# Planning (Chapter 2 from Lavalle book)

# Abstraction of a planning problem

- 1. State space  $s \in \mathcal{S}$ . For example, 2D coordinate of a grid s = (x, y).
- 2. Action space per state  $u \in \mathcal{U}(s)$ . For example, up, down, left right movement can be encoded as  $\mathcal{U}(s_t) = \{(0, -1), (0, 1), (1, 0), (-1, 0)\}$ .
- 3. State transition function  $s_{t+1}=f(s_t,u_t)$ . For example, the up-down-left-right action can be combined as addition to get the next state  $s_{t+1}=s_t+u_t$ .
- 4. Initial State  $\mathrm{s}_{\mathit{I}} \in \mathcal{S}$
- 5. Goal states  $\mathrm{s}_G \subseteq \mathcal{S}$

# A Graph

A graph  $\mathcal{G}=\{\mathcal{V},\mathcal{E}\}$  is defined by a set of vertices  $\mathcal{V}$  and a set of edges  $\mathcal{E}$  such that each edge  $e\in\mathcal{E}$  is formed by a pair of start and end vertices  $e=(v_s,v_e),v_s\in\mathcal{V},v_e\in\mathcal{V}$ . The first vertex is called the start of the edge  $v_s=\operatorname{start}(e)$  and second vertex is called the end  $v_e=\operatorname{end}(e)$ .

A discrete planning problem can be converted into a graph by definiting

- 1. Vertices as the state space  $\mathcal{V} = \mathcal{S}$ .
- 2. The action space at each state as the edges connected to that vertex/state,  $\mathcal{U}(s_t) = \{(s_t, s_j) \mid (s_t, s_j) \in \mathcal{E}\}.$
- 3. State transition function is the other end of th edge,

$$\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{u}_t) = \mathrm{end}(\mathbf{u}_t)$$
, where  $\mathbf{s}_t = \mathrm{start}(\mathbf{u}_t)$ .

# Representations of Graphs

Undirected graph

```
In [1]: # Programmatically you can represent a adjacency list as python lists
        # Python lists are not linked lists, they are arrays under the hood.
        G adjacency list = {
            1: [2, 5],
            2: [1, 5, 3, 4],
            3:[2,4],
            4: [2, 5, 3],
            5: [4, 1, 2]
        # Prefer to represent a matrix in python either as a list of lists or a nump
        import numpy as np
        G adjacency matrix = np.array([
            [0, 1, 0, 0, 1],
            [1, 0, 1, 1, 1],
            [0, 1, 0, 1, 0],
            [0, 1, 1, 0, 1],
            [1, 1, 0, 1, 0]
        ])
        # Edge list is another possible representation
        G edge list = [
            (1, 2), (1, 5),
            (2, 1), (2, 5), (2, 3), (2, 4),
            (3, 2), (3, 4),
            (4, 2), (4, 5), (4, 3),
            (5, 4), (5, 1), (5, 2)
        ]
```

Directed graph representation

```
[0, 0, 0, 0, 1, 1],
            [0, 1, 0, 0, 0, 0],
            [0, 0, 0, 1, 0, 0],
            [0, 0, 0, 0, 0, 1]
        ])
        # Edge list is another possible representation
        G_edge_list = [
            (1, 2), (1, 4),
            (2, 5),
            (3, 6), (3, 5),
            (4, 2),
            (5, 6)
In [3]: # Exercise 1
        # Write a function that converts a graph in adjacency list format to adjacen
        def adjacency list to matrix(G adj list):
            G adj mat = None # TODO: Write code to convert to adj mat
            return G_adj_mat
        def adjacency matrix to list(G adj mat):
            G adj list = None # TODO: Write code to convert to adj mat
            return G adj list
        # Use the above graphs to test
        print(adjacency_list_to_matrix(G_adjacency_list))
        print(adjacency matrix to list(G adjacency matrix))
        None
        None
```

# Graph Search algorithms

1. Breadth First Search

**b**fs.png

2. Depth First Search

**dfs.png** 

Breadth first search (BFS)

**b**fs-states

```
In [4]: from queue import Queue, LifoQueue, PriorityQueue

graph = {
    's' : ['w', 'r'],
    'r' : ['v'],
    'w' : ['t', 'x'],
    'x' : ['y' ],
```

```
't' : ['u'],
         'u' : ['y']
        def bfs(graph, start, debug=False):
            seen = set() # Set for seen nodes (contains both frontier and dead state
            # Frontier is the boundary between seen and unseen (Also called the aliv
            frontier = Queue() # Frontier of unvisited nodes as FIFO
            node2dist = {start : 0} # Keep track of distances
            search order = []
            seen.add(start)
            frontier.put(start)
            i = 0 \# step number
            while not frontier.empty():
                                                 # Creating loop to visit each node
                if debug: print("%d) Q = " % i, list(frontier.queue), end='; ')
                if debug: print("dists = " , [node2dist[n] for n in frontier.queue])
                m = frontier.get() # Get the oldest addition to frontier
                search order.append(m)
                for neighbor in graph.get(m, []):
                    if neighbor not in seen:
                         seen.add(neighbor)
                         frontier.put(neighbor)
                         node2dist[neighbor] = node2dist[m] + 1
                    else:
                         assert node2dist[neighbor] <= node2dist[m] + 1, 'this should</pre>
                         node2dist[neighbor] = min(node2dist[neighbor], node2dist[m]
                i += 1
            if debug: print("%d) Q = " % i, list(frontier.queue))
            return search order, node2dist
In [5]: print("Following is the Breadth-First Search order")
        print(bfs(graph, 's', debug=True)) # function calling
        Following is the Breadth-First Search order
        0) Q = ['s']; dists = [0]
        1) Q = ['w', 'r']; dists = [1, 1]
        2) Q = ['r', 't', 'x']; dists = [1, 2, 2]
        3) Q = ['t', 'x', 'v']; dists = [2, 2, 2]
        4) Q = ['x', 'v', 'u']; dists = [2, 2, 3]
5) Q = ['v', 'u', 'y']; dists = [2, 3, 3]
        6) Q = ['u', 'y']; dists = [3, 3]
        7) Q = ['y']; dists = [3]
        8) Q = []
        (['s', 'w', 'r', 't', 'x', 'v', 'u', 'y'], {'s': 0, 'w': 1, 'r': 1, 't': 2,
        'x': 2, 'v': 2, 'u': 3, 'y': 3})
        Depth first search
        image.png
```

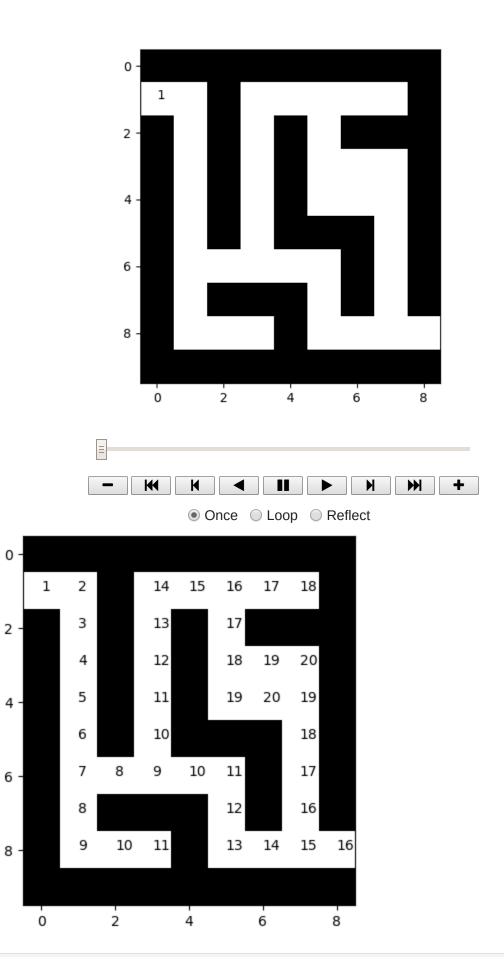
**₽**bfs-states

```
In [6]: graph = {
         's' : ['w', 'r'],
          'r' : ['v'],
          'w' : ['t', 'x'],
          'x' : ['y' ],
          't' : ['u'],
          'u' : ['y']
        def dfs(graph, start, debug=False):
            seen = set([start]) # List for seen nodes (contains both frontier and de
            # Frontier is the boundary between seen and unseen (Also called the aliv
            frontier = LifoQueue() # Frontier of unvisited nodes as FIF0
            node2dist = {start : 0} # Keep track of distances
            search order = [] # Keep track of search order
            frontier.put(start)
            i = 0 \# step number
            while not frontier.empty(): # Creating loop to visit each node
                if debug: print("%d) Q = " % i, list(frontier.queue), end='; ')
                if debug: print("dists = " , [node2dist[n] for n in frontier.queue])
                m = frontier.get() # Get the oldest addition to frontier
                search order.append(m)
                for neighbor in graph.get(m, []):
                    if neighbor not in seen:
                        seen.add(neighbor)
                        frontier.put(neighbor)
                        node2dist[neighbor] = node2dist[m] + 1
                    else:
                        node2dist[neighbor] = min(node2dist[neighbor], node2dist[m]
            if debug: print("%d) Q = " % i, list(frontier.queue))
            return search order, node2dist
In [7]: # Driver Code
        print("Following is the Depth-First Search path")
        print(dfs(graph, 's', debug=True)) # function calling
        Following is the Depth-First Search path
        0) Q = ['s']; dists = [0]
        1) Q = ['w', 'r']; dists = [1, 1]
        2) Q = ['w', 'v']; dists = [1, 2]
        3) Q = ['w']; dists = [1]
        4) Q = ['t', 'x']; dists = [2, 2]
        5) Q = ['t', 'y']; dists = [2, 3]
        6) Q = ['t']; dists = [2]
        7) Q = ['u']; dists = [3]
        8) Q = []
        (['s', 'r', 'v', 'w', 'x', 'y', 't', 'u'], {'s': 0, 'w': 1, 'r': 1, 'v': 2,
        't': 2, 'x': 2, 'y': 3, 'u': 3})
```

# Converting a maze search to a graph search

```
In [8]: # Skip these utilities for the class
        def batched(iterable, n):
            "Batch data into tuples of length n. The last batch may be shorter."
            # batched('ABCDEFG', 3) --> ABC DEF G
            if n < 1:
                raise ValueError('n must be at least one')
            it = iter(iterable)
            while batch := tuple(islice(it, n)):
                yield batch
        def draw path(self, path, visited='*'):
            new maze lines = [list(l) for l in self.maze_lines]
            for (r, c) in path:
                new maze lines[r][c] = visited
                print('\n'.join([''.join(l) for l in new maze lines]))
                print('\n\n\n')
        def init plots(self, reinit=False):
            if self.fig is None or reinit:
                self.fig, self.ax = plt.subplots()
        def plot maze(self):
            self.init plots()
            replace = { ' ' : 1, '+': 0}
            maze mat = np.array([[replace[c] for c in line]
                                  for line in self.maze lines])
            return [self.ax.imshow(maze mat, cmap='gray')]
        def plot step(self, i node):
            i, (r, c) = i \text{ node}
            return [self.ax.text(c, r, '%d' % (i+1))]
        def plot path(self, path):
            self.plot maze()
            return [self.plot step((i, (r,c)))
                    for i, (r, c) in enumerate(path)]
        def animate search path(maze, search path, node2dist):
            maze.init plots()
            return animation.FuncAnimation(maze.fig, maze.plot step, frames=[(node2c
                                                                               for n
                                           init func=maze.plot maze, blit=True, repea
In [9]: import matplotlib.pyplot as plt
        import numpy as np
        maze str = \
        ++++++++
         +
        + + + +++
        + + + +
        ++++
```

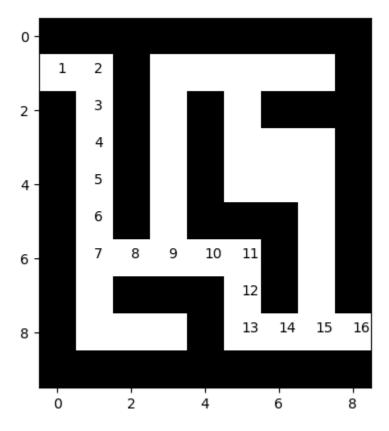
```
+ + +++ +
         + + +
         + +++ + +
         + +
         ++++++++
         class Maze:
             def init (self, maze str, freepath=' '):
                 self.maze lines = [l for l in maze str.split("\n")
                                     if len(l)]
                 self.FREEPATH = freepath
                 self.fig = None
             def get(self, node, default):
                 (r, c) = node
                 m row = self.maze lines[r]
                 nbrs = []
                 if c-1 >= 0 and m row[c-1] == self.FREEPATH:
                     nbrs.append((r, c-1))
                 if c+1 < len(m row) and m row[c+1] == self.FREEPATH:</pre>
                     nbrs.append((r, c+1))
                 if r-1 \ge 0 and self.maze lines[r-1][c] == self.FREEPATH:
                     nbrs.append((r-1, c))
                 if r+1 < len(self.maze lines) and self.maze lines[r+1][c] == self.FF</pre>
                     nbrs.append((r+1, c))
                 return nbrs if len(nbrs) else default
             init plots = init plots
             plot maze = plot maze
             plot step = plot step
             plot path = plot path
             animate search path = animate_search_path
In [10]: import matplotlib.pyplot as plt
         import matplotlib.animation as animation
         import matplotlib as mpl
         %matplotlib inline
         mpl.rc('animation', html='jshtml')
         maze = Maze(maze str)
         search path, node2dist = bfs(maze, (1, 0)) # prints the order of search all
         maze.plot maze()
         maze.animate search path(search path, node2dist)
```



```
In [11]: def bfs path(graph, start, goal):
             Returns success and node2parent
             success: True if goal is found otherwise False
             node2parent: A dictionary that contains the nearest parent for node
             seen = [start] # List for seen nodes.
             # Frontier is the boundary between seen and unseen
             frontier = Queue() # Frontier of unvisited nodes as FIFO
             node2parent = dict() # Keep track of nearest parent for each node (requi
             frontier.put(start)
             while not frontier.empty():
                                                  # Creating loop to visit each node
                 m = frontier.get() # Get the oldest addition to frontier
                 if m == goal:
                     return True, node2parent
                 for neighbor in graph.get(m, []):
                     if neighbor not in seen:
                         seen.append(neighbor)
                         frontier.put(neighbor)
                         node2parent[neighbor] = m
             return False, []
In [12]: def backtrace path(node2parent, start, goal):
             c = goal
             r path = [c]
             parent = node2parent.get(c, None)
             while parent != start:
                 r path.append(parent)
                 c = parent
                 parent = node2parent.get(c, None) # Keep getting the parent until yd
                 #print(parent)
             r path.append(start)
             return reversed(r path) # Reverses the path
         maze = Maze(maze str)
         start = (1, 0)
         goal = (8, 8)
         success, node2parent = bfs path(maze, (1, 0), (8, 8))
         path = backtrace path(node2parent, (1, 0), (8, 8))
         #print(list(path))
```

maze.plot path(path) # Draws all the searched nodes

plt.show()
#node2parent



### Dijkstra algorithm

ijkstra-step-by-step

# **PriorityQueue**

PriorityQueue returns the smallest (or the largest) item in the queue faster than other data structures

```
In [13]: #from queue import PriorityQueue
from hw2_solution import PriorityQueueUpdatable
from dataclasses import dataclass, field
from typing import Any

# https://docs.python.org/3/library/queue.html#queue.PriorityQueue
@dataclass(order=True)
class PItem:
    dist: int
    node: Any=field(compare=False)

# Make the PItem hashable
# https://docs.python.org/3/glossary.html#term-hashable
def __hash__(self):
    return hash(self.node)

graph = {
    's': [('x', 5), ('u', 10)],
```

```
'u' : [('v', 1), ('x', 2)],
    'x' : [('u', 3), ('v', 9), ('y', 2)],
    'y' : [('v', 6), ('s', 7)],
    'v' : [('y', 4)]
def dijkstra(graph, start, goal, debug=False):
   edgecost: cost of traversing each edge
   Returns success and node2parent
   success: True if goal is found otherwise False
   node2parent: A dictionary that contains the nearest parent for node
   seen = set([start]) # Set for seen nodes.
   # Frontier is the boundary between seen and unseen
   frontier = PriorityQueueUpdatable() # Frontier of unvisited nodes as a F
   node2parent = {start : None} # Keep track of nearest parent for each nod
   node2dist = {start: 0} # Keep track of cost to arrive at each node
   search order = []
    frontier.put(PItem(0, start))
   while not frontier.empty():
                                        # Creating loop to visit each node
        dist m = frontier.get() # Get the smallest addition to the frontier
        if debug: print("%d) Q = " % i, list(frontier.queue), end='; ')
        if debug: print("dists = " , [node2dist[n.node] for n in frontier.qu
       m = dist m.node
       m dist = node2dist[m]
        search order.append(m)
        if goal is not None and m == goal:
            return True, search order, node2parent, node2dist
        for neighbor, edge cost in graph.get(m, []):
            old dist = node2dist.get(neighbor, float("inf"))
            new dist = edge_cost + m_dist
            if neighbor not in seen:
                seen.add(neighbor)
                frontier.put(PItem(new dist, neighbor))
                node2parent[neighbor] = m
                node2dist[neighbor] = new dist
            elif new dist < old dist:</pre>
                node2parent[neighbor] = m
                node2dist[neighbor] = new dist
                # ideally you would update the dist of this item in the pric
                # as well. But python priority queue does not support fast u
                old item = PItem(old dist, neighbor)
                if old item in frontier:
                    frontier.replace(old item, PItem(new dist, neighbor))
        i += 1
   if goal is not None:
        return False, [], {}, node2dist
   else:
        return True, search order, node2parent, node2dist
```

```
In [14]: success, search path, node2parent, node2dist = dijkstra(graph, 's', None, de
                        print(success, node2parent, node2dist)
                       0) Q = []; dists = []
                        1) Q = [PItem(dist=10, node='u')]; dists = [10]
                       2) Q = [PItem(dist=8, node='u'), PItem(dist=14, node='v')]; dists = [8, 1]
                       4]
                       3) Q = [PItem(dist=13, node='v')]; dists = [13]
                       4) Q = []; dists = []
                       True {'s': None, 'x': 's', 'u': 'x', 'v': 'u', 'y': 'x'} {'s': 0, 'x': 5,
                        'u': 8, 'v': 9, 'y': 7}
In [15]: import itertools
                        class MazeD(Maze):
                                 def get(self, node, default):
                                           nbrs = Maze.get(self, node, default)
                                            return zip(nbrs, itertools.repeat(1))
                       maze = MazeD(maze str)
                        success, search path, node2parent, node2dist = dijkstra(maze, (1, 0), (8, 8)
                        print(success, node2parent)
                        if success:
                                 path = backtrace path(node2parent, (1, 0), (8, 8))
                                 maze.plot path(path) # Draws all the searched nodes
                       True \{(1, 0): None, (1, 1): (1, 0), (2, 1): (1, 1), (3, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (2, 1), (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): (4, 1): 
                        (3, 1), (5, 1): (4, 1), (6, 1): (5, 1), (6, 2): (6, 1), (7, 1): (6, 1), (6, 1)
                       3): (6, 2), (8, 1): (7, 1), (6, 4): (6, 3), (5, 3): (6, 3), (8, 2): (8, 1),
                        (6, 5): (6, 4), (4, 3): (5, 3), (8, 3): (8, 2), (7, 5): (6, 5), (3, 3): (4, 5)
                       3), (8, 5): (7, 5), (2, 3): (3, 3), (8, 6): (8, 5), (1, 3): (2, 3), (8, 7):
                        (8, 6), (1, 4): (1, 3), (8, 8): (8, 7), (7, 7): (8, 7), (1, 5): (1, 4)
                         0 -
                                                 2
                                     1
                                                 3
                         2 -
                                                 5
                         4 -
                                                 6
                                                                                                11
                                                 7
                                                                         9
                                                                                     10
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                                                                                                12
                                                                                                13
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                                                                                                                        15
                                                                                                                                    16
                         8 -
                                    0
                                                            2
                                                                                                           6
                                                                                                                                  8
```

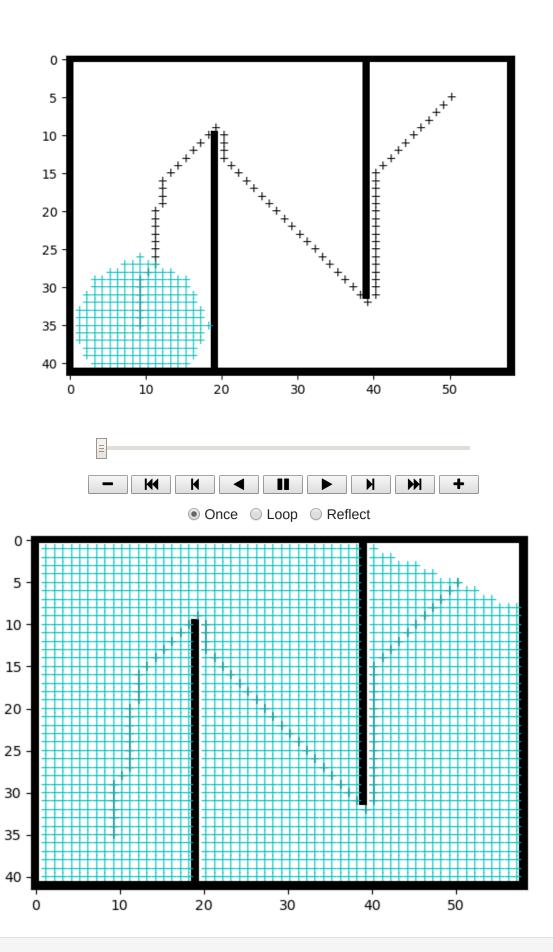
```
In [16]: maze_str = \
   start_pos, goal_pos = (35, 9), (5, 50)
```

```
In [17]: import math
from itertools import islice

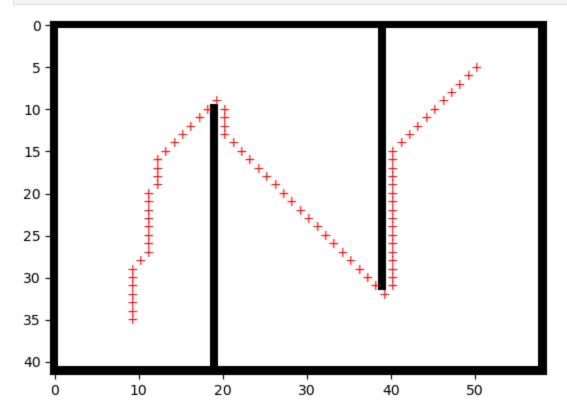
class Maze8(MazeD):
    def get(self, node, default):
        (r, c) = node
        rmax = len(self.maze_lines)
        cmax = len(self.maze_lines[0])
```

```
m row = self.maze lines[r]
    possible nbrs = [
        ((r, c-1), 1),
        ((r, c+1), 1),
        ((r-1, c), 1),
        ((r+1, c), 1),
        ((r-1, c-1), math.sqrt(2)),
        ((r-1, c+1), math.sqrt(2)),
        ((r+1, c-1), math.sqrt(2)),
        ((r+1, c+1), math.sqrt(2))
    free nbrs = []
    for (ri, ci), dist in possible nbrs:
        if (ri >= 0 and ci >= 0 and ri < rmax and ci < cmax</pre>
               and self.maze lines[ri][ci] == self.FREEPATH):
            free nbrs.append(((ri, ci), dist))
    return free nbrs if len(free nbrs) else default
def plot path(self, path, char='+', color='c'):
    return [self.ax.text(c-0.5, r+0.5, char, color=color)
           for (r, c) in path]
def plot path(self, path, **kw):
    self.plot maze()
    return self. plot path(path, **kw)
def animate(self, path, batch size=200):
    self.init plots()
    anim = animation.FuncAnimation(self.fig, self. plot path,
                                   frames=batched(search path, batch siz
                                   init func=self.plot maze, blit=True, r
    return anim
```

```
In [18]: maze = Maze8(maze_str)
    success, search_path, node2parent, node2dist = dijkstra(maze, start_pos, goa
    #print(success, search_path)
    assert success
    anim = maze.animate(search_path)
    path = backtrace_path(node2parent, start_pos, goal_pos)
    #maze.init_plots(reinit=True)
    path_plot = maze.plot_path(path, color='k') # Draws the traced shortest path
    anim
```

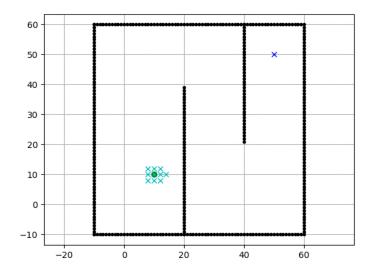


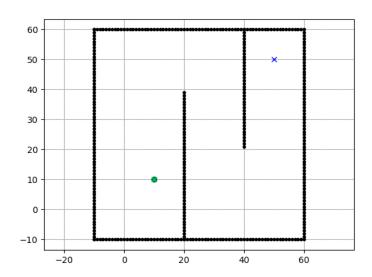
```
In [19]: path = backtrace_path(node2parent, (35, 9), (5, 50))
    maze.init_plots(reinit=True)
    maze.plot_path(path, color='r') # Draws the traced shortest path
    plt.show()
```



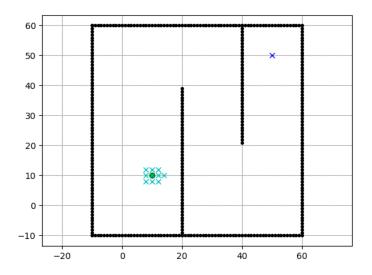
Search order in BFS vs DFS vs Dijkstra

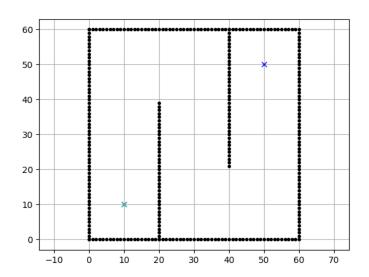
Breadth first search vs Depth first search





Breadth first search vs Dijkstra





# Computational complexity of BFS

```
In [20]: # Write down the computational complexity of each line in big-O notation O()
# Assume the graph has |V| nodes and |E| edges
def bfs_barebones(graph, start):
    seen = {start} # Set for seen nodes (contains both frontier and dead sta
    # Frontier is the boundary between seen and unseen (Also called the aliv
    frontier = Queue() # Frontier of unvisited nodes as FIFO # O(1)
    frontier.put(start) # O(1)

while not frontier.empty(): # Creating loop to visit each node # O(|V|)
    m = frontier.get() # Get the oldest addition to frontier # O(|V| * 1

for neighbor in graph.get(m, []): # O(|V| * |E|/|V|) = O(|E|)
    if neighbor not in seen: # O(|E| * 1)
        seen.add(neighbor) # O(|E| * 1)
        frontier.put(neighbor) # O(|E| * 1)
```

```
# The computational complexity of BFS is O(|E|). Some books write it as O(|V|) # where O(|V|) is the cost of initializing states of different nodes
```

# Computational complexity of Dijkstra

```
In [21]: # Write down the computational complexity of each line in big-0 notation O()
         # Assume the graph has |V| nodes and |E| edges
         def dijkstra barebones(graph, start):
             seen = {start} # Set for seen nodes (contains both frontier and dead sta
             # Frontier is the boundary between seen and unseen (Also called the aliv
             frontier = PriorityQueue() # Frontier of unvisited nodes as PriorityQueu
             frontier.put(PItem(0, start)) # 0(1)
             node2dist = {start: 0} # Keep track of cost to arrive at each node # 0(1
             while not frontier.empty(): # Creating loop to visit each node
                 dist and node = frontier.qet() # Get the smallest dist node # O(|V|)
                 m dist = dist and node.dist
                 m = dist and node.node
                 for neighbor, edge dist in graph.get(m, []): # O(|V| * |E|/|V|) = O(|V| * |E|/|V|)
                     if neighbor not in seen: # O(|E| * 1)
                          seen.add(neighbor) # O(|E| * 1)
                          frontier.put(neighbor) # # O(|E| * log(1)) # for fibonacci h
                          node2dist[neighbor] = m_dist + edge_dist # 0(1)
                     elif node2dist[neighbor] > m dist + edge dist: # 0(1)
                          node2dist[neighbor] = m dist + edge dist # 0(1)
         # The computational complexity of Dijkstra is O(|V|\log(|V|) + |E|) when impl
         # using a Fibonacci heap based PriorityQueue
```

# PriorityQueue (Heaps Chapter 7 of Carmen's intro to algorithms)



### Heap property

- 1.  $H[Parent(i)] \ge H[i]$
- 2. Parent(i) = ceil(i/2)
- 3. LeftChild(i) = 2i
- 4. RightRight(i) = 2i+1

### Heapify



# Heapify pseudocode

heapify-psuedocode

#### Heap Insert



### Heap runtimes

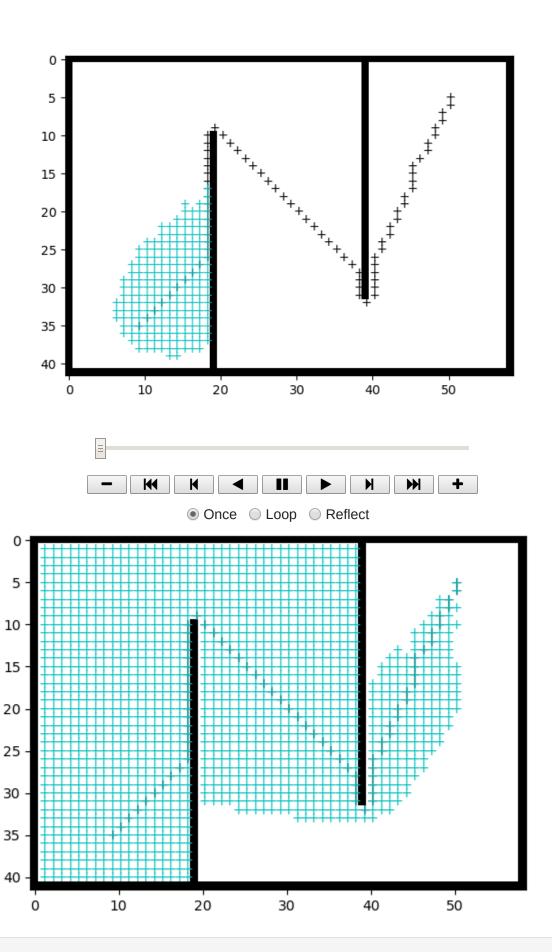
heap-runtimes

# A-star (A\*) algorithm

```
In [22]: from hw2 solution import PriorityQueueUpdatable
         import sys
         def astar(graph, heuristic dist fn, start, goal, debug=False, debugf=sys.stc
             edgecost: cost of traversing each edge
             Returns success and node2parent
             success: True if goal is found otherwise False
             node2parent: A dictionary that contains the nearest parent for node
             seen = set([start]) # Set for seen nodes.
             # Frontier is the boundary between seen and unseen
             frontier = PriorityQueueUpdatable() # Frontier of unvisited nodes as a F
             node2parent = {start : None} # Keep track of nearest parent for each nod
             hfn = heuristic dist fn # make the name shorter
             node2dist = {start: 0 } # Keep track of cost to arrive at each node
             search order = []
             frontier.put(PItem(0 + hfn(start, goal), start)) # <----- Diff</pre>
             if debug: debugf.write("goal = " + str(goal) + '\n')
             while not frontier.empty():
                                                 # Creating loop to visit each node
                 dist m = frontier.get() # Get the smallest addition to the frontier
                 if debug: debugf.write("%d) Q = " % i + str(list(frontier.queue)) +
                 if debug: debugf.write("%d) node = " % i + str(dist m) + '\n')
                 #if debug: print("dists = " , [node2dist[n.node] for n in frontier.q
                 m = dist m.node
                 m dist = node2dist[m]
                 search order.append(m)
                 if goal is not None and m == goal:
                     return True, search order, node2parent, node2dist
                 for neighbor, edge cost in graph.get(m, []):
                     old dist = node2dist.get(neighbor, float("inf"))
                     new dist = edge cost + m dist
                     if neighbor not in seen:
                         seen.add(neighbor)
                         frontier.put(PItem(new dist + hfn(neighbor, goal), neighbor
                         node2parent[neighbor] = m
                         node2dist[neighbor] = new dist
                     elif new dist < old dist:</pre>
                         node2parent[neighbor] = m
                         node2dist[neighbor] = new dist
```

```
# ideally you would update the dist of this item in the pric
# as well. But python priority queue does not support fast u
# ------- Different from dijkstra ------
old_item = PItem(old_dist + hfn(neighbor, goal), neighbor)
if old_item in frontier:
    frontier.replace(
        old_item,
        PItem(new_dist + hfn(neighbor, goal), neighbor))
i += 1
if goal is not None:
    return False, [], {}, node2dist
else:
    return True, search_order, node2parent, node2dist
```

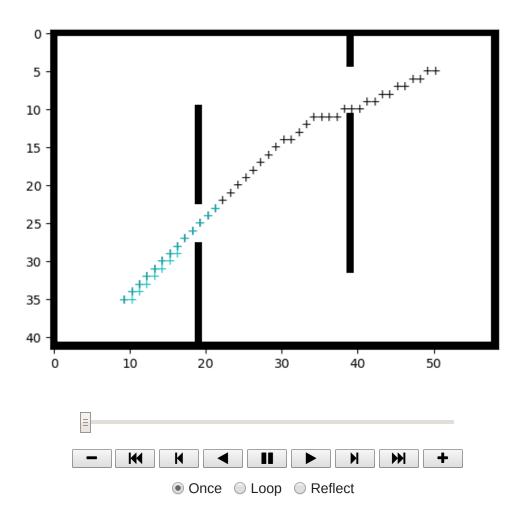
```
In [23]: import math
         from functools import partial
         def euclidean heurist dist(node, goal, scale=1):
             x n, y n = node
             x g, y g = goal
             return scale*math.sqrt((x_n-x_g)**2 + (y_n - y_g)**2)
         maze = Maze8(maze str)
         debugf=open('log.txt', 'w')
         success, search path, node2parent, node2dist = astar(
             maze, partial(euclidean heurist dist, scale=1),
             start_pos, goal_pos, debug=True, debugf=debugf)
         debugf.close()
         #print(success, search path)
         assert success
         anim = maze.animate(search path)
         path = backtrace path(node2parent, start pos, goal pos)
         #maze.init plots(reinit=True)
         path plot = maze.plot path(path, color='k') # Draws the traced shortest path
         anim
```

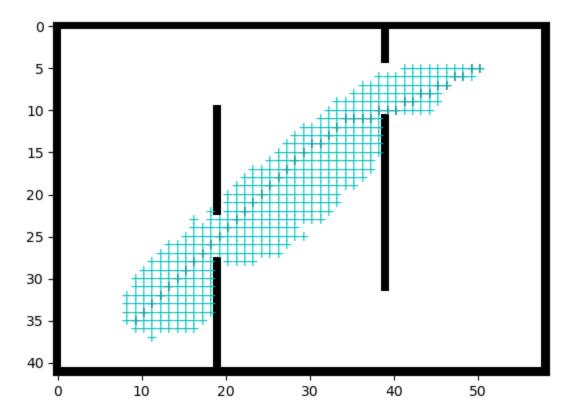


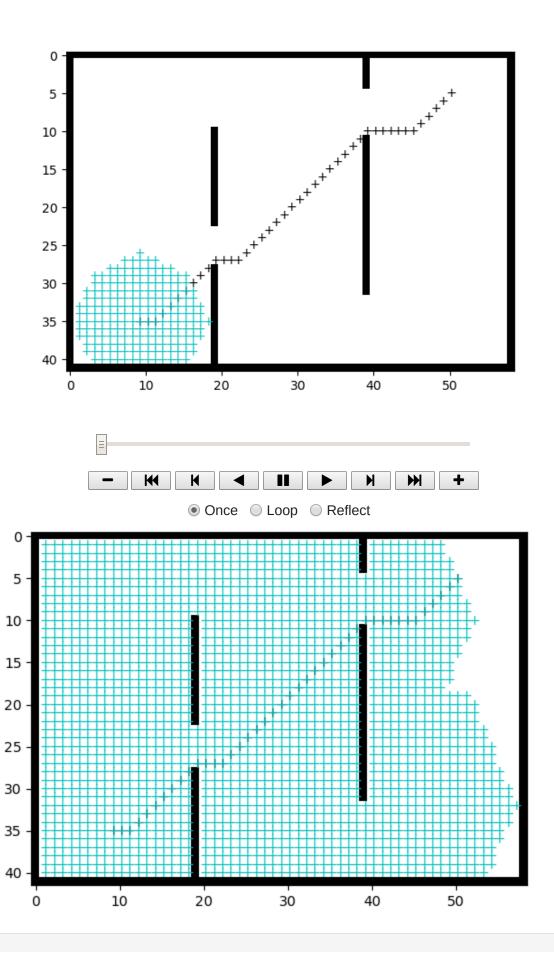
```
In [30]: maze_str = \
   +
                  +
                          +
   +
                  +
                          +
   start_pos, goal_pos = (35, 9), (5, 50)
```

```
#maze.init_plots(reinit=True)
path_plot = maze.plot_path(path, color='k') # Draws the traced shortest path
anim
```

### Out[31]:



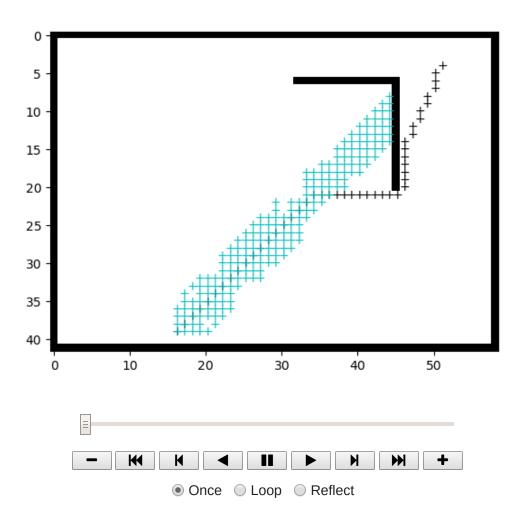


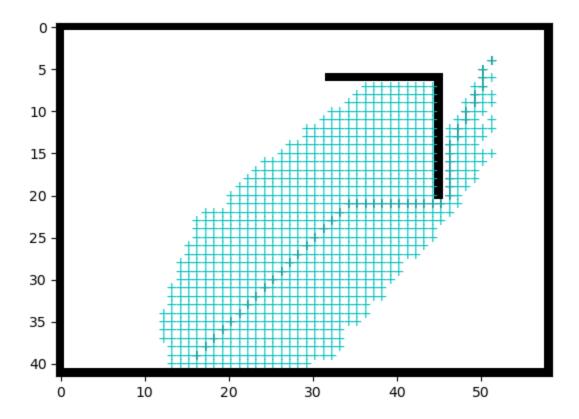


```
In [33]: maze_str = \
    0.000
    +
    +
                              +
                  +++++++++++++
    goal pos = (9-5, 46+5)
    start pos = (9+30, 46-30)
```

```
#maze.init_plots(reinit=True)
path_plot = maze.plot_path(path, color='k') # Draws the traced shortest path
anim
```

### Out[34]:





In [ ]: