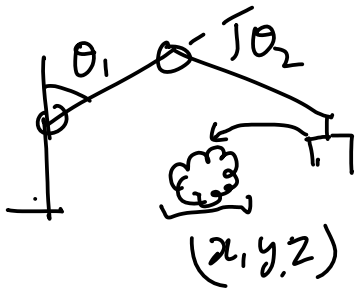
Forward and inverse kinematics

What should the joint ~~angles~~ state/conf. of the robot be so that the end-effector reaches a desired pose?



How to move the end-effector to a desired pose [position + orientation]

← end-effector gripper or suction cup

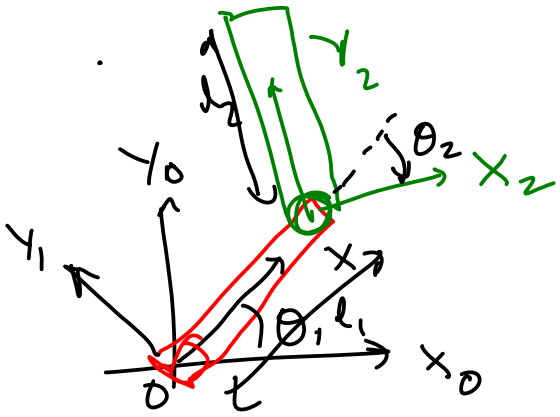
Forward kine.

If my joint ~~angles~~ state/conf. are given what would the pose of end-effector be?

Forward kinematics

$${}^0T_2 = {}^0T_1(\theta_1, l_1)\,{}^1T_2(\theta_2, l_2)$$

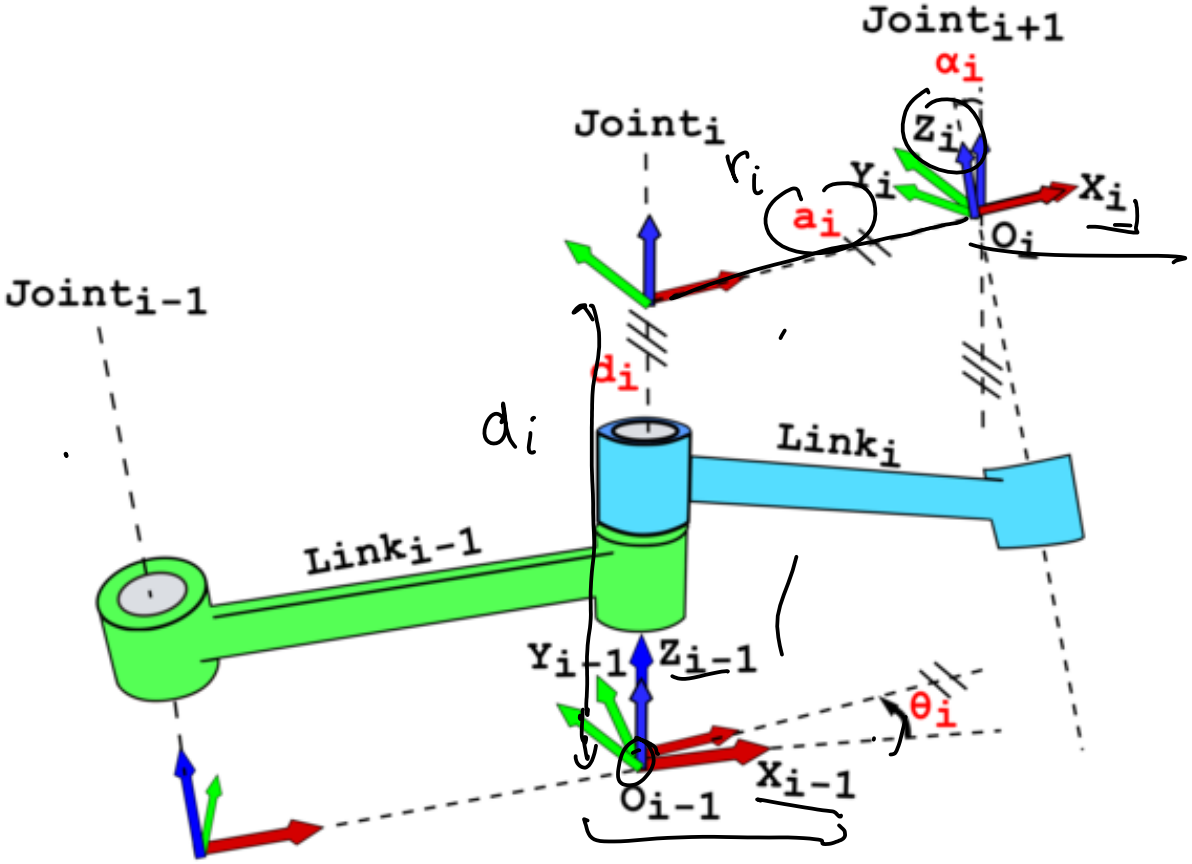
in terms of θ_1 and θ_2 (Given)



(Denavit Hartenberg Parameters / Convention)

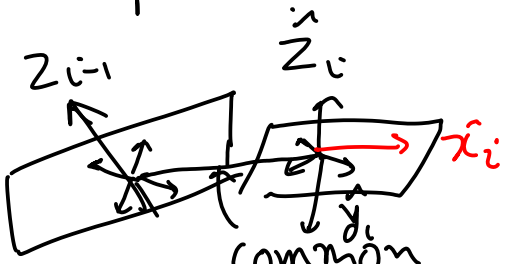
Denavit Hartenberg parameters

https://www.youtube.com/watch?v=rA9tm0gTln8



① $\hat{Z}_{i-1}$, $\hat{Z}_i$ aligned along the axis of rotation

② Choose $\hat{x}_i$ along the common normal between $\hat{Z}_{i-1}$, $\hat{Z}_i$



common normal between Z_{i-1} and Z_i

③ $\hat{y}_i = \hat{Z}_i \times \hat{x}_i$

a) θ_i = Rotation along Z_{i-1} (to align x_{i-1} with x_i)

b) d_i = translation along Z_{i-1} (to align the origins)

c) α_i = Rotation along x_i (to align Z_{i-1} with Z_i)

d) r_i/a_i = translation along x_i (to align the origin)

(a) and (b) can be swapped
(c) and (d)

But Tranformation along z goes first
followed by " " x

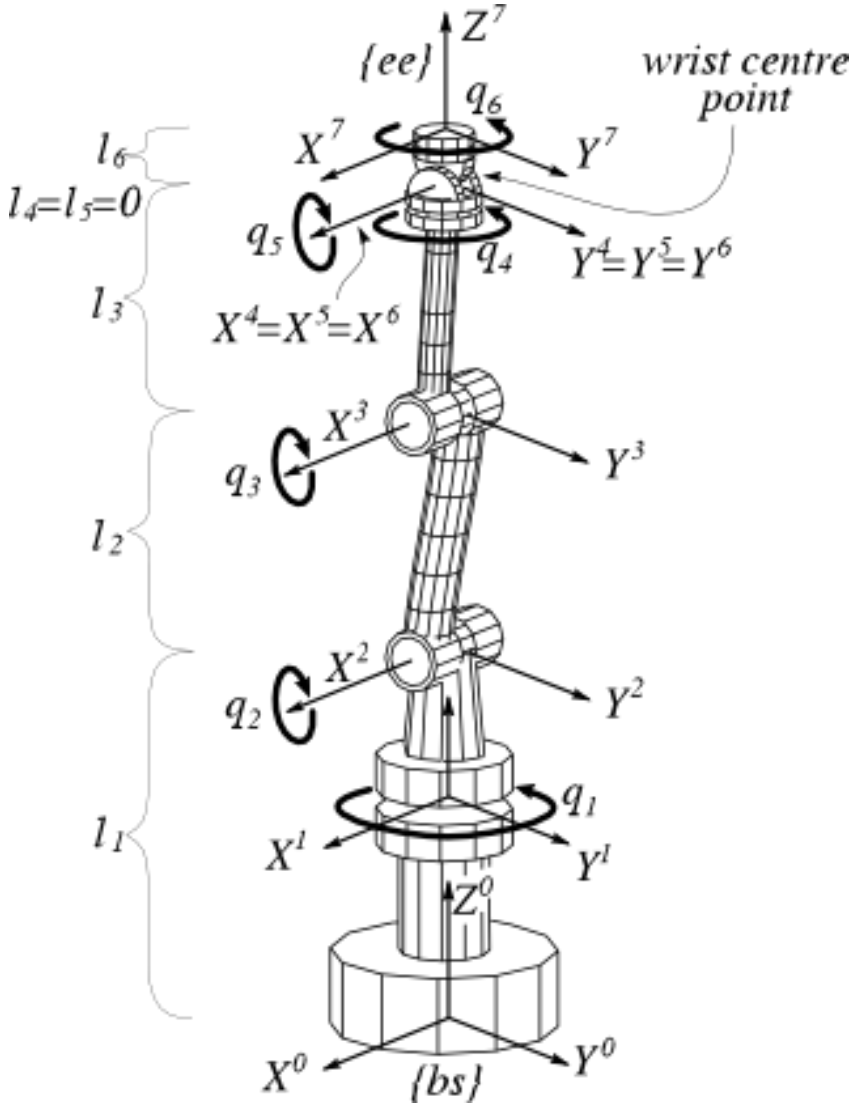
$$\underset{\substack{\uparrow \\ \text{target}}}{{}^{i-1}} T_{\underset{\substack{\uparrow \\ \text{source}}}{i}} = {}^{i-1}T_{zi} \; {}^{i-1}T_{xi}$$

Tranformations are applied right to left

$$ {}^{i-1}T_{xi} = \begin{bmatrix} 1 & 0 & 0 & r_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 1} \quad \begin{matrix} 3\times 3 & 3\times 1 \end{matrix} $$

$$ {}^{i-1}T_{zi} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 1} $$

θ_i, d_i


Z^7
{ee}
wrist centre point
q_6
l_6
X^7
Y^7
$l_4=l_5=0$
q_5
q_4
$Y^4=Y^5=Y^6$
l_3
$X^4=X^5=X^6$
X^3
Y^3
q_3
l_2
X^2
Y^2
q_2
q_1
l_1
X^1
Y^1
Z^0
X^0
Y^0
{bs}

Numerical solutions to IK problems: Jacobian inverse technique