RRT

September 20, 2024

0.1 Rapidly exploring dense trees

SIMPLE_RDT(q_0)
$\mathcal{G}$.init(q_0);
for $i = 1$ **to** k **do**
  $\mathcal{G}$.add_vertex($\alpha(i)$);
  $q_n \leftarrow$ NEAREST($S(\mathcal{G}), \alpha(i)$);
  $\mathcal{G}$.add_edge($q_n, \alpha(i)$);

45 iterations   2345 iterations

Figure 5.19: In the early iterations, the RRT quickly reaches the unexplored parts. However, the RRT is dense in the limit (with probability one), which means that it gets arbitrarily close to any point in the space.

LateX macros $ % Calligraphic fonts $

$% Sets: $ $% Vectors $

$ $

$% Matrices $

$ $

$ % Blackboard Bold: $

[1]:
```
!pip install requests ipympl
```

```
Requirement already satisfied: requests in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (2.32.3)
Requirement already satisfied: ipympl in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (0.9.4)
Requirement already satisfied: charset-normalizer<4,>=2 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from requests)
(3.3.2)
Requirement already satisfied: idna<4,>=2.5 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from requests)
(3.7)
Requirement already satisfied: urllib3<3,>=1.21.1 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from requests)
(2.2.2)
Requirement already satisfied: certifi>=2017.4.17 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from requests)
(2024.7.4)
Requirement already satisfied: ipython-genutils in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from ipympl)
(0.2.0)
Requirement already satisfied: ipython<9 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from ipympl)
(8.26.0)
Requirement already satisfied: ipywidgets<9,>=7.6.0 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from ipympl)
(8.1.3)
Requirement already satisfied: matplotlib<4,>=3.4.0 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from ipympl)
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Requirement already satisfied: numpy in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from ipympl)
(2.0.0)
Requirement already satisfied: pillow in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from ipympl)
(10.4.0)
Requirement already satisfied: traitlets<6 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from ipympl)
(5.14.3)
Requirement already satisfied: decorator in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipython<9->ipympl) (5.1.1)
Requirement already satisfied: jedi>=0.16 in
```

```
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipython<9->ipympl) (0.19.1)
Requirement already satisfied: matplotlib-inline in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipython<9->ipympl) (0.1.7)
Requirement already satisfied: prompt-toolkit<3.1.0,>=3.0.41 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipython<9->ipympl) (3.0.47)
Requirement already satisfied: pygments>=2.4.0 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipython<9->ipympl) (2.18.0)
Requirement already satisfied: stack-data in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipython<9->ipympl) (0.6.3)
Requirement already satisfied: exceptiongroup in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipython<9->ipympl) (1.2.2)
Requirement already satisfied: typing-extensions>=4.6 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipython<9->ipympl) (4.12.2)
Requirement already satisfied: pexpect>4.3 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipython<9->ipympl) (4.9.0)
Requirement already satisfied: comm>=0.1.3 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipywidgets<9,>=7.6.0->ipympl) (0.2.2)
Requirement already satisfied: widgetsnbextension~=4.0.11 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipywidgets<9,>=7.6.0->ipympl) (4.0.11)
Requirement already satisfied: jupyterlab-widgets~=3.0.11 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
ipywidgets<9,>=7.6.0->ipympl) (3.0.11)
Requirement already satisfied: contourpy>=1.0.1 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
matplotlib<4,>=3.4.0->ipympl) (1.2.1)
Requirement already satisfied: cycler>=0.10 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
matplotlib<4,>=3.4.0->ipympl) (0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
matplotlib<4,>=3.4.0->ipympl) (4.53.1)
Requirement already satisfied: kiwisolver>=1.3.1 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
matplotlib<4,>=3.4.0->ipympl) (1.4.5)
Requirement already satisfied: packaging>=20.0 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
matplotlib<4,>=3.4.0->ipympl) (24.1)
Requirement already satisfied: pyparsing>=2.3.1 in
```

```
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
matplotlib<4,>=3.4.0->ipympl) (3.1.2)
Requirement already satisfied: python-dateutil>=2.7 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
matplotlib<4,>=3.4.0->ipympl) (2.9.0.post0)
Requirement already satisfied: parso<0.9.0,>=0.8.3 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
jedi>=0.16->ipython<9->ipympl) (0.8.4)
Requirement already satisfied: ptyprocess>=0.5 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from
pexpect>4.3->ipython<9->ipympl) (0.7.0)
Requirement already satisfied: wcwidth in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from prompt-
toolkit<3.1.0,>=3.0.41->ipython<9->ipympl) (0.2.13)
Requirement already satisfied: six>=1.5 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from python-
dateutil>=2.7->matplotlib<4,>=3.4.0->ipympl) (1.16.0)
Requirement already satisfied: executing>=1.2.0 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from stack-
data->ipython<9->ipympl) (2.0.1)
Requirement already satisfied: asttokens>=2.1.0 in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from stack-
data->ipython<9->ipympl) (2.4.1)
Requirement already satisfied: pure-eval in
/home/vdhiman/.local/venvs/ece417/lib/python3.10/site-packages (from stack-
data->ipython<9->ipympl) (0.2.2)
```

[2]:

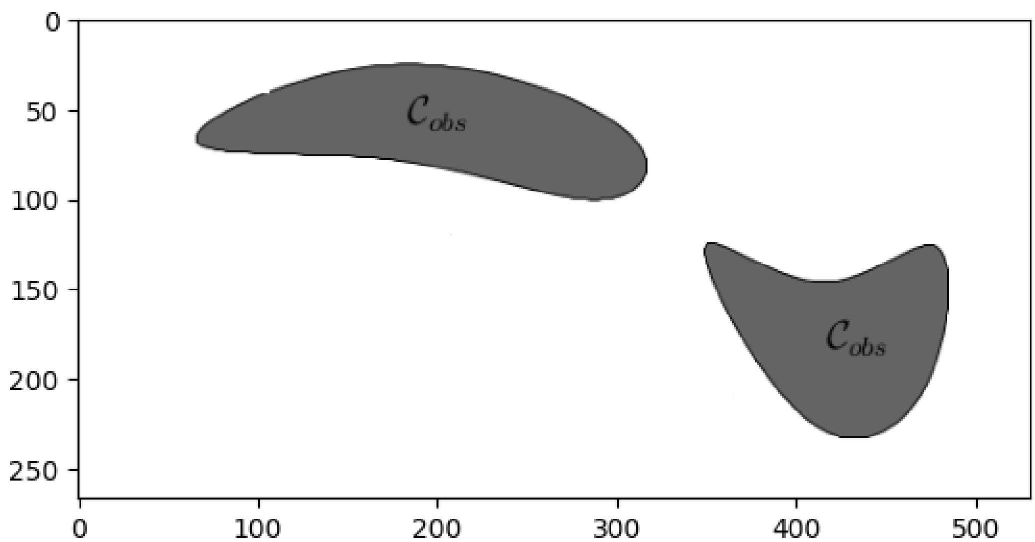
```python
import requests
import os
def wget(url, filename):
    r = requests.get(url)
    os.makedirs(os.path.dirname(filename), exist_ok=True)
    with open(filename, 'wb') as fd:
        for chunk in r.iter_content():
            fd.write(chunk)
url = 'https://vikasdhiman.info/ECE498-Mobile-Robots/notebooks/
 ↪01-1901-discrete-planning/imgs/RRT-map.png'
filename = 'imgs/RRT-map.png'
wget(url, filename)
```

[3]:

```python
%matplotlib inline
import numpy as np
import random
# random.seed(1004)
# np.random.seed(1004)
from PIL import Image
import matplotlib.pyplot as plt
```

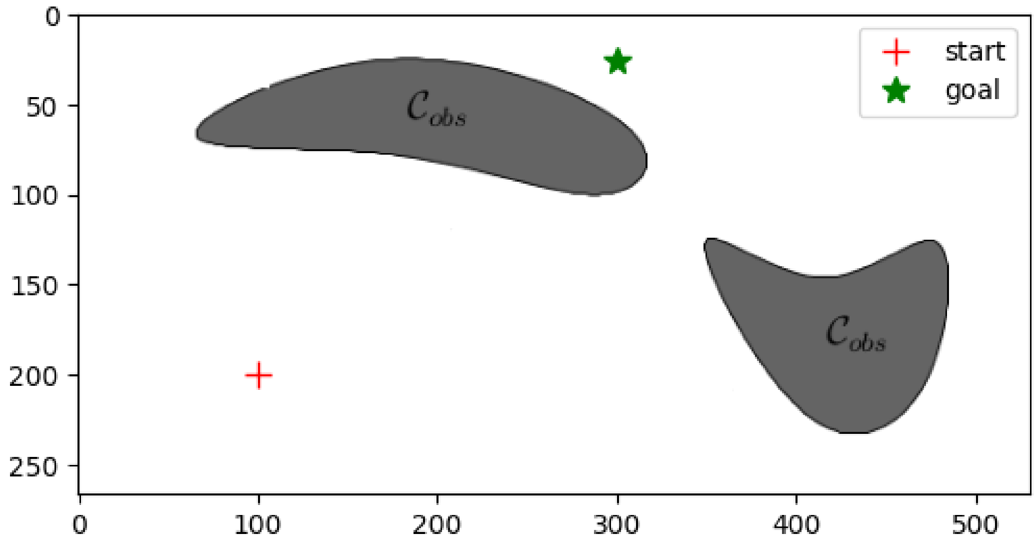
```python
# I removed the graph lines from the map above using photoshop and
# saved only the obstacles. Load that map as a png file.
# It is color image; convert it to grayscale.
img_gray = Image.open("imgs/RRT-map.png").convert('L')
# convert the image to a numpy array
img = np.asarray(img_gray)
fig, ax = plt.subplots()
ax.imshow(img, cmap='gray') # plot the image
```

[3]: <matplotlib.image.AxesImage at 0x779920279360>



[4]:
```python
%matplotlib inline
# Pick some arbitray start and goal points
goal = (300., 25.)
start = (100., 200.)
fig, ax = plt.subplots()
ax.imshow(img, cmap='gray') # Plot the image again
ax.plot(start[0], start[1], 'r+', markersize=10, label='start')
ax.plot(goal[0], goal[1], 'g*', markersize=10, label='goal')
ax.legend()
```

[4]: <matplotlib.legend.Legend at 0x779914aee860>



0.2 We have a problem to solve

We want to find the shortest path from start to goal in the continuous domain while avoiding obstacles.

0.2.1 Rapidly exploring random trees

The main idea of the algorithm is: 1. Initialize an empty graph with the start point 2. While not done:

```
a. Sample points on the chosen area. If the point is obstacle area, continue to the next itera
```

```
b. Connect the sampled point to the nearest point (vertex or edge) on the graph, as long as the
```

[40]:
```
from dataclasses import dataclass
# Need img as the map representation
assert img is not None

@dataclass
class Vertex:
    """
    Class to encode a graph vertex with a unique idx : a number
    and its coordinates as a numpy array.
    """
    idx: int
    coord: np.ndarray

    # Make the PItem hashable
    # https://docs.python.org/3/glossary.html#term-hashable
```

```python
    def __hash__(self):
        return self.idx

    def __eq__(self, other):
        return self.idx == other.idx

class Graph:
    """
    Keeps track of nodes and their 2D coordinates.
    The datastructure of choice here is an adjacency list.
    """
    def __init__(self):
        self.adjacency_list = {}
        self.vertex_list = []

    @classmethod
    def from_adjacency_matrix(cls, vertex_coords, G_adjacency_matrix):
        """
        Generate the graph from an adjacency matrix and vertex coordinates
        """
        self = cls()
        self.vertex_coordinates = vertex_coords
        for vi, v in enumerate(vertex_coords):
            vert = Vertex(idx=vi, coord=v)
            self.vertex_list.append(vert)
            self.adjacency_list[vert] = [
                Vertex(idx=pnj, coord=pn)
                for pnj, pn in enumerate(vertex_coords)
                if (G_adjacency_matrix[vi, pnj])]
        return self

    def get(self, v, default=[]):
        """
        Interface with path planning algorithms like astar using
        .get function.

        This function returns a list of neighbors along with
        edge-cost which is the euclidean distance between the
        coordinates of this ndoe and the neighbors.
        """
        vcoord = np.array(v.coord)
        return [(nbr, np.linalg.norm(vcoord-nbr.coord))
                for nbr in self.adjacency_list[v]]

    def add_vertex(self, coordinate):
        """
        Add new vertex to the graph. Assume it does not exists.
```

```
        """
        idx = len(self.vertex_list)
        vert = Vertex(idx=idx, coord=coordinate)
        self.adjacency_list[vert] = []
        self.vertex_list.append(vert)
        return vert

    def add_edge_directed(self, vi : Vertex, vj : Vertex):
        """
        Add a new edge to the graph from vi -> vj
        """
        assert isinstance(vi, Vertex)
        assert isinstance(vj, Vertex)
        self.adjacency_list.setdefault(vi, []).append(vj)

    def add_edge(self, vi, vj, undirected=True):
        """
        Add an undirected or directed edge to the graph.
        """
        self.add_edge_directed(vi, vj)
        if undirected:
            self.add_edge_directed(vj, vi)

    def remove_edge_directed(self, vi, vj):
        vjidx = self.adjacency_list[vi].index(vj)
        del self.adjacency_list[vi][vjidx]

    def remove_edge(self, vi, vj, undirected=True):
        self.remove_edge_directed(vi, vj)
        if undirected:
            self.remove_edge_directed(vj, vi)

    def vertices(self):
        """
        Return all vertices
        """
        return self.vertex_list

    def get_vertex(self, idx):
        """
        Get a perticular Vertex object by Vertex.idx
        """
        return self.vertex_list[idx]

    def vertex_coords(self):
        """
        Return the vertex coordinates as a numpy array
```

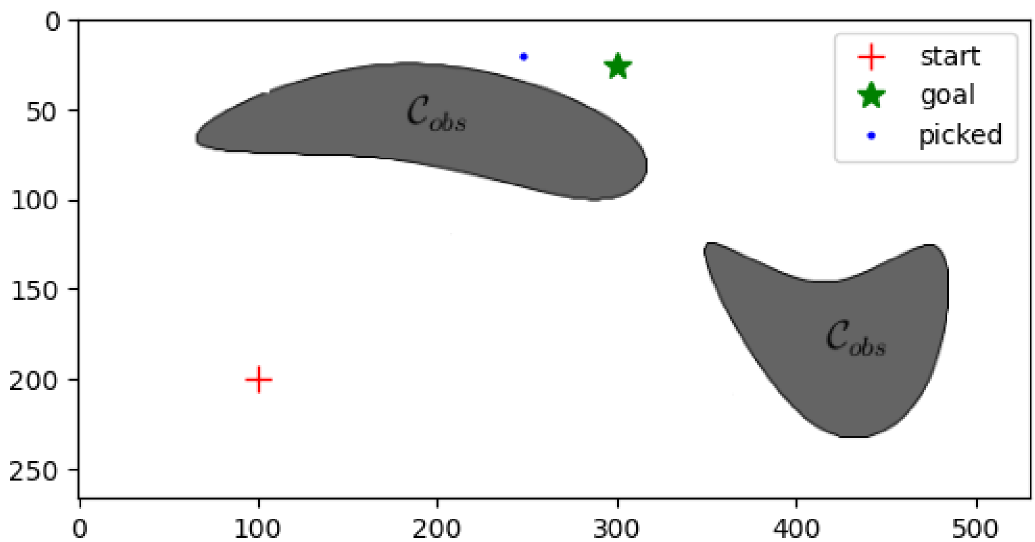
```
        """
        return np.asarray([vert.coord
                           for vert in self.vertex_list])

    def vertices_no_nbrs(self):
        """
        Return isolated vertices that do not have any
        neighbors.
        """
        return [vid for vid, nbrsid in self.adjacency_list.items()
                if not len(nbrsid)]

    def edges_coords(self):
        """
        Return edge_ids and edge_coords as lists where

        edge_ids = [(v1s.idx, v1e.idx),
                    (v2s.idx, v2e.idx), ...]
        edge_coords = [(v1s.coord, v2e.coord),
                       (v2s.coord, v2e.coord), ...]

        edge_ids contain the vertex indices as start and end pairs
        edge_coords contain the vertex coordinates for each edge with
        start and end pairs.
        """
        edge_ids = []
        edge_list = []
        for vid, nbrsid in self.adjacency_list.items():
            for nid in nbrsid:
                edge_ids.append((vid.idx, nid.idx))
                edge_list.append((vid.coord, nid.coord))
        return edge_ids, edge_list

    def plot(self, ax : plt.Axes, vertexids=False, marker='k*-'):
        """
        Plot the graph on the matplotlib axes object
        """
        ax.axis('equal')
        edge_ids, edge_coords = self.edges_coords()
        for (vid, nid), (v, n) in zip(edge_ids, edge_coords):
            ax.plot([v[0], n[0]], [v[1], n[1]], marker)
            if vertexids:
                ax.text(v[0], v[1], str(vid))
                ax.text(n[0], n[1], str(nid))

    def plot_path(self, ax : plt.Axes, path, color='r'):
        """
```

```
        Plat the path on the matplotlib axes
        """
        xs = []
        ys = []
        for vert in path:
            xs.append(vert.coord[0])
            ys.append(vert.coord[1])
        ax.plot(xs, ys, '-', color=color)

# 1. Initialize an empty graph with the start point
G_adjacency_list = Graph()
G_adjacency_list.add_vertex(start)

Npts = 1 # we are going to sample 100 points, but start with 1 point
pt_min, pt_max = np.array([0, 0]), np.array([img.shape[1], img.shape[0]])
# 2. While not done:
for i in range(Npts):
    # 2.a Sample points on the chosen area.
    # If the point is obstacle area, continue to the next iteration.
    random_pt = np.random.rand(2) * (pt_max - pt_min) + pt_min

random_pt
```

[40]:
```
array([247.40935162,  20.28991334])
```

[41]:
```
%matplotlib inline
# Let's plot this point
def plot_map(ax, img, goal, start):
    ax.imshow(img, cmap='gray') # Plot the image again
    ax.plot(start[0], start[1], 'r+', markersize=10, label='start')
    ax.plot(goal[0], goal[1], 'g*', markersize=10, label='goal')
    ax.legend()
    return ax
fig, ax = plt.subplots()
plot_map(ax, img, goal, start)
picked_pt, = ax.plot(random_pt[0], random_pt[1], 'bo', markersize=2,␣
 ↪label='picked')
ax.legend()
```

[41]:
```
<matplotlib.legend.Legend at 0x7798e9445510>
```



[42]:
```
# check the color of image at the random_pt
# Note that I have used y-coordinate for rows and
# x-coordinate for cols
random_pt_int = np.round(random_pt).astype(dtype=np.int64)
img[random_pt_int[1], random_pt_int[0]]
```

[42]: np.uint8(255)

0.2.2 Distinguishing obstacles from free area

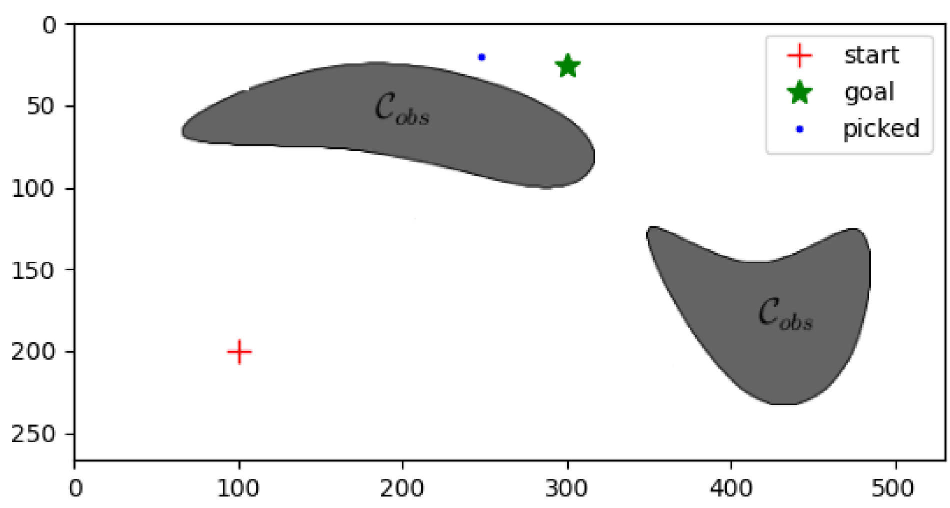
We will find the color values inside the gray obstacles and in the white area. The following code creates an interactive widget that you can click on to find the color of the pixel at the clicked point.

[43]:
```
# Click anywere on the gray area in the image to find the color of that point
import ipywidgets as widgets # Make the print statement interactive
# Make the matplotlib figure interactive
%matplotlib widget

# Draw the map
fig, ax = plt.subplots()
plot_map(ax, img, goal, start)
picked_pt, = ax.plot(random_pt[0], random_pt[1], 'bo', markersize=2,␣
 ↪label='picked')
ax.legend()

# Create a textarea to display the interactive message
txtwidget = widgets.Textarea(
    value='You have not clicked on the figure yet',
```

```
    placeholder='You have not clicked on the figure yet',
    description='Color:  ',
    disabled=True,
    width=200
)
display(txtwidget)

# This function will be called whenever you click anywhere on the map
def onclick(event):
    x, y = event.xdata, event.ydata
    picked_pt.set_xdata([x])
    picked_pt.set_ydata([y])
    # Change the display message in the figure
    txtwidget.value = "%d" % (img[int(y), int(x)])

fig.canvas.mpl_connect('button_release_event', onclick)
```

```
Textarea(value='You have not clicked on the figure yet', description='Color:  ',␣
↪disabled=True, placeholder='Y…
```

[43]: 15



Note that the gray areas have color value 100 while the white areas have color 255. We will treat 200 as threshold. Any color value smaller than 200 will be considered as an obstacle. Image boundaries are also an obstacle. The robot cannot go through gray areas or outside the image.

[44]:
```python
# 100 is darker than 255.
# Our collision check is checking for the color.
# I pick the threshold between 100 and 255 arbitrarily as
# 200
def do_points_collide(img, pts):
    """
    Returns true or false per point,

    If the point is out of the image on
    """
    # threshold between white (255) and gray (100) color
    threshold = 200 # chose the threshold as 200
    pts = np.round(pts).astype(dtype=np.int64) # convert the points to integers

    # Test if the points are inside the iamge or not
    # We are using numpy boolean operators
    # https://numpy.org/doc/stable/reference/generated/numpy.logical_and.html
    in_img = ((0 <= pts) & (pts < np.array((img.shape[1], img.shape[0])))).
↪all(axis=-1)
    out_of_img = ~in_img # Numpy not operator

    # Convert all the out of image points in image so that we can use them to␣
↪index
    # in img
    in_img_pts = pts.copy()
    # it does not matter what the value is as long as it is inside the img␣
↪bounds
    in_img_pts[out_of_img, :] = 0

    # Index the image using pts. Y-coordinate is the row and X-coordinate is␣
↪the column
    colors_per_pt = img[in_img_pts[..., 1], in_img_pts[..., 0]]
    # The points collide if they are out of the image or below the grayness␣
↪threshold
    return (out_of_img) | (colors_per_pt < threshold)

def does_point_collide(img, pt):
    return do_points_collide(img, pt)

# Lets check our function again
# For a collision free point
assert does_point_collide(img, np.array([20.68332004, 228.68439464])) == False
# For a collision point
```

```python
assert does_point_collide(img, np.array([200., 50.])) == True
```

We are going to go back to our incomplete algorithm and add collision check

[45]:
```python
# Need img as the map representation
assert img is not None

# 1. Initialize an empty graph with the start point
G_adjacency_list = Graph()
G_adjacency_list.add_vertex(start)

Npts = 1 # we are going to sample 100 points, but start with 1 point
# Specify the bounds of the map
pt_min = np.array([0, 0])
pt_max = np.array([img.shape[1], img.shape[0]])

# 2. While not done
for i in range(Npts):
    # 2.a Sample points on the chosen area.
    random_pt = np.random.rand(2) * (pt_max - pt_min) + pt_min

    # If the point is obstacle area, continue to the next iteration.
    if does_point_collide(img, random_pt):
        continue

    # 2.B Connect the sampled point to the nearest point (vertex or edge)
    # on the graph, as long as the connecting line does not pass through the␣
↪obstacle.
```

1 Finding the nearest vertex or edge on graph

There are faster algorithms to do this where we can maintain a k-d tree and lookup the nearest vertex in $O(log(|V|))$ time.

However, we are going to go with brute force approach and loop over all the vertices to find the closest vertex, which is $O(|V|)$.

[46]:
```python
vertices_np = G_adjacency_list.vertex_coords() # np.array of size N x 2
diff_vec = (vertices_np - random_pt) # np.array of size N x 2
dists_per_vec = np.sqrt((diff_vec**2).sum(axis=-1)) # np.array of size N
closest_vertex = vertices_np[np.argmin(dists_per_vec)] # np.array of size 2
closest_vertex
```

[46]: array([100., 200.])

Let's make the above code a function and stress test it a bit.

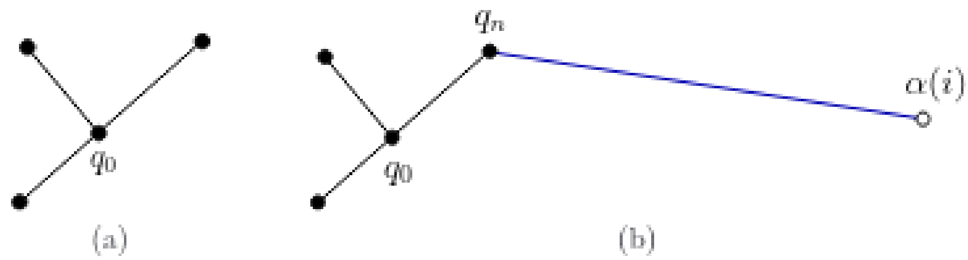


Figure 5.17: (a) Suppose inductively that this tree has been constructed so far using the algorithm in Figure 5.16. (b) A new edge is added that connects from the sample $\alpha(i)$ to the nearest point in S, which is the vertex q_n.

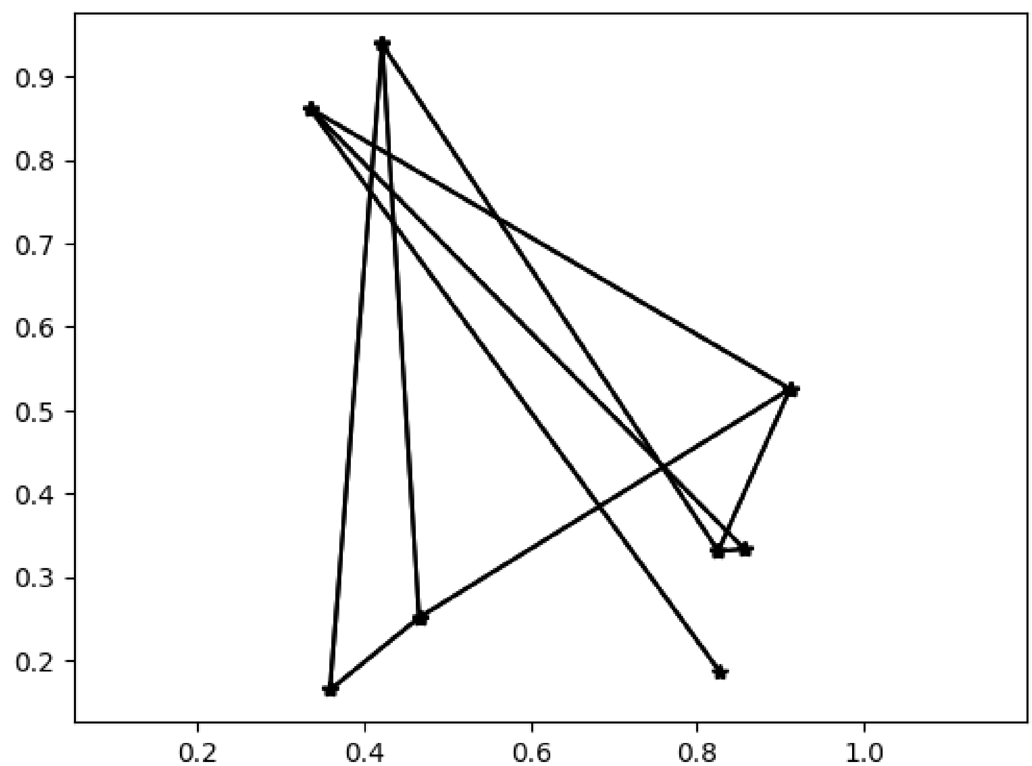
[47]:
```
def find_nearest_vertex(G_adjacency_list, pt):
    """
    Find the nearest vertex to the point pt in the graph G_adjacency_list.
    """
    vertices_np = G_adjacency_list.vertex_coords() # np.array of size N x 2
    diff_vec = (vertices_np - pt) # np.array of size N x 2
    dists_per_vec = np.sqrt((diff_vec**2).sum(axis=-1)) # np.array of size N
    closest_vertex = vertices_np[np.argmin(dists_per_vec)] # np.array of size 2
    return closest_vertex
```

[48]:
```
# Create a random graph to stress test the function
def generate_random_graph(nvertices=10, # How many vertices
                          # Fraction of vertices connected to each other
                          # 1 means fully connected
                          # 0 means none connected
                          edge_density=0.2,
                          selfedges=False, # allow self edges
                          undirected=True, # is the graph undirected
                          pt_min=np.array([0., 0.]), # range of points
                          pt_max=np.array([1., 1.])):
    """
    Generate a random graph with given
    """
    D = pt_min.shape[0] # dimensions
    vertices = np.random.rand(nvertices, D) * (pt_max - pt_min) + pt_min
    G_adjacency_matrix_samples = np.random.rand(
        nvertices, nvertices)

    if undirected:
        matrix_edge_density = edge_density / 2
        G_adjacency_matrix_samples = np.tril(G_adjacency_matrix_samples, k=1)
        G_adjacency_matrix_samples += G_adjacency_matrix_samples.T
```

```
        G_adjacency_matrix_samples /= 2.
    # Pick the edge if the uniformly sampled prob is below edge_density
    G_adjacency_matrix = G_adjacency_matrix_samples < matrix_edge_density
    if not selfedges:
        np.fill_diagonal(G_adjacency_matrix, 0)
    G_adjacency_list = Graph.from_adjacency_matrix(vertices.tolist(),␣
 ↪G_adjacency_matrix)
    return G_adjacency_list

generate_random_graph()
```

[48]: <__main__.Graph at 0x7798e4bdefb0>

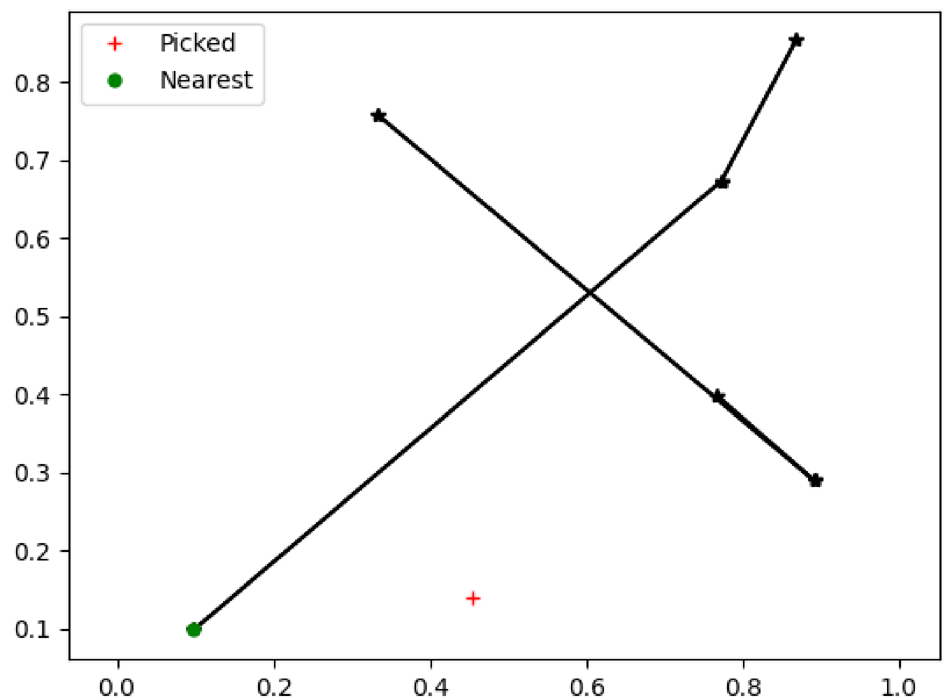
[50]:
```
%matplotlib inline
fig, ax = plt.subplots()
graph = generate_random_graph()
graph.plot(ax)
```



Let's pick a test point near different nodes by clicking on the figure and test the find_nearest_vertex function.

[51]:
```
%matplotlib widget
fig, ax = plt.subplots()
graph = generate_random_graph()
graph.plot(ax)
picked_pt = np.random.rand(2)
nearest_pt = find_nearest_vertex(graph, picked_pt)
pickedline, = ax.plot(picked_pt[0], picked_pt[1], 'r+', label='Picked')
nearestline, = ax.plot(nearest_pt[0], nearest_pt[1], 'go', markersize=5,␣
 ↪label='Nearest')
ax.legend()

# useful for debuging
# # Create a textarea to display the interactive message
# txtwidget = widgets.Textarea(
#     value='Picked: (000.0, 000.0); Nearest: (000.0, 000.0)',
#     placeholder='Picked: (000.0, 000.0); Nearest: (000.0, 000.0)',
#     description='',
#     disabled=True
# )
# display(txtwidget)

def onclick_nearest(event):
    pickedline.set_xdata([event.xdata])
    pickedline.set_ydata([event.ydata])

    nearest_pt = find_nearest_vertex(graph, np.array([event.xdata, event.
 ↪ydata]))
    nearestline.set_xdata(nearest_pt[:1])
    nearestline.set_ydata(nearest_pt[1:])
    # txtwidget.value = f'Picked: ({event.xdata:0.3f}, {event.ydata:0.3f});␣
 ↪Nearest: {nearest_pt}'

fig.canvas.mpl_connect('button_release_event', onclick_nearest)
```

[51]: 15



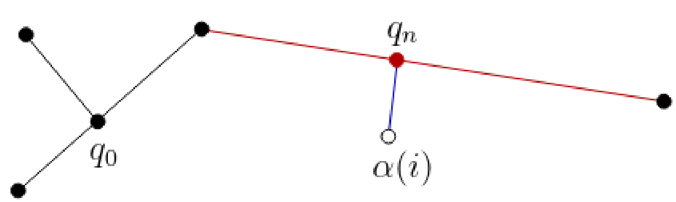


Figure 5.18: If the nearest point in S lies in an edge, then the edge is split into two, and a new vertex is inserted into $\mathcal{G}$.

Finding nearest point on edges What if the nearest point lies on an edge rather than a vertex? To compute this we need to find a formula for nearest point to a line. Consider a point $\mathbf{x} = (x, y) = \begin{bmatrix} x \\ y \end{bmatrix}$ and an edge $(\mathbf{v}_s, \mathbf{v}_e)$ where $\mathbf{v}_s = (v_{xs}, v_{ys})$ is the start vertex and $\mathbf{v}_e$ is the end vertex for the edge. Find the shortest distance to the edge.

1. Representation of a line passsing through two points. Let $t \in \mathbb{R}$ be a free parameter. Then the line passinging through $\mathbf{v}_s$ and $\mathbf{v}_e$ is a set of all points

$$L = \{\mathbf{l}(t) = \mathbf{v}_s + (\mathbf{v}_e - \mathbf{v}_s)t | \forall t \in \mathbb{R}\}$$

Moreover, if $t \in [0, 1]$ then the line point $\mathbf{l}(t)$ lies between the two end points $\mathbf{v}_s$ and $\mathbf{v}_e$. If $t < 0$, then the point $\mathbf{l}(t)$ lies before $\mathbf{v}_s$ and if $t > 1$ then it lies after $\mathbf{v}_e$.

2. The shortest distance between a point $\mathbf{x}$ and a line $\mathbf{l}(t)$ is along the perpendicular to the line that passes through $\mathbf{x}$. Let $\mathbf{l}(t_x)$ be such a point where the perpendicular from $\mathbf{x}$ meets the line $\mathbf{l}(t)$. Then we have,

$$(\mathbf{l}(t_x) - \mathbf{x})^\top(\mathbf{v}_e - \mathbf{v}_s) = 0 \tag{1}$$

3. This is one equation to solve for one variable t_x,

$$\begin{align} (\mathbf{l}(t_x) - \mathbf{x})^\top(\mathbf{v}_e - \mathbf{v}_s) = 0 \tag{2}\\ \implies (\mathbf{v}_s + (\mathbf{v}_e - \mathbf{v}_s)t_x - \mathbf{x})^\top(\mathbf{v}_e - \mathbf{v}_s) = 0 \tag{3}\\ \implies (\mathbf{v}_s - \mathbf{x})^\top(\mathbf{v}_e - \mathbf{v}_s) + (\mathbf{v}_e - \mathbf{v}_s)^\top(\mathbf{v}_e - \mathbf{v}_s)t_x = 0 \tag{4}\\ \implies t_x = \frac{(\mathbf{x} - \mathbf{v}_s)^\top(\mathbf{v}_e - \mathbf{v}_s)}{\|\mathbf{v}_e - \mathbf{v}_s\|^2} \tag{5} \end{align}$$

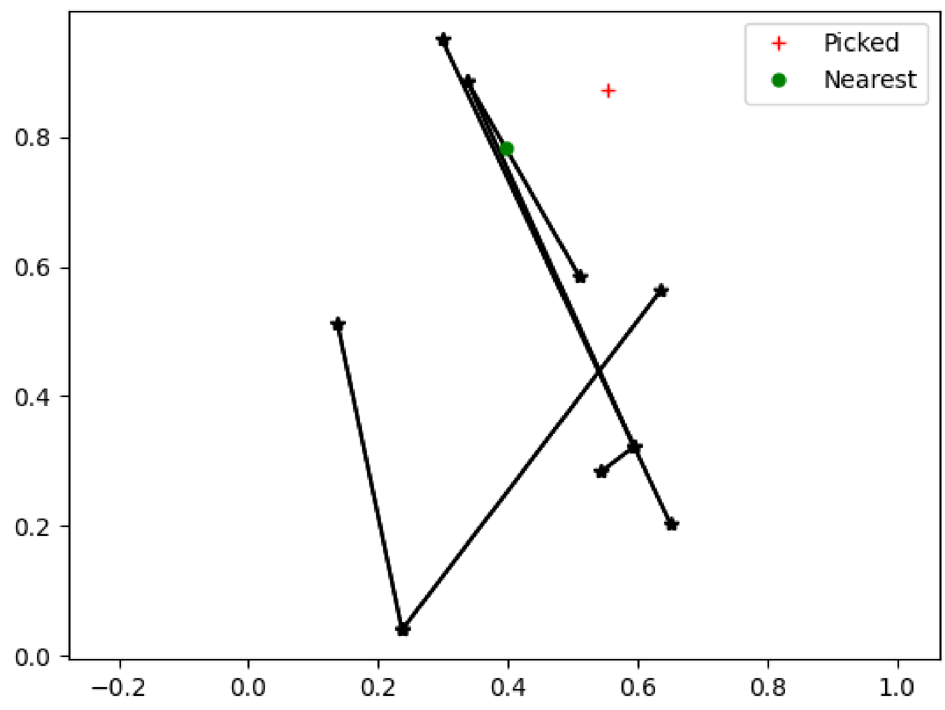
[52]:
```
def closest_point_on_line_segs(edges, x):
    """
    Find the closest point to x on all the edges
    """
    assert edges.shape[-2] == 2
    *N, _, D = edges.shape
    vs, ve = edges[:, 0, :], edges[:, 1, :]
    # edge_vec = ve - vs # *N x D
    # edge_mag = np.linalg.norm(edge_vec, axis=-1, keepdims=True) #  *N
    # edge_unit = edge_vec / edge_mag # *N x D

    # closest pt on edge = l(t) = vs + t * (ve - vs)
    # t = (x - vs) @ (ve - vs) / ||ve - vs||^2
    edge_vec = (ve - vs)
    edge_vec_mag_sq = (edge_vec * edge_vec).sum(axis=-1, keepdims=True) # N x 1
    t = ((x - vs) * edge_vec).sum(axis=-1, keepdims=True) / edge_vec_mag_sq # N␣
↪x 1

    # l(t) = vs + t * (ve - vs)
    lt = vs + t * edge_vec # *N x D

    # Perpendicular distance from the edge
    dist_e = np.linalg.norm(x - lt, axis=-1)

    # Distance from the end vertices
    dist_vs = np.linalg.norm(x - vs, axis=-1)
    dist_ve = np.linalg.norm(x - ve, axis=-1)
    # The minimum of the two is the closer one
    dist_v = np.minimum(dist_vs, dist_ve)
```

```
    # Is the point inside the edge?
    is_pt_inside_edge = ((0 <= t) & (t <= 1))[..., 0]


    # Take the edge distance only if the perpendicular falls
    # within the edge bounds otherwise take the minimumm
    # of the vertex distance
    dist = np.where(is_pt_inside_edge,
                    dist_e,
                    dist_v)
    min_idx = np.argmin(dist)
    closest_point, point_type =  (
        (lt[min_idx], slice(0, 2)) if is_pt_inside_edge[min_idx]
        else (vs[min_idx], slice(0, 1)) if (dist_vs[min_idx] < dist_ve[min_idx])
        else (ve[min_idx], slice(1, 2))
    )
    return closest_point, dist[min_idx], (min_idx, point_type)
```

[53]:
```
%matplotlib widget
fig, ax = plt.subplots()
graph = generate_random_graph()
graph.plot(ax)
picked_pt =  np.random.rand(2)
nearest_pt, _, _ = closest_point_on_line_segs(np.asarray(graph.
 ↪edges_coords()[1]), picked_pt)
pickedline, = ax.plot(picked_pt[0], picked_pt[1], 'r+', label='Picked')
nearestlines, = ax.plot(nearest_pt[0], nearest_pt[1], 'go', markersize=5,␣
 ↪label='Nearest')
ax.legend()

# # useful for debuging
# # Create a textarea to display the interactive message
# txtwidget = widgets.Textarea(
#     value='Picked: (000.0, 000.0); Nearest: (000.0, 000.0)',
#     placeholder='Picked: (000.0, 000.0); Nearest: (000.0, 000.0)',
#     description='',
#     disabled=True
# )
# display(txtwidget)

def onclick_nearest(event):
    pickedline.set_xdata([event.xdata])
    pickedline.set_ydata([event.ydata])
    picked_pt = np.array([event.xdata, event.ydata])
    nearest_pt, _, _ = closest_point_on_line_segs(np.asarray(graph.
 ↪edges_coords()[1]), picked_pt)
    # txtwidget.value = str(nearest_pt)
```

```
    nearestlines.set_xdata([nearest_pt[0]])
    nearestlines.set_ydata([nearest_pt[1]])

fig.canvas.mpl_connect('button_release_event', onclick_nearest)
```

[53]: 15



[54]:
```
def points_within_circle(edges, x, radius=None):
    """
    Find all the edges that lie within a given radius of a point x
    """
    assert edges.shape[-2] == 2
    *N, _, D = edges.shape
    vs, ve = edges[:, 0, :], edges[:, 1, :]
    # edge_vec = ve - vs # *N x D
    # edge_mag = np.linalg.norm(edge_vec, axis=-1, keepdims=True) #  *N
    # edge_unit = edge_vec / edge_mag # *N x D

    # closest pt on edge = l(t) = vs + t * (ve - vs)
    # t = (x - vs) @ (ve - vs) / ||ve - vs||^2
    edge_vec = (ve - vs)
```

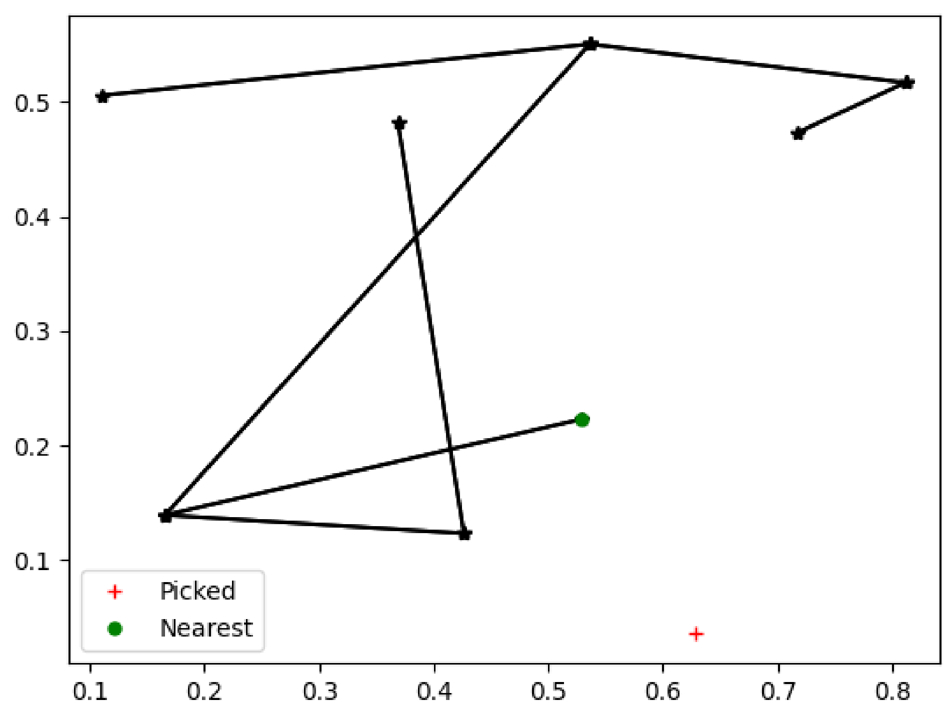
```
    edge_vec_mag_sq = (edge_vec * edge_vec).sum(axis=-1, keepdims=True) # N x 1
    t = ((x - vs) * edge_vec).sum(axis=-1, keepdims=True) / edge_vec_mag_sq # N␣
↪x 1

    # l(t) = vs + t * (ve - vs)
    lt = vs + t * edge_vec # *N x D

    # Perpendicular distance from the edge
    dist_e = np.linalg.norm(x - lt, axis=-1)

    # Distance from the end vertices
    dist_vs = np.linalg.norm(x - vs, axis=-1)
    dist_ve = np.linalg.norm(x - ve, axis=-1)
    # The minimum of the two is the closer one
    dist_v = np.minimum(dist_vs, dist_ve)

    # Is the point inside the edge?
    is_pt_inside_edge = ((0 <= t) & (t <= 1))[..., 0]


    # Take the edge distance only if the perpendicular falls
    # within the edge bounds otherwise take the minimumm
    # of the vertex distance
    dist = np.where(is_pt_inside_edge,
                    dist_e,
                    dist_v)

    closest_points = np.where(is_pt_inside_edge,
                              lt,
                              np.where(dist_vs < dist_ve,
                                       vs, ve))
    point_type = np.where(is_pt_inside_edge,
                          slice(0, 2),
                          np.where(dist_vs < dist_ve,
                                   slice(0, 1),
                                   slice(1, 2)))
    if radius is None:
        radius = np.min(dist)

    within_radius = dist < radius # a boolean per edge
    dists_within_radius = dist[within_radius]
    closest_points_within_radius = closest_points[within_radius]
    indices_within_radius = np.arange(len(dist))[within_radius]
    point_types_within_radius = point_type[within_radius]
    return closest_points_within_radius, dists_within_radius,␣
↪(indices_within_radius, point_types_within_radius)
```

[55]:
```python
def closest_point_on_graph(graph, pt):
    assert len(graph.vertices())
    edge_ids, edge_list = map(np.asarray, graph.edges_coords())
    if len(edge_list):
        closest_point_e, min_dist_e, min_idx_pt_type =␣
 ↪closest_point_on_line_segs(edge_list, pt)
        min_idx_e, pt_type = min_idx_pt_type
        vids = edge_ids[min_idx_e, pt_type]
        vertices = ((graph.get_vertex(vids[0]), graph.get_vertex(vids[1]))
                    if len(vids) == 2
                    else
                    (graph.get_vertex(vids[0]),))
    else:
        min_dist_e = np.inf

    vertices_no_nbrs = graph.vertices_no_nbrs()
    if len(vertices_no_nbrs):
        verticesnp = np.array([vid.coord for vid in vertices_no_nbrs])
        dists_v = np.linalg.norm(verticesnp - pt, axis=-1)
        min_idx_v = np.argmin(dists_v)
        closest_point_v = verticesnp[min_idx_v]
        min_dist_v = dists_v[min_idx_v]
    else:
        min_dist_v  = np.inf

    return ((closest_point_v, min_dist_v, (vertices_no_nbrs[min_idx_v],))
            if  min_dist_v < min_dist_e
            else (closest_point_e, min_dist_e, vertices))
```

[56]:
```python
%matplotlib widget
fig, ax = plt.subplots()
graph = generate_random_graph()
graph.plot(ax)
picked_pt =  np.random.rand(2)
nearest_pt, _, _ = closest_point_on_graph(graph, picked_pt)
pickedline, = ax.plot(picked_pt[0], picked_pt[1], 'r+', label='Picked')
nearestlines, = ax.plot(nearest_pt[0], nearest_pt[1], 'go', markersize=5,␣
 ↪label='Nearest')
ax.legend()

# # useful for debuging
# # Create a textarea to display the interactive message
# txtwidget = widgets.Textarea(
#     value='Picked: (000.0, 000.0); Nearest: (000.0, 000.0)',
#     placeholder='Picked: (000.0, 000.0); Nearest: (000.0, 000.0)',
```

```
#     description='',
#     disabled=True
# )
# display(txtwidget)

def onclick_nearest(event):
    pickedline.set_xdata([event.xdata])
    pickedline.set_ydata([event.ydata])
    picked_pt = np.array([event.xdata, event.ydata])
    nearest_pt, _, _ = closest_point_on_graph(graph, picked_pt)
    # txtwidget.value = str(nearest_pt)
    nearestlines.set_xdata([nearest_pt[0]])
    nearestlines.set_ydata([nearest_pt[1]])

fig.canvas.mpl_connect('button_release_event', onclick_nearest)
```

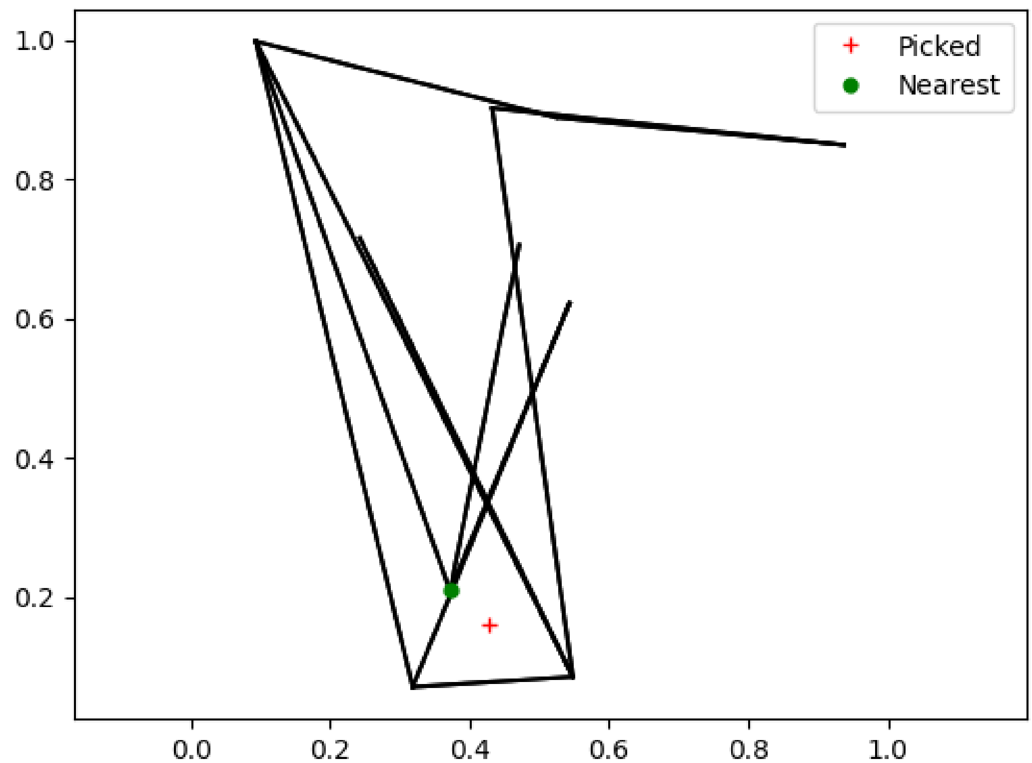
[56]: 15

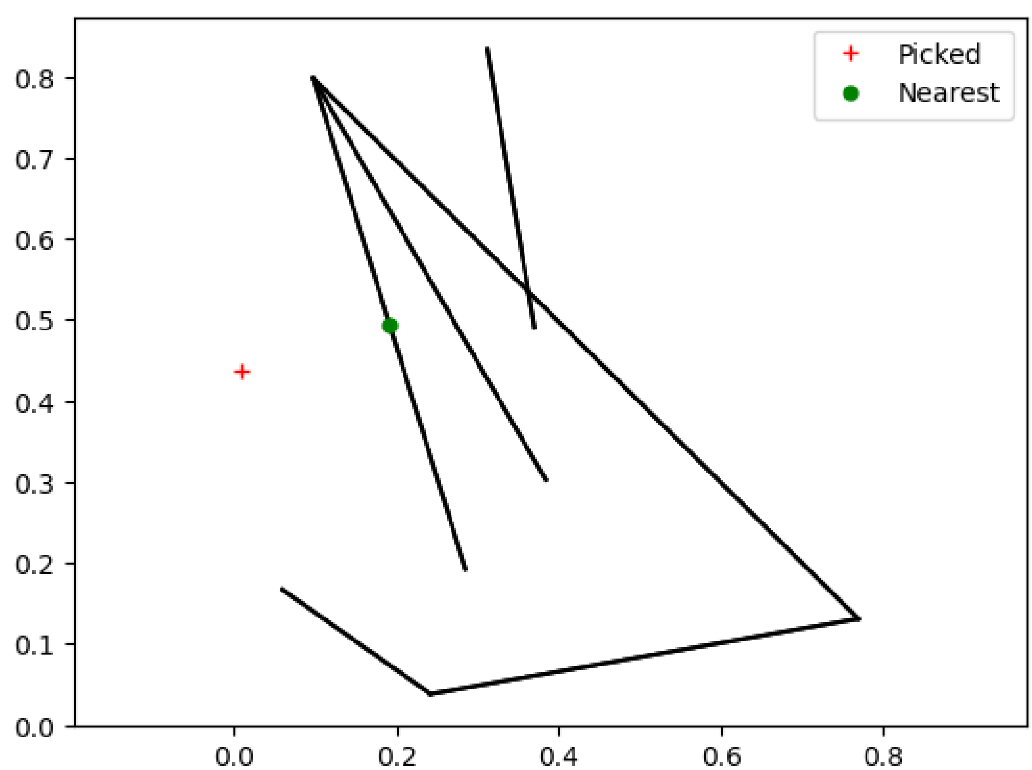


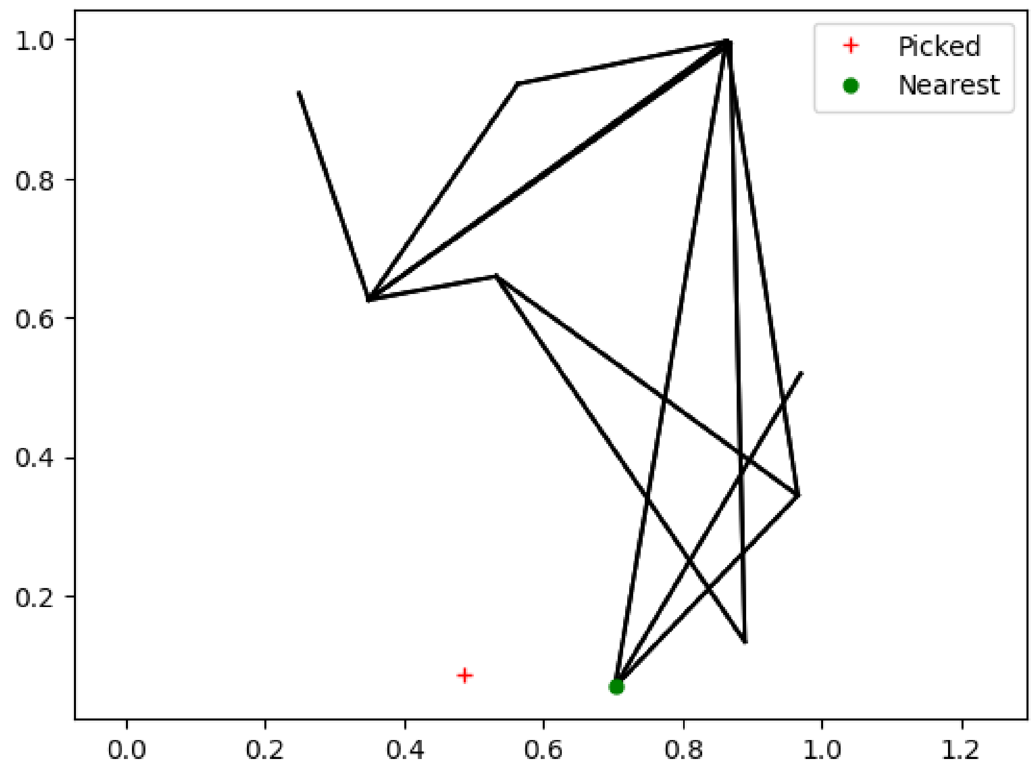
[21]:
```
def expand_graph(graph, pt, nearest_pt, nearest_pt_verts):
    if len(nearest_pt_verts) == 2:
        vs, ve = nearest_pt_verts
```

```
        graph.remove_edge(vs, ve)
        npt_vert = graph.add_vertex(nearest_pt)
        #print(npt_vert.coord)
        graph.add_edge(vs, npt_vert)
        graph.add_edge(npt_vert, ve)
    elif len(nearest_pt_verts) == 1:
        npt_vert = nearest_pt_verts[0]
    else:
        raise ValueError("Invalid nearest_pt_vids")

    fid = graph.add_vertex(pt)
    #print(fid.coord)
    graph.add_edge(npt_vert, fid)
    return fid
```

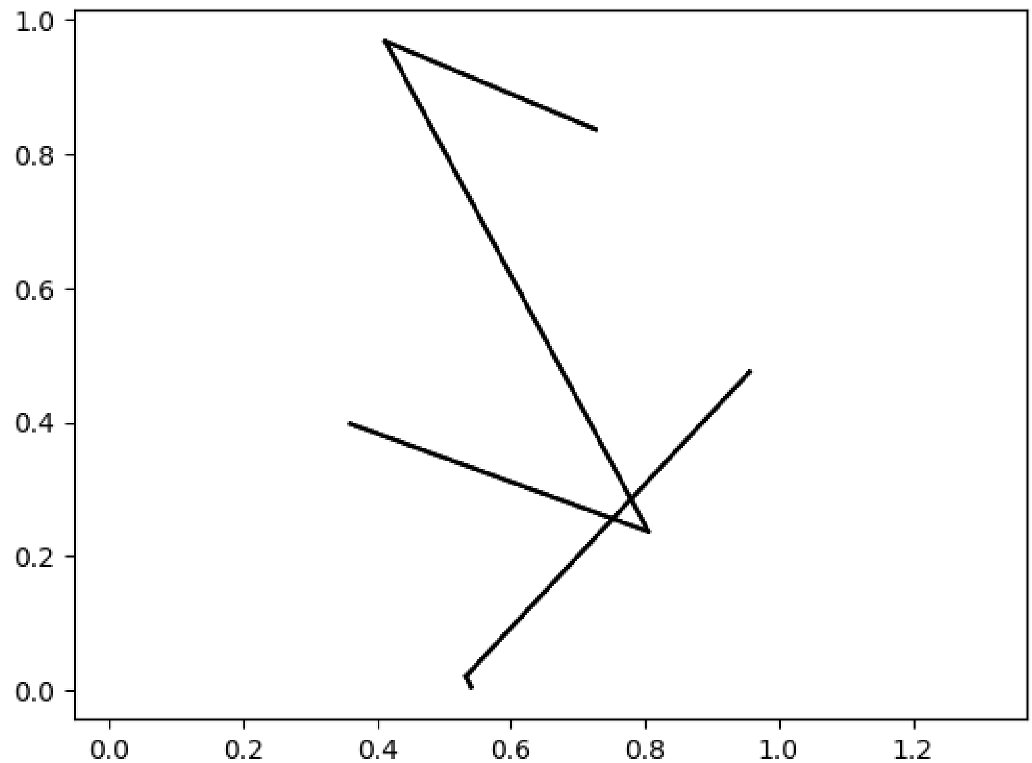
[22]:
```
%matplotlib inline
fig, ax = plt.subplots()
graph = generate_random_graph()
graph.plot(ax)

fig, ax = plt.subplots()
picked_pt = np.random.rand(2)
nearest_pt, dist, nearest_pt_verts = closest_point_on_graph(graph, picked_pt)
expand_graph(graph, picked_pt, nearest_pt, nearest_pt_verts)
graph.plot(ax)
pickedline, = ax.plot(picked_pt[0], picked_pt[1], 'r+', label='Picked')
```


Picked
Nearest
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

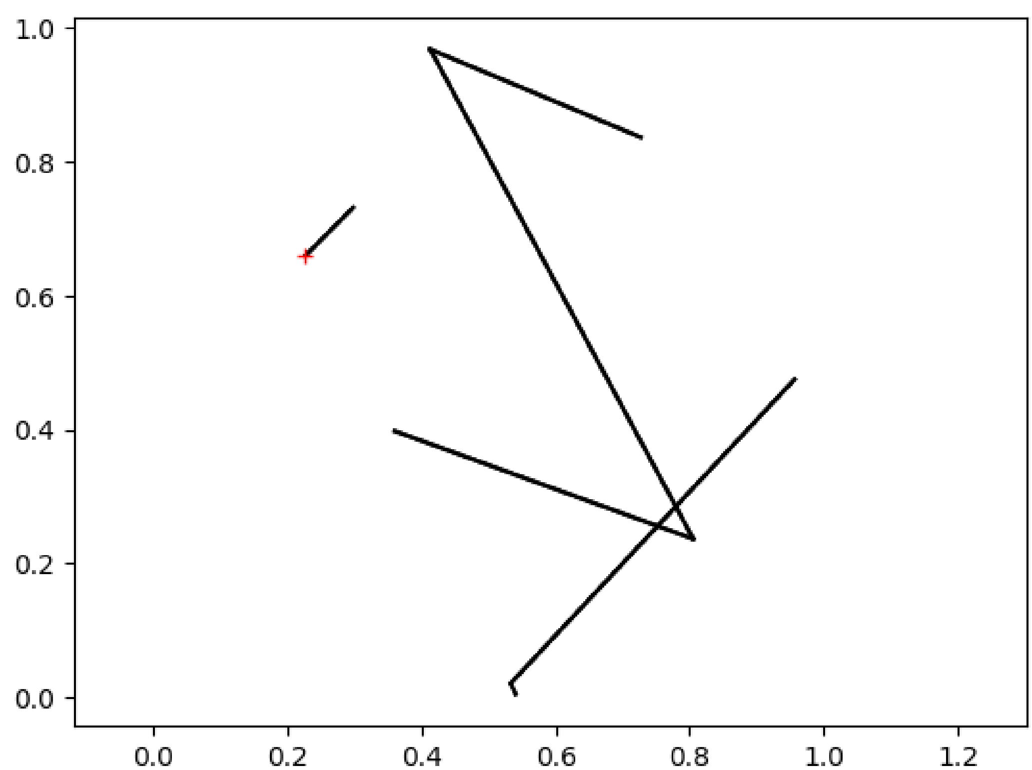

Picked
Nearest
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.2
0.4
0.6
0.8



Picked
Nearest
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2



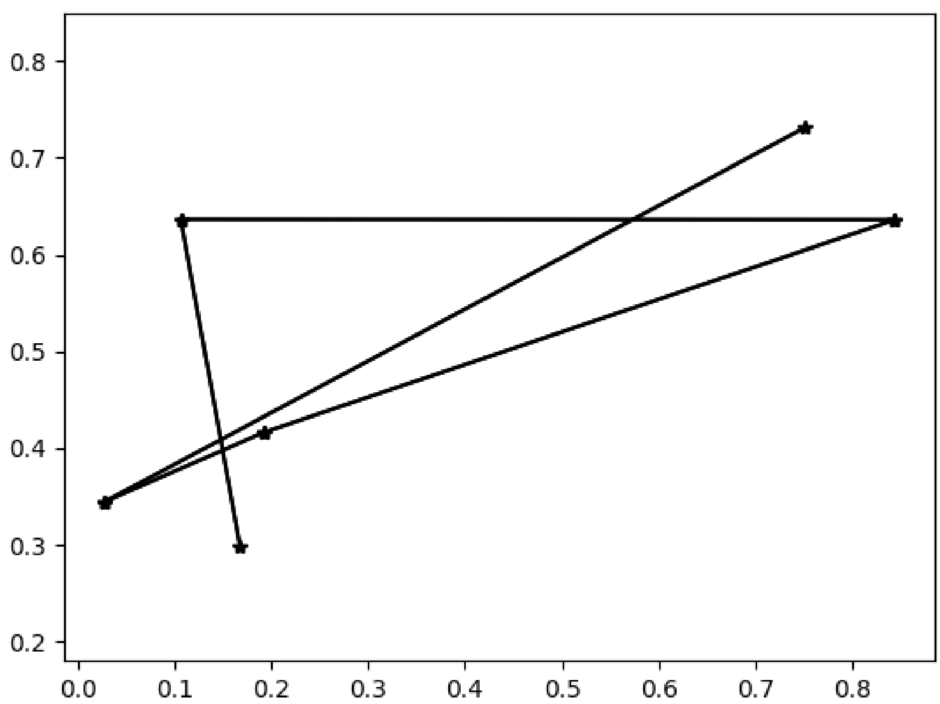
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2

[57]:
```
%matplotlib widget
fig, ax = plt.subplots()
graph = generate_random_graph()
graph.plot(ax)

def onclick_nearest(event):
    picked_pt = np.array([event.xdata, event.ydata])
    nearest_pt, dist, nearest_pt_verts = closest_point_on_graph(graph,␣
 ↪picked_pt)
    expand_graph(graph, picked_pt, nearest_pt, nearest_pt_verts)
    ax.clear()
    graph.plot(ax)
    pickedline, = ax.plot(picked_pt[0], picked_pt[1], 'r+', label='Picked')
    ax.legend()

fig.canvas.mpl_connect('button_release_event', onclick_nearest)
```

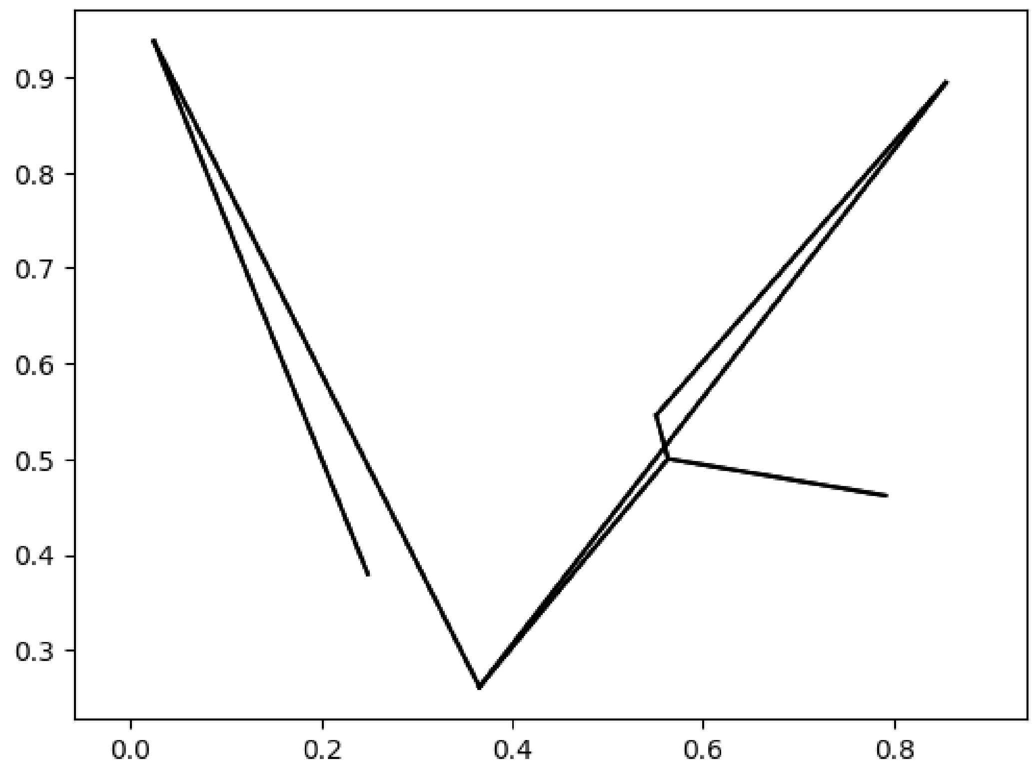
[57]: 15

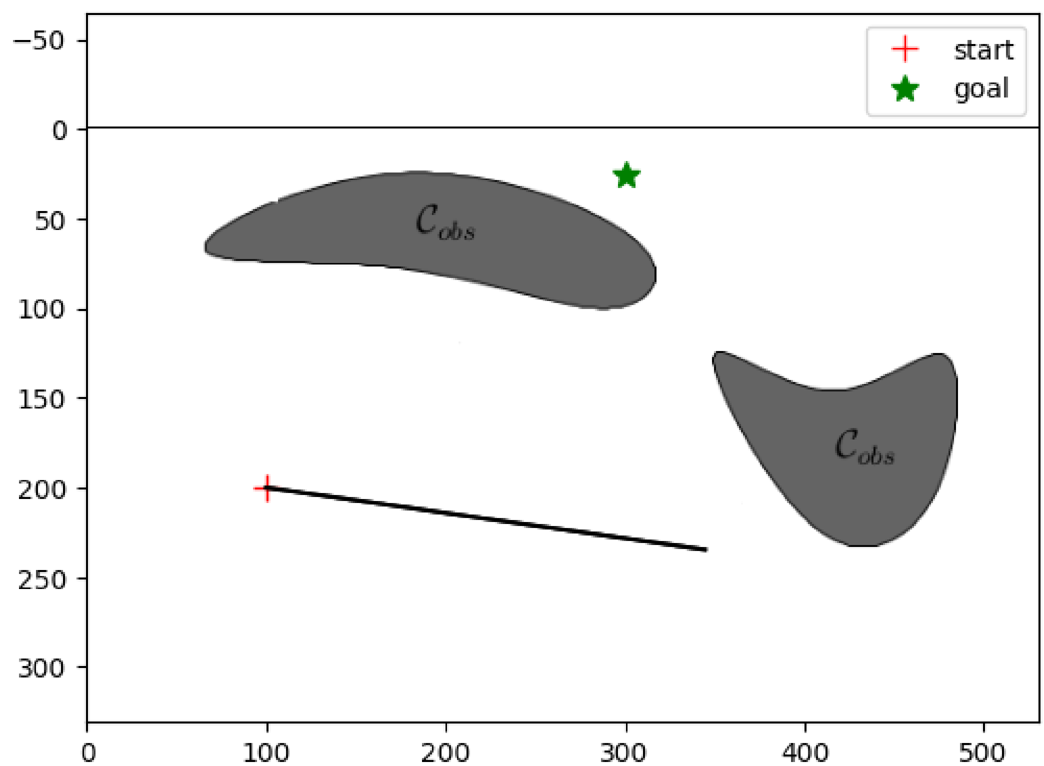


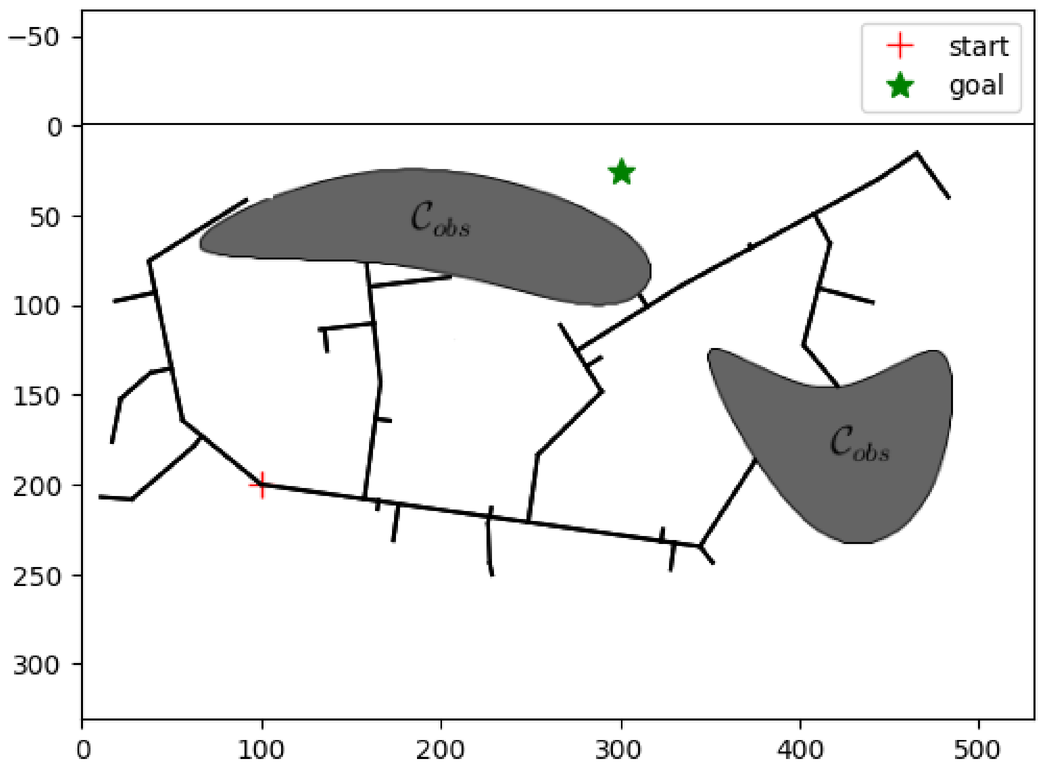
[24]:
```
def does_edge_collide(graph, random_pt, nearest_pt, stepsize):
    steps = int(np.floor(dist / stepsize))
    if steps <= 0:
        return True, None
    direction = (random_pt - nearest_pt) / np.linalg.norm(random_pt -␣
↪nearest_pt)
    all_points = np.arange(1, steps + 1)[:, None]*stepsize*direction+␣
↪nearest_pt[None, :]
    collisions = do_points_collide(img, all_points)
    if collisions[0]:
        return True, None
    indices, = np.nonzero(collisions)
    first_non_colliding = all_points[indices[0]-1] if len(indices)  else␣
↪random_pt
    return False, first_non_colliding
```

[25]:
```
%matplotlib inline
# Need img as the map representation
assert img is not None

Npts = 401 # we are going to sample 100 points, but start with 1 point
# Specify the bounds of the map
pt_min = np.array([0, 0])
pt_max = np.array([img.shape[1], img.shape[0]])

stepsize = 1


# 1. Initialize an empty graph with the start point
graph = Graph()
graph.add_vertex(start)

# 2. While not done
for i in range(Npts):
    # 2.a Sample points on the chosen area.
    random_pt = np.random.rand(2) * (pt_max - pt_min) + pt_min
    nearest_pt, dist, nearest_pt_vids = closest_point_on_graph(graph, random_pt)

    # 2.B Connect the sampled point to the nearest point (vertex or edge)
    # on the graph, as long as the connecting line does not pass through the␣
↪obstacle.
    collision, first_non_colliding = does_edge_collide(graph, random_pt,␣
↪nearest_pt, stepsize)
    if collision:
        continue
    expand_graph(graph, first_non_colliding, nearest_pt, nearest_pt_vids)
```

```
    if i % 50 == 0:
        fig, ax = plt.subplots()
        plot_map(ax, img, goal, start)
        graph.plot(ax)
        plt.show()


# 2.a Sample points on the chosen area.
random_pt = goal

nearest_pt, dist, nearest_pt_vids = closest_point_on_graph(graph, random_pt)

# 2.B Connect the sampled point to the nearest point (vertex or edge)
# on the graph, as long as the connecting line does not pass through the␣
 ↪obstacle.
collision, first_non_colliding = does_edge_collide(graph, random_pt,␣
 ↪nearest_pt, stepsize)
assert collision is False
goal_vert = expand_graph(graph, first_non_colliding, nearest_pt,␣
 ↪nearest_pt_vids)

fig, ax = plt.subplots()
plot_map(ax, img, goal, start)
graph.plot(ax)
plt.show()
```


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.0
0.2
0.4
0.6
0.8


start
goal
$\mathcal{C}_{obs}$
$\mathcal{C}_{obs}$
−50
0
50
100
150
200
250
300
0
100
200
300
400
500


start
goal
$\mathcal{C}_{obs}$
$\mathcal{C}_{obs}$
−50
0
50
100
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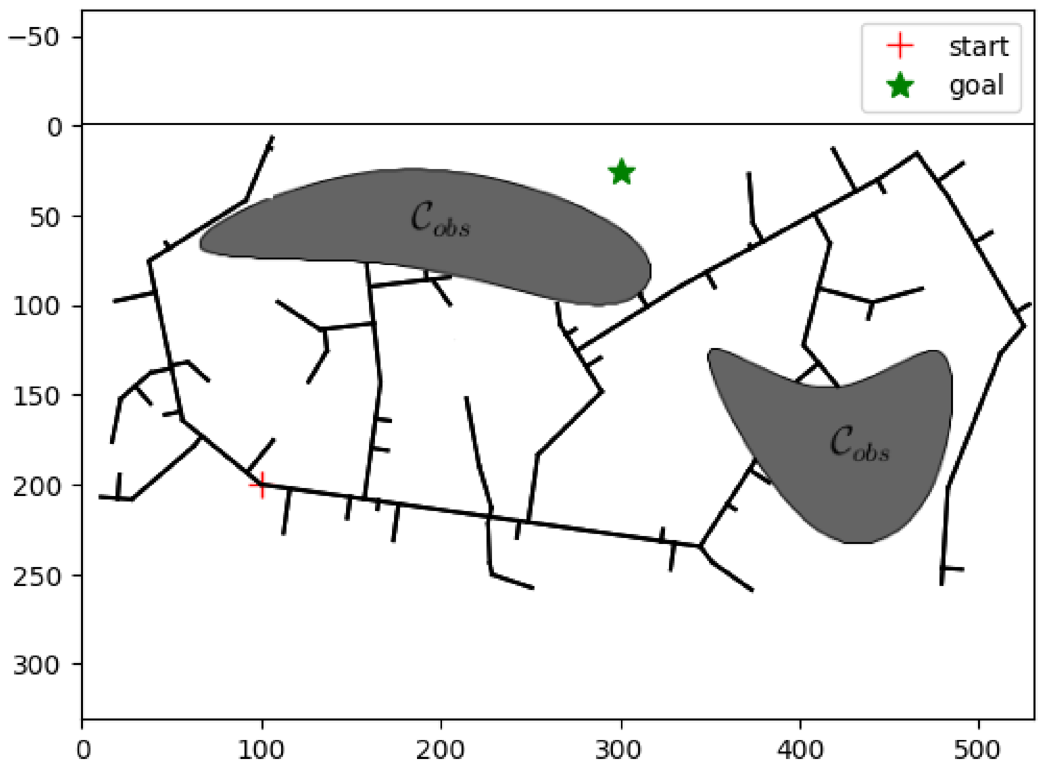
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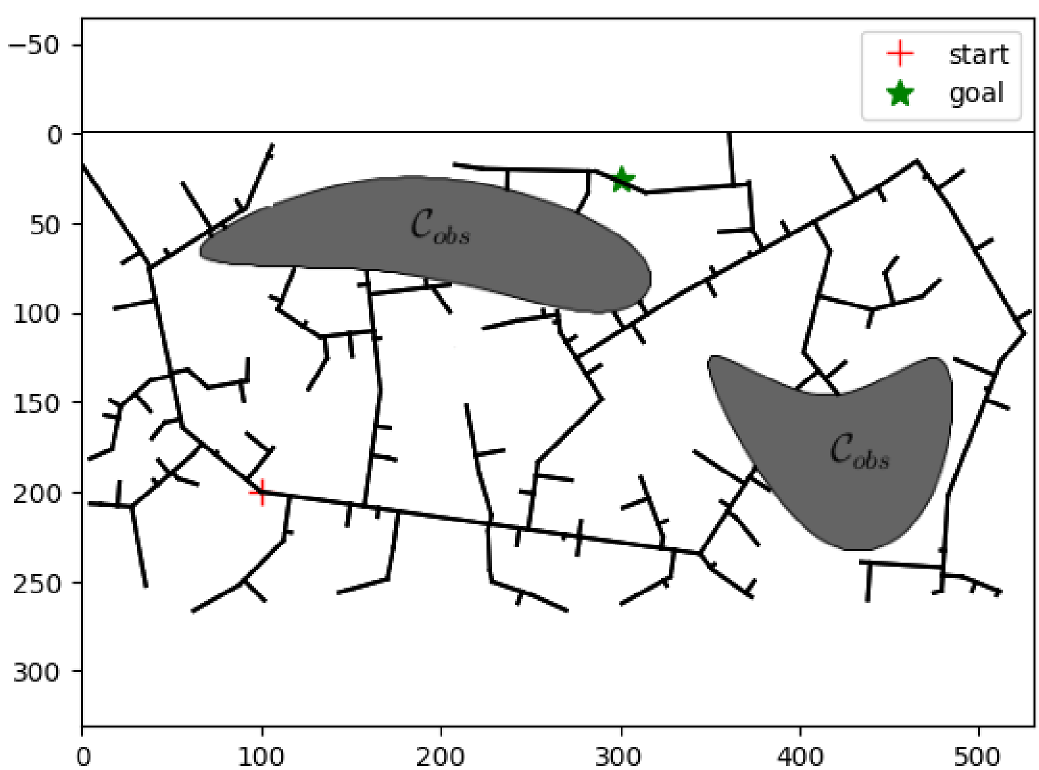
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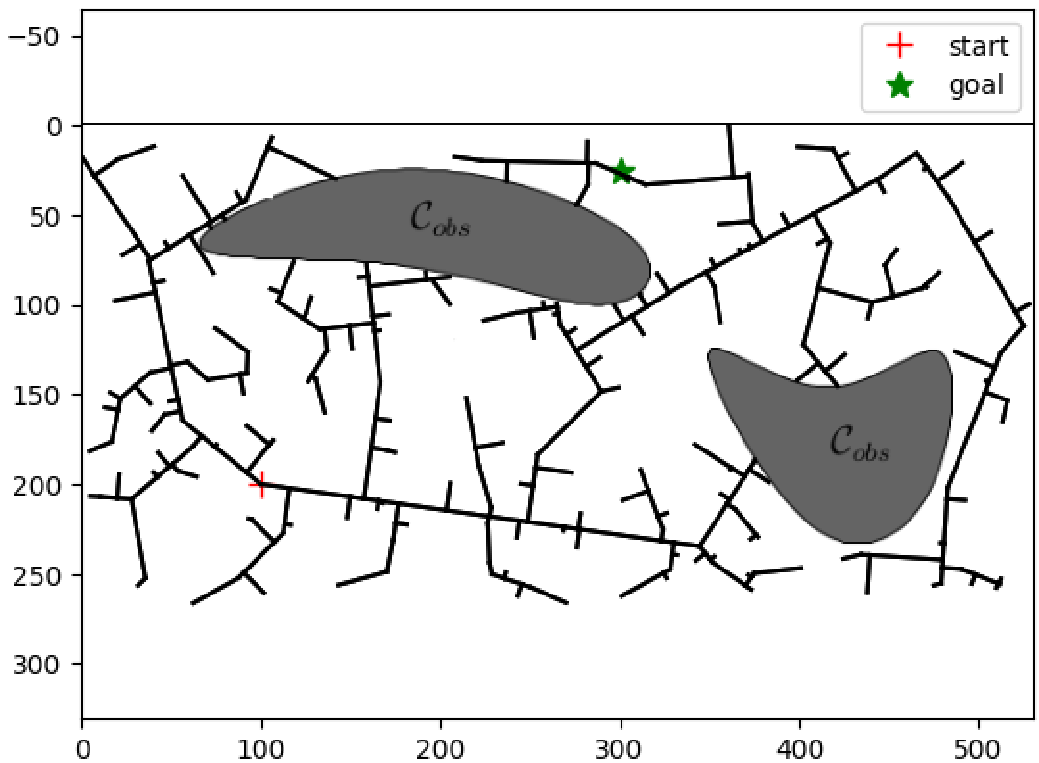
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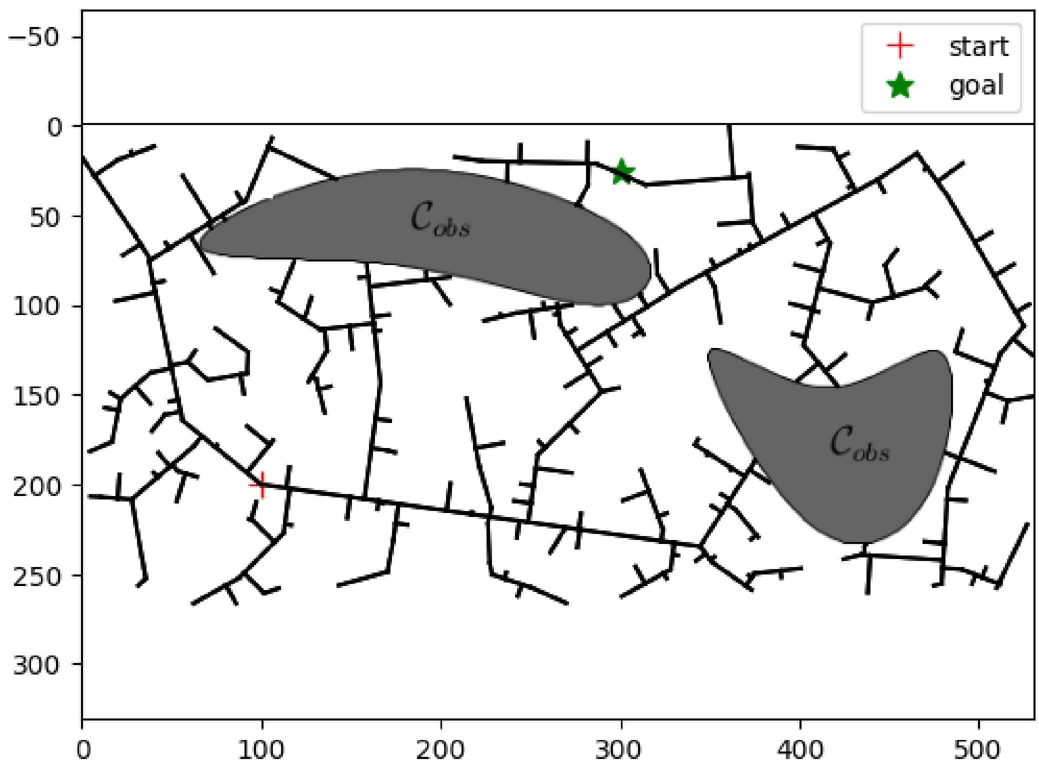
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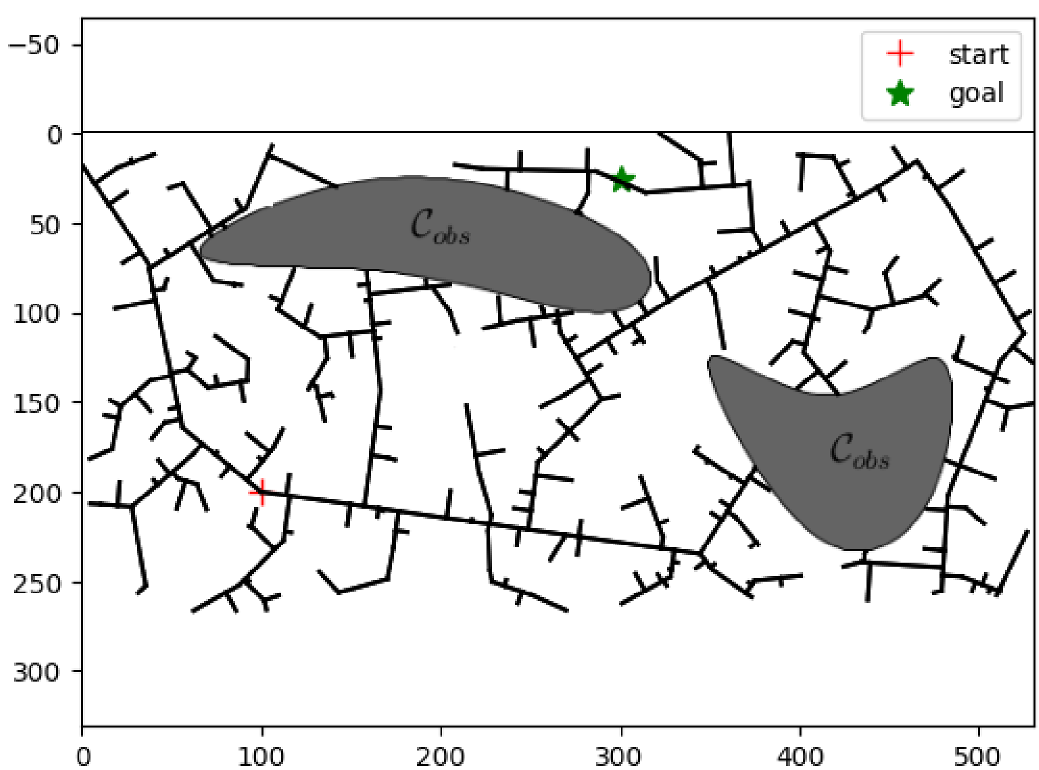
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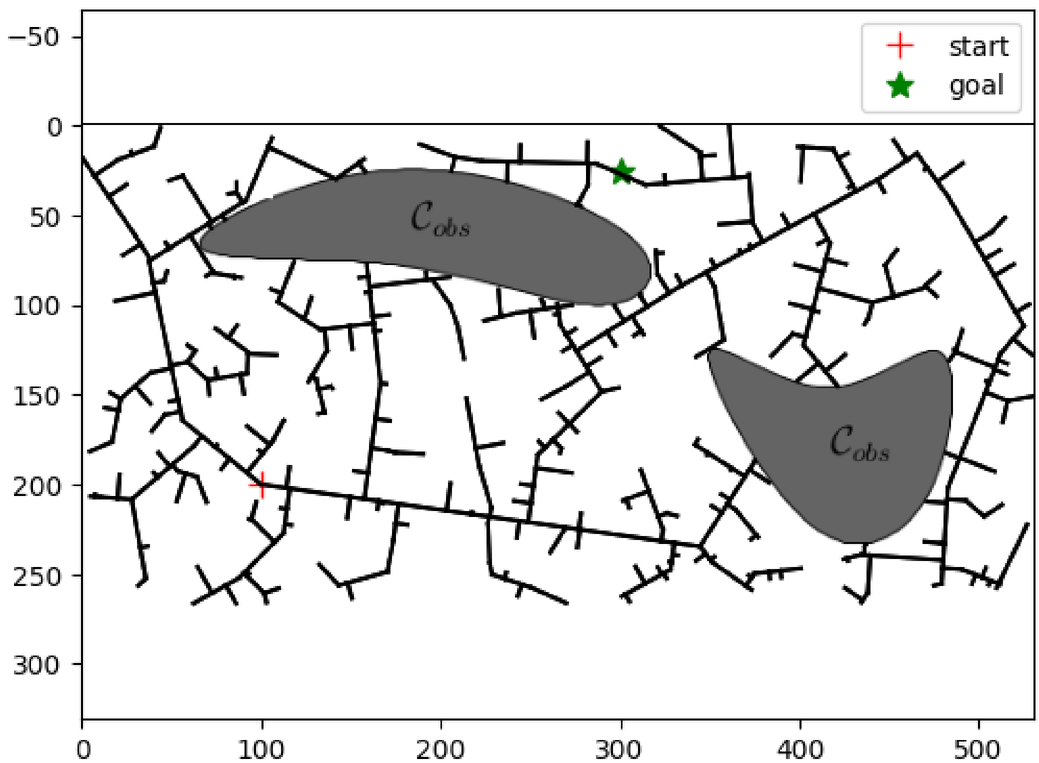
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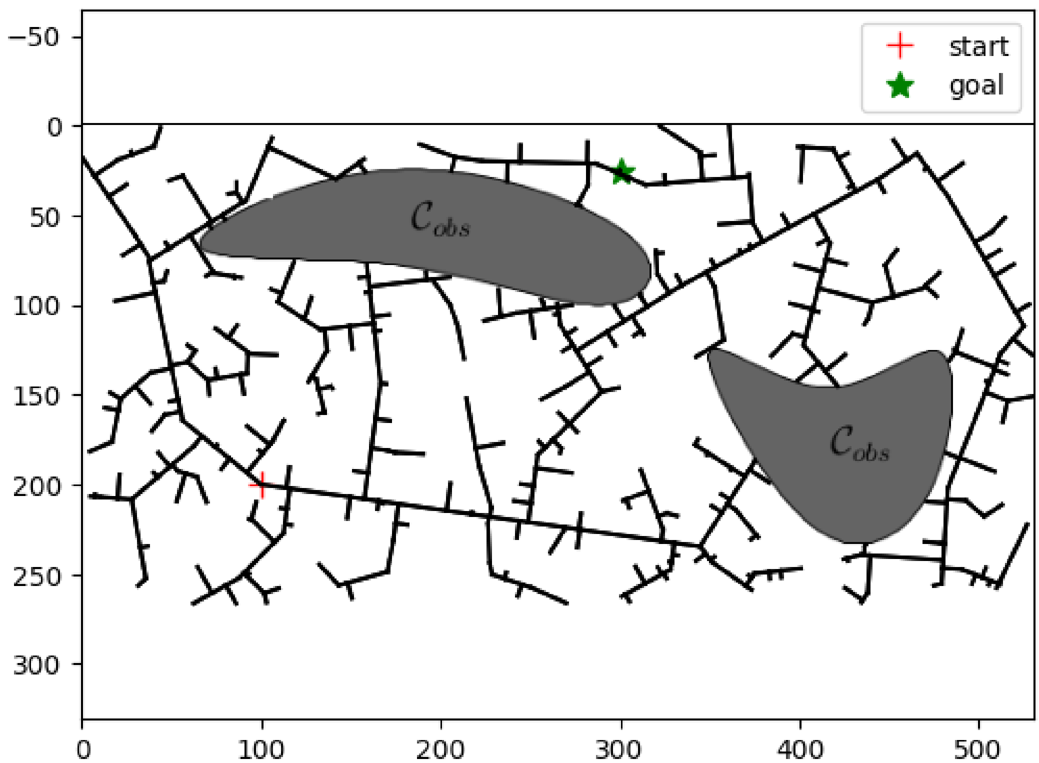
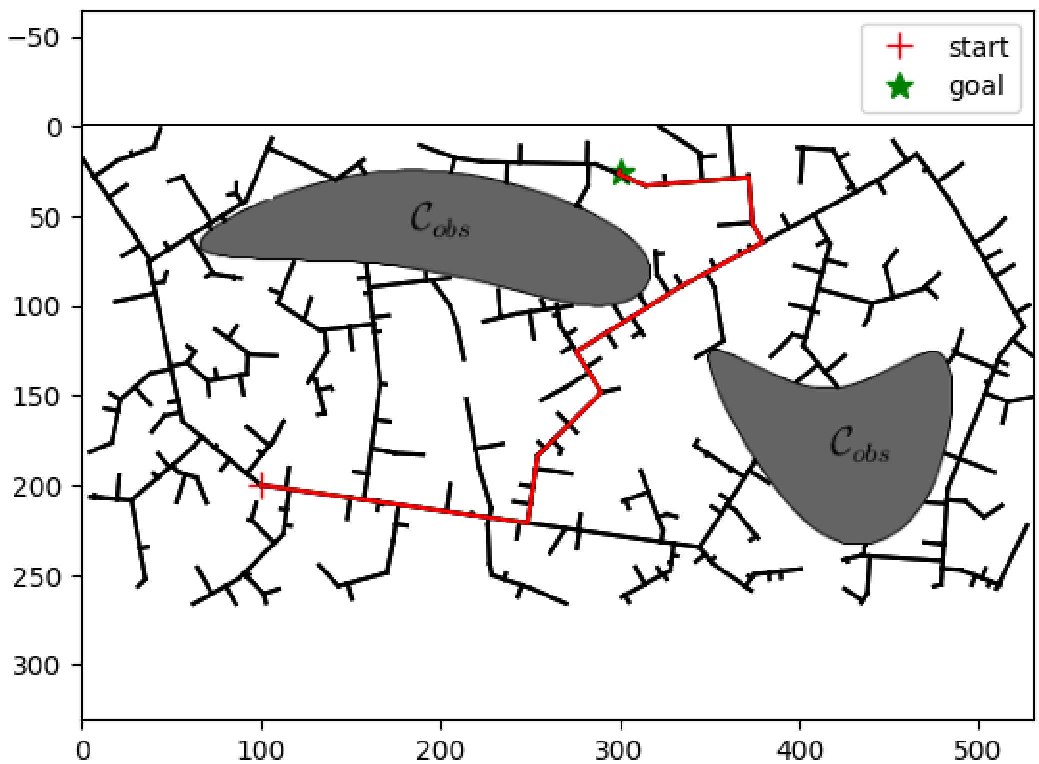


[26]:
```
from astar import astar, backtrace_path
from functools import partial
import math

def euclidean_heurist_dist(node, goal, scale=1):
    x_n, y_n = node.coord
    x_g, y_g = goal.coord
    return scale*math.sqrt((x_n-x_g)**2 + (y_n - y_g)**2)


debugf=open('log.txt', 'w')
start_vert = graph.get_vertex(0)

success, search_path, node2parent, node2dist = astar(
    graph, partial(euclidean_heurist_dist, scale=1),
    start_vert, goal_vert, debug=True, debugf=debugf)
debugf.close()
```

[27]:
```
#print(success, search_path)
assert success
#anim = maze.animate(search_path)
#anim.save(filename='astar-anim.gif', writer='pillow')
```

```
path = list(backtrace_path(node2parent, start_vert, goal_vert))
#maze.init_plots(reinit=True)
#print(path)

fig, ax = plt.subplots()
plot_map(ax, img, goal, start)
graph.plot(ax)
path_plot = graph.plot_path(ax, path, color='r') # Draws the traced shortest␣
 ↪path
plt.savefig('rrt-maze.pdf')
```




```
[ ]:
```