Trajectory in Terms of state space

Control : Proportional Integral Derivative (PID)

2. LQR: Linear Quadratic Regulator

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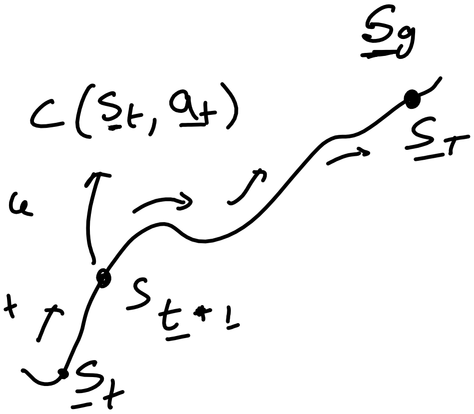
Planning/Control ⟶ Trajectory in terms of action space




$$\min_{\{a_t\}_{t=1}^{T}} \sum_{t=1}^{T} c(\underline{s}_t, \underline{a}_t)$$

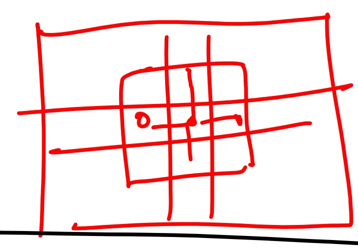
subject to $\underline{s}_{t+1} = f(\underline{s}_t, \underline{a}_t)$

Examples

$$\overbrace{\begin{bmatrix} x_{t+1} \\ y_{t+1} \\ \theta_{t+1} \end{bmatrix}}^{s_{t+1}} = \overbrace{\begin{bmatrix} v_t \cos\theta_t \Delta t + x_t \\ v_t \sin\theta_t \Delta t + y_t \\ \omega_t \Delta t + \theta_t \end{bmatrix}}^{f(\underline{s}_t, \underline{a}_t)}$$

$$\underline{c(\underline{s}_t, \underline{a}_t)} = \|\underline{s}_g - \underline{s}_t\|^2 + \frac{1}{2} m v^2 + \frac{1}{2} I_m \omega^2$$

$$\underline{s}_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} \quad \underline{a}_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}$$



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System dynamics to be Linear

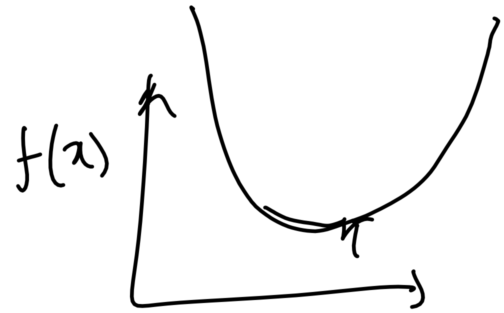
Cost function to be Quadratic

$$\underline{s}_{t+1} = f(\underline{s}_t, \underline{a}_t) = \underset{n \times n}{A_t} \underset{n \times 1}{\underline{s}_t} + \underset{n \times m}{B_t} \underset{m \times 1}{\underline{a}_t}$$

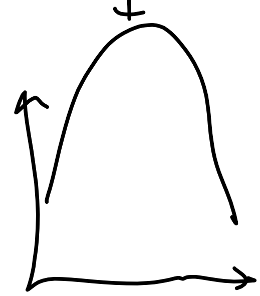
$$c(s_t, a_t) = \underline{s}_t^T \underset{n \times n}{Q} \underline{s}_t + \underline{a}_t^T \underset{m \times m}{R} \underline{a}_t + \underbrace{\underline{b}^T \underline{s}_t + \underline{c}^T \underline{a}_t + \underline{s}_t^T N \underline{a}_t}_{\text{optional}}$$

$$\underline{x}^T A \underline{x} = x^2 a_{11} + y^2 a_{22} + a_{12} xy + $$

$+2x^2 - 5x + 6$




$-2x^2 + 5x + 6$



$Q \succ 0$

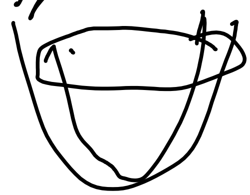
Positive definite

$Q \prec 0$

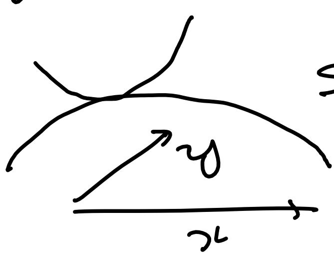
negative definite

Q can be indefinite

$Q \succeq 0$



Positive semi definite




saddle point

$Q \preceq 0$

Negative semi definite

Defn: $Q \succ 0$

for any $\underline{x}$

$\underline{x}^T Q \underline{x} > 0$

Test: all eigenvalues $> 0$

$\underline{x}^T (E \Lambda E^T) \underline{x} > 0$

matrix of eigen vector

Diagonal matrix of eigen values

$\lambda_1 y_1^2 + \lambda_2 y_2^2 + \dots + \lambda_n y_n^2 = \underline{y}^T \Lambda \underline{y}$

$\underline{y} = E \underline{x}$

# LQR

$$\min_{\{\underline{a}_t\}} \sum_{t=1}^{T} \underline{s}_t^T Q \underline{s}_t + \underline{a}_t^T R \underline{a}_t$$

$$\text{s.t.} \quad \underline{s}_{t+1} = A_t \underline{s}_t + B_t \underline{u}_t$$

Dynamic programming (mathematical Induction)

$$\min_{\{\underline{a}_t\}_{t=1}^{T-1}} \left( \sum_{t=1}^{T-1} \underline{s}_t^T Q \underline{s}_t + \underline{a}_t^T R \underline{a}_t \right) + \left( \min_{a_T} \underline{s}_T^T Q \underline{s}_T + \underline{a}_T^T R \underline{a}_t \right)$$