

Digital circuit design notes

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¹The notes are from the following books [1, 2]. Not intended for distribution outside the class.

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Chapter 1

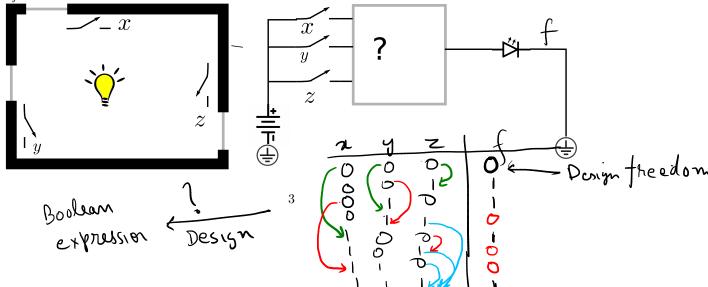
Boolean Algebra

1.1 Learning objectives

- 1. Representing digital circuits
- 2. Converting between different notations: Boolean expression, logic networks and switching circuits
- 3. Converting between different logic network specifications: truth table, minterm, maxterms, product of sums canonical form and sum of product canonical form.
- 4. Introduce truth tables as Behavioral Verilog
- 5. This handout has 11 homework problems totaling to 140 marks

1.2 Motivating Problem

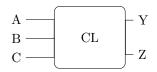
Example 1. Assume that a large room has three doors and that a switch near each door controls a light in the room. It has to be possible to turn the light on or off by changing the state of any one of the switches.



Venn diagram		x	x_1 x_1
(ANSI) symbol	x_1 x_2 x_3	$x_1 \xrightarrow{L(x_1, x_2)} x_2$	x_1
Switching circuit	Power S S S Light	Prower Supply Tright	Power W A Light
Truth Table	$\begin{array}{c cccc} x_1 & x_2 & x_1 \cdot x_2 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{c cc c} x_1 & x_2 & x_1 + x_2 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{c c} x_1 & \overline{x_1} \\ \hline 0 & 1 \\ 1 & 0 \\ \end{array}$
Boolean expr.	$L = x_1 \cdot x_2 = x_1 x_2$	$L = x_1 + x_2$	$L=\bar{x}_1=x_1'$
C/Verilog	AND Gate L = x1 & x2	L = x1 x2	
Name	AND Gate	OR Gate	NOT Gate

5

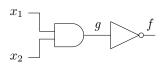
1.3 Digital circuits or networks



$$Y = F(A, B, C)$$
 $Z = G(A, B, C)$

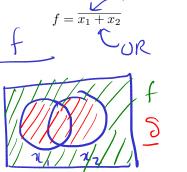
1.4 Two input networks

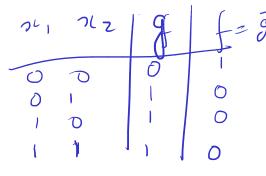
Example 2. Convert the following (ANSI) network into a Boolean expression, a truth table and a Venn diagram.



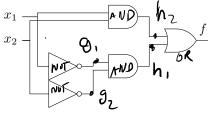
Example 3. Convert the following Boolean expression into a (ANSI) network, a truth table and a Venn diagram:

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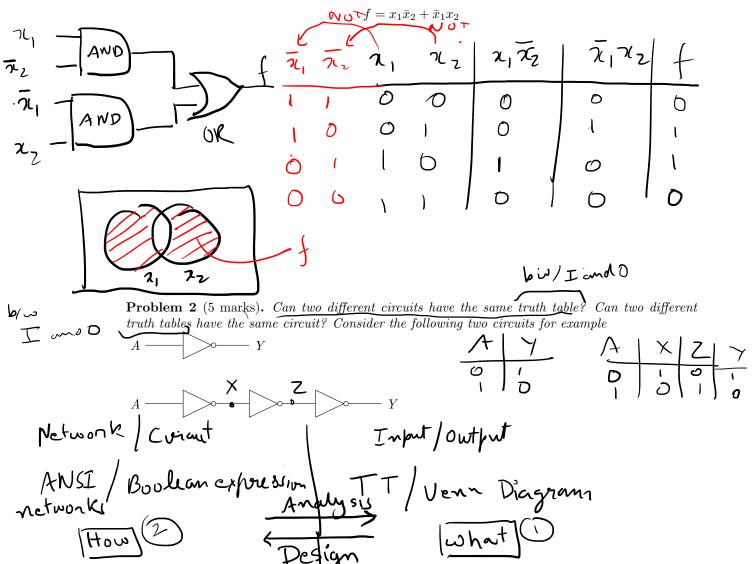
Problem 1 (10 marks). Convert the following (ANSI) network into a Boolean expression, a truth table and a Venn diagram.



$$9_1 = 21_1$$

$$9_2 = 22_2$$

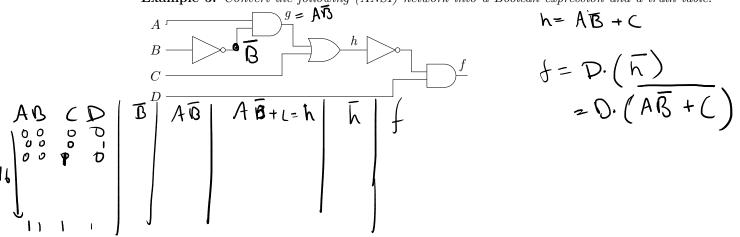
Example 4. Convert the following Boolean expression into a network, a truth table and a Venn diagram:



Remark 1. Truth tables and Venn diagrams define what the combinational circuit should do. Truth tables define output for every input. Boolean expression and networks define how to achieve the desired input output relationship.

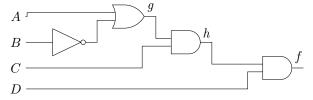
1.5 Multi-input networks

Example 5. Convert the following (ANSI) network into a Boolean expression and a truth table.



Decrmal to Bunary Repeated division by 2 and noting all the gumaindes

Problem 3 (20 marks). Convert the following (ANSI) network into a Boolean expression and a



truth table.

1.6 Minterms and Maxterms

1.6.1 Minterms

Minterm is a product involving all inputs (or complements) to a function. Every row of a truth table has a corresponding minterm. Minterm is true if and only if the corresponding row in the table is active.

Minterms defined as follows for each row of a two input truth table:

	/11116	erms denne	d as follows for	each row of a two input	tiutii tabie		-	70	
A	В	$\min term$	minterm			A BO	AB	413	
			name	_	Mo	00	1	0	
- 0	0	\sqrt{AB}	$\sum m_0$		1	Oi	٥]	
0 1	0	$AB \over Aar{B}$	$m_1 \ m_2$		3	10	0	10	
1	1	AB	m_3		.a.	1 1	D	5	
			,	_	5	,	_	U	
Consider a two input circuit whose output V is given by the truth table:									

Consider a two input circuit whose output Y is given by the truth table:

A	В	Y	minterm	minterm
				name
0	0	0	$ar{A}ar{B}$	m_0
0	1	1	$ar{A}B$	m_1
1	0	0	$Aar{B}$	m_2
1	1	1	AB	m_3

then
$$Y = \bar{A}B + AB = m_1 + m_3 = \sum (1, 3)$$
.

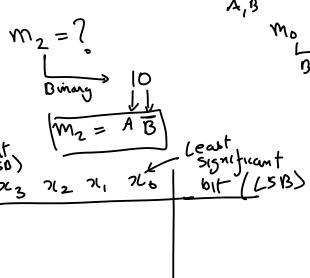
A A A A

- L) (1) Are product, terms
 - (2) Contain all inputs invented on not NOT gate
 - 3) In the truth table of a mintern one and only one now is 1, all other nows are zero
 - 4) They are denoted by small in with on index of the row number that is 1 in the Truth table mi= mi

Two mbuts

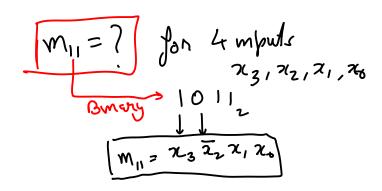
	ABI	M3=00	MO= AB	ĀB	AB
0 1 2 3	0 1 1 0 1 1	000	0000	000	00-0

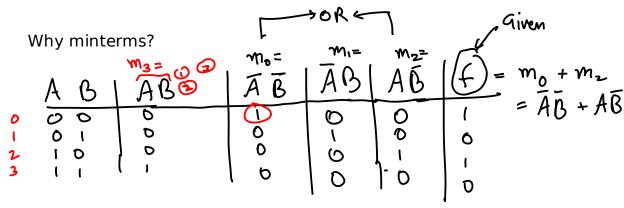
AB



$$M_{13} = \lambda_3 \chi_2 \bar{\chi}_1 \chi_0$$

$$M_{13} = A B \bar{C} D$$





Maxterms (Dual of minterm) $M_0 = M_2 = M_3 = M_3 = M_4 = M_4 = M_5 =$

Shortcut conversion of Maxtern into bodeon expression

$$M_2 = ?$$
Binary
 $A + B$

Maxtonm

1 Sum of all imput inverted or not

(Not discussed yet)

- @ contains all imputs
- (3) one and only one row of TT is O all others are 1
- (4) Denote by capital M, subscripted by the row that is O

How to write Boolean expression from TT using Maxterns

Maxterm notation
$$f = M_1 \cdot M_3 = TTM(1, 3)$$

$$= (A + \overline{B}) \cdot (\overline{A} + \overline{B})$$

Product of sums canonical form

This also gives the sum of products canonical form.

Example 6. What is the minterm m_{13} for a 4-input circuit with inputs x, y, z, w (ordered from MSB to LSB).

$$m_{13} = xyzw$$
 because $13_{10} = 1101_2$

Problem 4 (5 marks). What is the minterm m_{23} for a 5-input circuit with inputs a, b, c, d, e (ordered from MSB to LSB).

Example 7. Convert the following 4-input truth table into sum of minterms and sum of products canonical form.

minterm	A	B	C	D	\int		
name						$f = m_1 + m_5 + m_{13} = A$	ABCD + ABCD + ABCD
m_0	0	0	0	0	0	Sum of minterms	
m_1	0	0	0	1	1	Sum of militerins	Sum of products
m_2	0	0	1	0	0		canonical form
m_3	0	0	1	1	0		earrornear rorn
m_4	0	1	0	0	0		
m_5	0	1	0	1	1		
m_6	0	1	1	0	0		
m_7	0	1	1	1	0		
m_8	1	0	0	0	0		
m_9	1	0	0	1	0		
m_{10}	1	0	1	0	0		
m_{11}	1	0	1	1	0		
m_{12}	1	1	0	0	0		
m_{13}	1	1	0	1	1		
m_{14}	1	1	1	0	0		
m_{15}	1	1	1	1	0		

Problem 5 (10 marks). Convert the following 4-input truth table into sum of minterms and sum of products canonical form.

minterm	A	В	C	D	f
name					
m_0	0	0	0	0	0
m_1	0	0	0	1	0
m_2	0	0	1	0	0
m_3	0	0	1	1	1
m_4	0	1	0	0	0
m_5	0	1	0	1	0
m_6	0	1	1	0	0
m_7	0	1	1	1	1
m_8	1	0	0	0	0
m_9	1	0	0	1	0
m_{10}	1	0	1	0	0
m_{11}	1	0	1	1	1
m_{12}	1	1	0	0	0
m_{13}	1	1	0	1	1
m_{14}	1	1	1	0	1
m_{15}	1	1	1	1	0

1.6.2 Maxterms

Maxterm is a sum involving all inputs (or complements) to a function. Every row of a truth table has a corresponding maxterm. Minterm is false if and only if the corresponding row in the table is active.

Maxterms are defined as follows for each row of a two input truth table:

A	В	maxterm	maxterm
			name
0	0	A + B	$\overline{M_0}$
0	1	$A + \bar{B}$	M_1
1	0	$\bar{A} + B$	M_2
_1	1	$\bar{A} + \bar{B}$	M_3

Consider a two input circuit whose output Y is given by the truth table:

A	В	Y	\max term	\max term
				name
0	0	0	A + B	M_0
0	1	1	$A + \bar{B}$	M_1
1	0	0	$\bar{A} + B$	M_2
1	1	1	$\bar{A} + \bar{B}$	M_3

then $Y = (A + B)(\bar{A} + B) = M_0 M_2$.

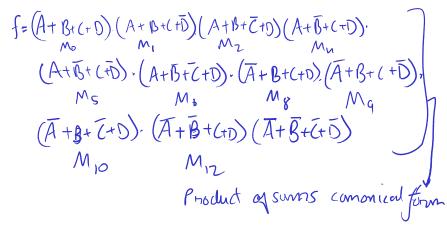
Writing a functional specification in terms of minterms is also called product of sums canonical form.

Example 8. Write the maxterm M_{11} for 4-input Boolean function with the ordered inputs A, B, C, D. $M_{11} = A + B + C + D$

Example 9. Convert the following 4-input truth table into product of maxterns and product of sums canonical form.

maxterm	A	В	C	D	f
name					
M_0	0	0	0	0	0
M_1	0	0	0	1	0
M_2	0	0	1	0	0
M_3	0	0	1	1	1
M_4	0	1	0	0	0
M_5	0	1	0	1	0
M_6	0	1	1	0	0
M_7	0	1	1	1	1
M_8	1	0	0	0	0
M_9	1	0	0	1	0
M_{10}	1	0	1	0	0
M_{11}	1	0	1	1	1
M_{12}	1	1	0	0	0
M_{13}	1	1	0	1	1
M_{14}	1	1	1	0	1
M_{15}	1	1	1	1	0

 $f = M_0 \ M_1 \ M_2 \ M_4 M_5 M_6 M_8 M_9 M_{10} M_{12} M_{15}$ Product of Maxterms



Problem 6 (10 marks). Convert the following 4-input truth table into product of maxterms and products of sums canonical form.

maxterm	A	В	C	D	<i>f</i>
name					
$\overline{M_0}$	0	0	0	0	0
M_1	0	0	0	1	1
M_2	0	0	1	0	1
M_3	0	0	1	1	1
M_4	0	1	0	0	1
M_5	0	1	0	1	0
M_6	0	1	1	0	1
M_7	0	1	1	1	1
M_8	1	0	0	0	0
M_9	1	0	0	1	1
M_{10}	1	0	1	0	1
M_{11}	1	0	1	1	1
M_{12}	1	1	0	0	0
M_{13}	1	1	0	1	1
M_{14}	1	1	1	0	1
M_{15}	1	1	1	1	0

Example 10. Write the 3-input truth table for the function $f = m_2 + m_3 + m_7$.

Problem 7 (10 marks). Write the 3-input truth table for the function $f = M_4 M_5 M_7$.

Problem 8 (10 marks). Write the truth table for the function $f = \bar{A}B\bar{C} + AB\bar{C}$.

1.7 Karnaugh maps

Two input K-maps

Three input K-maps

Four input K-maps

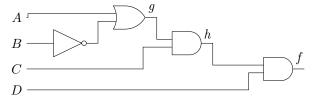
Five input K-maps

1.8 More Gates and notations summary

K-map	B 0 1 1 0 V 1 V 1 V 1 V 1 V 1 V 1 V 1 V 1		B 0 1 1 0 1 1 4 0 1 1 4 0 1 1 1 1 1 1 1 1	
(ANSI) symbol	\bigcap_{B}^{O}	\bigcap_{B}^{A}	$\bigoplus_{B} \forall$	$\stackrel{A}{\longrightarrow}$ $\stackrel{A}{\longrightarrow}$ $\stackrel{Out}{\longrightarrow}$
Truth Table	$\begin{array}{c c c} x_1 & x_2 & \overline{x_1 \cdot x_2} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ \end{array}$	$ \begin{array}{c c c} x_1 & x_2 & \overline{x_1 + x_2} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ \end{array} $	$ \begin{array}{c c c} x_1 & x_2 & x_1 \oplus x_2 \\ \hline 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ \hline \end{array} $	$\begin{array}{c c c} x_1 & x_2 & \overline{x_1 \oplus x_2} \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$
Boolean expr.	$Q = \overline{x_1 \cdot x_2} = \overline{x_1 x_2}$	$Q = \overline{x_1 + x_2}$	$Q = x_1 \oplus x_2$	$Q = \overline{x_1 \oplus x_2}$
C/Verilog	Q = ~(x1 & x2)	Q = ~(x1 x2)	$Q = x1 ^ x2$	Q = ~(x1 ^ x2)
Name	NAND Gate	NOR Gate	XOR Gate	XNOR Gate

Example 11. Convert the following Boolean expression into a K-map. $f = \overline{AB} + \overline{CD}$

Problem 9. Convert the following logic circuit into a K-map.



1.9 Boolean Algebra

1.9.1 Axioms of Boolean algebra

1.
$$0 \cdot 0 = 0$$

2.
$$1+1=1$$

- 3. $1 \cdot 1 = 1$
- 4. 0+0=0
- 5. $0 \cdot 1 = 1 \cdot 0 = 0$
- 6. $\bar{0} = 1$
- 7. $\bar{1} = 0$
- 8. $x = 0 \text{ if } x \neq 1$
- 9. $x = 1 \text{ if } x \neq 0$

1.9.2 Single variable theorems (Prove by drawing K-maps)

- 1. $x \cdot 0 = 0$
- 2. x + 1 = 1
- 3. $x \cdot 1 = x$
- 4. x + 0 = x
- 5. $x \cdot x = x$
- 6. x + x = x
- 7. $x \cdot \bar{x} = 0$

8.
$$x + \bar{x} = 1$$

9.
$$\bar{\bar{x}} = x$$

Remark 2 (Duality). $Swap + with \cdot and 0$ with 1 to get another theorem

1.9.3 Two and three variable properties (Prove by K-maps)

1. Commutative:
$$x \cdot y = y \cdot x$$
 , $x + y = y + x$

2. Associative:
$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$
, $x + (y + z) = (x + y) + z$

3. Distributive:
$$x \cdot (y+z) = x \cdot y + x \cdot z, \ x+y \cdot z = (x+y) \cdot (y+z)$$

4. Absorption:
$$x + x \cdot y = x$$
, $x \cdot (x + y) = x$

5. Combining: $x \cdot y + x \cdot \bar{y}$, $(x + y) \cdot (x + \bar{y}) = x$

6. DeMorgan's theorem: $\overline{x\cdot y} = \bar{x} + \bar{y}, \ \overline{x+y} = \bar{x}\cdot \bar{y}.$

7. Concensus:

(a)
$$x + \bar{x} \cdot y = x + y$$

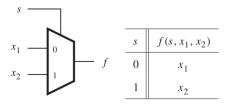
(b)
$$x \cdot (\bar{x} + y) = x \cdot y$$

(c)
$$x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$$

(d)
$$(x+y) \cdot (y+z) \cdot (\bar{x}+z) = (x+y) \cdot (\bar{x}+z)$$

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Example 12 (Multiplexer). Multiplexer is a circuit used to select one of the input lines x_1 and x_2 based only select input s. When s = 0, x_1 is selected, x_2 is selected otherwise. Find a boolean expression and a circuit for multiplexer



Example 13. Simplify $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C}$ using boolean algebra.

Problem 10 (30 marks, Exercise 2.14 [1]). Simplify the following Boolean equations using Boolean theorems.

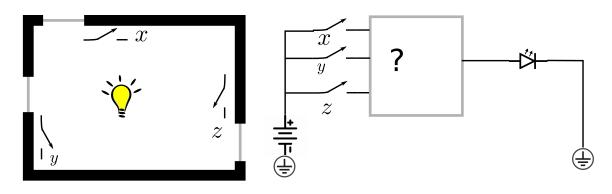
$$Y = \bar{A}BC + \bar{A}B\bar{C} \tag{1.1}$$

$$Y = A\bar{B}C + A\bar{B} \tag{1.2}$$

$$Y = ABC\bar{D} + AB\bar{C}D + (A + B + C + D) \tag{1.3}$$

Example 14. Simplify $f = \bar{A}\bar{A}\bar{C} + \bar{A}\bar{B}C$ using K-maps.

Example 15. Assume that a large room has three doors and that a switch near each door controls a light in the room. It has to be possible to turn the light on or off by changing the state of any one of the switches.



Problem 11 (20 marks, Exercise 2.38 [1]). An M-bit thermometer code for the number k consists of k 1's in the least significant bit positions and M-k 0's in all the more significant bit positions. A binary-to-thermometer code converter has N inputs and 2^{N-1} outputs. It produces a 2^N-1 bit thermometer code for the number specified by the input. For example, if the input is 110, the output should be 0111111. Design a 3:7 binary-to-thermometer code converter. Give a simplified Boolean equation for each output.

Bibliography

- [1] Sarah L Harris and David Harris. Digital design and computer architecture. Morgan Kaufmann, 2022.
- [2] Brown Stephen and Vranesic Zvonko. Fundamentals of digital Logic with Verilog design. McGraw Hill, 2022.