ECE 417 Midterm 1

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About the exam

1. There are total 5 problems. You must attempt all 5.
2. Maximum marks: 50 (70 with bonus).
3. Maximum time allotted: 50 min
4. Calculators are allowed.
5. One US Letter size or A4 size cheat sheet (both-sides) is allowed.

Problem 1 *What are the two criteria for a 3×3 matrix A to be a valid rotation matrix? (5 min, 5 marks)*

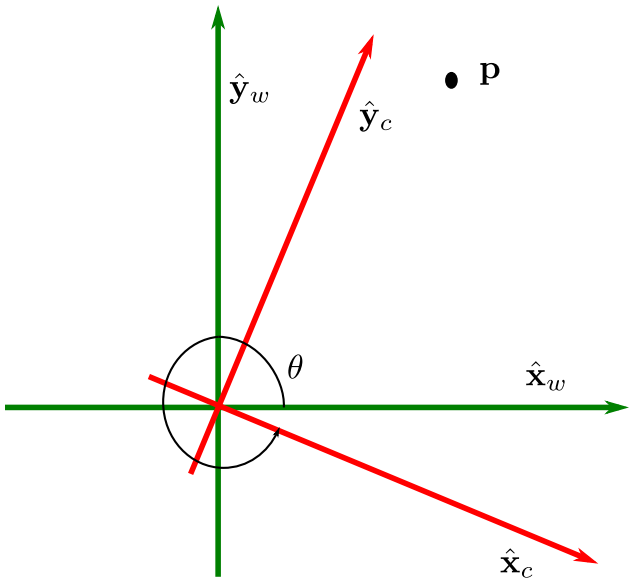

Figure 1: Rotation between red frame and green coordinate frame.

Problem 2 *For Fig 1, write the rotation matrix $R(\theta)$ that converts between coordinates of point from red coordinate frame $\mathbf{p}_c$ to green coordinate frame $\mathbf{p}_w$ such that $\mathbf{p}_w = R(\theta)\mathbf{p}_c$. (Optional part) Test your formula if it looks correct for $\theta = -30°$, $\mathbf{p}_c = \begin{bmatrix}1\\3\end{bmatrix}$. (5 min, 5 marks)*

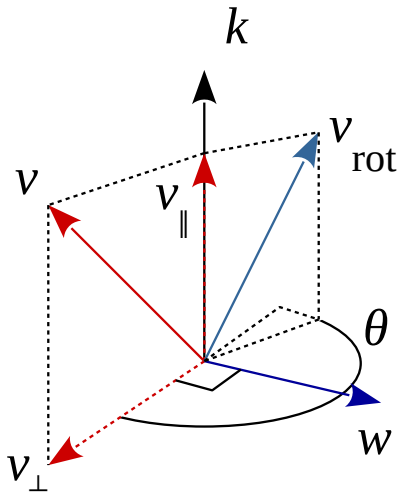

Figure 2: Rotation of point $\mathbf{v}$ around axis $\mathbf{k}$ by angle θ

Problem 3 *In Figure 2, we are rotating point* $\mathbf{v}$ *around axis unit-vector* $\hat{\mathbf{k}}$ *by an angle* θ*.* $\mathbf{v}_\perp$ *lies in the plane of* $\mathbf{v}$ *and* $\hat{\mathbf{k}}$ *and is orthogonal (perpendicular) to* $\mathbf{k}$*.* $\mathbf{w}$ *is perpendicular to the plane of* $\mathbf{v}$ *and* $\hat{\mathbf{k}}$*. First, write the unit-vector* $\hat{\mathbf{w}}$ *in terms of* $\mathbf{v}$ *and* $\hat{\mathbf{k}}$*. Then write unit-vector* $\hat{\mathbf{v}}_\perp$ *in terms of* $\mathbf{v}$ *and* $\hat{\mathbf{k}}$*. (10 min, 10 marks)*

Problem 4 *Using the fact that at minima (or maxima) of a differentiable function $f(\mathbf{x})$, $\nabla_{\mathbf{x}} f(\mathbf{x}) = 0$, find the minima of the following function,*

$$f(\mathbf{x}) = \mathbf{x}^\top A^\top A\mathbf{x} + 2\mathbf{b}^\top \mathbf{x} + c \tag{1}$$

(10 min, 10 marks)

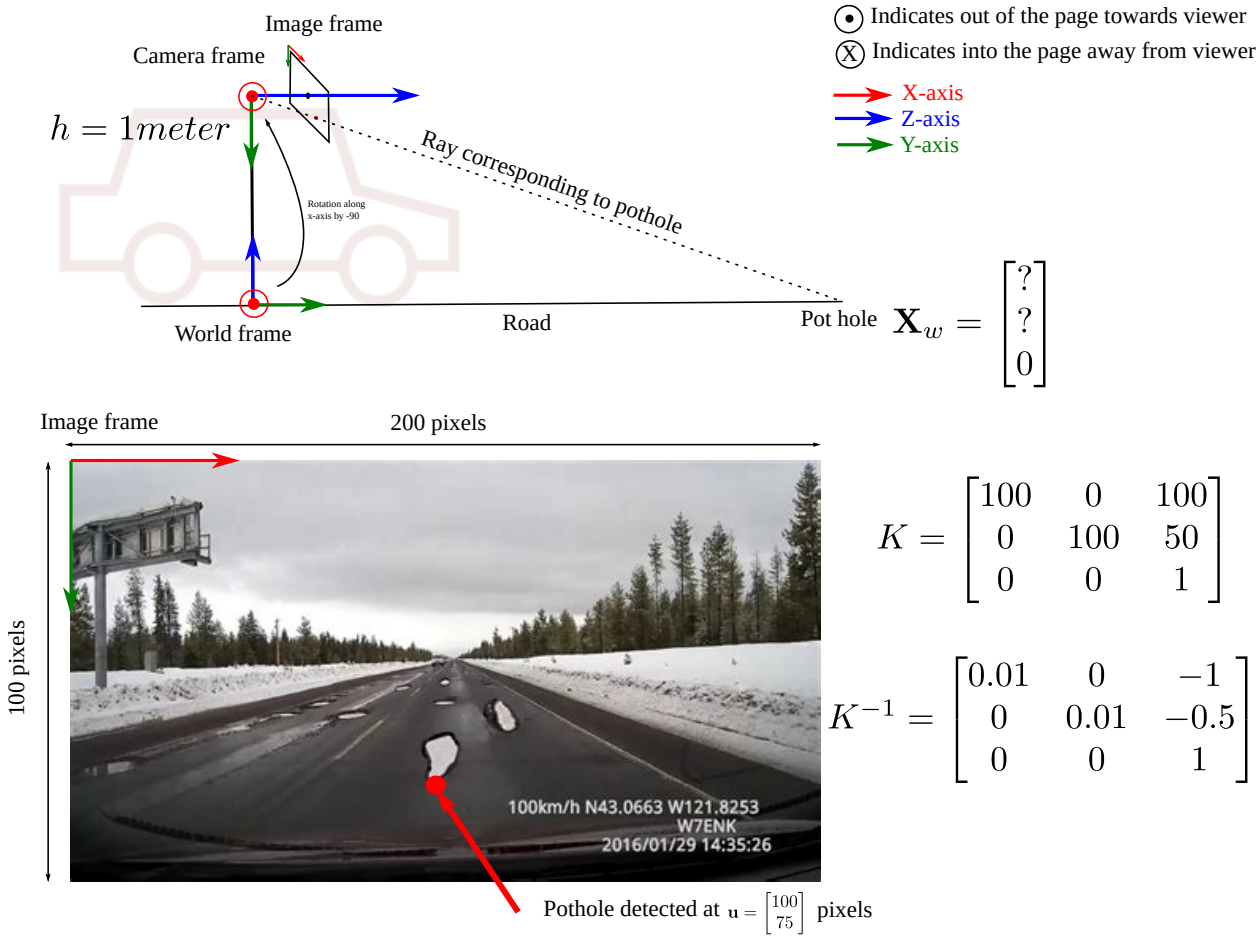

Figure 3: Image road triangulation

Problem 5 *In figure 3 find the 3D position of pothole in the World coordinate frame, in terms of h and K. The Camera mounted directly on top of the world frame. The road is a perfect plane and the pothole lies on the road plane. You do not need to substitute in the values, providing a formula or pseudo-code for computing the pothole coordinates is enough. (20 min, 20 marks, Bonus marks: 20)*