sub>	S <sub>0</sub>	S <sub>0</sub>	S <sub>0</sub>	S <sub>0</sub>	1 0
15	0	S <sub>4</sub>	S <sub>4</sub>	S <sub>6</sub>	S <sub>3</sub>	S <sub>8</sub>	0 0
30	5	S <sub>5</sub>	S <sub>3</sub>	S <sub>3</sub>	S <sub>3</sub>	S <sub>3</sub>	0 1
20	0	S <sub>6</sub>	S <sub>6</sub>	S <sub>5</sub>	S <sub>5</sub>	S <sub>9</sub>	0 0
35	10	S <sub>7</sub>	S <sub>5</sub>	S <sub>5</sub>	S <sub>5</sub>	S <sub>5</sub>	0 1
40	15	S <sub>8</sub>	S <sub>7</sub>	S <sub>7</sub>	S <sub>7</sub>	S <sub>7</sub>	0 1
45	20	S <sub>9</sub>	S <sub>8</sub>	S <sub>8</sub>	S <sub>8</sub>	S <sub>8</sub>	0 1

0 0000  
 1 0001  
 2 0010  
 3 0011  
 4 0100  
 5 0101  
 6 0110  
 7 0111  
 8 1000  
 9 1001

**Problem 2.** Realize a BCD to excess-3 code converter using a 4-to-16 decoder with active low outputs and a minimum number of gates. (The excess-3 code is obtained from the binary numbers (0-9) by adding 3 (0011) to each of the binary numbers.) (Active low outputs mean that the selected output pin is 0, while all others are 1.) (10 marks)

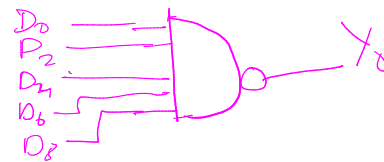
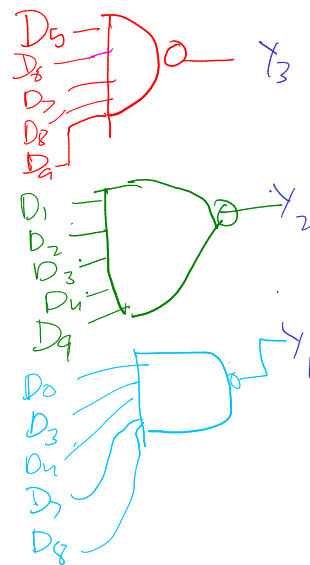
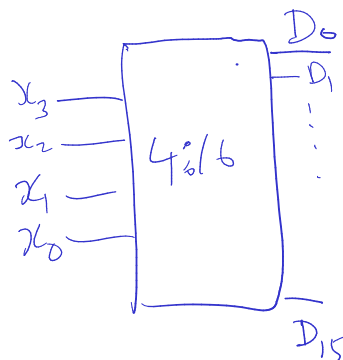
$$Y_3 = \overline{D_5} \cdot \overline{D_6} \cdot \overline{D_7} \cdot \overline{D_8} \cdot \overline{D_9}$$

$$Y_3 = \overline{D_5 + D_6 + D_7 + D_8 + D_9}$$

Count

	B	C	D
$x_3$	$x_2$	$x_1$	$x_0$
0	0	0	0
1	0	0	1
2	0	0	1
3	0	1	1
4	0	1	0
5	0	1	0
6	0	1	1
7	0	1	1
8	1	0	0
9	1	0	0

	$y_3$	$y_2$	$y_1$	$y_0$
0	0	0	1	1
1	0	1	0	0
2	0	1	0	1
3	0	1	1	0
4	0	1	1	1
5	1	0	0	0
6	1	0	0	1
7	1	0	1	0
8	1	0	1	1
9	1	1	0	0



20 marks x 2  
40 marks

**Problem 3.** Reduce the following state table to a minimum number of states using implication charts (20 marks).

Present State	Next State		Output Z
	X = 0	1	
A	A	B	1
B	C	E	0
C	F	G	1
D	C	A	0
E	<del>B</del> <sup>B</sup>	G	1
F	<del>F</del> <sup>F</sup> <del>H</del> <sup>H</sup>	<del>B</del> <sup>B</sup>	1
G	C	F	0
<del>H</del>	<del>F</del>	<del>B</del>	<del>1</del>
<del>I</del>	<del>C</del>	<del>E</del>	<del>0</del>

X due to output difference

X due to Round 1

X due to Round 2

X due to Round 3

X due to Round 4

	A	B	C	D	E	F	G	H	I
A									
B	X								
C	<del>A=F</del> <del>B=G</del>	X							
D	X	<del>E=A</del>	X						
E	<del>A=E</del> <del>B=G</del>	X	<del>E=A</del>	X					
F	<del>A=H</del> <del>B=I</del>	X	<del>F=H</del> <del>G=I</del>	X	<del>E=H</del> <del>G=I</del>				
G	X	<del>E=F</del>	X	<del>A=E</del>	X	X			
H	<del>A=F</del>	X	<del>G=B</del>	X	<del>E=F</del> <del>G=H</del>	<del>I=F</del> <del>I=B</del>	X		
I	X	<del>C=E</del> <del>C=E</del>	X	<del>A=E</del>	X	X	<del>E=E</del>	X	

$$\underbrace{B \equiv I}_B$$

$$\underbrace{F \equiv H}_F$$

**Problem 4.** 1. For the following state table, use the three guidelines to determine which of the three possible nonequivalent state assignments should give the best solution (20 marks).

2. Using your answer to (a), derive J-K flip-flop input equations and the output equations (20 marks).

(40 min)

x2  
(80 min)

Present State	Next state				Outputs			
	$X_1X_2 = 00$	$01$	$11$	$10$	$X_1X_2 = 00$	$01$	$11$	$10$
A	A	A	C	C	01	01	01	01
B	B	D	B	D	11	11	11	11
C	A	A	B	D	11	11	00	00
D	D	B	A	C	01	01	01	01

Guidelines

Highest priority:  $\alpha \rightarrow \beta$ :  $(A, C)^2, (B, C)^2, (A, D)$

Medium priority:  $\alpha \rightarrow \beta$ :  $(A, C), (B, D), (A, B, D), (A, B, C, D)$

Lowest priority:  $\alpha \rightarrow \beta$ :  $00/01 (A, D), 00/11 (B, C)$

4 states . 2 ff  
 $2^1 < 4 \leq 2^2$

State map

$Q_1$	$Q_0$
C	B
A	D

State	State assignment $Q_1Q_0$
A	01
B	10
C	00
D	11

Present State	Next state				Outputs			
	$X_1X_2 = 00$	$01$	$11$	$10$	$X_1X_2 = 00$	$01$	$11$	$10$
A	A	A	C	C	01	01	01	01
B	B	D	B	D	11	11	11	11
C	A	A	B	D	11	11	00	00
D	D	B	A	C	01	01	01	01

PS	$Q_1Q_0$	NS				Outputs ( $Z_1Z_2$ )			
		$X_1X_2 = 00$	$01$	$11$	$10$	$X_1X_2 = 00$	$01$	$11$	$10$
C	00	01	01	10	11	11	11	00	00
A	01	01	01	00	00	01	01	01	01
D	11	11	10	01	00	01	01	01	01
B	10	10	11	10	11	11	11	11	11

$Q_1^+$	$Q_0^+$	$X_1X_2 = 00$	$01$	$11$	$10$
00	00	0	0	1	1
01	01	0	0	0	0
11	11	1	1	0	0
10	10	1	1	1	1

$Q_1^+$	$Q_0^+$	$X_1X_2 = 00$	$01$	$11$	$10$
00	00	0	0	1	1
01	01	0	0	0	0
11	11	1	1	0	0
10	10	1	1	1	1

$Q_1^+$	$Q_0^+$	$X_1X_2 = 00$	$01$	$11$	$10$
00	00	1	1	1	1
01	01	1	1	1	1
11	11	0	0	1	1
10	10	0	0	0	0

JK excitation

$Q_1$	$Q_1^+$	$J_1$	$K_1$
0	0	0	d
0	1	1	d
1	0	d	0
1	1	d	d

$Q_0$

	$x_1 x_2$	00	01	11	10
$Q_0$	00	1	1	0	1
	01	1	1	0	0
	11	1	0	1	0
	10	0	1	0	1

$Q_0$

$J_0$

	$x_1 x_2$	00	01	11	10
$Q_0$	00	1	1	0	1
	01	d	d	d	d
	11	d	d	d	d
	10	0	1	0	1

$Q_0$

$K_0$

	$x_1 x_2$	00	01	11	10
$Q_0$	00	d	d	d	d
	01	0	0	1	1
	11	0	1	0	1
	10	d	d	d	d

$Q_0$

$$J_0 = x_1 \bar{x}_2 + \bar{x}_1 x_2 + \bar{x}_1 \bar{Q}_1$$

$$\bar{J}_0 = x_1 x_2 + \bar{x}_1 \bar{x}_2 Q_1$$

$$J_0 = (\bar{x}_1 + \bar{x}_2)(x_1 + x_2 + \bar{Q}_1)$$

$$K_0 = x_1 \bar{Q}_1 + x_1 \bar{x}_2 + \bar{x}_1 x_2 Q_1$$

$Q_0$	$K_0$
0	d
1	$\bar{Q}_1$

$Z_1$

	$x_1 x_2$	00	01	11	10
$Q_0$	00	1	1	0	0
	01	0	0	0	0
	11	0	0	0	0
	10	1	1	1	1

$Q_0$

$Q_0$	$J_0$
0	$Q_0^+$
1	d

$Z_2$

	$x_1 x_2$	00	01	11	10
$Q_0$	00	1	1	0	0
	01	1	1	1	1
	11	1	1	1	1
	10	1	1	1	1

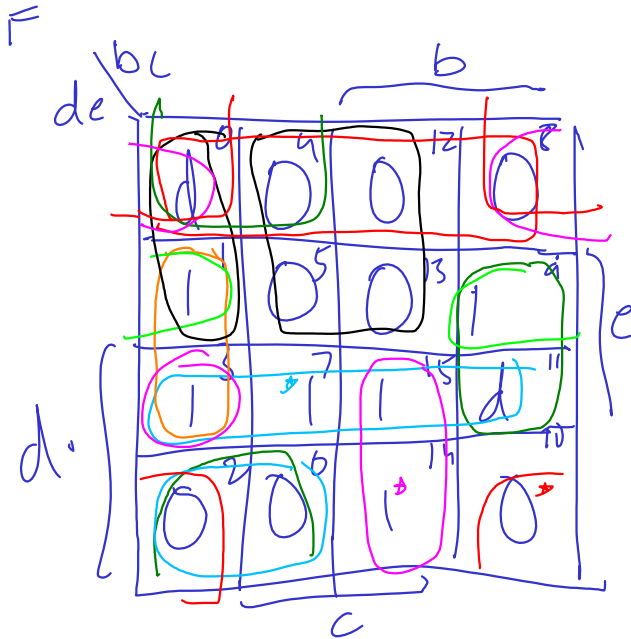
$Q_0$

$$Z_1 = Q_1 \bar{Q}_0 + \bar{x}_1 \bar{Q}_0$$

$$Z_2 = \bar{x}_1 + Q_0 + Q_1$$

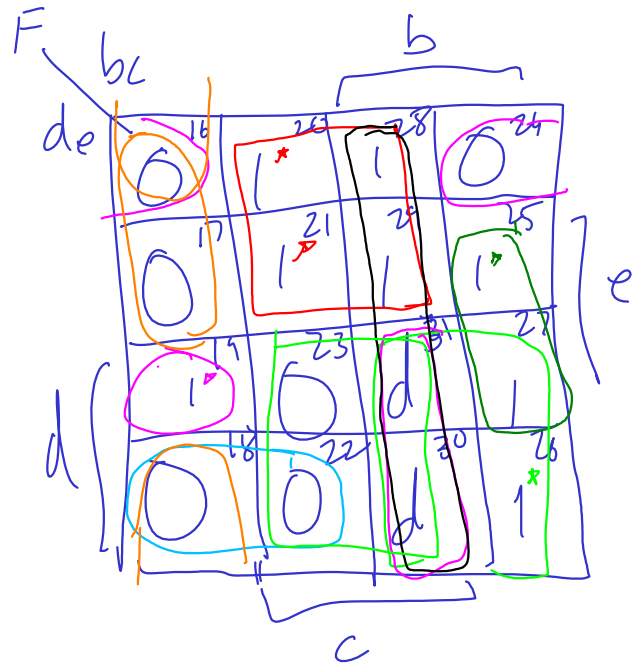
**Problem 7.** Find the minimum cost circuit for the following function using K-map. Find both ~~sum-of-products~~ and ~~product-of-sum~~ forms and find the minimum cost one.

$$F(a, b, c, d, e) = \prod M(2, 4, 5, 6, 8, 10, 12, 13, 16, 17, 18, 22, 23, 24) + \prod D(0, 11, 30, 31) \quad (20 \text{ marks}).$$



$$a=0$$

$$\bar{a}=1$$



$$a=1$$

At Maxterms  $F=0$

SOP form

$$F = bcd + \bar{a}de + \bar{a}\bar{c}\bar{d}e + b\bar{c}e + ac\bar{d} + \bar{b}\bar{c}de + abd$$

$$\text{Cost} = 7 + 1 + 3 \times 5 + 4 \times 2 + 7 = 38$$

POS form

$$\bar{F} = \bar{a}\bar{c}\bar{e} + \bar{a}c\bar{d} + \bar{b}d\bar{e} + acd + \bar{c}\bar{d}\bar{e} + a\bar{b}\bar{c}\bar{d}$$

$$F = (a+c+e)(a+\bar{c}+d)(b+\bar{d}+e)(\bar{a}+\bar{c}+\bar{d})(c+d+e)(\bar{a}+b+c+d)$$

$$\text{Cost} = 6 + 1 + 3 \times 5 + 4 \times 1 + 6 = 32$$

POS is minimum cost