ECE 417/598: Plane to points and DLT

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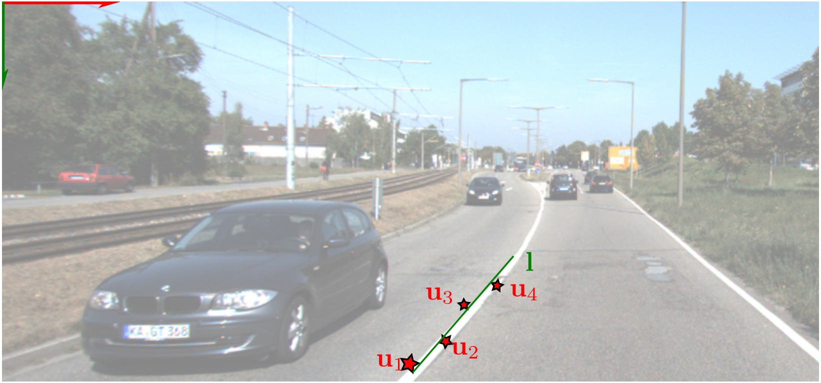


$$\underline{\mathbf{u}}_1 = [100, 98, 1]^\top$$
$$\underline{\mathbf{u}}_2 = [105, 95, 1]^\top$$
$$\underline{\mathbf{u}}_3 = [107, 90, 1]^\top$$
$$\underline{\mathbf{u}}_4 = [110, 85, 1]^\top$$

Find the line $\mathbf{l}$ such that it is the "closest line" passing through $\mathbf{u}_1, \ldots, \mathbf{u}_4$.

$$A = \begin{bmatrix} \mathbf{u}_1^\top \\ \mathbf{u}_2^\top \\ \mathbf{u}_3^\top \\ \mathbf{u}_4^\top \end{bmatrix}$$

We want to solve for $\mathbf{l}$ such that

$$A\mathbf{l} = 0$$

Singular Value Decomposition (SVD)

$$A = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^\top$$

$$A^\top A = V \Sigma^2 V^{-1}$$

$$A^\top A \mathbf{v}_i = \lambda_i \mathbf{v}_i \qquad \lambda_i = \sigma_i^2, \Sigma = \mathrm{diag}([\sigma_1, \ldots, \sigma_r])$$

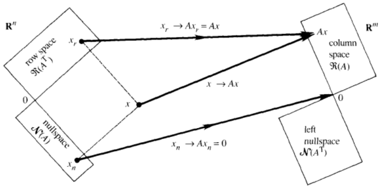
$$\mathbf{u}_i = \frac{A\mathbf{v}_i}{\sigma_i}$$

$$U = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \ldots & \mathbf{u}_m \end{bmatrix}$$

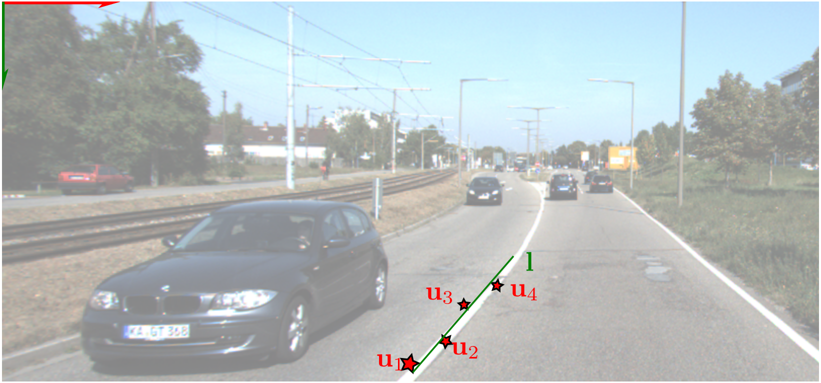
If $A \in \mathbb{R}^{m \times n}$ and the rank of A is r, then

$$A = \begin{bmatrix} U_{m \times r} & U_{m \times (m-r),\perp} \end{bmatrix} \begin{bmatrix} \Sigma_{r \times r} & 0_{r \times (n-r)} \\ 0_{(m-r) \times r} & 0_{(m-r) \times (n-r)} \end{bmatrix} \begin{bmatrix} V_{n \times r}^\top \\ V_{n \times (n-r),\perp}^\top \end{bmatrix}$$

$$A = U_{m \times r} \Sigma_{r \times r} V_{n \times r}^\top + 0 * U_{m \times (m-r),\perp} V_{n \times (n-r),\perp}^\top$$



$$\mathcal{N}(A) = V_{n\times(n-r),\perp}$$
$$\mathcal{R}(A) = U_{m\times r}$$
$$\mathcal{N}(A^\top) = U_{m\times(m-r),\perp}$$
$$\mathcal{R}(A^\top) = V_{n\times r}$$

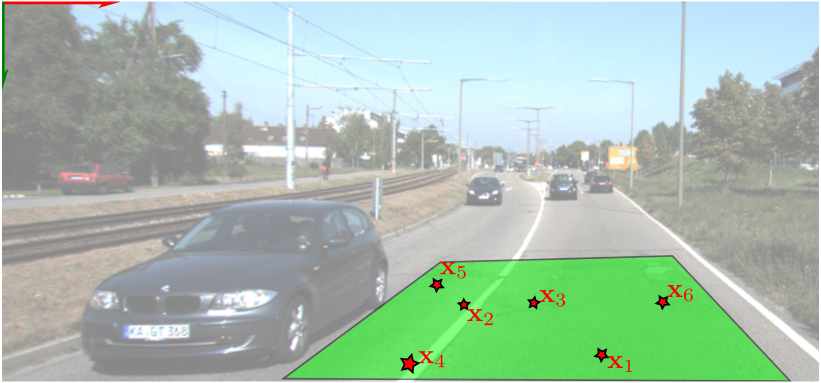


We want to solve for $\mathbf{l}$ such that

$$A\mathbf{l} = 0$$

A is $m \times 3$ and has rank 2. Solution

$$\begin{aligned} U\Sigma V^\top &= A \\ V &= [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3] \\ \mathbf{l} &= \mathbf{v}_3 \end{aligned}$$

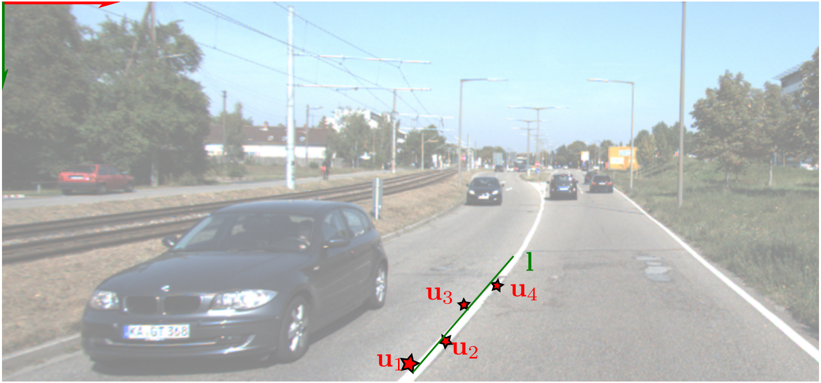


$\underline{\mathbf{x}}_1 = [-2.3, 1.04, 3.2, 1]^\top$ $\quad$ $\underline{\mathbf{x}}_2 = [-2.2, 1.02, 2.2, 1]^\top$

$\underline{\mathbf{x}}_3 = [-2.1, 1.01, 1.2, 1]^\top$ $\quad$ $\underline{\mathbf{x}}_4 = [2.1, 1.04, 1.2, 1]^\top$

$\underline{\mathbf{x}}_5 = [2.2, 1.03, 3.2, 1]^\top$ $\quad$ $\underline{\mathbf{x}}_6 = [2.3, 1.01, 4.2, 1]^\top$

Find the equation of plane $\mathbf{p} = [p_1, p_2, p_3, p_4]^\top$ such all points lie on the plane.

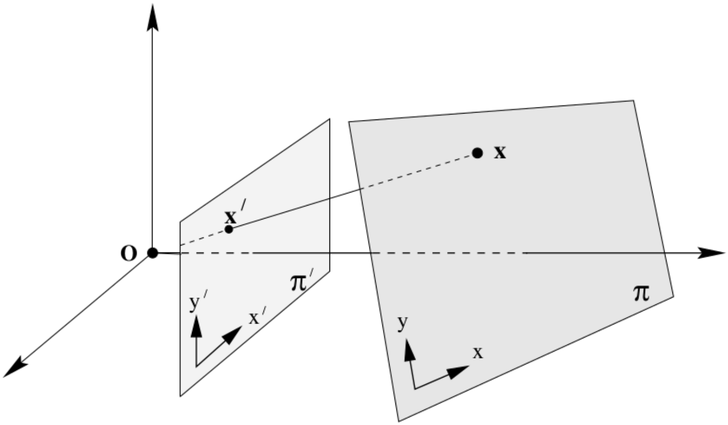


$$\underline{\mathbf{x}}_1 = [100, 98, 45, 1]^\top$$
$$\underline{\mathbf{x}}_2 = [105, 95, 46, 1]^\top$$
$$\underline{\mathbf{x}}_3 = [107, 90, 47, 1]^\top$$
$$\underline{\mathbf{x}}_4 = [110, 85, 43, 1]^\top$$

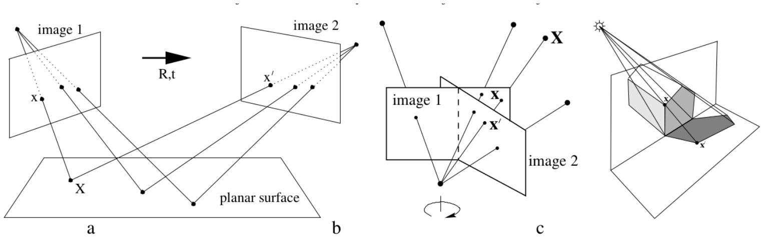
Find the 3D line such that it is the "closest line" passing through $\mathbf{x}_1, \ldots, \mathbf{x}_4 \in \mathbf{P}^3$.

Implicit and parameteric equations of lines and plane

Homography

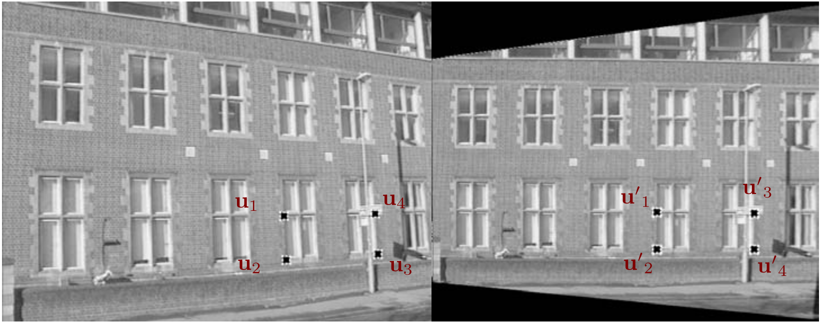


Examples of Homography




Top view
Graphic
Look! Safe to move?
Settings

Computing Homography



$$\underline{\mathbf{u}}_1 = [100, 98, 1]^\top \qquad \underline{\mathbf{u}}_2 = [102, 95, 1]^\top$$
$$\underline{\mathbf{u}}_3 = [107, 90, 1]^\top \qquad \underline{\mathbf{u}}_4 = [110, 85, 1]^\top$$
$$\underline{\mathbf{u}}'_1 = [100, 98, 1]^\top \qquad \underline{\mathbf{u}}'_2 = [102, 95, 1]^\top$$
$$\underline{\mathbf{u}}'_3 = [107, 98, 1]^\top \qquad \underline{\mathbf{u}}'_4 = [110, 85, 1]^\top$$

Find H such that $\underline{\mathbf{u}}' = H\underline{\mathbf{u}}$ for any point on one image to another image.

2D homography

Given a set of points $\underline{\mathbf{u}}_i \in \mathbb{P}^2$ and a corresponding set of points $\underline{\mathbf{u}}'_i \in \mathbb{P}^2$, compute the projective transformation that takes each $\underline{\mathbf{u}}_i$ to $\underline{\mathbf{u}}'_i$. In a practical situation, the points $\underline{\mathbf{u}}_i$ and $\underline{\mathbf{u}}'_i$ are points in two images (or the same image), each image being considered as a projective plane $\mathbb{P}^2$.

Solving for Homography

3D to 2D camera projection matrix estimation

Given a set of points $\mathbf{X}_i$ in 3D space, and a set of corresponding points $\mathbf{x}_i$ in an image, find the 3D to 2D projective $\mathbf{P}$ mapping that maps $\mathbf{X}_i$ to $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$.