Digital circuit design notes

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[1]The notes are from the following books [1, 2]. Not intended for distribution outside the class.

Contents

Chapter 1

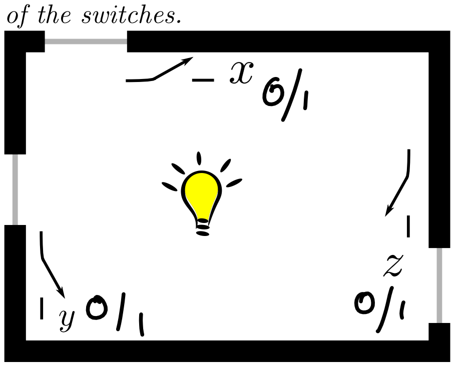
Boolean Algebra

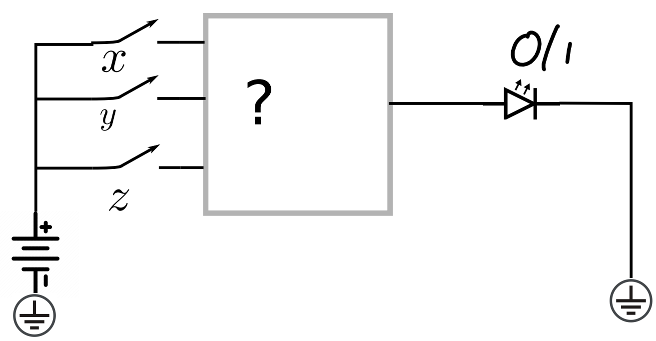
1.1 Learning objectives

1. Representing digital circuits
2. Converting between different notations: Boolean expression, logic networks and switching circuits
3. Converting between different logic network specifications: truth table, minterm, maxterms, product of sums canonical form and sum of product canonical form.
4. Introduce truth tables as Behavioral Verilog

1.2 Motivating Problem

Example 1. *Assume that a large room has three doors and that a switch near each door controls a light in the room. It has to be possible to turn the light on or off by changing the state of any one of the switches.*

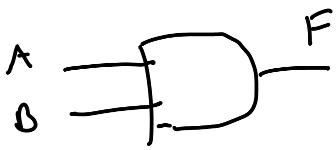




Basic Gates

Elementry or the building blocks of digital circuts!

AND Gate

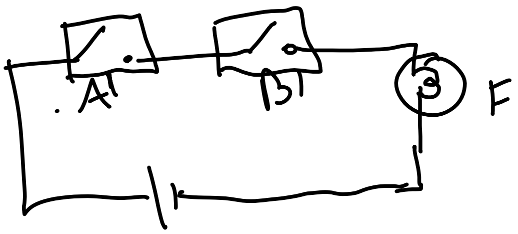


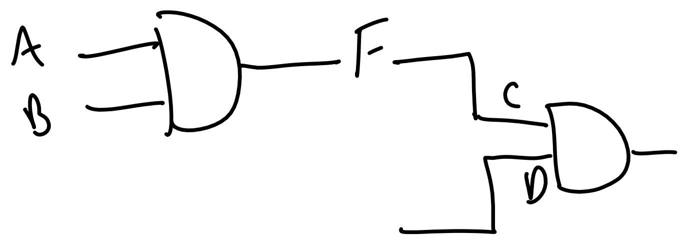
ANSI SYMBOL

TRUTH TABLE : All possible inputs and corresponding outputs

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

Switching circuit





A = Class in Williams 110

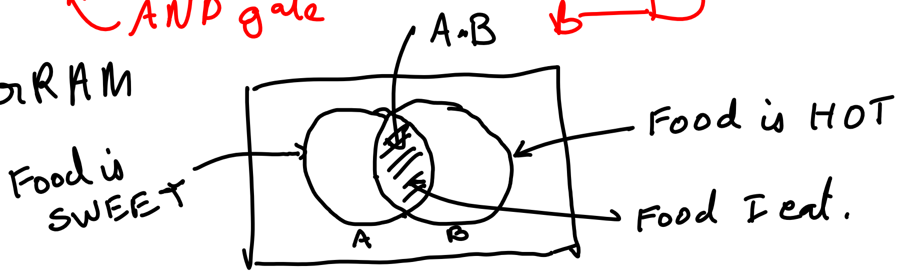
B = " on MWF 11-12

A AND B = F : ECE 275 class

1. ANSI SYMBOL
2. TRUTH TABLE
3. Boolean Algebra notation $A \cdot B = F$ ($\cdot$ AND gate) $AB = F$ $F = A \cdot B$
4. VERILOG / SYSTEM VERILOG / C bitwise AND gate

$F = A \mathbin{\&} B$ (& AND gate)

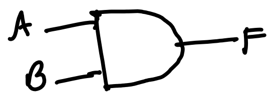
5. VENN DIAGRAM



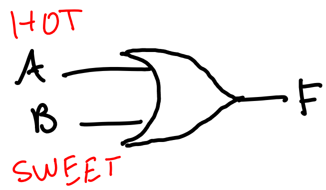
0 = FALSE / 1 = TRUE

	ANSI SYMBOL	TRUTH TABLE	Boolean Algebra	C/ Verilog	Venn Diagram

AND



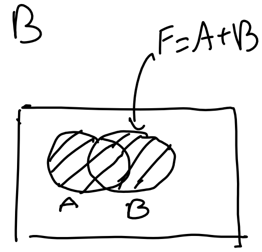
OR gate



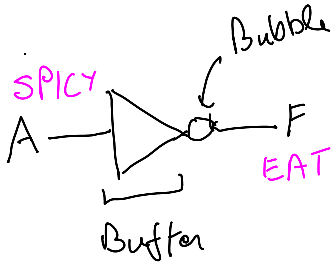
	A	B	F
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1

$F = A + B$

F = A|B



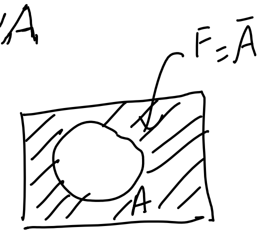
NOT Gate



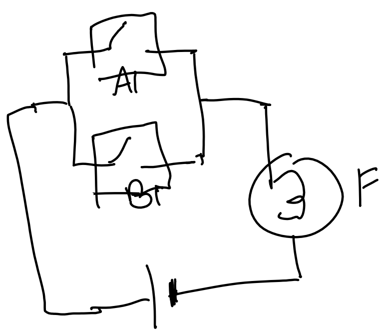
A	F
0	1
1	0

$F = \overline{A}$

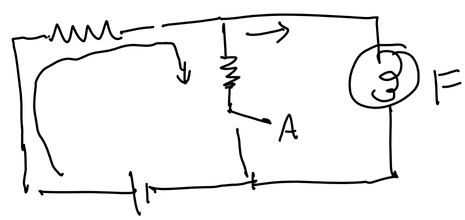
F = ~A



OR Gate Switching circuit

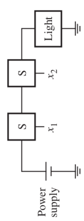
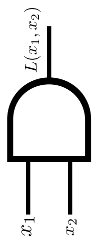
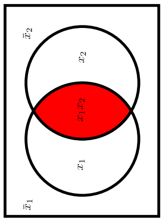
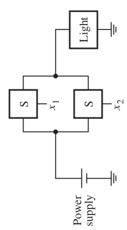
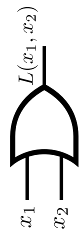
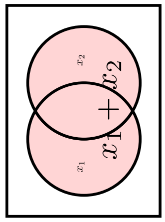
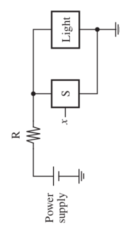
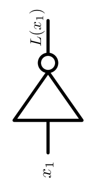
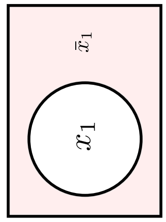


NOT Gate Switching Circuit

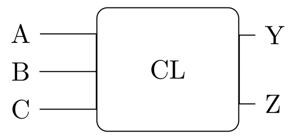


$F = \overline{A}$

1.3 Basic Gates and notations summary

<table>
<tr><th>Name</th><th>C/Verilog</th><th>Boolean expr.</th><th>Truth Table</th><th>Switching circuit</th><th>(ANSI) symbol</th><th>Venn diagram</th></tr>
<tr><td>AND Gate</td><td><code>L = x1 & x2</code></td><td>$L = x_1 \cdot x_2 = x_1 x_2$</td><td><table><tr><th>$x_1$</th><th>$x_2$</th><th>$x_1 \cdot x_2$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td><td></td><td></td><td></td></tr>
<tr><td>OR Gate</td><td><code>L = x1 | x2</code></td><td>$L = x_1 + x_2$</td><td><table><tr><th>x_1</th><th>x_2</th><th>$x_1 + x_2$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td><td></td><td></td><td></td></tr>
<tr><td>NOT Gate</td><td><code>L = ∼ x1</code></td><td>$L = \bar{x}_1 = x_1'$</td><td><table><tr><th>x_1</th><th>$\bar{x}_1$</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table></td><td></td><td></td><td></td></tr>
</table>

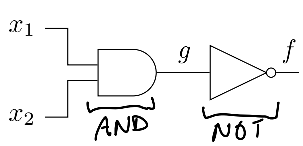
1.4 Digital circuits or networks



$$Y = F(A, B, C) \qquad Z = G(A, B, C)$$

1.5 Two input networks

Example 2. *Convert the following (ANSI) network into a Boolean expression, a truth table and a Venn diagram.*



AND

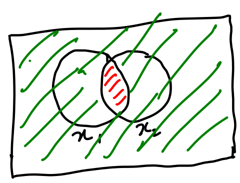
$g = x_1 \cdot x_2$

$f = \bar{g}$ ← NOT

$\boxed{f = \overline{x_1 \cdot x_2}}$

	x_1	x_2	g	$f = \bar{g}$
0	0	0	0	1
1	0	1	0	1
2	1	0	0	1
3	1	1	1	0

Venn Diagram



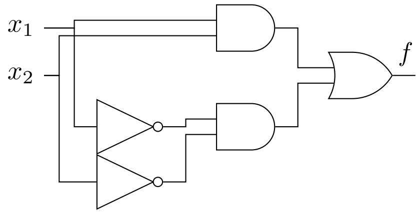
g = Red

$f = \bar{g}$

Example 3. *Convert the following Boolean expression into a (ANSI) network, a truth table and a Venn diagram:*

$$f = \overline{x_1 + x_2}$$

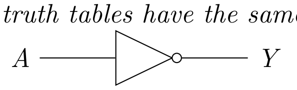
Problem 1 (10 marks)**.** *Convert the following (ANSI) network into a Boolean expression, a truth table and a Venn diagram.*

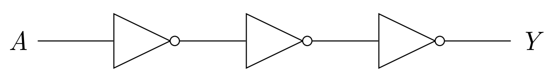


Example 4. *Convert the following Boolean expression into a network, a truth table and a Venn diagram:*

$$f = x_1\bar{x}_2 + \bar{x}_1 x_2$$

Problem 2 (5 marks)**.** *Can two different circuits have the same truth table? Can two different truth tables have the same circuit? Consider the following two circuits for example*

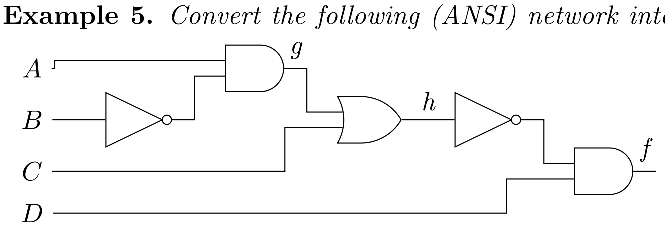




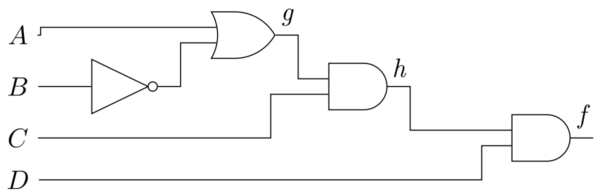
Remark 1. *Truth tables and Venn diagrams define* what *the combinational circuit should do. Truth tables define output for every input. Boolean expression and networks define* how *to achieve the desired input output relationship.*

1.6 Multi-input networks

Example 5. *Convert the following (ANSI) network into a Boolean expression and a truth table.*



Problem 3 (20 marks)**.** *Convert the following (ANSI) network into a Boolean expression and a truth table.*



1.7 Minterms and Maxterms

1.7.1 Minterms

Minterm is a product involving all inputs (or complements) to a function. Every row of a truth table has a corresponding minterm. Minterm is true if and only if the corresponding row in the table is active.

Minterms defined as follows for each row of a two input truth table:

A	B	minterm	minterm name
0	0	$\bar{A}\bar{B}$	m_0
0	1	$\bar{A}B$	m_1
1	0	$A\bar{B}$	m_2
1	1	AB	m_3

Consider a two input circuit whose output Y is given by the truth table:

A	B	Y	minterm	minterm name
0	0	0	$\bar{A}\bar{B}$	m_0
0	1	1	$\bar{A}B$	m_1
1	0	0	$A\bar{B}$	m_2
1	1	1	AB	m_3

then $Y = \bar{A}B + AB = m_1 + m_3 = \sum(1,3)$.

This also gives the *sum of products canonical form.*

Example 6. *What is the minterm m_{13} for a 4-input circuit with inputs x, y, z, w (ordered from MSB to LSB).*

Problem 4 (5 marks)**.** *What is the minterm m_{23} for a 5-input circuit with inputs a, b, c, d, e (ordered from MSB to LSB).*

Example 7. *Convert the following 4-input truth table into sum of minterms and sum of products canonical form.*

minterm name	*A*	*B*	*C*	*D*	*f*
m_0	0	0	0	0	0
m_1	0	0	0	1	1
m_2	0	0	1	0	0
m_3	0	0	1	1	0
m_4	0	1	0	0	0
m_5	0	1	0	1	1
m_6	0	1	1	0	0
m_7	0	1	1	1	0
m_8	1	0	0	0	0
m_9	1	0	0	1	0
m_{10}	1	0	1	0	0
m_{11}	1	0	1	1	0
m_{12}	1	1	0	0	0
m_{13}	1	1	0	1	1
m_{14}	1	1	1	0	0
m_{15}	1	1	1	1	0

Problem 5 (10 marks)**.** *Convert the following 4-input truth table into sum of minterms and sum of products canonical form.*

minterm name	A	B	C	D	f
m_0	*0*	*0*	*0*	*0*	*0*
m_1	*0*	*0*	*0*	*1*	*0*
m_2	*0*	*0*	*1*	*0*	*0*
m_3	*0*	*0*	*1*	*1*	*1*
m_4	*0*	*1*	*0*	*0*	*0*
m_5	*0*	*1*	*0*	*1*	*0*
m_6	*0*	*1*	*1*	*0*	*0*
m_7	*0*	*1*	*1*	*1*	*1*
m_8	*1*	*0*	*0*	*0*	*0*
m_9	*1*	*0*	*0*	*1*	*0*
m_{10}	*1*	*0*	*1*	*0*	*0*
m_{11}	*1*	*0*	*1*	*1*	*1*
m_{12}	*1*	*1*	*0*	*0*	*0*
m_{13}	*1*	*1*	*0*	*1*	*1*
m_{14}	*1*	*1*	*1*	*0*	*1*
m_{15}	*1*	*1*	*1*	*1*	*0*

1.7.2 Maxterms

Maxterm is a sum involving all inputs (or complements) to a function. Every row of a truth table has a corresponding maxterm. Minterm is false if and only if the corresponding row in the table is active.

Maxterms are defined as follows for each row of a two input truth table:

A	B	maxterm	maxterm name
0	0	$A+B$	M_0
0	1	$A+\bar{B}$	M_1
1	0	$\bar{A}+B$	M_2
1	1	$\bar{A}+\bar{B}$	M_3

Consider a two input circuit whose output Y is given by the truth table:

A	B	Y	maxterm	maxterm name
0	0	0	$A+B$	M_0
0	1	1	$A+\bar{B}$	M_1
1	0	0	$\bar{A}+B$	M_2
1	1	1	$\bar{A}+\bar{B}$	M_3

then $Y = (A+B)(\bar{A}+B) = M_0M_2$.

Writing a functional specification in terms of minterms is also called product of sums canonical form.

Example 8. *Write the maxterm M_{11} for 4-input Boolean function with the ordered inputs A, B, C, D.*

Example 9. *Convert the following 4-input truth table into product of maxterms and product of sums canonical form.*

maxterm name	A	B	C	D	f
M_0	0	0	0	0	0
M_1	0	0	0	1	0
M_2	0	0	1	0	0
M_3	0	0	1	1	1
M_4	0	1	0	0	0
M_5	0	1	0	1	0
M_6	0	1	1	0	0
M_7	0	1	1	1	1
M_8	1	0	0	0	0
M_9	1	0	0	1	0
M_{10}	1	0	1	0	0
M_{11}	1	0	1	1	1
M_{12}	1	1	0	0	0
M_{13}	1	1	0	1	1
M_{14}	1	1	1	0	1
M_{15}	1	1	1	1	0

Problem 6 (10 marks). *Convert the following 4-input truth table into product of maxterms and products of sums canonical form.*

maxterm name	A	B	C	D	f
M_0	0	0	0	0	0
M_1	0	0	0	1	1
M_2	0	0	1	0	1
M_3	0	0	1	1	1
M_4	0	1	0	0	1
M_5	0	1	0	1	0
M_6	0	1	1	0	1
M_7	0	1	1	1	1
M_8	1	0	0	0	0
M_9	1	0	0	1	1
M_{10}	1	0	1	0	1
M_{11}	1	0	1	1	1
M_{12}	1	1	0	0	0
M_{13}	1	1	0	1	1
M_{14}	1	1	1	0	1
M_{15}	1	1	1	1	0

Example 10. *Write the 3-input truth table for the function* $f = m_2 + m_3 + m_7$.

Problem 7 (10 marks). *Write the 3-input truth table for the function* $f = M_4 M_5 M_7$.

Problem 8 (10 marks)**.** *Write the truth table for the function* $f = \bar{A}B\bar{C} + AB\bar{C}$.

Bibliography

[1] Sarah L Harris and David Harris. *Digital design and computer architecture*. Morgan Kaufmann, 2022.

[2] Brown Stephen and Vranesic Zvonko. *Fundamentals of digital Logic with Verilog design*. McGraw Hill, 2022.