ECE 417/598: Direct Linear Transform

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Implicit equation and parameteric representation of 3D plane

Implicit equation of 3D plane

$\mathbf{p}^\top \underline{\mathbf{x}} = 0 \qquad \mathbf{p} \in \mathbb{P}^4, \underline{\mathbf{x}} \in \mathbb{P}^4$

Parameteric representation of 3D plane

$$\underline{\mathbf{x}} = \mathbf{v}_2 + t_1\mathbf{v}_3 + t_2\mathbf{v}_4$$

where $t_1, t_2 \in \mathbb{R}$ are the free parameters.

Implicit equation and parameteric representation of a 3D line

Parameter representation of a 3D line

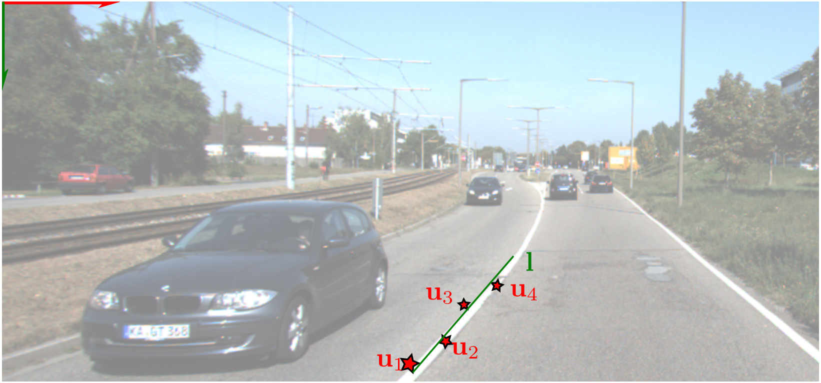
$$\underline{\mathbf{x}} = \lambda \underline{\mathbf{d}} + \underline{\mathbf{x}}_0,$$

where $\lambda \in \mathbb{R}$ is the free parameter, $\underline{\mathbf{x}}_0 \in \mathbb{P}^3$ is a point on the line and $\underline{\mathbf{d}} \in \mathbb{P}^3$ is the direction of the line.

Implicit equation of a 3D line

$$\mathbf{p}_1^\top \underline{\mathbf{x}} = 0, \quad \mathbf{p}_2^\top \underline{\mathbf{x}} = 0, \quad (1)$$

where $\mathbf{p}_1, \mathbf{p}_2, \underline{\mathbf{x}} \in \mathbb{P}^3$.

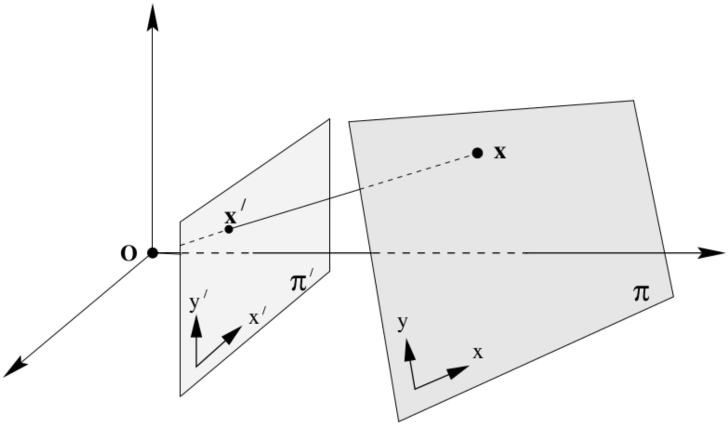


$$\underline{\mathbf{x}}_1 = [100, 98, 45, 1]^\top$$
$$\underline{\mathbf{x}}_2 = [105, 95, 46, 1]^\top$$
$$\underline{\mathbf{x}}_3 = [107, 90, 47, 1]^\top$$
$$\underline{\mathbf{x}}_4 = [110, 85, 43, 1]^\top$$

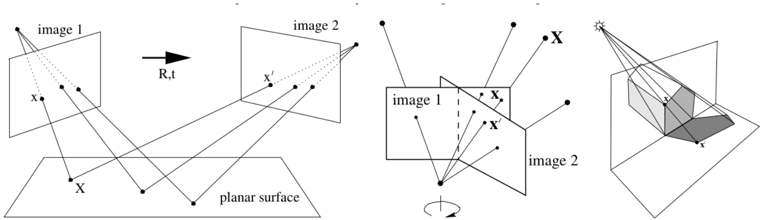
Find the 3D line such that it is the "closest line" passing through $\mathbf{x}_1, \ldots, \mathbf{x}_4 \in \mathbf{P}^3$.

Parameteric representation through Range space

Homography



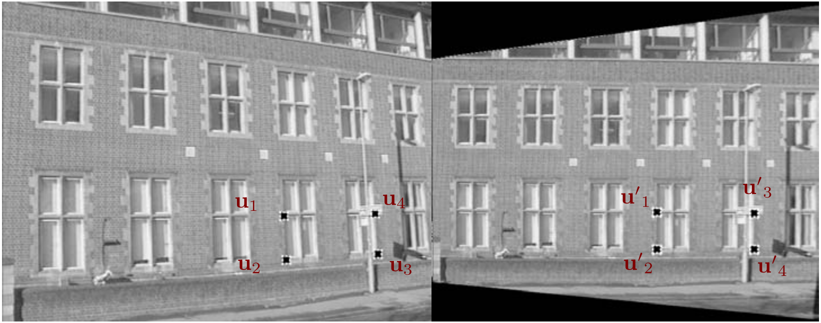
Examples of Homography



a b c


Top view
Graphic
Look! Safe to move?
Settings

Computing Homography



$$\underline{\mathbf{u}}_1 = [100, 98, 1]^\top \qquad \underline{\mathbf{u}}_2 = [102, 95, 1]^\top$$
$$\underline{\mathbf{u}}_3 = [107, 90, 1]^\top \qquad \underline{\mathbf{u}}_4 = [110, 85, 1]^\top$$
$$\underline{\mathbf{u}}'_1 = [100, 98, 1]^\top \qquad \underline{\mathbf{u}}'_2 = [102, 95, 1]^\top$$
$$\underline{\mathbf{u}}'_3 = [107, 98, 1]^\top \qquad \underline{\mathbf{u}}'_4 = [110, 85, 1]^\top$$

Find H such that $\underline{\mathbf{u}}' = H\underline{\mathbf{u}}$ for any point on one image to another image.

2D homography

Given a set of points $\underline{\mathbf{u}}_i \in \mathbb{P}^2$ and a corresponding set of points $\underline{\mathbf{u}}'_i \in \mathbb{P}^2$, compute the projective transformation that takes each $\underline{\mathbf{u}}_i$ to $\underline{\mathbf{u}}'_i$. In a practical situation, the points $\underline{\mathbf{u}}_i$ and $\underline{\mathbf{u}}'_i$ are points in two images (or the same image), each image being considered as a projective plane $\mathbb{P}^2$.

Solving for Homography

3D to 2D camera projection matrix estimation

Given a set of points $\mathbf{X}_i$ in 3D space, and a set of corresponding points $\mathbf{x}_i$ in an image, find the 3D to 2D projective $\mathbf{P}$ mapping that maps $\mathbf{X}_i$ to $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$.