Row	x_1	x_2	x_3	f
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

Table 2.1: Truth table for a 3-way light switch

Problem 2.10 (10 marks)**.** *Find the minimum-cost Sum of Products (SOP) and Product of Sums (POS) forms for the function* $f(x_1, x_2, x_3) = \sum m(1, 5, 7) + D(2, 4)$.

Problem 2.11 (10 marks)**.** *Find the minimum-cost Sum of Products (SOP) and Product of Sums (POS) forms for the function* $f(x_1, x_2, x_3, x_4) = \prod M(1, 2, 4, 5, 7, 8, 9, 10, 12, 14, 15)$. *Chose the minimum-cost expression by comparing Product of Sums (POS) and Sum of Products (SOP) forms.*

Problem 2.12 (10 marks)**.** *Derive a minimum-cost realization of the four-variable function that is equal to 1 if exactly two or exactly three of its variables are equal to 1; otherwise it is equal to 0.*

Chapter 3

Review

3.1 Syllabus covered

- ✓ Binary numbers
- ✓ Generate minterms, maxterms, SOP canonical form and POS canonical forms and convert between them
- ✓ Understand and use the laws and theorems of Boolean Algebra
- ✓ Perform algebraic simplification using Boolean algebra
- ✓ Simplification using K-maps
- ✓ Derive sum of product and product of sums expressions for a combinational circuit
- ✓ Convert combinational logic to NAND-NAND and NOR-NOR forms
- Hexadecimal, Sign-magnitude, One's-complement and Two's complement. Conversions between them.
- Design combinational circuits for positive and negative logic
- Design Hazard-free two level circuits and understand Hazards in multi-level circuits
- Compute noise margin of one device
- Describe how tri-state and open-collector outputs are different from totem-pole outputs.
- Different between and limitations of master-slave and edge-triggered flip-flops.
- Compute fan out and noise margin of one device driving the same time
- Know the differences and similarities between PAL, PLA, and ROMs and can use each for logic design
- Design combinational circuits using multiplexers and decoders
- Analyze a sequential circuit and derive a state-table and a state-graph
- Understand the difference between synchronous and asynchronous inputs
- Derive a state graph or state table from a word description of the problem

- Reduce the number of states in a state table using row reduction and implication tables
- Perform a state assignment using the guideline method
- Implement a design using JK, SR, D or T flip-flops
- Analyse and design both Mealy and Moore sequential circuits with multiple inputs and multiple outputs
- Convert between Mealy and Moore designs
- Partition a system into multiple state machines

3.1.1 Labs (not questioned in exams)

- Use computer tools to enter designs graphically and HDL
- Simulate designs using computer tools
- Use computer tools to program gate arrays logic and debug and test

Chapter 4

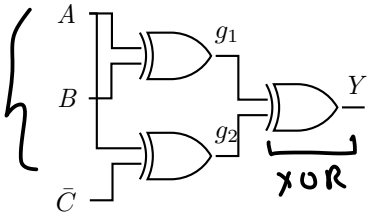
Sample midterm exam

Student Name: Student Email:

4.1 Instructions

- Time allowed is 50 minutes. (This sample exam might be lengthier than the actual exam. Same instruction will apply to the midterm.)
- In order to minimize distraction to your fellow students, you may not leave during the last 10 minutes of the examination.
- The examination is closed-book. One 8x11in cheatsheet is allowed.
- Non-programmable calculators are permitted.
- The maximum number of marks is 80, as indicated; the midterm examination amounts 10% toward the final grade.
- Please colored pen/pencils for K-maps, use a pen or heavy pencil to ensure legibility.
- Please show your work; where appropriate, marks will be awarded for proper and well-reasoned explanations.

Problem 4.1. *Consider the circuit below*



$Y = g_1 \oplus g_2$

$= g_1\bar{g}_2 + \bar{g}_1 g_2$

$= (\bar{A}B + A\bar{B})(\bar{A}\bar{C} + AC) + (A\bar{B} + \bar{A}B)(\bar{A}\bar{C} + AC)$

$g_1 = A \oplus B$

$g_2 = A \oplus \bar{C}$

A	B	g_1
0	0	0
0	1	1
1	0	1
1	1	0

$g_1 = m_1 + m_2 = \bar{A}B + A\bar{B}$

$g_2 = \bar{A}\bar{C} + A\bar{\bar{C}} = \bar{A}\bar{C} + AC$

By algebraic manipulation, prove or disprove that $Y = \bar{B}C + B\bar{C}$ (10 marks).

Problem 4.2. *Use the following 4-variable K-map for F (A, B, C, D), and find a minimal sum of products (SOP) expression for F (15 marks)*

$$Y = (\bar{A}B + A\bar{B})\overline{(\bar{A}\bar{C} + AC)} + \overline{(A\bar{B} + \bar{A}B)}(\bar{A}\bar{C} + AC)$$

Whenever you see a big not over an expression, apply DeMorgan's theorem.

$$= (\bar{A}B + A\bar{B})\left((A+C)\cdot(\bar{A}+\bar{C})\right) + \left((\bar{A}+B)\cdot(A+\bar{B})\right)(\bar{A}\bar{C} + AC)$$

$$= (\bar{A}B + A\bar{B})(A\bar{A} + C\bar{A} + A\bar{C} + C\bar{C}) + (\bar{A}A + BA + \bar{A}\bar{B} + B\bar{B})\cdot(\bar{A}\bar{C} + AC)$$

$$= \bar{A}B\cdot(C\bar{A} + A\bar{C}) + A\bar{B}\,(C\bar{A} + A\bar{C}) + BA\cdot(\bar{A}\bar{C} + AC) + \bar{A}\bar{B}(\bar{A}\bar{C} + AC)$$

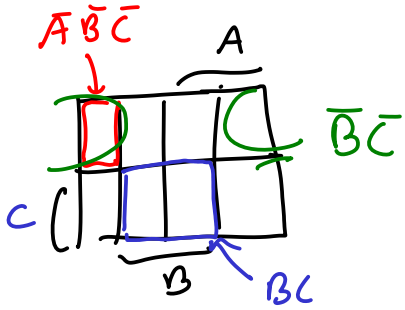
$$= \underset{①}{\bar{A}BC} + \bar{A}A\bar{B}\bar{C} + A\bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{A}\bar{B}\bar{C} + \underset{②}{ABC} + \bar{A}\bar{B}\bar{C} + \bar{A}A\bar{B}\bar{C}$$

$$= (\bar{A} + A)\,BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

$$= 1\cdot BC + (A + \bar{A})\,\bar{B}\bar{C}$$

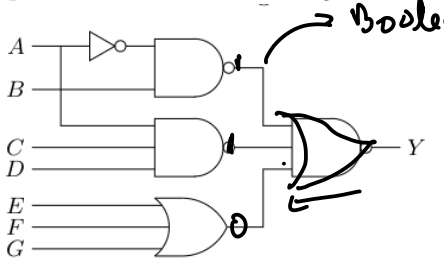
$$= BC + \bar{B}\bar{C}$$

$$= \text{RHS} \qquad \text{Hence Proved}$$



CD \ AB	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	0	1	0	0
10	0	1	1	0

Problem 4.3. *Use boolean algebra to find an SOP expression for Y in the circuit below. Draw an equivalent circuit beside the given circuit (5 marks).*



Boolean expr → K-maps → Boolean exp → circuit

$Y = \bar{A}B + ACD + \overline{(E+F+G)}$

$Y = \bar{A}B + ACD + \bar{E}\bar{F}\bar{G}$

Problem 4.4. *Consider the function Y given below.*

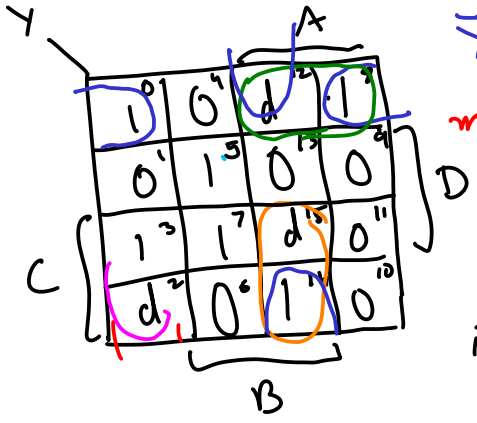
$$Y(A,B,C,D) = \sum m(0,3,5,7,8,14) + d(2,12,15)$$

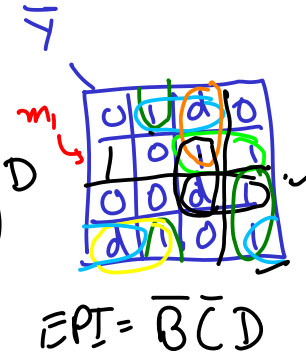
1. *Draw a K-maps to derive a minimum SOP and POS expressions for Y . Indicate all essential prime implicants for Y or $\bar{Y}$ in your K-maps (20 marks).*
2. *Sketch a two-level NOR-NOR circuit for Y. Assume that A, B, C, and D are available in true end complimentary forms (5 marks).*
3. *Write Y in Product of sums (POS)* canonical *form (5 marks).*

Problem 4.5. *Design a minimal SOP circuit to add two two-bit unsigned numbers. Denote the two bits of first number as A_1A_0 and the two bits of second number as B_1B_0. The result will be a 2-bit sum S_1S_0 and a carry C. Start with filling out the following truth table (3 example rows are provided) and then use K-maps to find minimal SOP for S_1, S_0 and a single carry bit C_1 (20 marks).*

$Y(A,B,C,D) = \sum m(0,3,5,7,8,14) + d(2,12,15)$

minterm — don't cares





EPI = $\bar{B}\bar{C}D$

Prime implicants ≡ [Biggest groups of 1's (and d's)]

Essential prime implicants

≡ Prime implicants that uniquely cover some 1's

= $\bar{A}BD$

$$Y = \underbrace{\bar{A}BD}_{m_5+m_7} + \underbrace{\bar{A}\bar{B}C}_{m_3+m_2} + \underbrace{\bar{B}\bar{C}\bar{D}}_{m_0+m_8} + \underbrace{ABC}_{m_{14}+m_{15}}$$

] cover all 1's, not all d's

$$\bar{Y} = \underbrace{\bar{B}\bar{C}D}_{m_1+m_9} + \underbrace{AB\bar{C}}_{m_{13}+m_{12}} + \underbrace{A\bar{B}C}_{m_{10}+m_{11}} + \underbrace{\bar{A}B\bar{D}}_{m_6+m_4}$$

A_1	A_0	B_1	B_0	C_1	S_1	S_0
0	*0*	*0*	*0*			
0	*0*	*0*	*1*			
0	*0*	*1*	*0*			
0	*0*	*1*	*1*			
0	*1*	*0*	*0*			
0	*1*	*0*	*1*	*0*	*1*	*0*
0	*1*	*1*	*0*			
0	*1*	*1*	*1*			
1	*0*	*0*	*0*			
1	*0*	*0*	*1*			
1	*0*	*1*	*0*			
1	*0*	*1*	*1*			
1	*1*	*0*	*0*			
1	*1*	*0*	*1*	*1*	*0*	*0*
1	*1*	*1*	*0*			
1	*1*	*1*	*1*	*1*	*1*	*0*

Chapter 5

Number System

5.1 Place value number system

- What is a place value number system?
- What are some examples?
- What are some non-examples? Roman numeral
- What is a radix (or base)? 23_{10}. 10111_2 ← radix / base
- How to convert between different radix in place value system? decimal ⟷ binary
- What are some commonly used number systems in computer engineering? symbols

	base	symbols
binary	2	0/1
decimal	10	0...9
octal	8	0....7
Hexadecimal	16	0.....9, A, B, C, D, E, F
Base 64 encoding	64	10+26+26+_2 addn symbols

Binary Representation ⟷ Decimal Value

multiply + add

int 23

char* [10111_2 , 23_{10}]

Repeated division

$1\ 0\ 1\ 1\ 1$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$16 + 0 + 4 + 2 + 1 = 23$

Binary → Decimal → Base3

$10111_2 \rightarrow 23_{10} \rightarrow 212_3$

3	23	
3	7	2
3	2	1
	0	2

5.1.1 Decimal number system

5.1.2 Binary numbers

5.1.3 Conversion between different radix

Example 5.1. *Convert the following binary numbers to decimal:* $(11110)_2$, $(100111)_2$.

Conversion from decimal to binary The value is in decimal because we find it easy to do calculations in decimal numbers. Decimal values can be converted back to Binary representation by *repeated division* by 2 while noting down the remainder. Allow me to use / sign to denote both quotient and remainder after division. Let's convert $(22)_{10}$ back to binary:

$22/2 = (11, 0)$	11 is the quotient and 0 is the remainder
$11/2 = (5, 1)$	5 is the quotient and 1 is the remainder
$5/2 = (2, 1)$	
$2/2 = (1, 0)$	
$1/2 = (0, 1)$	

Read the remainders from bottom to top and right them as left to right, to form the resultant binary number $(22)_{10} = (10110)_2$.

Example 5.2. *Find the binary representation for decimal numbers:* 123 *and* 89. *Show your work.*

For any radix/base that is a power of 2, for example octal has base 2^3 = 8 and Hexadecimal has base 2^4 = 16, you do not need repeated division for conversion from binary to HexD and vice-versa

$(10AD)_{16} =$ 0010_0000_1010_1101$_{16}$ 110_1011$_2$ =0110_1011$_2$ = 6B$_{16}$

5.2 Hexadecimal numbers

Numbers with base 16 are called Hexadecimal numbers. From 0 to 9 the symbols are same as decimal numbers. From 10 to 15, Hexadecimal numbers use A to F.

$$A = 10, B = 11, C = 12, D = 13, E = 14, F = 15$$

. Example, $(10AD)_{16} = 1 \times 16^3 + 10 \times 16^1 + 13 = 4096 + 160 + 13 = 4269$.

5.3 Octal numbers

Numbers with base 8 are called octal numbers. Example, $(354)_8 = 3\times8^2+5\times8+4 = 192+40+4 = 236$.

011_101_100$_2$

5.4 Hexadecimal/octal to binary and vice-versa

Normally, if you have to convert between a number of base r_1 to a number of base r_2, we will have to convert it via decimal numbers. Convert from base r_1 to decimal and then from decimal to r_2.

Since Hexadecimal base 16 is an exact power of 2 ($16 = 2^4$). Conversion between Hexadecimal to binary is easy. You can group 4 binary digits from right to left and convert each group of 4 binary digits to a single Hexadecimal digit and back. Example, $(10110)_2 = (0001\_0110)_2 = (16)_{16}$. To convert back. Take example, $(10AD)_{16} = (0001\_0000\_1010\_1101)_2 = (1\_0000\_1010\_1101)_2$.

Problem 5.1. *Find the binary and decimal values of the following Hexadecimal numbers* $(A25F)_{16}$, $(F0F0)_{16}$.

Similarly octal to binary can proceed by grouping 3-binary digits at a time. Example, $(354)_8 = (011\_101\_100)_2$.

Problem 5.2. *Find the binary and decimal values of the following Octal numbers* $(3751)_8$ *and* $(722)_8$.

5.5 Signed binary numbers

Signed numbers include both negative and positive numbers. There three common signed number representations

1. Sign magnitude representation
2. One's complement
3. Two's complement

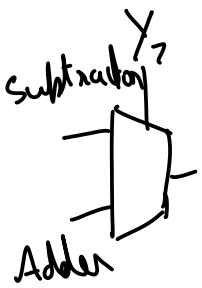
5.5.1 Sign-magnitude representation

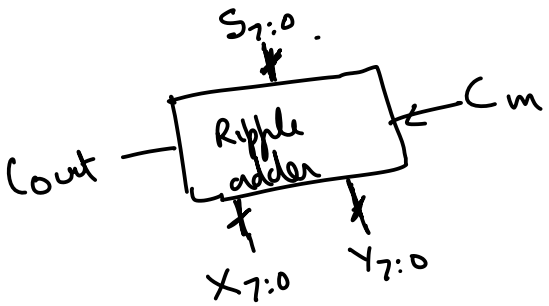
The Most significant (left most) *bit* (binary digit) represents sign (0 = + and 1 = −), the rest represent the magnitude. Example, a 5-bit number $(11010)_2$ in signed magnitude representation has the value of $(-1010)_2 = -10$. Note that +10 has to be represented by a leading 0 at the most significant bit (MSB) $+10 = (01010)_2$. Hence, the number of bits have to be specified.

Problem 5.3.
- *Write down all possible 4-digit binary numbers and corresponding decimal values if they are in signed magnitude format? What is the minimum and maximum value?*
- *What is the minimum and maximum value of n-digit signed binary number in sign-magnitude format?*

5.5.2 One's complement negation

You can convert a positive number (say +10) to negative number by applying a negative sign in front of it (-(+10) = -10). It is more evident from taking negative of a negative number (-(-10) = +10). In case of sign-magnitude representation, the "negative operator" flips the sign bit. The next two signed number representations (1's complement and 2's complement) are designed around specific negative operator definitions.
Negate $13_{10} = 01101_2$ using 5-bit one's complement.





Negate -13_{10} using 5-bit one's complement.

$+13_{10} = 01101_2$

11111_2
01101_2
10010

In 1's complement
$+13_{10} = 01101_2$
$-13_{10} = 10010_2$
$\rightarrow 11111_2 = 0_{10}$

5.5.3 One's complement binary numbers

In one's complement representation, the negative operation is obtained by flipping all the bits of the binary number. Example, a 5-bit one's complement of $+10 = (01010)_2$ is $(10101)_2 = -10$. Note that flipping bits is equivalent to subtracting the number from $(11111)_2$, hence the name. You can also confirm that double negative operator yields back the same number.

Problem 5.4.
- *Write down all possible ~~4-digit~~ but binary numbers and corresponding decimal values if they are in sign magnitude format? What is the minimum and maximum value?*
- *What is the minimum and maximum value of n-digit signed binary number in one's complement?*

	sign magnitude	1's complement
0000	0	0
0001	1	1
0010	2	2
⋮		
0111	7	7
1000	-0	-7
1001	-1	-6
⋮	⋮	
1111	-7	-0

$-6 = (1111_2 + 1)_2 + 0110_2$

$= 10000_2 - 0110_2$

$2^5 - 0110_2$

2's complement

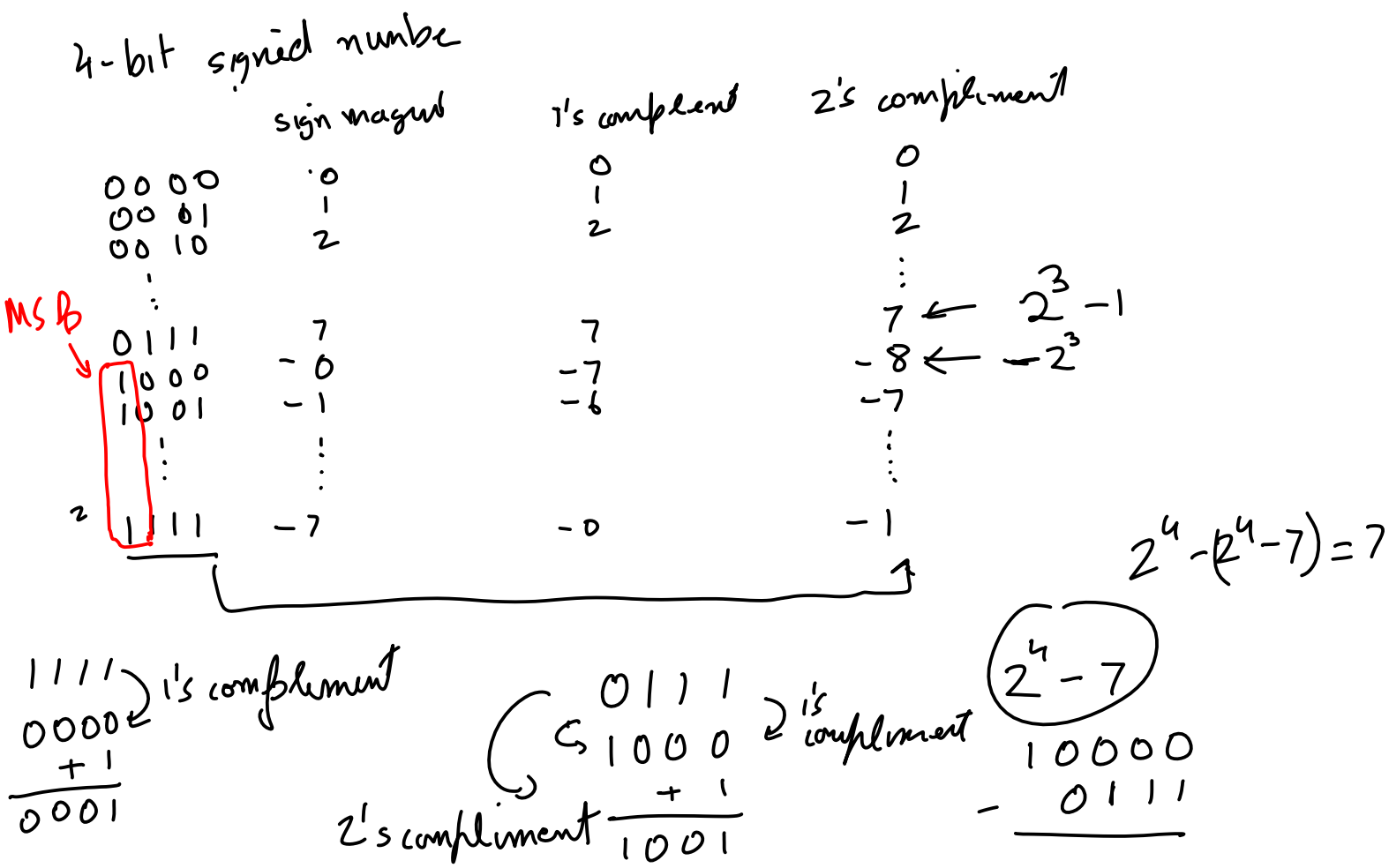
$7 + (-6) =$

```
  1 1
  0111
  1001
 10000
  1111
  0111
  1001
 0001   = +1
```

Addition of numbers of opposite sign requires ✓ and addition of +1

Problem 5.5. *Determine the decimal values of the following 1's complement 6-digit binary numbers*

1. *01101110*
2. *10101101*

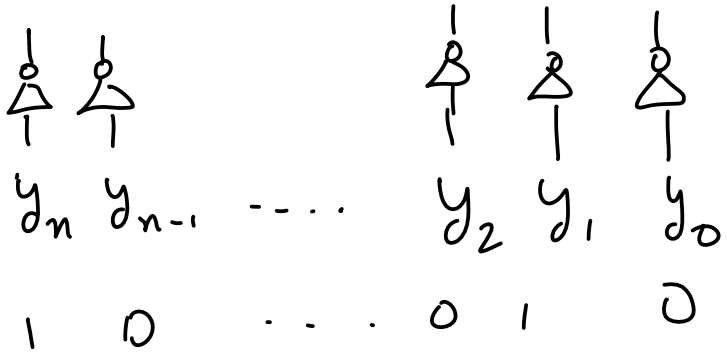


Converting to 2's compliment:
1. Convert to 1's compliment by inverting all the bits
2. Add one to the result

For n-bit 2's complement signed numbers what is the range?

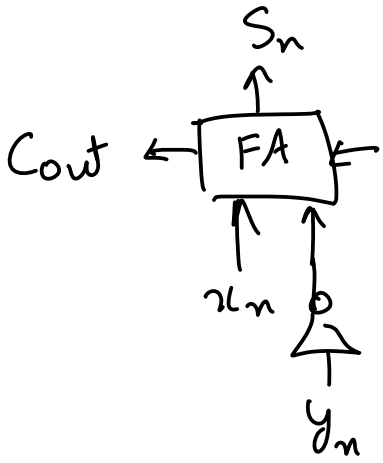
$$\text{min value} = -2^{n-1}$$

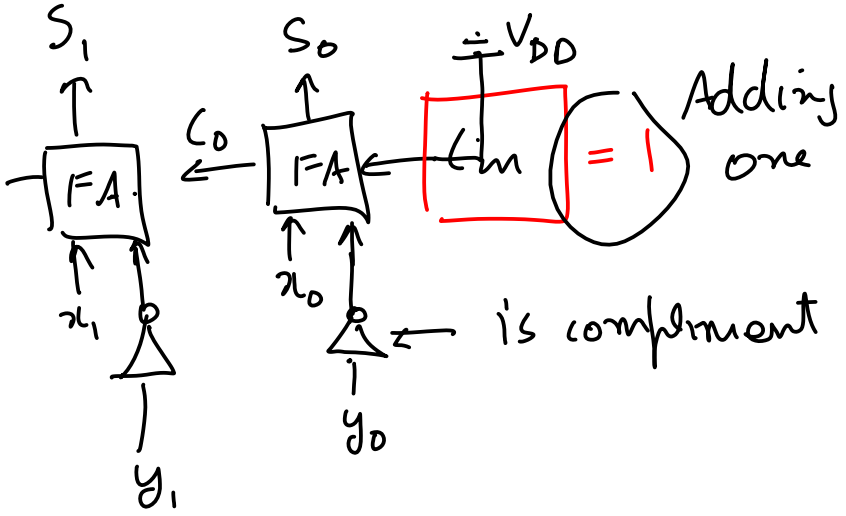
$$\text{max value} = +2^{n-1} - 1$$



Adder

Subtracter
$x - y$





Problem 5.6. *Convert the decimal numbers -17 and +23 into the 6-digit one's complement binary numbers and try adding them. What adjustments will you need to make to get the right result's (23-17=6) in binary representation.*

5.5.4 Two's complement negation

In two's complement representation, the n-digit negative number is obtained by subtracting the positive number from 2^n. Example, two's complement of 5-digit binary number $+10 = (01010)_2$ is $2^5 - 10 = 22 = (11000)_2$. An easier algorithm to get two's complement goes via one's complement. Note that $(11111)_2 = 2^5 - 1$. We can get two's complement by adding 1 to one's complement. To get two's complement:

1. Flip all the bits. (Same as taking one's complement).
2. Add 1 to the number.

Negate $13_{10} = 01101_2$ using 5-bit two's complement.

Negate -13_{10} using 5-bit two's complement.

How to convert one's complement number representation into sign-magnitude numbers?

1. Check if the number is positive or negative. Even for one's complement representation, or two's complement representation, if the MSB (Most-significant bit) is 1, then the number is negative, otherwise positive.

2. If positive: For positive numbers, two's complement, one's complement and sign magnitude are the same. No conversion between different representation is needed. 2.b If negative: For negative numbers. Flip the bits of 1's complement. Once you flip the 1's complement bits of a negative number, you get the corresponding positive number.

3. We still want to represent the original negative number. So we set the MSB of sign-magnitude representation to 1. Since the range (min and max) for both n-bit 1's complement and sign-magnitude are the same (between $-(2^{n-1}-1)$ and $2^{n-1}-1$), you can always represent 8-bit 1's complement numbers with needing to extend the 8-bit number to 9-bits.

Example: Convert 8-bit one's complement 10101010 to 8-bit sign-magnitude Let number n = 10101010

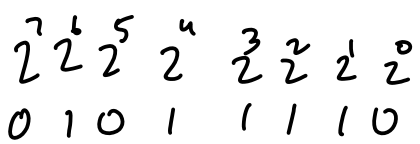
1. Is the number +ve or -ve: It is negative because it starts with 1.

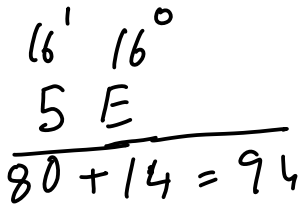
2. The number is not positive.

3. Take the 1's complement of the negative number to get the positive part. i.e. Flip the bits: -n = 01010101 or n = -(01010101)

4. We got the positive part of the number, but we want to represent the original negative number, so we set the MSB bit one. Hence, the equivalent sign-magnitude representation is: n = 11010101

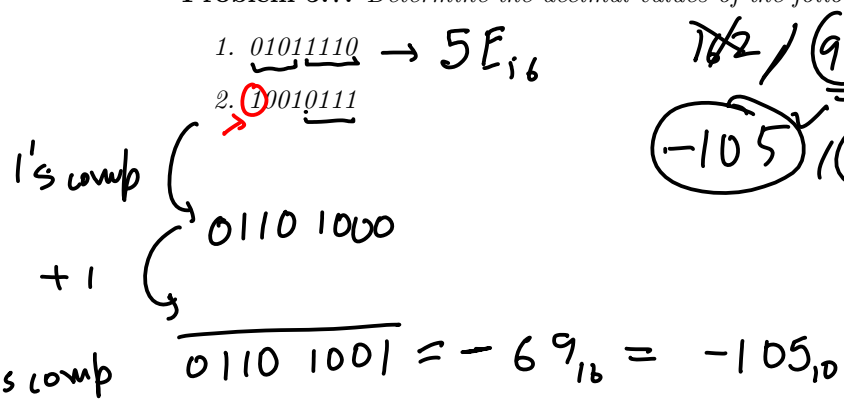
5.5.5 Two's complement representation





Problem 5.7. *Determine the decimal values of the following 2's complement 6-digit numbers :*

1. *01011110* → $5E_{16}$
2. *10010111*



-32 to +31
-2^{6-1} to $2^{6-1}+1$

Problem 5.8. *Convert the decimal numbers -17 and +23 into the 6-digit two's complement binary numbers and try adding them. What adjustments will you need to make to get the right result's (23-17=6) in binary representation.*

$23_{10} = 16+7 = 010000 + 000111$
$= 010111_2$

$+17_{10} = 16+1 = 010001_2$

$-17_{10} = 101110 + 1 = 101111_2$

$23+9_{10} = ?$ Arithmetic overflow?

carry out bit Overflow

$23_{10} = 010111$
$-17_{10} = 101111$
$+6_{10} = 001110_2$

Problem 5.9. *Convert the decimal numbers 73, 23, -17, and -163 into signed 8-bit numbers in the following representations:*

1. *Sign and magnitude*
2. *1's complement*
3. *2's complement*

5.5.6 Arithmetic overflow

bit -2^3 to $+2^3-1 = -8$ to 7

Problem 5.10. *Consider addition of 4-digit two's complement binary numbers*

1. $1010_2 + 1101_2$
2. $1011_2 + 1100_2$

$= -0110_2 + (-0011_2)$
-6_{10} -3_{10}

1010 = -6₁₀
1111 = -1₁₀
1001

0110 = 6₁₀
0001 = 1₁₀
0111 = 7₁₀

0110 = 6₁₀
0011 = 3₁₀
1001 = -7₁₀
0111

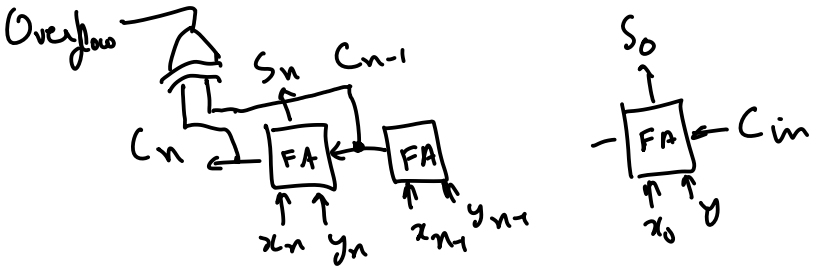
10000
1010 = -6₁₀
1101 = -3₁₀
0111 = 7₁₀

0001 → 1101

Carry out

C_n	C_{n-1}	Overflow?
0	0	0
0	1	1
1	0	1
1	1	0

Overflow = $C_n \oplus C_{n-1}$



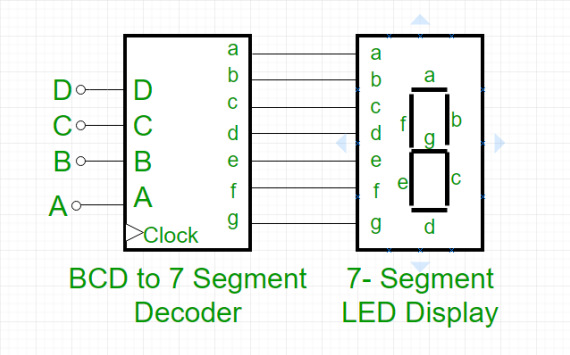
In which of the two case overflow happens? Can you come up with a rule to "easily" detect overflow?

Rules for detecting arithmetic overflow:

1. Adding numbers of different signs never produces an overflow.
2. Adding numbers of the same sign may produce an overflow
   (a) Wrong approach: Adding two negative 2's complement numbers always produces an additional carry-over 1, but that in itself isn't an overflow. An example, the range of 4-bit 2's complement numbers is between -8 to +7. Adding -3 to -4 in 2's complement is 1101 + 1100 produces an additional carry over 1. You can ignore the additional carry-over 1 to get the correct answer 1001 = -7 which is within range -8 to 7.
   (b) Approach 1: The easiest way for now to detect overflow is if adding two -ve numbers results in a +ve number, or adding +ve numbers results in a -ve number.
   (c) Approach 2: You can also do a range test in decimal based range test. The range of n-bit 2's complement numbers is between -2^{n-1} and $2^{n-1}-1$. For 5-bit 2's complement numbers, it is between -16 and 15. For 6-bit 2's complement numbers, it is between -32 and 31.
   (d) Approach 3: You can also check the carry-overs of the most significant two bits. If they match, i.e. 0 and 0, or 1 and 1, then there is no overflow. If they do not match, i.e. 0 and 1 or 1 and 0, then there is an overflow.

5.6 Binary coded decimal

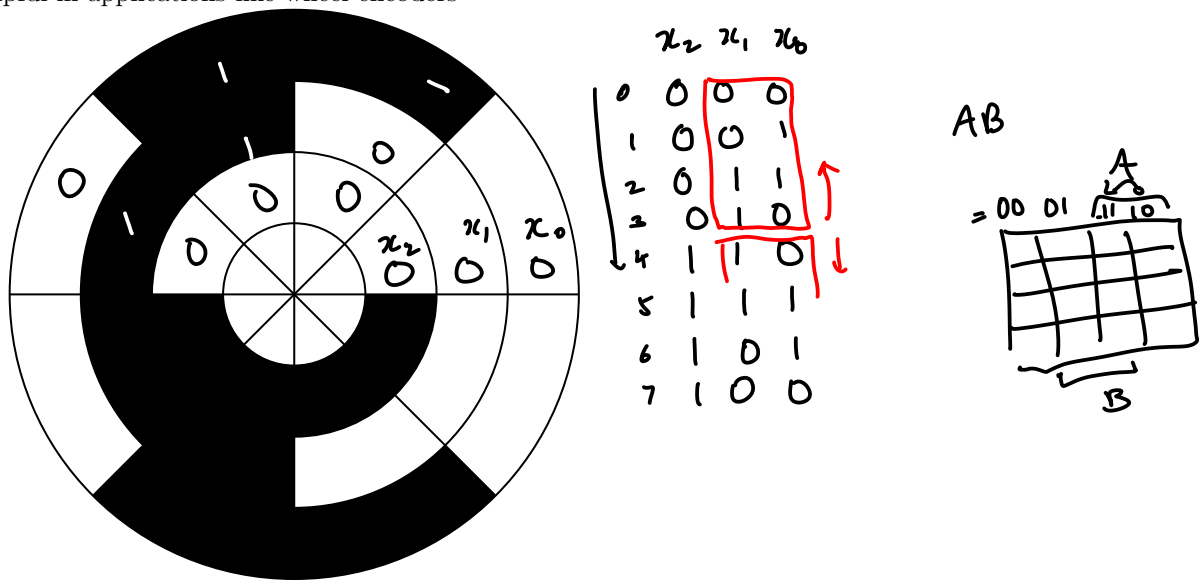
In Binary coded decimal (BCD), each decimal digit is represented by 4 bits. For example, $1047 = (0001\_0000\_0100\_0111)_{\text{BCD}}$. It is useful in input-output applications where the number has to be either displayed as decimal or received as decimal.



Problem 5.11. *Convert 11, 23, 35, 57 and 103897 to BCD?*

5.7 Gray code

A sequence of binary numbers where only one bit changes when the number increases by 1. It is helpful in applications like wheel encoders



Problem 5.12. *Write all possible 3-bit binary numbers in gray-code*

Always show your work/process. Correct final answer is worth less than the correct process. Submit digitally via brightspace.

Problem 5.13. *Convert the each of the following numbers into binary, decimal, hexadecimal, octal numbers. Show your work. Just filling in the values is not enough. (*8×6 *marks)*

	Binary	*Decimal*	*Hexadecimal*	*Octal*
a)	1010_2			
b)	10_0110_2			
c)		329_{10}		
d)		741_{10}		
e)			$7D_{16}$	
f)			$EC3A_{16}$	
g)				351_8
h)				2563_8

Problem 5.14. *Convert the each of the following numbers into decimal, 8-bit sign-magnitude binary, 8-bit one's complement binary and 8-bit two's complement binary. Show your work. (*6×6 *marks)*

	Decimal	*Sign-magnitude*	*One's complement*	*Two's complement*
a)	-79_{10}			
b)	-110_{10}			
c)				0110_1110_2
d)				1011_1101_2
e)			0110_1101_2	
f)			1001_1010_2	

Problem 5.15. *Convert the decimal numbers to 6-bit two's complement binary and then add them. Check if the addition causes overflow (*3×6 *marks).*

1. $-16_{10} - 7_{10}$
2. $19_{10} - 5_{10}$
3. $-4_{10} - 29_{10}$

Problem 5.16.

1. *Convert* 299_{10} *to binary coded decimal (BCD). (2 marks)*
2. *Convert* $1001\_0111\_0101_{BCD}$ *to decimal. (2 marks)*
3. *Convert* 0110_1101_{BCD} *to binary. (4 marks)*