ECE 417/598: Image formation

.

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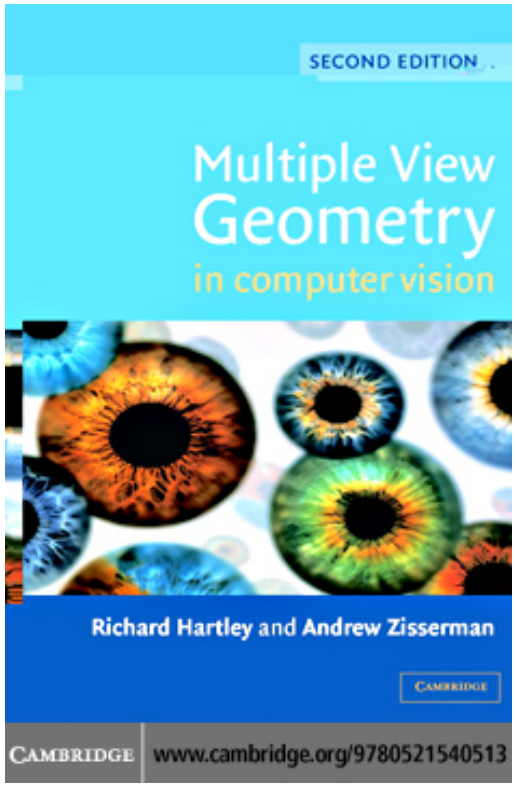
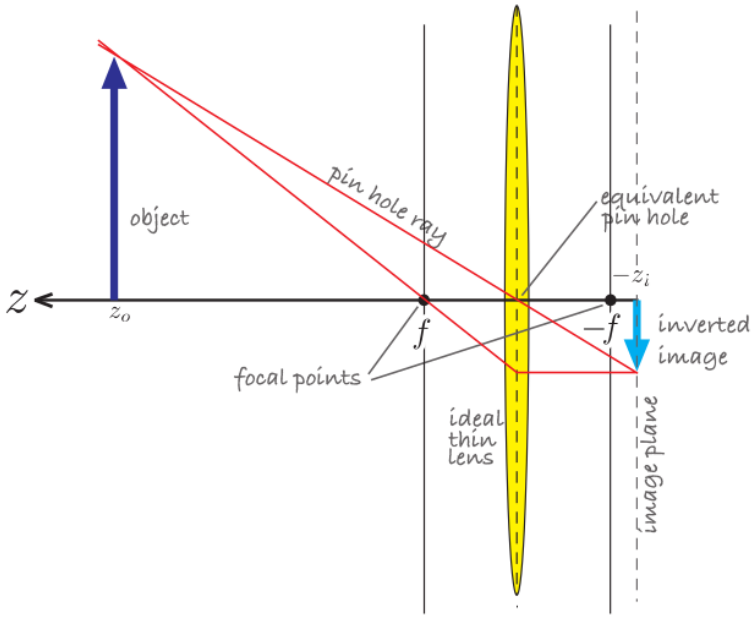
Feb 2, 2022

Additional reference

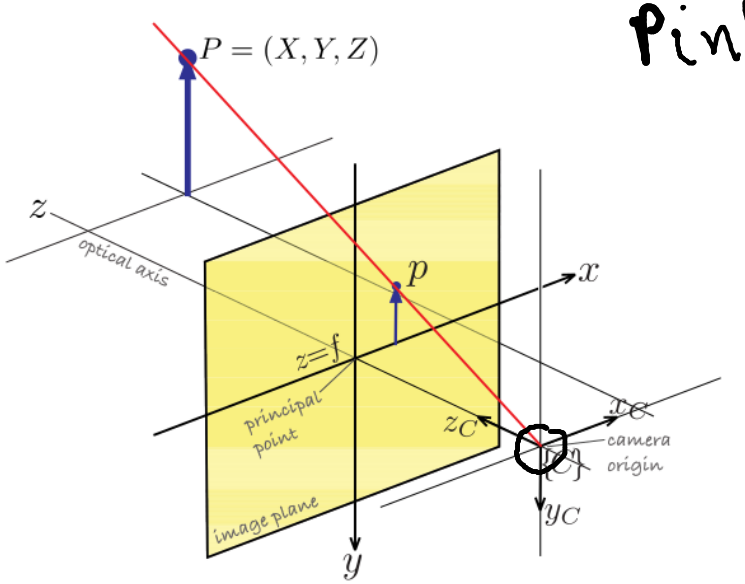
Chapter 6, 7, 8 of 

[1]

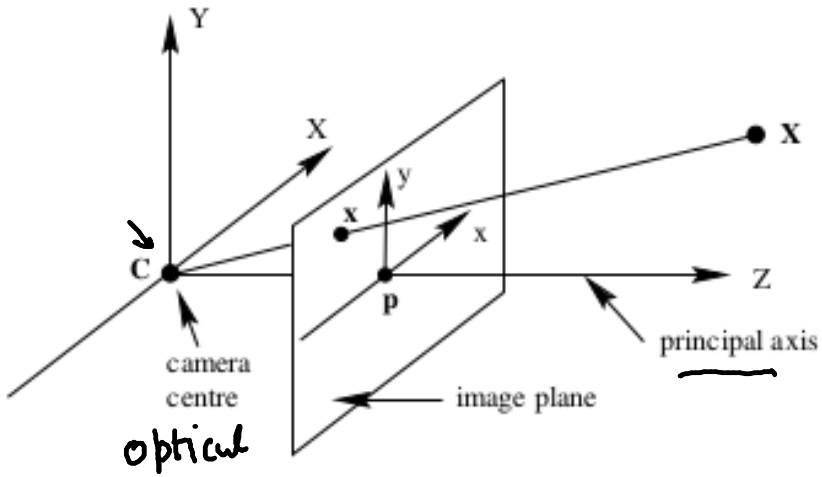
[1] Lookup on libgen.rs

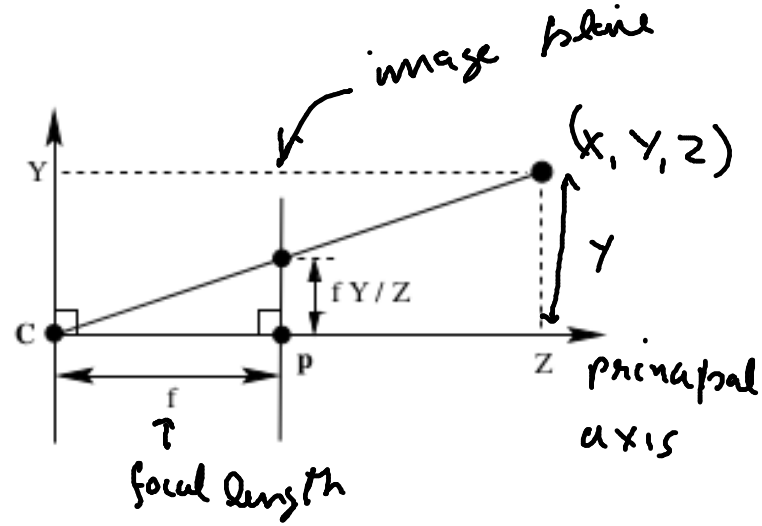


Pinhole camera model



[2]Chapter 11. Corke.





3

$$\frac{y}{f} = \frac{Y}{Z}$$

$$y = \frac{f_y Y}{Z} \qquad x = \frac{f_x X}{Z}$$

[3] Chapter 6. Hartley and Zisserman

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{f_x X}{Z} \\ \frac{f_y Y}{Z} \end{bmatrix} = \begin{bmatrix} f_x & 0 \\ 0 & f_y \end{bmatrix} \begin{bmatrix} \frac{X}{Z} \\ \frac{Y}{Z} \end{bmatrix}$$

Homogeneous coordinates

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$\rightarrow \underline{x} = \lambda \underline{x}$

$$\underline{x} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

K - camera intrinsic matrix

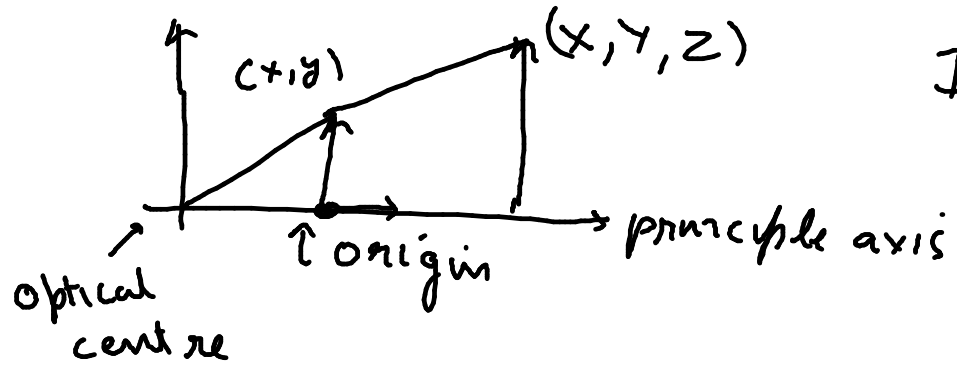
$$\underline{x} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

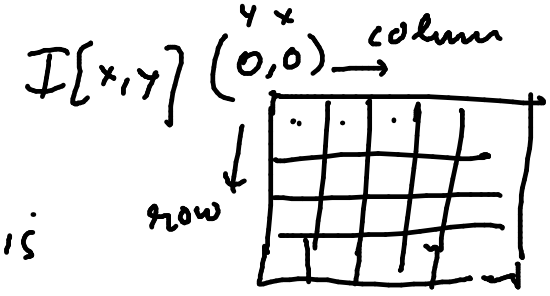
$$\begin{bmatrix} X_c \\ Y_c \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

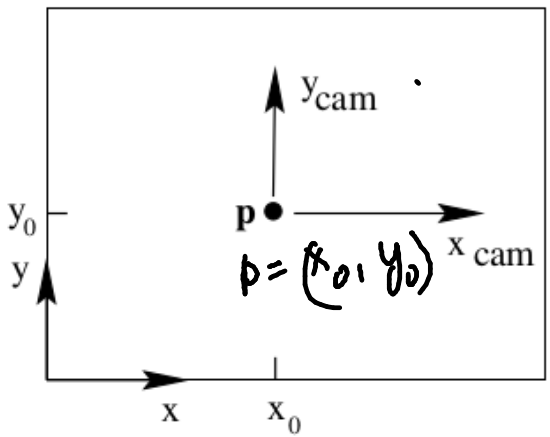
$$= \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\underline{x} = K [I_3 | \underline{0}] \underline{X}$$

$$K = \begin{bmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



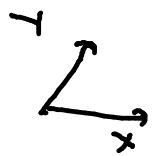




$$x = \frac{f_x X}{Z} + x_0$$

$$y = \frac{f_y Y}{Z} + y_0$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & x_0 & 0 \\ 0 & f_y & y_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

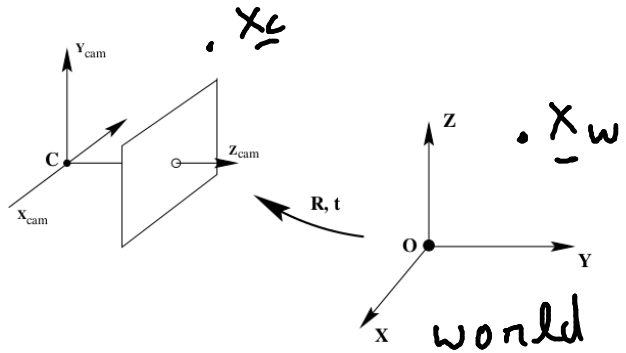


$$K = \begin{bmatrix} f_x & s & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

skew parameter $= s$

[4] Chapter 6. Hartley and Zisserman

Camera extrinsic matrix



[5]

$$\underline{X}_c = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \underline{X}_w \qquad \Leftarrow \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\underline{x} = K[I_3 | 0] X_c$$

$$= K[I_3 | 0] \begin{bmatrix} R_3 & t_3 \\ 0 & 1 \end{bmatrix} X_w$$

$$= K[R_{3\times 1} | t_3] X_w$$

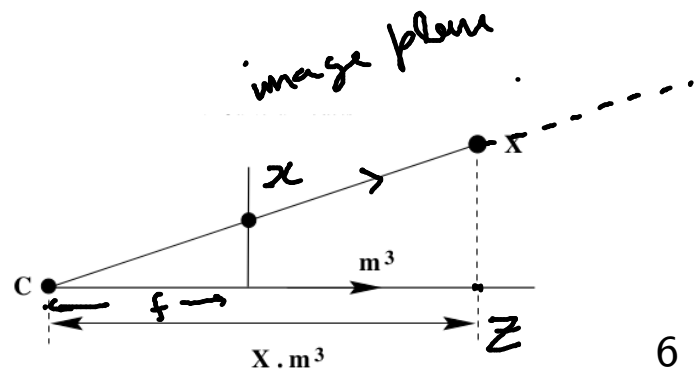
$$\underline{x} = K[{}^cR_w | {}^ct_w] X_w$$

extrinsic camera matrix

[5] Chapter 6. Hartley and Zisserman

3D pt → image 2D pt

image 2D pt → 3D pt



6

$$\frac{X}{Z} = \frac{x}{f}$$

$$X = \lambda \frac{x}{f} \quad , \lambda > 0$$

$$\underline{x} = K[R|t]\underline{X} \qquad \vec{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\underline{x} = KR\vec{X} + Kt$$

QR-decomposit

K = upper triangular

$$[\underline{x} - Kt] = KR\vec{X}$$

$$\vec{X} = (KR)^{-1}[\underline{x} - Kt]$$

R = orthogonal

$$\underline{x} = \lambda \underline{x}$$

$$\vec{X} = (KR)^{-1}[\lambda \underline{x} - Kt]$$

$$= \lambda (KR)^{-1}\underline{x} - (KR)^{-1}Kt$$

[6] Chapter 6. Hartley and Zisserman

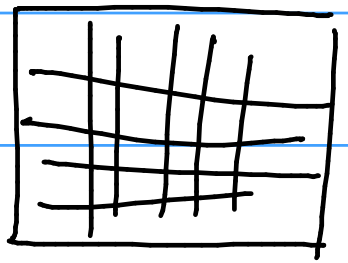
$$\underline{x} = \underbrace{K[R|t]}_{P} \underline{X}$$

$P \in \mathbb{R}^{3\times 4}$

$P \leftarrow$ projection matrix

$$\underline{x} = P\underline{X} \quad \rightarrow \quad \underline{X} = P^{+}\underline{x} \approx x P^{+}\underline{x}$$

Camera calibration

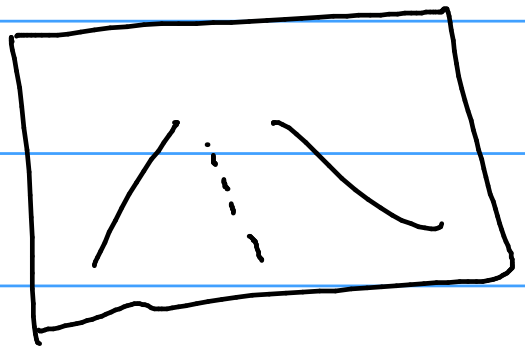


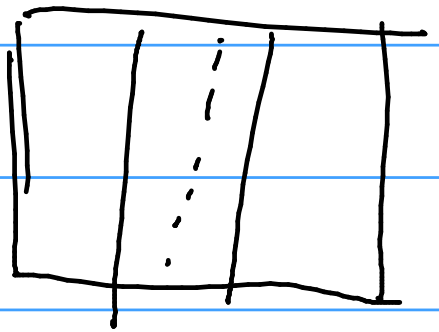
$\rightarrow K$

$\rightarrow K?$

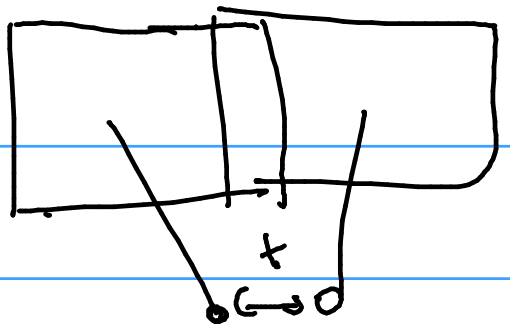
pts

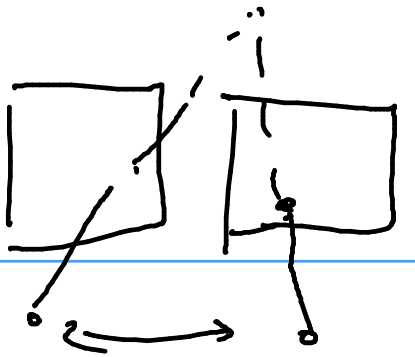
Homography





Stereo dept estimation





Pseudo-Inverse

$\det(M) \neq 0$ non-singular square M

$$M M^{-1} = I = M^{-1} M$$

$P^+ P P^+ = P$

$Px = b$

$x = P^+ b$

over determined or under determined

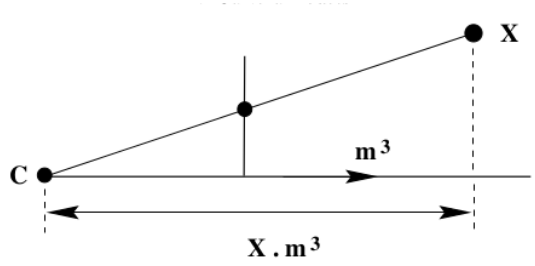
over determined # eq > # var.

$$\#\text{eq} \left\{ \overbrace{\left[\quad \right]}^{\#\text{var}} x = b \right.$$

$Px = b$

$$\min_x \|Px - b\|_2^2$$

Points as rays: aka Prospective geometry



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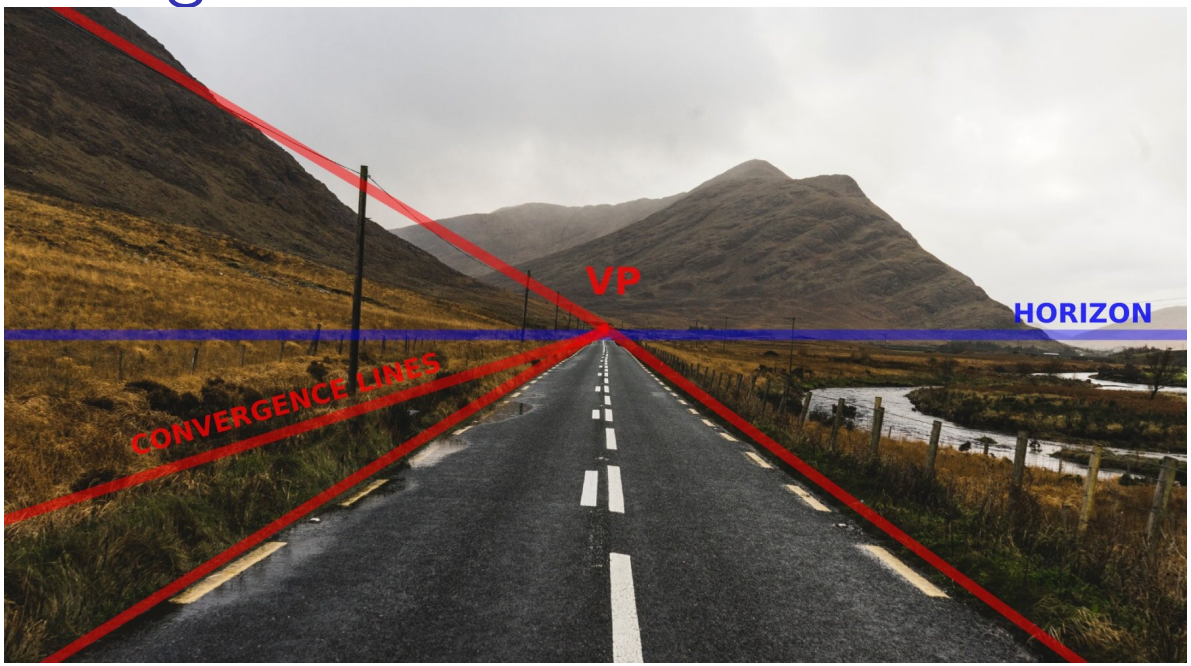
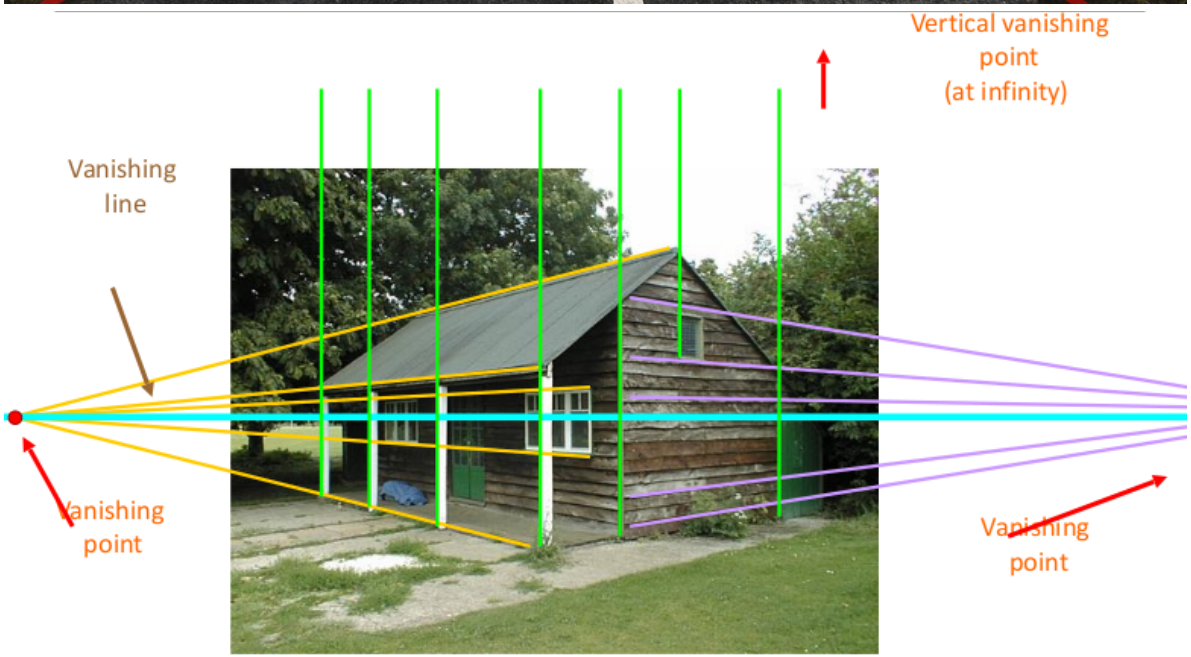
[7] Chapter 6. Hartley and Zisserman

Vanishing Point





e Courtesy Antonio Criminisi

Vanishing Point