ECE 498/598: Homework 3

Max marks: 210

October 19, 2023

You can also use the following template to fill in your answers: transformations.py. There are many solutions to this on the internet. Please do not use them. Use only the math taught in the class to finish this assignment.

Problem 1 *Write a program in Python that checks if a given 3x3 matrix is a valid Rotation matrix is a valid Rotation matrix (check for orthogonality and determinant = 1). You may use numpy's matrix multiplication and determinant() function. (10 marks. Used in the following problems. Estimated time: 15 min).*

Problem 2 *In class, we proved the expression to convert roll (θ), pitch (ϕ), yaw (ψ) from Euler Angles to Rotation matrix,*

$$R(\theta,\phi,\psi) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = R_z(\psi)R_y(\phi)R_x(\theta) \tag{1}$$

What if we want to do the inverse? Prove that given a proper 3x3 rotation matrix ($R^\top R = I$ and $det(R) = 1$) , the Euler angles are given by

$$\begin{bmatrix} \theta(R) \\ \phi(R) \\ \psi(R) \end{bmatrix} = \begin{bmatrix} \arctan2(r_{32}, r_{33}) \\ -\arcsin(r_{31}) \\ \arctan2(r_{21}, r_{11}) \end{bmatrix} \tag{2}$$

where r_{ij} is the element in ith row and jth column of the rotation matrix R. (10 marks. Used in the following problems. Estimated time: 15 min).

Problem 3 *Write a pair of functions in Python that converts rotation matrix from XYZ Euler angles (roll, pitch, yaw) and vice versa. Test the pair of functions with randomly generated Euler angles. And check if the converted rotation matrix is orthonormal. What happens when pitch = $\pi/2$, are you able to convert from rotation matrix to Euler angle? Why or why not? (50 marks. Estimated time: 30 min)*

Problem 4 *Write a pair of functions in Python that converts rotation matrix from axis angle representation and vice versa. Test the pair of functions with randomly generated Euler angles. (50 marks. Estimated time: 30 min)*

Problem 5 *Write a pair of functions in Python that converts rotation matrix from quaternion representation and vice versa. (50 marks. Estimated time: 30 min)*

Problem 6 *Write a function in Python that generates a 4x4 transformation matrix given the 3x3 rotation matrix and translation. (20 marks. Estimated time: 15 min).*

Problem 7 *Recall the definition of Denavit-Hartenberg parameters from the video. Recall that transformation between two joints for the defined parameters d, θ, r, α is given by,*

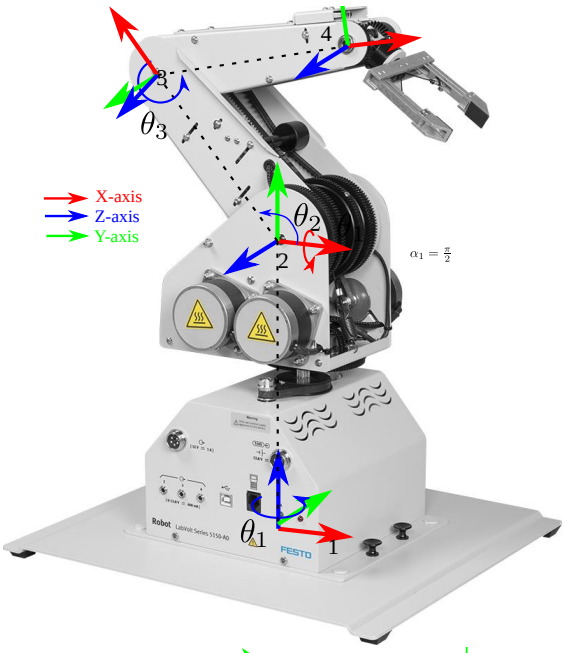
$$T = T_z(\theta, d)T_x(\alpha, r), \tag{3}$$

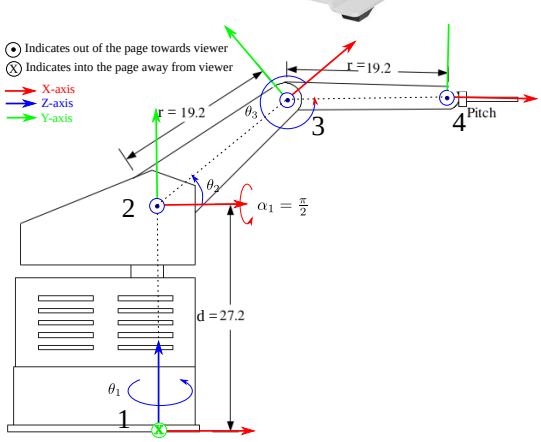
where

$$T_x(\alpha, r) = \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$T_z(\theta, d) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

For the robot given below find transformation matrix from joint 4 to joint 1 assuming the joint angles to be θ_1, θ_2, θ_3 respectively. Write the expression for ${}^3T_4(\theta_3)$, ${}^2T_3(\theta_2)$, ${}^1T_2(\theta_1)$ and then ${}^1T_4(\theta_1, \theta_2, \theta_3)$ in terms of the first three transformations. You do not need to expand the expression of ${}^1T_4(\theta_1, \theta_2, \theta_3)$.





(20 marks. 15 min)