

ECE 417/598: Review Homework 4

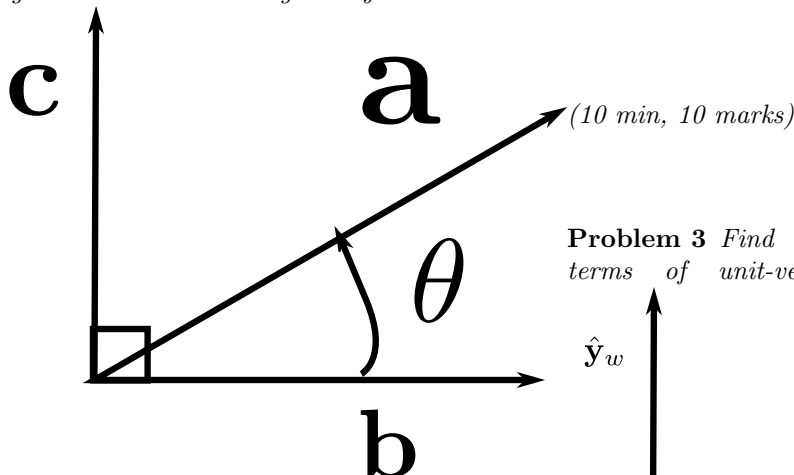
Max marks: 100 marks

Due on March 9th, 2021, midnight, 11:59 PM.

All notes so far are [linked here](#).

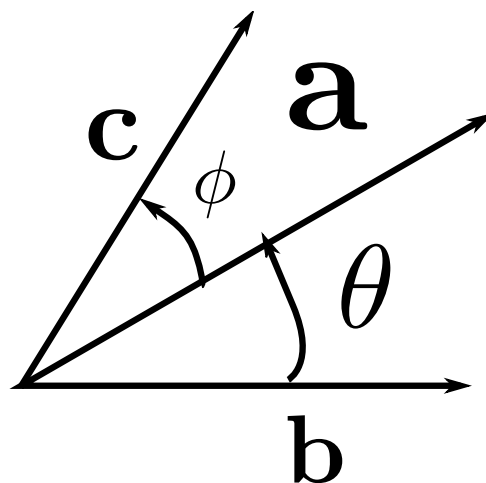
1 Trigonometry and triangle laws of vector addition

Problem 1 The magnitude of vector $\mathbf{a} \in \mathbb{R}^n$ is given to be $\|\mathbf{a}\| = \alpha$. Using the following figure, write \mathbf{a} in terms of α , θ , vector $\mathbf{b} \in \mathbb{R}^n$ and $\mathbf{c} \in \mathbb{R}^n$. All three vectors lie in the same plane. \mathbf{b} and \mathbf{c} are perpendicular to each other. The angle between \mathbf{a} and \mathbf{b} is given by θ .

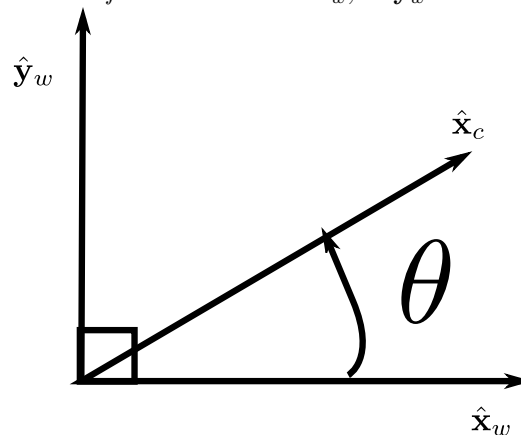


(5 min, 5 marks)

Problem 2 The magnitude of vector $\mathbf{a} \in \mathbb{R}^n$ is given to be $\|\mathbf{a}\| = \alpha$. Using the following figure, write \mathbf{a} in terms of α , θ , ϕ , vector $\mathbf{b} \in \mathbb{R}^n$ and $\mathbf{c} \in \mathbb{R}^n$. All three vectors lie in the same plane. The angle between \mathbf{a} and \mathbf{b} is given by θ . The angle between \mathbf{a} and \mathbf{c} is given by ϕ . Assume $\theta + \phi \neq 0$. When $\theta + \phi = \frac{\pi}{2}$, is the solution is same as Problem 1? (Hint: You can convert this to Problem 1, by drawing a unit-vector perpendicular to \mathbf{b} . Call it $\hat{\mathbf{d}}$. First write $\hat{\mathbf{d}}$ in terms of \mathbf{c} and others knowns and then write \mathbf{a} in terms of $\hat{\mathbf{d}}$ and other knowns. You might want to use [trigonometric identities](#). The simplest form is not required.).



Problem 3 Find unit-vector $\hat{\mathbf{x}}_c$ in terms of unit-vectors $\hat{\mathbf{x}}_w$, $\hat{\mathbf{y}}_w$ and θ .



(5 min, 5 marks)

Problem 4 Find unit-vector $\hat{\mathbf{y}}_c$ in terms of unit-vectors $\hat{\mathbf{x}}_w$, $\hat{\mathbf{y}}_w$ and θ .

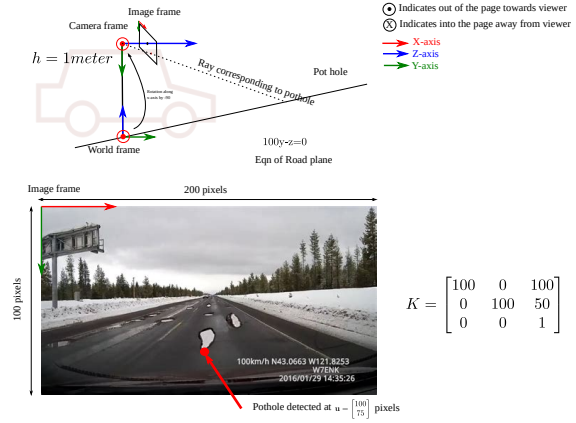
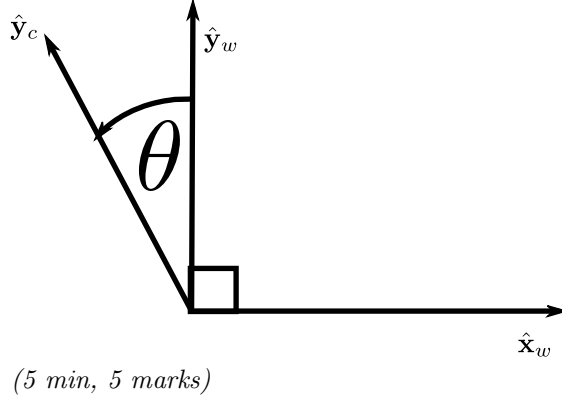
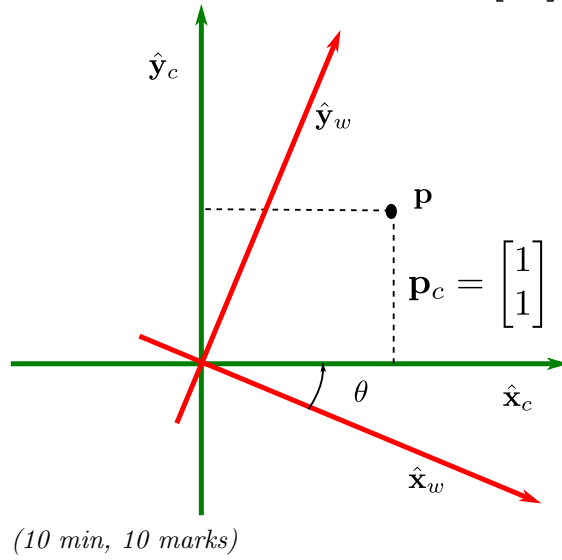
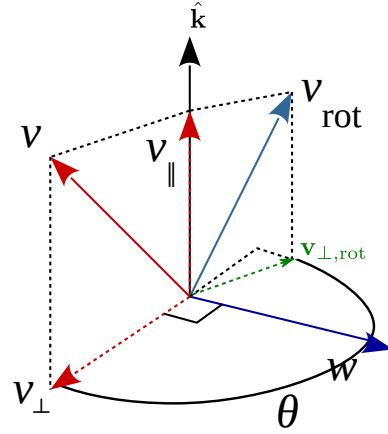


Figure 1: Point-plane triangulation

Problem 5 Let the coordinates of a vector \mathbf{p} in terms of $\hat{\mathbf{x}}_c$ and $\hat{\mathbf{y}}_c$ be $\mathbf{p}_c = \begin{bmatrix} p_{cx} \\ p_{cy} \end{bmatrix}$, so that: $\mathbf{p} = p_{cx}\hat{\mathbf{x}}_c + p_{cy}\hat{\mathbf{y}}_c$. Using the results from Prob 3 and Prob 4, write \mathbf{p} in terms of $\hat{\mathbf{x}}_w$ and $\hat{\mathbf{y}}_w$. Thus derive the formula for rotation matrix $R(\theta)$ that converts coordinates from \mathbf{p}_c to $\mathbf{p}_w = \begin{bmatrix} p_{wx} \\ p_{wy} \end{bmatrix}$.



Problem 6 We know that $\|\mathbf{v}_{\perp, \text{rot}}\| = \|\mathbf{v}_{\perp}\|$. Write $\mathbf{v}_{\perp, \text{rot}}$ in terms of \mathbf{v}_{\perp} , \mathbf{w} and θ . \mathbf{v}_{\perp} and \mathbf{w} are known to be orthogonal to each other.



(5 min, 5 marks)

Problem 7 In figure 1 find the 3D position of the pothole the World coordinate frame, in terms of $h = 1$ (the height of the camera), image-coordinates of the pothole \mathbf{u} (provided in figure), camera matrix K (provided in figure). The Camera is mounted directly on top of the world frame, both of which are aligned to the gravity vector. The road is a perfect plane with a slope such that the equation of road plane in world-coordinate frame is given by $100Y_w - Z_w = 0$ and the pothole lies on the road plane. Provide the formula or pseudo-code for computing the pothole coordinates, and also substitute in the values. (20 min, 20 marks)

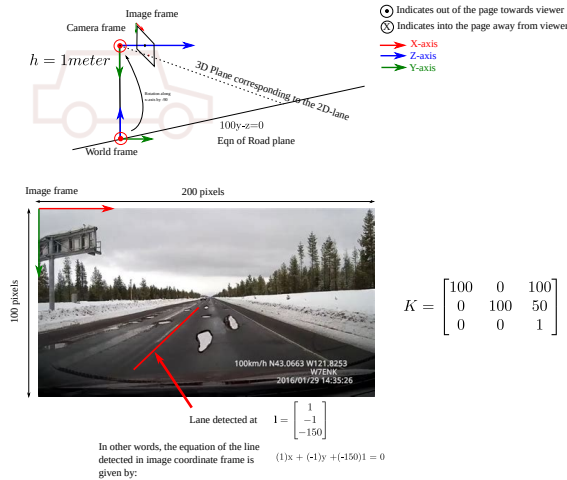


Figure 2: Line-plane triangulation

Problem 8 In figure 2 find the 3D representation of the lane the World coordinate frame, in terms of h (the height of the camera), image-representation of the line \mathbf{l} (provided in figure), camera matrix K (provided in figure). Assume the lane to be a straight line. The Camera is mounted directly on top of the world frame, both of which are aligned to the gravity vector. The road is a perfect plane with a slope such that the equation of road plane in world-coordinate frame is given by $100Y_w - Z_w = 0$ and the pothole lies on the road plane. Provide the formula or pseudo-code for computing the pothole coordinates, and also substitute in the values. (20 min, 20 marks)

Problem 9 You are a part of Tesla self-driving team. Team 1 provides you with lane-detection algorithms and their output. Team 2 provides you with detailed maps of road conditions. Your task is to write a function that solves problem 8 for arbitrary lanes detected by team 1 and for arbitrary plane provided by team 2. (Hint: Equation of a plane 3D is very similar to equation of line in 2D). What input representations of lane and plane would you ask for? Write a general algorithm or pseudo-code that solves problem 8. (30 min, 10 marks)

Line is a set of points that satisfy $ax + by + c = 0$

$$\mathbf{l}^T \mathbf{x}_c = 0$$

$$\mathbf{x}_c = K \mathbf{x}_w$$

$$\mathbf{x}_c = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\mathbf{l}^T (K \mathbf{x}_c) = 0$$

$$\mathbf{x}_c = ? \Rightarrow \mathbf{l}^T K \mathbf{x}_c = 0$$

Eqn of a plane in 3D

$$ax + by + c = 0$$

$$ax + by + cz + w = 0$$

$$\mathbf{l}^T \mathbf{x}_c = 0 \Rightarrow \mathbf{l} = K^T \mathbf{l}$$