

# ECE 417/598: Homework 1

Max marks: 100

Due on Jan 28, 2021, before class.

You are allowed to use any matrix or linear algebra library (Eigen or xtensor), but no library that implements rotation matrices. You are not allowed to use Eigen/Geometry. You can use the following code for generating random rotation matrices: `random_rotation.cpp`.

$[[r_{ij}]_{i=1}^3]_{j=1}^3$  ( $R^\top R = I$  and  $\det(R) = 1$ ), the Euler angles are given by

$$\begin{bmatrix} \theta(R) \\ \phi(R) \\ \psi(R) \end{bmatrix} = \begin{bmatrix} \arctan2(r_{23}, r_{33}) \\ -\arcsin(r_{13}) \\ \arctan2(r_{12}, r_{11}) \end{bmatrix} \quad (2)$$

where  $r_{ij}$  is the element in  $i$ th row and  $j$ th column of the rotation matrix  $R$ . (10 marks. Used in the following problems. Estimated time: 15 min).

## 1 Jan 24 Lecture: 3D transformations

**Problem 1** Degrees of Freedom of a quantity is the number independent scalar variables needed to represent that quantity. What is degrees of freedom required to

1. Position and orientation in 1-D
2. Position and orientation in 2-D
3. Position and orientation in 3-D
4. Position and orientation in 4-D

(10 marks. Estimated time: 15 min) Justify your answer.

**Problem 2** Write a program in C++ that checks if a given  $3 \times 3$  matrix is a valid Rotation matrix is a valid Rotation matrix (check for orthonormality i.e. orthogonality and determinant = 1). You may use Eigen's `matrix multiplication` and `determinant()` function. (10 marks. Used in the following problems. Estimated time: 15 min).

**Problem 3** In class, we proved the expression to convert roll ( $\theta$ ), pitch ( $\phi$ ), yaw ( $\psi$ ) from Euler Angles to Rotation matrix,

$$R(\theta, \phi, \psi) = R_z(\psi)R_y(\phi)R_x(\theta). \quad (1)$$

What if we want to do the inverse? Prove that given a proper  $3 \times 3$  rotation matrix  $R =$

**Problem 4** Write a pair of functions in C++ that converts rotation matrix from XYZ Euler angles (roll, pitch, yaw) and vice versa. Test the pair of functions with randomly generated Euler angles. And check if the converted rotation matrix is orthonormal. What happens when pitch =  $\pi/2$ , are you able to convert from rotation matrix to Euler angle? Why or why not? (50 marks. Estimated time: 30 min)

**Problem 5** Write a function in C++ that generates a  $4 \times 4$  transformation given axis-angle representation and translation. (20 marks. Estimated time: 15 min).