ECE 417/598: Review Homework 4

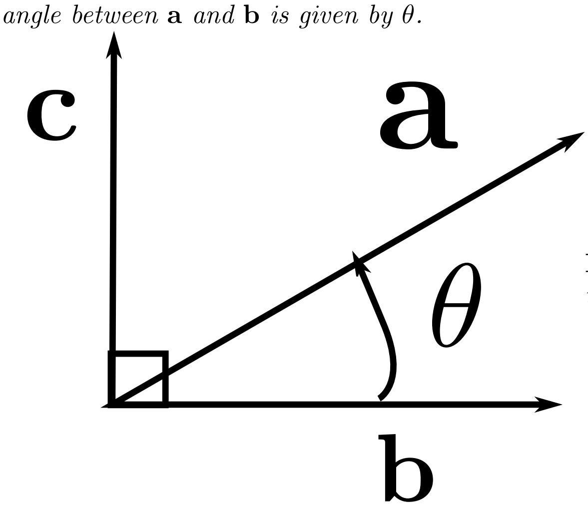
Max marks: 30 marks (more problems coming)

Due on March 9th, 2021, midnight, 11:59 PM.

All notes so far are linked here.

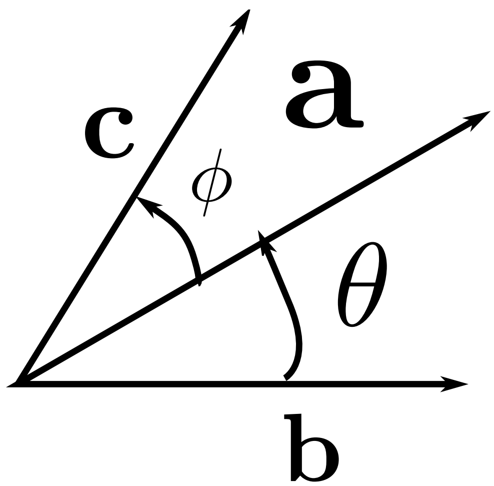
1 Trignometry and triangle laws of vector addition

Problem 1 *The magnitude of vector* $\mathbf{a} \in \mathbb{R}^n$ *is given to be* $\|\mathbf{a}\| = \alpha$. *Using the following figure, write* $\mathbf{a}$ *in terms of* α, θ, *vector* $\mathbf{b} \in \mathbb{R}^n$ *and* $\mathbf{c} \in \mathbb{R}^n$. *All three vectors lie in the same plane.* $\mathbf{b}$ *and* $\mathbf{c}$ *are perpendicular to each other. The angle between* $\mathbf{a}$ *and* $\mathbf{b}$ *is given by* θ.



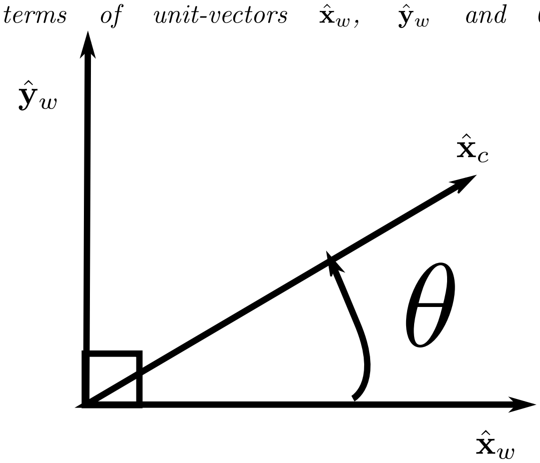
(5 min, 5 marks)

Problem 2 *The magnitude of vector* $\mathbf{a} \in \mathbb{R}^n$ *is given to be* $\|\mathbf{a}\| = \alpha$. *Using the following figure, write* $\mathbf{a}$ *in terms of* α, θ, ϕ *vector* $\mathbf{b} \in \mathbb{R}^n$ *and* $\mathbf{c} \in \mathbb{R}^n$. *All three vectors lie in the same plane. The angle between* $\mathbf{a}$ *and* $\mathbf{b}$ *is given by* θ. *The angle between* $\mathbf{a}$ *and* $\mathbf{c}$ *is given by* ϕ. *Assume* $\theta+\phi \neq 0$. *When* $\theta+\phi = \frac{\pi}{/}2$, *is the solution is same as Problem 1? (Hint: You can convert this to Problem 1, by drawing a unit-vector perpendicular to* $\mathbf{b}$. *Call it* $\hat{\mathbf{d}}$. *First write* $\hat{\mathbf{d}}$ *in terms of* $\mathbf{c}$ *and others knowns and then write* $\mathbf{a}$ *in terms of* $\hat{\mathbf{d}}$ *and other knowns. You might want to use trignometric identities. The simplest form is not required.).*



(10 min, 10 marks)

Problem 3 *Find unit-vector* $\hat{\mathbf{x}}_c$ *in terms of unit-vectors* $\hat{\mathbf{x}}_w$, $\hat{\mathbf{y}}_w$ *and* θ.



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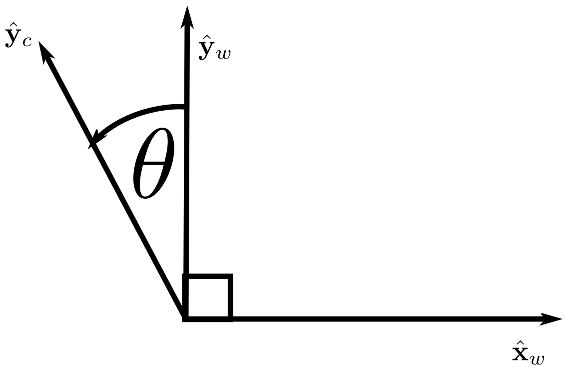
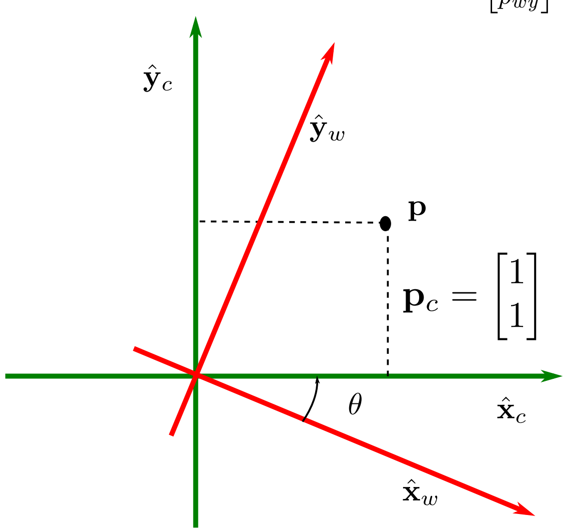
Problem 4 *Find unit-vector* $\hat{\mathbf{y}}_c$ *in terms of unit-vectors* $\hat{\mathbf{x}}_w$, $\hat{\mathbf{y}}_w$ *and* θ.



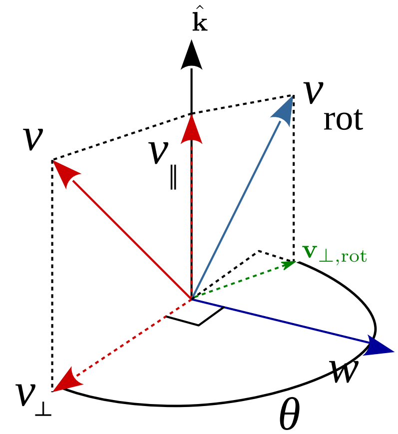
(5 min, 5 marks)

Problem 5 *Let the coordinates of a vector* $\mathbf{p}$ *in terms of* $\hat{\mathbf{x}}_c$ *and* $\hat{\mathbf{y}}_c$ *be* $\mathbf{p}_c = \begin{bmatrix} p_{cx} \\ p_{cy} \end{bmatrix}$, *so that:* $\mathbf{p} = p_{cx}\hat{\mathbf{x}}_c + p_{cy}\hat{\mathbf{y}}_c$. *Using the results from Prob 3 and Prob 4, write* $\mathbf{p}$ *in terms of* $\hat{\mathbf{x}}_w$ *and* $\hat{\mathbf{y}}_w$. *Thus derive the formula for rotation matrix* $R(\theta)$ *that converts coordinates from* $\mathbf{p}_c$ *to* $\mathbf{p}_w = \begin{bmatrix} p_{wx} \\ p_{wy} \end{bmatrix}$.



(10 min, 10 marks)

Problem 6 *We know that* $\|\mathbf{v}_{\perp,rot}\| = \|\mathbf{v}_\perp\|$. *Write* $\mathbf{v}_{\perp,rot}$ *in terms of* $\mathbf{v}_\perp$, $\mathbf{w}$ *and* θ. $\mathbf{v}_\perp$ *and* $\mathbf{w}$ *are known to be orthogonal to each other.*



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