ECE 417/598: K,R,t from P matrix

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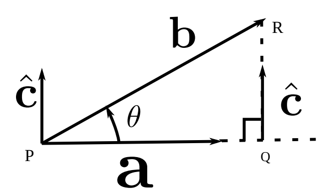
Rules

1. Let unit vectors of $\mathbf{a}$ be denoted by $\hat{\mathbf{a}} = \frac{\mathbf{a}}{\|\mathbf{a}\|}$.
2. The projection of $\mathbf{b}$ on $\mathbf{a}$ is $\overrightarrow{PQ}$. The magnitude of the projection is given by dot product,
$$|\overrightarrow{PQ}| = \mathbf{b}^\top \hat{\mathbf{a}} = \hat{\mathbf{a}}^\top \mathbf{b} = \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\theta) = \|\mathbf{b}\| \cos(\theta)$$
3. Since $\overrightarrow{PQ}$ is in the direction of $\hat{\mathbf{a}}$, the vector $\overrightarrow{PQ}$ is given by,
$$\overrightarrow{PQ} = |\overrightarrow{PQ}| \hat{\mathbf{a}} = (\mathbf{b}^\top \hat{\mathbf{a}})\hat{\mathbf{a}}$$
4. Similarly the projection of $\mathbf{b}$ on $\hat{\mathbf{c}}$ is $\overrightarrow{QR}$
$$|\overrightarrow{QR}| = \mathbf{b}^\top \hat{\mathbf{c}} = \hat{\mathbf{c}}^\top \mathbf{b} = \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\frac{\pi}{2} - \theta) = \|\mathbf{b}\| \cos(\frac{\pi}{2} - \theta)$$
$$\overrightarrow{QR} = (\mathbf{b}^\top \hat{\mathbf{c}})\hat{\mathbf{c}}$$
5. By triangle law $\mathbf{b} = \overrightarrow{PQ} + \overrightarrow{QR}$, or
$$\mathbf{b} = (\mathbf{b}^\top \hat{\mathbf{a}})\hat{\mathbf{a}} + (\mathbf{b}^\top \hat{\mathbf{c}})\hat{\mathbf{c}}$$

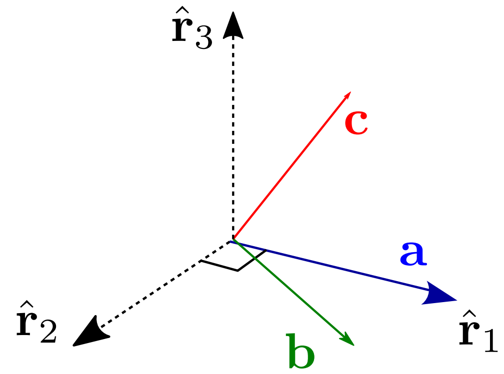


Problem 1

We want to find a pair of orthonormal vectors in the same plane as $\mathbf{a}$ *and* $\mathbf{b}$. *First vector is* $\hat{\mathbf{a}}$. *What is the second vector? (Call it* $\hat{\mathbf{c}}$ *and find it in terms of* $\mathbf{a}$ *and* $\mathbf{b}$.*)*

Problem 2 *Express the above relationship in terms of matrix vector multiplication so that the matrix* $M = \begin{bmatrix} \mathbf{b}^\top \\ \mathbf{a}^\top \end{bmatrix}$ *can be written in terms of an upper triangular matrix and an orthonormal matrix.*

Problem 3 *Repeat the process for 3 vectors,* $\mathbf{a}$, $\mathbf{b}$ *and* $\mathbf{c}$, *and then matrix* $M = \begin{bmatrix} \mathbf{c}^\top \\ \mathbf{b}^\top \\ \mathbf{a}^\top \end{bmatrix}$. *In other words, find* $\hat{\mathbf{r}}_1$, $\hat{\mathbf{r}}_2$, $\hat{\mathbf{r}}_3$ *and write them in (upper triangular matrix) (orthonormal matrix) factorization form, also known as QR factorization.*



1. Write $\hat{\mathbf{r}}_1$ in terms of $\mathbf{a}$.

2. Write $\hat{\mathbf{r}}_2$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\hat{\mathbf{r}}_1$.

3. Write $\hat{\mathbf{r}}_3$ in terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\hat{\mathbf{r}}_1$ and $\hat{\mathbf{r}}_2$.

4. Write the above equations in matrix multiplication form.

Problem 4 *Assuming a QR factorization algorithm is given, find* $K, R, \mathbf{t}$ *from* $P \in \mathbb{R}^{3\times4}$ *matrix such that*

$$P = \begin{bmatrix} KR & K\mathbf{t} \end{bmatrix}$$

and $K \in \mathbb{R}^{3\times3}$ *is an upper triangular matrix, and* $R \in \mathbb{R}^{3\times3}$ *is a rotation matrix (thus orthonormal) and* $\mathbf{t} \in \mathbb{R}^{3\times1}$ *is a translation vector.*