ECE 417/598: Pseudo-Inverse review

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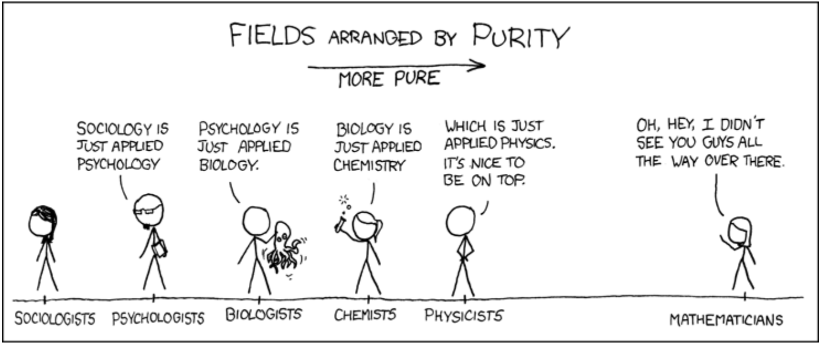
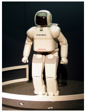
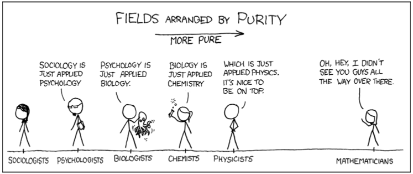

FIELDS ARRANGED BY PURITY
MORE PURE
SOCIOLOGY IS JUST APPLIED PSYCHOLOGY
PSYCHOLOGY IS JUST APPLIED BIOLOGY.
BIOLOGY IS JUST APPLIED CHEMISTRY
WHICH IS JUST APPLIED PHYSICS. IT'S NICE TO BE ON TOP.
OH, HEY, I DIDN'T SEE YOU GUYS ALL THE WAY OVER THERE.
SOCIOLOGISTS
PSYCHOLOGISTS
BIOLOGISTS
CHEMISTS
PHYSICISTS
MATHEMATICIANS

Robotics is applied everything





Link to github

Pseudo-Inverse

$$\mathbf{A}\mathbf{A}^{\dagger}\mathbf{A} = \mathbf{A} \tag{1}$$

$$\text{If SVD of } \mathbf{A} \text{ is given by } \mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^{\top} \text{ then } \mathbf{A}^{\dagger} = \mathbf{U}\Sigma^{-1}\mathbf{V}^{\top} \tag{2}$$

$$\text{if } \mathbf{A} \text{ is tall, then } \mathbf{A}^{\dagger} = (\mathbf{A}^{\top}\mathbf{A})^{-1}\mathbf{A}^{\top} \tag{3}$$

$$\text{if } \mathbf{A} \text{ is fat, then } \mathbf{A}^{\dagger} = \mathbf{A}^{\top}(\mathbf{A}\mathbf{A}^{\top})^{-1} \tag{4}$$

[1]

[1] See Appendix A of Gilbert Strang (1988): Linear Algebra and Its Applications

Pseudo-Inverse for tall matrix by Optimization

$$
\begin{aligned}
&\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2^2 && (5)\\
&= \min_{\mathbf{x}} (A\mathbf{x} - \mathbf{b})^\top (A\mathbf{x} - \mathbf{b}) && (6)\\
&= \min_{\mathbf{x}} (\mathbf{x}^\top A^\top - \mathbf{b}^\top)(A\mathbf{x} - \mathbf{b}) && (7)\\
&= \min_{\mathbf{x}} (\mathbf{x}^\top A^\top - \mathbf{b}^\top)(A\mathbf{x} - \mathbf{b}) && (8)\\
&= \min_{\mathbf{x}} \mathbf{x}^\top A^\top A\mathbf{x} - \mathbf{b}^\top A\mathbf{x} - \mathbf{x}^\top A^\top \mathbf{b} + \mathbf{b}^\top \mathbf{b} && (9)
\end{aligned}
$$

2

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[2] Also see Chapter 3 of Gilbert Strang (1988): Linear Algebra and Its Applications

Recall that a minimum (or maximum) point of a differentiable function $f(\mathbf{x})$, $f'(\mathbf{x}) = 0$. Let us define vector derivative as

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f(\mathbf{x})}{\partial x_1} \\ \frac{\partial f(\mathbf{x})}{\partial x_2} \\ \vdots \\ \frac{\partial f(\mathbf{x})}{\partial x_n} \end{bmatrix} \tag{10}$$

You can verfiy that

$$\frac{\partial}{\partial \mathbf{x}} \mathbf{x}^\top Q \mathbf{x} = 2Q\mathbf{x} \tag{11}$$

$$\frac{\partial}{\partial \mathbf{x}} \mathbf{b}^\top \mathbf{x} = \mathbf{b} \tag{12}$$

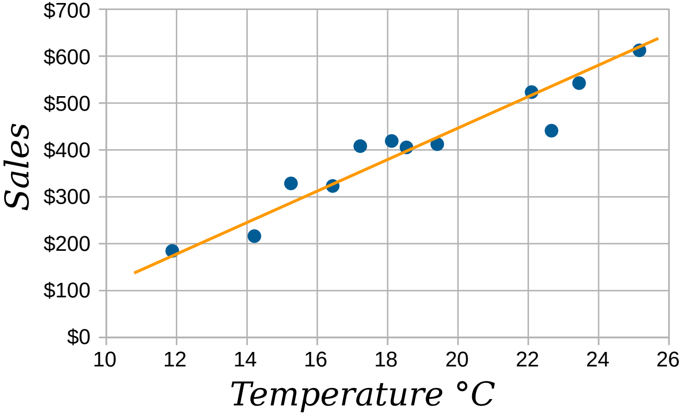
At a minimum point $\mathbf{x}$,

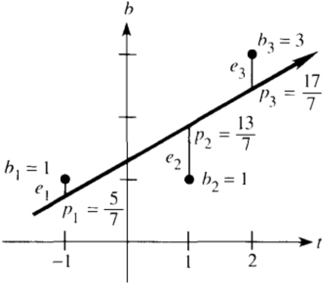
$$\frac{\partial}{\partial \mathbf{x}} \mathbf{x}^\top A^\top A \mathbf{x} - \mathbf{b}^\top A \mathbf{x} - \mathbf{x}^\top A^\top \mathbf{b} + \mathbf{b}^\top \mathbf{b} = 0 \tag{13}$$

$$\text{or } 2A^\top A \mathbf{x} - 2A^\top \mathbf{b} = 0 \tag{14}$$

$$\text{or } \mathbf{x} = \underbrace{(A^\top A)^{-1} A^\top}_{A^\dagger} \mathbf{b} \tag{15}$$

Application





b
$b_3 = 3$
e_3
$p_3 = \frac{17}{7}$
$p_2 = \frac{13}{7}$
e_2
$b_1 = 1$
e_1
$b_2 = 1$
$p_1 = \frac{5}{7}$
t
−1
1
2

Homogeneous representation of lines

$$ax + by + c = 0$$

Projective space

$$\mathbb{P}^2 = \mathbb{R}^3 - (0,0,0)^\top$$

Homogeneous representation of points

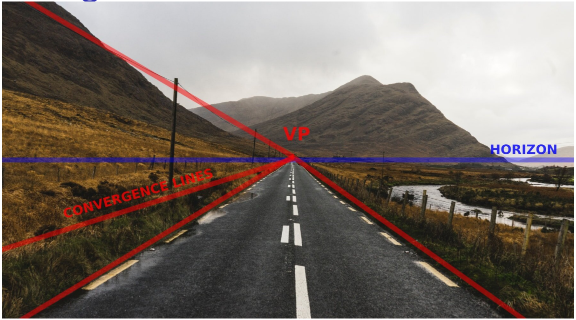
$$ax + by + c = 0$$

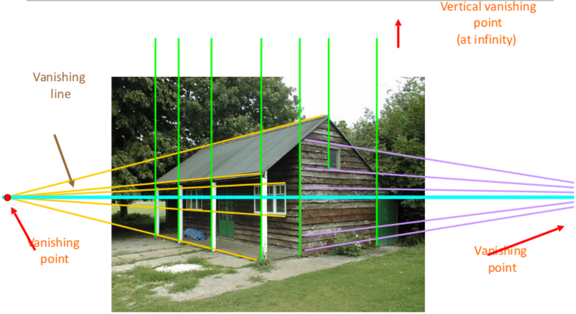
Eq of line in Projective space

The point $\mathbf{x} \in \mathbb{P}^2$ lies on a line $\mathbf{l}$ if and only if $\mathbf{x}^\top \mathbf{l} = 0$.

Intersection of lines

Vanishing Point





Courtesy Antonio Criminisi

Vanishing Point