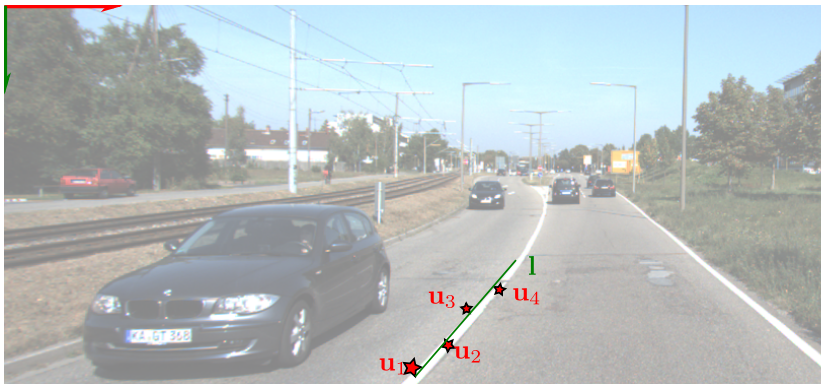


ECE 417/598: Line fitting using null spaces

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$$\underline{\mathbf{u}}_1 = [100, 98, 1]^\top$$

$$\underline{\mathbf{u}}_2 = [105, 95, 1]^\top$$

$$\underline{\mathbf{u}}_3 = [107, 90, 1]^\top$$

$$\underline{\mathbf{u}}_4 = [110, 85, 1]^\top$$

Find the line \mathbf{l} such that it is the “closest line” passing through $\underline{\mathbf{u}}_1, \dots, \underline{\mathbf{u}}_4$.

$$U = \begin{bmatrix} \mathbf{u}_1^\top \\ \mathbf{u}_2^\top \\ \mathbf{u}_3^\top \\ \mathbf{u}_4^\top \end{bmatrix}$$

We want to solve for \mathbf{l} such that

$$U\mathbf{l} = \mathbf{0}$$

Singular Value Decomposition (SVD)

$$A = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^{\top}$$

$$A^{\top} A = V \Sigma^2 V^{-1}$$

$$A^{\top} A \mathbf{v}_i = \lambda_i \mathbf{v}_i \qquad \lambda_i = \sigma_i^2$$

$$\mathbf{u}_i = \frac{A \mathbf{v}_i}{\sigma_i}$$

$$U = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \dots & \mathbf{u}_m \end{bmatrix}$$

$$A = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^{\top} \quad (1)$$

If $A \in \mathbb{R}^{m \times n}$

$$A = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \dots & \mathbf{u}_m \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^{\top} \\ \mathbf{v}_2^{\top} \\ \vdots \\ \mathbf{v}_n^{\top} \end{bmatrix} \quad (2)$$

Coding example

$$\underline{\mathbf{u}}_1 = [100, 98, 1]^\top$$

$$\underline{\mathbf{u}}_2 = [105, 95, 1]^\top$$

$$\underline{\mathbf{u}}_3 = [107, 90, 1]^\top$$

$$\underline{\mathbf{u}}_4 = [110, 85, 1]^\top$$

Find the line \mathbf{l} such that it is the “closest line” passing through $\underline{\mathbf{u}}_1, \dots, \underline{\mathbf{u}}_4$.

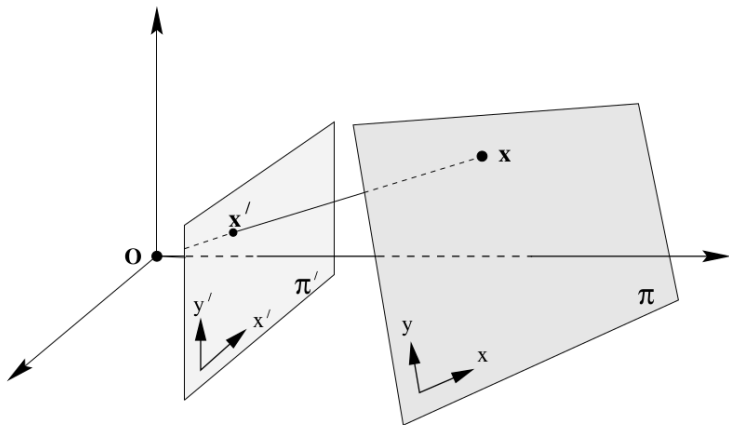
<https://github.com/wecacuee/ECE417-Mobile-Robots/tree/master/docs/slides/03-09-svd-null-space>

$$A = \begin{bmatrix} \mathbf{u}_1^\top \\ \mathbf{u}_2^\top \\ \mathbf{u}_3^\top \\ \mathbf{u}_4^\top \end{bmatrix}$$

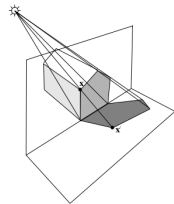
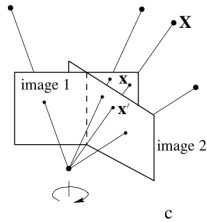
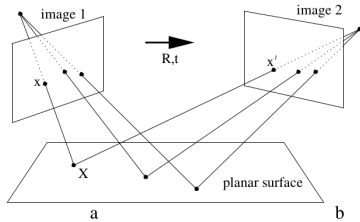
We want to solve for \mathbf{l} such that

$$U\mathbf{l} = 0$$

Homography



Examples of Homography





Computing Homography



Computing Homography



Solving for Homography derivation

Direct Linear Transformation (DLT) algorithm

Objective

Given $n \geq 4$ 2D to 2D point correspondences $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$, determine the 2D homography matrix \mathbf{H} such that $\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$.

Algorithm

- (i) For each correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ compute the matrix \mathbf{A}_i from (4.1). Only the first two rows need be used in general.
- (ii) Assemble the $n \times 9$ matrices \mathbf{A}_i into a single $2n \times 9$ matrix \mathbf{A} .
- (iii) Obtain the SVD of \mathbf{A} (section A4.4(p585)). The unit singular vector corresponding to the smallest singular value is the solution \mathbf{h} . Specifically, if $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^T$ with \mathbf{D} diagonal with positive diagonal entries, arranged in descending order down the diagonal, then \mathbf{h} is the last column of \mathbf{V} .
- (iv) The matrix \mathbf{H} is determined from \mathbf{h} as in (4.2).

2D homography

Given a set of points $\mathbf{x}_i \in \mathbb{P}^2$ and a corresponding set of points $\mathbf{x}'_i \in \mathbb{P}^2$, compute the projective transformation that takes each \mathbf{x}_i to \mathbf{x}'_i . In a practical situation, the points \mathbf{x}_i and \mathbf{x}'_i are points in two images (or the same image), each image being considered as a projective plane \mathbb{P}^2 .

3D to 2D camera projection matrix estimation

Given a set of points \mathbf{X}_i in 3D space, and a set of corresponding points \mathbf{x}_i in an image, find the 3D to 2D projective \mathbf{P} mapping that maps \mathbf{X}_i to $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$.