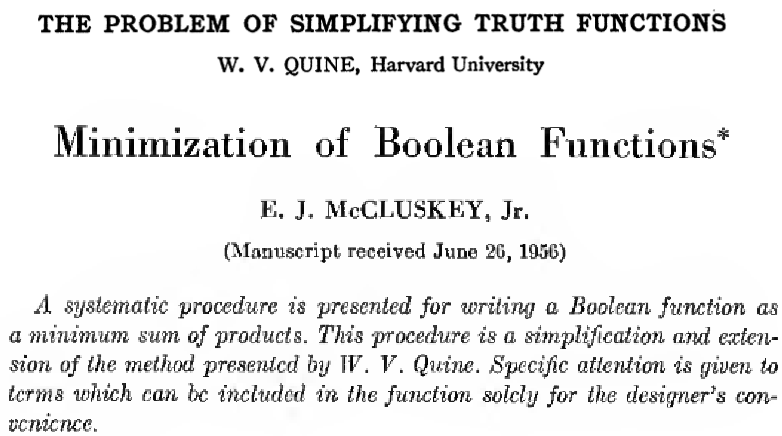
# Chapter 12

# Quine McCluskey

## 12.1 Quine-McCluskey PI Table reduction and Petrick's method

This is not in the text-book. For additional reading, please refer to the linked resources on the website.



**THE PROBLEM OF SIMPLIFYING TRUTH FUNCTIONS**

W. V. QUINE, Harvard University

**Minimization of Boolean Functions***

E. J. McCLUSKEY, Jr.

(Manuscript received June 26, 1956)

*A systematic procedure is presented for writing a Boolean function as a minimum sum of products. This procedure is a simplification and extension of the method presented by W. V. Quine. Specific attention is given to terms which can be included in the function solely for the designer's convenience.*

| $x$ | $y$ | $z$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |

not prime

$\bar{x}y\bar{z}$ implicant

$\bar{x}yz$ implicant

$\bar{x}y\bar{z} + \bar{x}yz$

$\bar{x}y(\bar{z}+z) = \bar{x}y$

implicant

prime

includes minterms

**Definition 12.1** (Implicant). *Given a function $f$ of $n$ variables, a product term $P$ is an implicant of $f$ if and only if for every combination of values of the $n$ variables for which $P = 1$, $f$ is also equal to 1.*

biggest grouping

**Definition 12.2** (Prime Implicant). *A prime implicant of a function $f$ is an implicant which is no longer an implicant if any literal is removed from it.*

There are 4 main steps in the Quine-McCluskey algorithm/PI Table reduction and Petrick's method:

1. Generate Prime Implicants
2. Construct Prime Implicant Table. PIs as columns, and minterms as rows (don't cares are excluded).
3. Reduce Prime Implicant Table by repeating following steps until they it cannot be reduced further
   (a) Remove Essential Prime Implicants

(b) Row Dominance: Remove *dominating* rows. (i.e. unnecessary minterms)

(c) Column Dominance: Remove *dominated* columns. (i.e. remove unnecessary PIs)

4. Solve Prime Implicant Table by Petrick's method

### 12.1.1 Generate Prime Implicants

**Example 12.1.** *Generate prime implicants of the function*

$x_5, x_4, x_3, x_2, x_1$

$$F(\cancel{A, B, C, D}, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 12, 13, 14, 16, 18, 19, 29, 30)$$

*using Quine-McCluskey method*

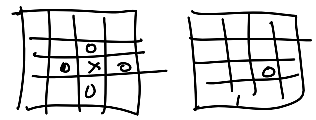


TABLE II — DETERMINATION OF PRIME IMPLICANTS FOR TRANSMISSION

$T = \sum (0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$ ← minterm form

$\boxed{\bar{x}_5 \bar{x}_4 \bar{x}_1}$

PI = prime implicant

Group

(a) I

| Group | | $x_5x_4x_3x_2x_1$ | |
|---|---|---|---|
| 0 | 0 | 0 0 0 0 0 | ✓ ← NOT PI |
| 1 | 2 | 0 0 0 1 0 | ✓ |
| | 4 | 0 0 1 0 0 | ✓ |
| | 8 | 0 1 0 0 0 | ✓ |
| | 16 | 1 0 0 0 0 | ✓ |
| 2 | 6 | 0 0 1 1 0 | ✓ |
| | 10 | 0 1 0 1 0 | ✓ |
| | 12 | 0 1 1 0 0 | ✓ |
| | 18 | 1 0 0 1 0 | ✓ |
| 3 | 7 | 0 0 1 1 1 | ✓ |
| | 11 | 0 1 0 1 1 | ✓ |
| | 13 | 0 1 1 0 1 | ✓ |
| | 14 | 0 1 1 1 0 | ✓ |
| | 19 | 1 0 0 1 1 | ✓ |
| 4 | 29 | 1 1 1 0 1 | ✓ |
| | 30 | 1 1 1 1 0 | ✓ |
| 5 | 31 | | |

(b) II ← implicant

| | $x_5x_4x_3x_2x_1$ | |
|---|---|---|
| 0 2 | 0 0 0 – 0 | ✓ |
| 0 4 | 0 0 – 0 0 | ✓ |
| 0 8 | 0 – 0 0 0 | ✓ |
| 0 16 | – 0 0 0 0 | ✓ |
| 2 6 | 0 0 – 1 0 | ✓ |
| 2 10 | 0 – 0 1 0 | ✓ |
| 2 18 | – 0 0 1 0 | ✓ |
| 4 6 | 0 0 1 – 0 | ✓ |
| 4 12 | 0 – 1 0 0 | ✓ |
| 8 10 | 0 1 0 – 0 | ✓ |
| 8 12 | 0 1 – 0 0 | ✓ |
| 16 18 | 1 0 0 – 0 | ✓ ← NOT PI |
| 6 7 | 0 0 1 1 – | ← PI |
| 6 14 | 0 – 1 1 0 | ✓ |
| 10 11 | 0 1 0 1 – | |
| 10 14 | 0 1 – 1 0 | ✓ |
| 12 13 | 0 1 1 0 – | |
| 12 14 | 0 1 1 – 0 | ✓ |
| 18 19 | 1 0 0 1 – | |
| 13 29 | – 1 1 0 1 | |
| 14 30 | – 1 1 1 0 | |

(c) III

| | $x_5x_4x_3x_2x_1$ | |
|---|---|---|
| 0 2 4 6 | 0 0 – – 0 | ✓ |
| 0 2 8 10 | 0 – 0 – 0 | ✓ |
| 0 2 16 18 | – 0 0 – 0 | ← PI $\bar{x}_4 \bar{x}_3 \bar{x}_1$ |
| 0 4 8 12 | 0 – – 0 0 | ✓ |
| 2 6 10 14 | 0 – – 1 0 | ✓ |
| 4 6 12 14 | 0 – 1 – 0 | ✓ |
| 8 10 12 14 | 0 1 – – 0 | ✓ |

0 8 2 10

(d) IV

| | $x_5x_4x_3x_2x_1$ |
|---|---|
| 0 2 4 6 8 10 12 14 | 0 – – – 0 ← $\bar{x}_5 \bar{x}_1$ |

0 $\bar{x}_5 \bar{x}_4 \bar{x}_3 \bar{x}_2 \bar{x}_1$

2 $\bar{x}_5 \bar{x}_4 \bar{x}_3 x_2 \bar{x}_1$

$\boxed{0\text{-}2}$ $\bar{x}_5 \bar{x}_4 \bar{x}_3 \bar{x}_1$

0 0 0 – 0

$\boxed{0\text{-}4}$ $\boxed{2\text{-}6}$

Steps:

1. Start with writing minterms in binary format (include don't cares as minterms).
2. Create potential groups of minterms that can be combined (merged). The only minterms that can be combined differ only be single 1. Create a new list of combined minterms as n-1 literal implicants.
3. Check off the minterms that could be combined. Unchecked minterms are prime implicants (PIs).
4. Repeat the grouping process with n-1 literal implicants.

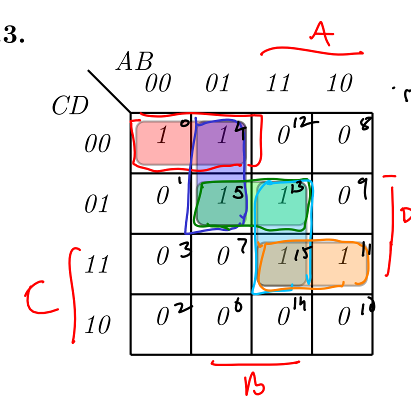
### 12.1.2 Prime Implicants table and reduction

**Example 12.2.** *Reduce the prime implicants* $\{\bar{B}\bar{D}, C\bar{D}, BD, BC, A\bar{D}, AB\}$ *using prime implicants table.*

Prime implicant

How to find the smallest set of PIs that cover all the minterms
1. PI table reduction
2. Petrick's method

**Example 12.3.**




PI →

| minterms | $\bar{A}\bar{C}\bar{D}$ | $\bar{A}B\bar{C}$ | $B\bar{C}D$ | $ABD$ | $ACD$ |
|---|---|---|---|---|---|
| 0 | X | | | | |
| 4 | X | X | | | |
| 5 | | X | X | | |
| 11 | | | | | X |
| 13 | | | X | X | |
| 15 | | | | X | X |

2 EPI → $\bar{A}\bar{C}\bar{D} + ACD$

$m_0 + m_4$ $\quad$ $m_{11} + m_{15}$

| | $\bar{A}B\bar{C}$ | $B\bar{C}D$ | $ABD$ |
|---|---|---|---|
| 5 | X | X | |
| 13 | | X | X |

min cover = $\bar{A}\bar{C}\bar{D} + ACD + B\bar{C}D$

$\boxed{\text{column dominance}}$

$B\bar{C}D > \bar{A}B\bar{C}$

$B\bar{C}D > ABD$

## PI table reduction

1. EPI

dominatee — dominated

2. column dominance (PI) $\bar{A}D > \bar{C}D$
(remove the dominated)

3. row dominance (minterm)
(remove the dominatee)

$m_1 > m_5$

$m_3 > m_7$

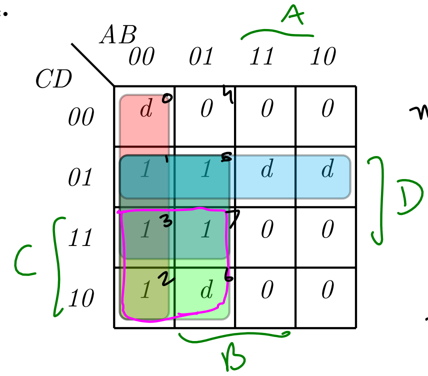
D

| minterm / PI | $\bar{A}\bar{B}$ | $\bar{C}D$ | $\bar{A}D$ | $\bar{A}C$ |
|---|---|---|---|---|
| 1 | x | x | x | |
| 2 | x | | | x |
| 3 | x | | x | x |
| 5 | | x | x | |
| 7 | | | x | x |

min cover = $\bar{A}D + \bar{A}\bar{B}$

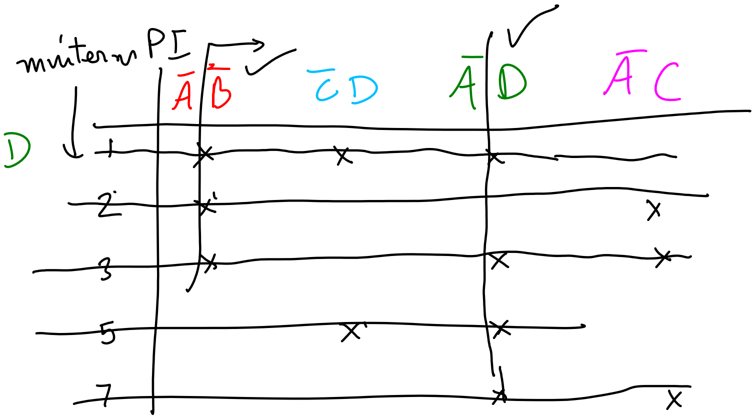


**Example 12.4.** Simpler version




No EPIs

PI table reduction




Petrick's method

if the PI covers include that minterm

A B C D

Does $\bar{A}\bar{D}$ include minterm 0? $\bar{A}\bar{B}\bar{C}\bar{D}$

**Example 12.5.** *Reduce the following PI table* (f)

Prime implicant

minterms

| | $\bar{A}\bar{D}$ | $\bar{B}\bar{D}$ | $\bar{C}\bar{D}$ | $\bar{A}C$ | $\bar{B}C$ | $\bar{A}B$ | $B\bar{C}$ | $A\bar{B}$ | $A\bar{C}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | X | X | X | | | | | | |
| 2 | X | X | | X | X | | | | |
| 3 | | | | X | X | | | | |
| 4 | X | | X | | | X | X | | |
| 5 | | | | | | X | X | | |
| 6 | X | | | X | | X | | | |
| 7 | | | | X | | X | | | |
| 8 | | X | X | | | | | X | X |
| 9 | | | | | | | | X | X |
| 10 | | X | | | X | | | X | |
| 11 | | | | | X | | | X | |
| 12 | | | X | | X | | X | | X |
| 13 | | | | | | | X | | X |

minterms

EPI
Essential PI -

0 2 4 6
0000 + 0010 + 0100 + 0110

$\bar{A}\bar{D} = \bar{A}\left(\bar{B}\bar{C} + B\bar{C} + \bar{B}C + BC\right)\bar{D}$

$= \underbrace{\bar{A}\bar{B}\bar{C}\bar{D}}_{\text{minterm } 0} + \_\_ \quad \_\_\_$

$\bar{B}C = \bar{A}\bar{B}C\bar{D} = m_2$
$+ \bar{A}\bar{B}CD = m_3$
$+ A\bar{B}C\bar{D} = m_{10}$
$+ A\bar{B}CD = m_{11}$

What are the minterms covered by $\bar{A}\bar{D}$? How do I find them

4-variable
A B C D
MSB $\longrightarrow$ LSB

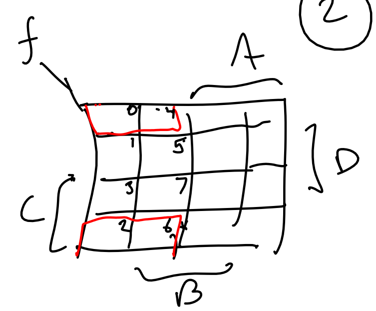
(1)

$\bar{A}\bar{D} = 0\text{-}\text{-}0 \rightarrow$
0000 $m_0$
0010 $m_2$
0100 $m_4$
0110 $m_6$

$\bar{A}\bar{D} = \bar{A}\left((B+\bar{B})(C+\bar{C})\right)\bar{D}$

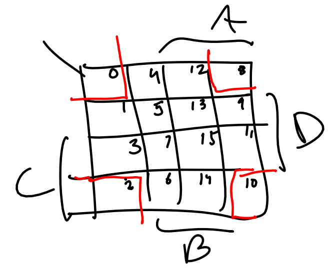
$= \underset{m_0}{\bar{A}\bar{B}\bar{C}\bar{D}} + \underset{m_2}{\bar{A}\bar{B}C\bar{D}} + \underset{m_4}{\bar{A}B\bar{C}\bar{D}} + \underset{m_6}{\bar{A}BC\bar{D}}$

(2)



$\overline{B}\,\overline{D} = ?$

Find the minterms covered by $\overline{B}\,\overline{D}$




$$\overline{B}\,\overline{D} = m_0 + m_2 + m_8 + m_{10}$$

$$\overline{B}\,\overline{D} = (A+\overline{A})\cdot\overline{B}\cdot(C+\overline{C})\cdot\overline{D}$$

$$= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,C\,\overline{D} + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,C\,\overline{D}$$

$$= m_0 + m_2 + m_8 + m_{10}$$

**Table IV — Prime Implicant Table for the Transmission of Table II**

| | 0 | 2 | 4 | 8 | 16 | 6 | 10 | 12 | 18 | 7 | 11 | 13 | 14 | 19 | 29 | 30 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | x | x | x | x | | x | x | x | | | | | x | | | | * |
| B | x | x | | | x | | | | x | | | | | | | | * |
| C | | | | | | | | | | | | | x | | | x | * |
| D | | | | | | | | | | | | x | | | x | | * |
| E | | | | | | | | | x | | | | | x | | | * |
| F | | | | | | | | x | | | | x | | | | | |
| G | | | | | | | x | | | | x | | | | | | * |
| H | | | | | | x | | | | x | | | | | | | * |

**Example 12.6.**

### 12.1.3 Petrick's method

**Example 12.7.** *Solve the Prime Implicant table using Petrick's method*

| | $p_1 = \bar{A}C$ | $p_2 = \bar{B}C$ | $p_3 = \bar{A}B$ | $p_4 = B\bar{C}$ | $p_5 = A\bar{B}$ | $p_6 = A\bar{C}$ |
|---|---|---|---|---|---|---|
| *3* | $X$ | $X$ | | | | |
| *5* | | | $X$ | $X$ | | |
| *7* | $X$ | | $X$ | | | |
| *9* | | | | | $X$ | $X$ |
| *11* | | $X$ | | | $X$ | |
| *13* | | | | $X$ | | $X$ |