State space

↳ Set of all states

What is a state

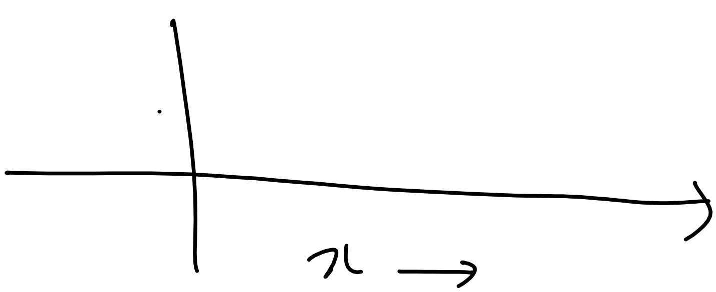
Example;

(x, y) coordinate of the robot in the maze

$S_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix}$

State contains all information needed to plan the future course of action without having to look in the past.

$\underline{S_t} = \begin{bmatrix} x_t \\ v_t \end{bmatrix}$



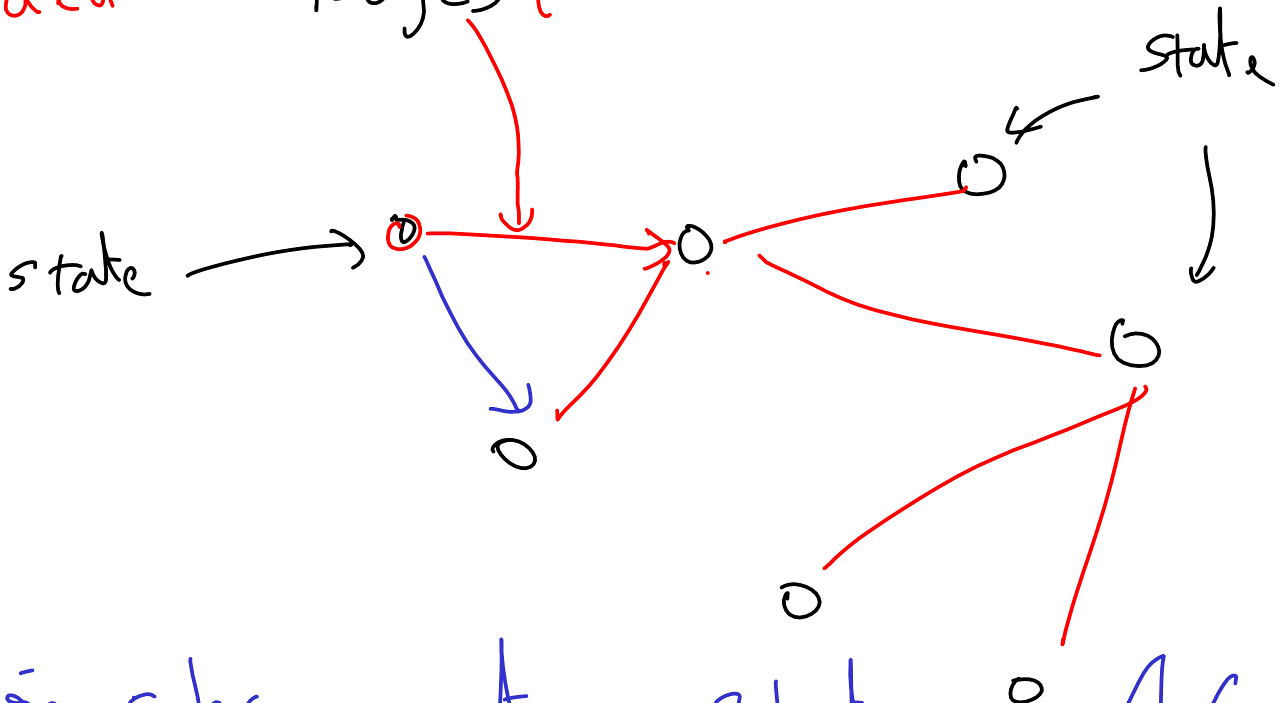
$v = \dot{x}$

$\frac{F}{m} = a = \dot{v}$

$x_{t+\Delta t} = x_t + v_t \Delta t + \frac{1}{2} a \Delta t^2$

Directed Graph → Vertices ≡ States

→ Edges (Directions)



Action space at a states $\mathcal{A}(s_t)$

set of actions you can take at a given state s_t

Denote sets with curly capital letters

Actions?

Example: maze

$A(s_t) = \{Left, Right, Top, Bottom\}$

① States ② Actions

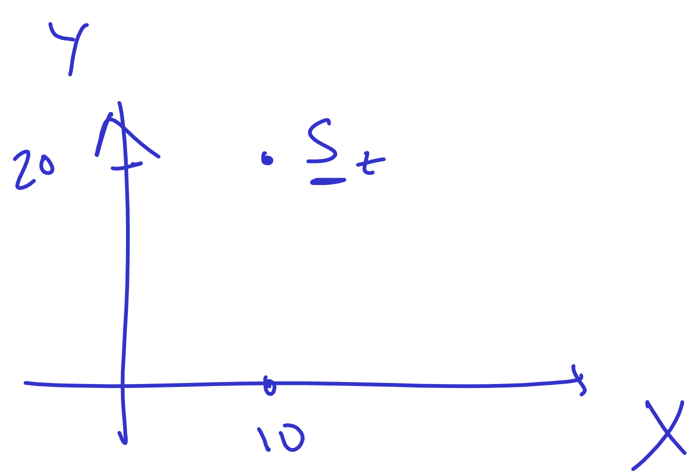
③ Cost function ④ <u>Transition function</u>

$$T(\underbrace{s_t}_{\text{current state}}, \underbrace{a_t}_{\text{action taken}}) \longmapsto \underbrace{s_{t+1}}_{\text{next state}}$$

Example: maze

$$s_t = \begin{array}{c} \begin{bmatrix} 10 \\ 20 \end{bmatrix} \end{array} \begin{array}{l} \to x \\ \to y \end{array}$$

a_t = Left by 1 step



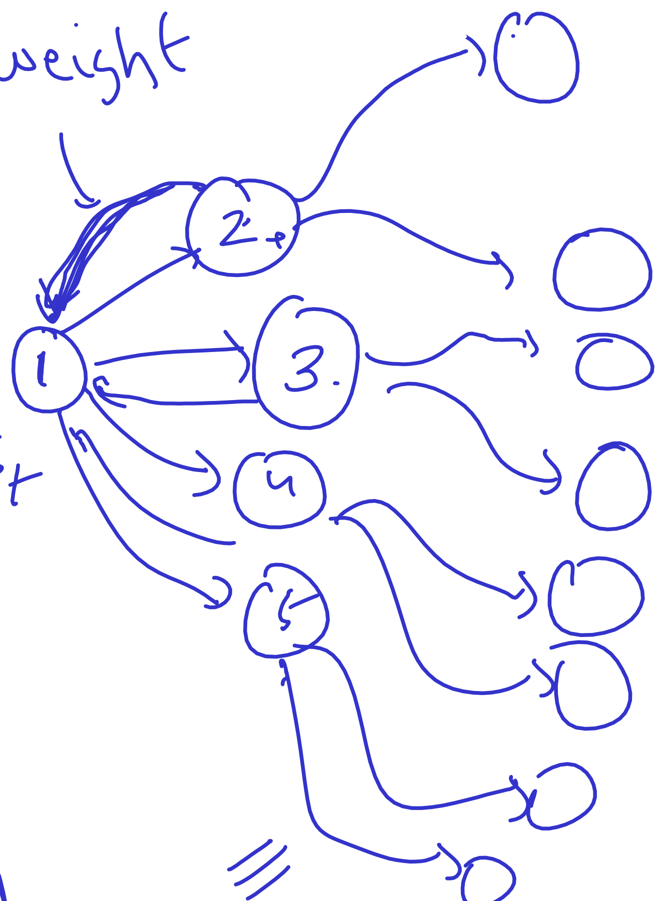
$$s_{t+1} = \begin{bmatrix} 9 \\ 20 \end{bmatrix}$$

```
def transition_fn(s_t, a_t):

    s_{t+1} = s_t + a_t

    return s_{t+1}
```

$$\underline{a}_t = \underbrace{\begin{bmatrix} -1 \\ 0 \end{bmatrix}}_{\text{Left}} \quad \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\text{Right}} \quad \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\text{Top}} \quad \underbrace{\begin{bmatrix} 0 \\ -1 \end{bmatrix}}_{\text{Bottom}}$$

edge weight $\equiv$ cost function

vertices $\equiv$ $\underline{s}_t$



edges $\equiv$ action space at a state + Transition function

④ Cost function

$$c(\underline{s}_t, \underline{a}_t) \longmapsto \mathbb{R}$$

State Action

Real number

Cost of taking this action $\underline{a}_t$ at state $\underline{s}_t$

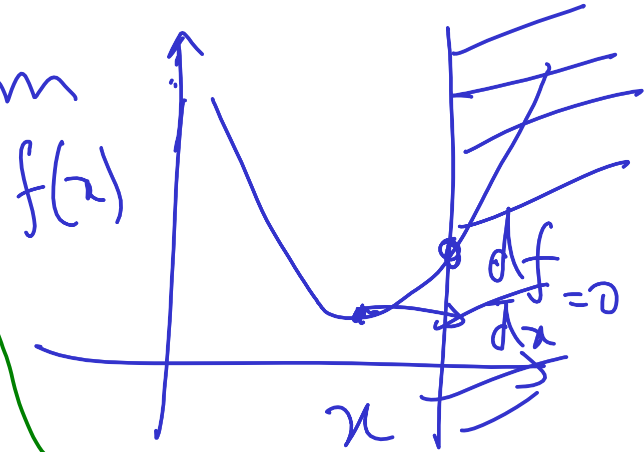
⑥ Initial state $\underline{s}_I$

⑦ Goal state $\underline{s}_G$

Planning problem

Find a sequence of actions to take that takes ~~us~~ the robot from $\underline{s}_I$ to $\underline{s}_G$ with minimum cost.

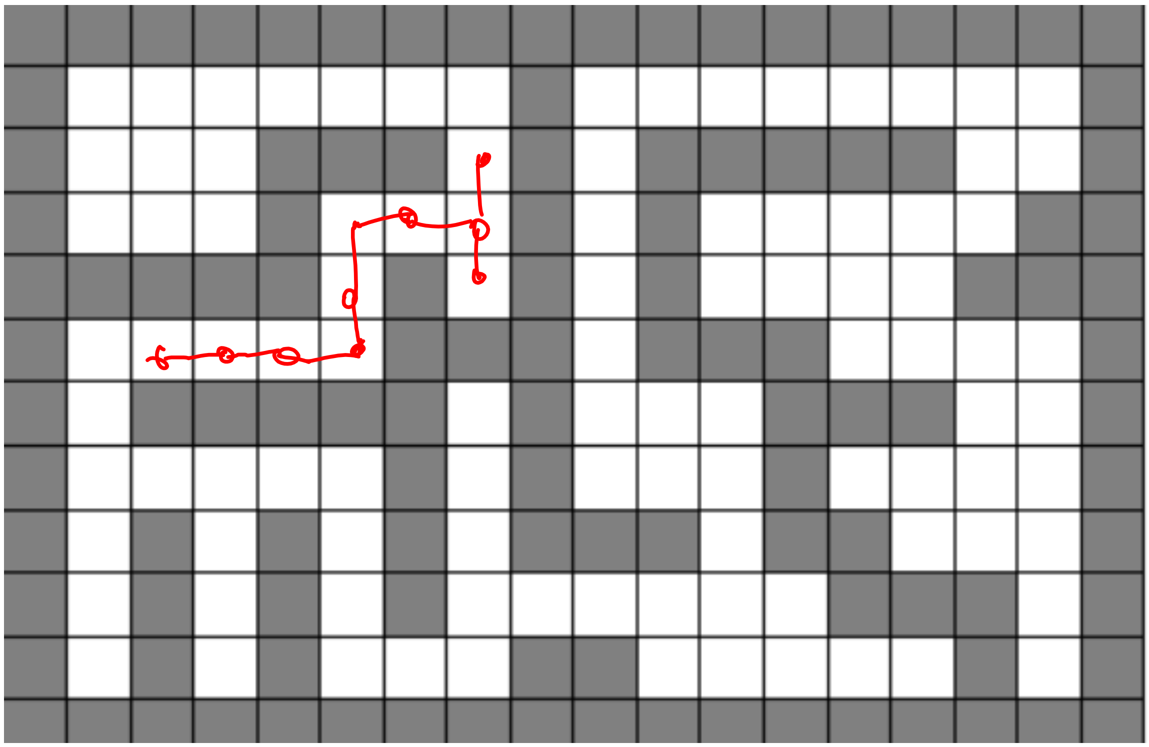
Planning problem as an optimization problem



$$\boxed{\begin{aligned} &\underset{\{\underline{a}_t\}_{t=0}^{T}}{\text{minimize}} \sum_{t=0}^{T} c(\underline{s}_t, \underline{a}_t) \\ &\text{such that} \quad \underline{s}_{t+1} = T(\underline{s}_t, \underline{a}_t) \end{aligned}}$$

The category of approaches is called Dynamic programming

(intuitively similar to mathematical induction)



Graph Data structures

① Adjacency list

② Adjacency matrix

Planning (Chapter 2 from Lavalle book)

.

Abstraction of a planning problem

1. State space $\mathbf{s} \in \mathcal{S}$. For example, 2D coordinate of a grid $\mathbf{s} = (x, y)$.
2. Action space per state $\mathbf{u} \in \mathcal{U}(\mathbf{s})$. For example, up, down, left right movement can be encoded as $\mathcal{U}(\mathbf{s}_t) = \{(0, -1), (0, 1), (1, 0), (-1, 0)\}$.
3. State transition function $\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{u}_t)$. For example, the up-down-left-right action can be combined as addition to get the next state $\mathbf{s}_{t+1} = \mathbf{s}_t + \mathbf{u}_t$.
4. Initial State $\mathbf{s}_I \in \mathcal{S}$
5. Goal states $\mathbf{s}_G \subseteq \mathcal{S}$

A Graph

A graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ is defined by a set of vertices $\mathcal{V}$ and a set of edges $\mathcal{E}$ such that each edge $e \in \mathcal{E}$ is formed by a pair of start and end vertices $e = (v_s, v_e), v_s \in \mathcal{V}, v_e \in \mathcal{V}$. The first vertex is called the start of the edge $v_s = \text{start}(e)$ and second vertex is called the end $v_e = \text{end}(e)$.

A discrete planning problem can be converted into a graph by definiting

1. Vertices as the state space $\mathcal{V} = \mathcal{S}$.
2. The action space at each state as the edges connected to that vertex/state, $\mathcal{U}(\mathbf{s}_t) = \{(\mathbf{s}_t, \mathbf{s}_j) \mid (\mathbf{s}_t, \mathbf{s}_j) \in \mathcal{E}\}$.
3. State transition function is the other end of th edge, $\mathbf{s}_{t+1} = f(\mathbf{s}_t, \mathbf{u}_t) = \text{end}(\mathbf{u}_t)$, where $\mathbf{s}_t = \text{start}(\mathbf{u}_t)$.

Representations of Graphs

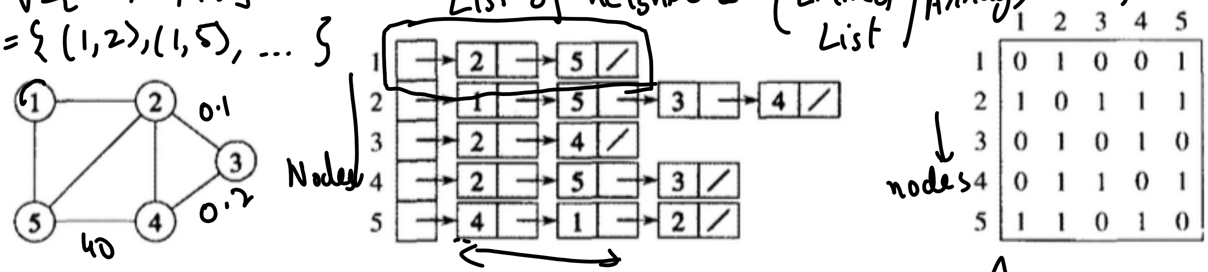
Operation : Given a vertex find its neighbors

✓ a) cost of an edge
✓ b) Check if an edge exists b/w two vertices

$V = [1, 2, 3, 4 \; 5]$

$E = \{(1,2), (1,5), \ldots\}$

List of neighbors (Linked List / Array) $\xrightarrow{\text{nodes}}$



(a) (b) (c)

Adjacency matrix

Figure 23.1 Two representations of an undirected graph. **(a)** An undirected graph G having five vertices and seven edges. **(b)** An adjacency-list representation of G. **(c)** The adjacency-matrix representation of G.

$$A[v_1, v_2] = A[v_2, v_1]$$

(Chapter 23 of Introduction to Algorithms by Carmen et al)

Undirected graph

In [1]:
```
# Programmatically you can represent a adjacency list as python lists
# Python lists are not linked lists, they are arrays under the hood.
G_adjacency_list = {
    1 : [2, 5],
    2 : [1, 5, 3, 4],
    3 : [2, 4],
    4 : [2, 5, 3],
    5 : [4, 1, 2]
}

# Prefer to represent a matrix in python either as a list of lists or a nump
import numpy as np
G_adjacency_matrix = np.array([
    [0, 1, 0, 0, 1],
    [1, 0, 1, 1, 1],
    [0, 1, 0, 1, 0],
    [0, 1, 1, 0, 1],
    [1, 1, 0, 1, 0]
])

# Edge list is another possible representation
G_edge_list = [
    (1, 2), (1, 5),
    (2, 1), (2, 5), (2, 3), (2, 4),
    (3, 2), (3, 4),
    (4, 2), (4, 5), (4, 3),
    (5, 4), (5, 1), (5, 2)
]
```

Directed graph representation

In [2]:
```
# Programmatically you can represent a adjacency list as python lists
# Python lists are not linked lists, they are arrays under the hood.
G_adjacency_list = {
    1 : [2, 4],
    2 : [5],
    3 : [6, 5],
    4 : [2],
    5 : [4],
    5 : [6]
}

# Prefer to represent a matrix in python either as a list of lists or a nump
import numpy as np
G_adjacency_matrix = np.array([
    [0, 1, 0, 1, 0, 0],
```

```
    [0, 0, 0, 0, 1, 0],
    [0, 0, 0, 0, 1, 1],
    [0, 1, 0, 0, 0, 0],
    [0, 0, 0, 1, 0, 0],
    [0, 0, 0, 0, 0, 1]
])

# Edge list is another possible representation
G_edge_list = [
    (1, 2), (1, 4),
    (2, 5),
    (3, 6), (3, 5),
    (4, 2),
    (5, 6)
]
```

In [3]:
```
# Exercise 1

# Write a function that converts a graph in adjacency list format to adjacer
def adjacency_list_to_matrix(G_adj_list):
    G_adj_mat = None # TODO: Write code to convert to adj_mat
    return G_adj_mat

def adjacency_matrix_to_list(G_adj_mat):
    G_adj_list = None # TODO: Write code to convert to adj_mat
    return G_adj_list

# Use the above graphs to test
print(adjacency_list_to_matrix(G_adjacency_list))
print(adjacency_matrix_to_list(G_adjacency_matrix))
```

```
None
None
```

Graph Search algorithms

1. Breadth First Search bfs.png

2. Depth First Search

dfs.png

Breadth first search (BFS)

bfs-states

In [4]:
```
from queue import Queue, LifoQueue, PriorityQueue

graph = {
  's' : ['w', 'r'],
  'r' : ['v'],
  'w' : ['t', 'x'],
  'x' : ['y' ],
  't' : ['u'],
```

Breadth first search (BFS)

Not visited nodes have distance = ∞

distance from start node

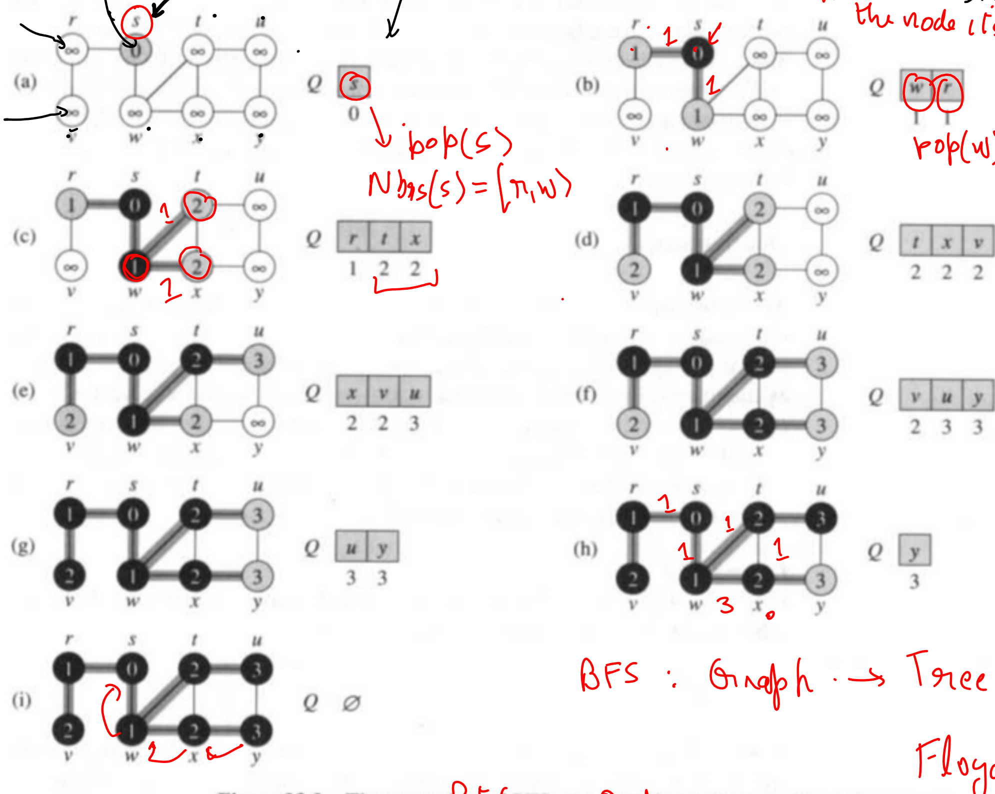
A queue of active nodes but not the node itself

Nodes whose parent has been visited

White nodes = unvisited

Active nodes = gray

Black nodes = visited



pop(s)

Nbrs(s) = {r, w}

pop(w)

Nbrs(w) = {s, t, x}

Queue

BFS → FIFO First in first out

DFS → FILO First in Last out

w (parent)

children

BFS : Graph → Tree

Floyd Warshall

BFS : Only works for equally weighted edges

```python
    'u' : ['y']
}

def bfs(graph, start, debug=False):
    seen = set() # Set for seen nodes (contains both frontier and dead state
    # Frontier is the boundary between seen and unseen (Also called the aliv
    frontier = Queue() # Frontier of unvisited nodes as FIFO
    node2dist = {start : 0} # Keep track of distances
    search_order = []
    seen.add(start)
    frontier.put(start)


    i = 0 # step number
    while not frontier.empty():          # Creating loop to visit each node
        if debug: print("%d) Q = " % i, list(frontier.queue), end='; ')
        if debug: print("dists = " , [node2dist[n] for n in frontier.queue])
        m = frontier.get() # Get the oldest addition to frontier
        search_order.append(m)


        for neighbor in graph.get(m, []):
            if neighbor not in seen:
                seen.add(neighbor)
                frontier.put(neighbor)
                node2dist[neighbor] = node2dist[m] + 1
            else:
                assert node2dist[neighbor] <= node2dist[m] + 1, 'this shoulc
                node2dist[neighbor] = min(node2dist[neighbor], node2dist[m]

        i += 1
    if debug: print("%d) Q = " % i, list(frontier.queue))
    return search_order, node2dist
```

set() → { start : 1 }

visited

active nodes

Stack ≡ FILO

seen = [start]

frontier = [

for( )

Stack

{ start for( )y

FILO

FIFO

dict

graph[m]

class Graph:

def get(self, m, d):

return nbrs of m

In [5]:

```python
print("Following is the Breadth-First Search order")
print(bfs(graph, 's', debug=True))    # function calling
```

```
Following is the Breadth-First Search order
0) Q =  ['s']; dists =  [0]
1) Q =  ['w', 'r']; dists =  [1, 1]
2) Q =  ['r', 't', 'x']; dists =  [1, 2, 2]
3) Q =  ['t', 'x', 'v']; dists =  [2, 2, 2]
4) Q =  ['x', 'v', 'u']; dists =  [2, 2, 3]
5) Q =  ['v', 'u', 'y']; dists =  [2, 3, 3]
6) Q =  ['u', 'y']; dists =  [3, 3]
7) Q =  ['y']; dists =  [3]
8) Q =  []
(['s', 'w', 'r', 't', 'x', 'v', 'u', 'y'], {'s': 0, 'w': 1, 'r': 1, 't': 2,
'x': 2, 'v': 2, 'u': 3, 'y': 3})
```

Depth first search

image.png

bfs-states

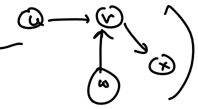
Depth first search

BFS - FIFO queue

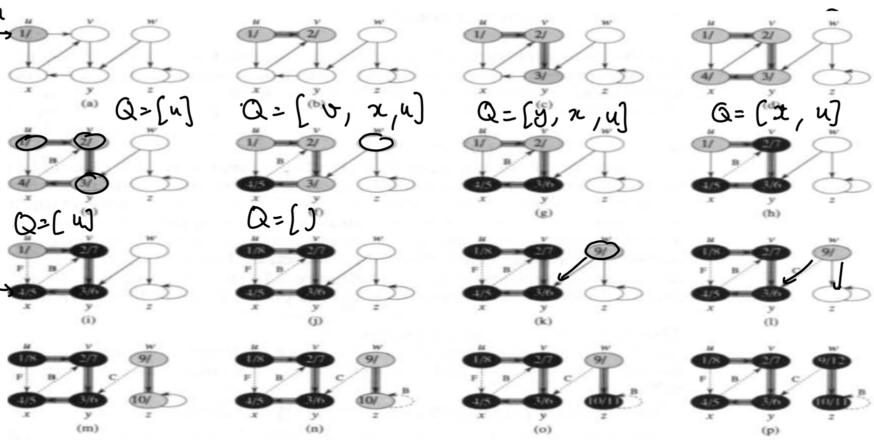
DFS - LIFO queue O

(Topological sorting

u > w > v > x



Frontier



Q = [u]

Q = [v, x, u]

Q = [y, x, u]

Q = [x, u]

Q = [u]

Q = [ ]

Visited

In [6]:
```
graph = {
  's' : ['w', 'r'],
  'r' : ['v'],
  'w' : ['t', 'x'],
  'x' : ['y' ],
  't' : ['u'],
  'u' : ['y']
}

def dfs(graph, start, debug=False):
    seen = set([start]) # List for seen nodes (contains both frontier and de
    # Frontier is the boundary between seen and unseen (Also called the aliv
    frontier = LifoQueue() # Frontier of unvisited nodes as FIFO
    node2dist = {start : 0} # Keep track of distances
    search_order = [] # Keep track of search order
    frontier.put(start)


    i = 0 # step number
    while not frontier.empty():          # Creating loop to visit each node
        if debug: print("%d) Q = " % i, list(frontier.queue), end='; ')
        if debug: print("dists = " , [node2dist[n] for n in frontier.queue])
        m = frontier.get() # Get the oldest addition to frontier
        search_order.append(m)


        for neighbor in graph.get(m, []):
            if neighbor not in seen:
                seen.add(neighbor)
                frontier.put(neighbor)
                node2dist[neighbor] = node2dist[m] + 1
            else:
                node2dist[neighbor] = min(node2dist[neighbor], node2dist[m]
        i += 1
    if debug: print("%d) Q = " % i, list(frontier.queue))
    return search_order, node2dist
```

In [7]:
```
# Driver Code
print("Following is the Depth-First Search path")
print(dfs(graph, 's', debug=True))    # function calling
```

```
Following is the Depth-First Search path
0) Q =  ['s']; dists =  [0]
1) Q =  ['w', 'r']; dists =  [1, 1]
2) Q =  ['w', 'v']; dists =  [1, 2]
3) Q =  ['w']; dists =  [1]
4) Q =  ['t', 'x']; dists =  [2, 2]
5) Q =  ['t', 'y']; dists =  [2, 3]
6) Q =  ['t']; dists =  [2]
7) Q =  ['u']; dists =  [3]
8) Q =  []
(['s', 'r', 'v', 'w', 'x', 'y', 't', 'u'], {'s': 0, 'w': 1, 'r': 1, 'v': 2,
't': 2, 'x': 2, 'y': 3, 'u': 3})
```

Converting a maze search to a graph search

In [8]:
```
# Skip these utilities for the class

def batched(iterable, n):
    "Batch data into tuples of length n. The last batch may be shorter."
    # batched('ABCDEFG', 3) --> ABC DEF G
    if n < 1:
        raise ValueError('n must be at least one')
    it = iter(iterable)
    while batch := tuple(islice(it, n)):
        yield batch

def draw_path(self, path, visited='*'):
    new_maze_lines = [list(l) for l in self.maze_lines]
    for (r, c) in path:
        new_maze_lines[r][c] = visited
        print('\n'.join([''.join(l) for l in new_maze_lines]))
        print('\n\n\n')

def init_plots(self, reinit=False):
    if self.fig is None or reinit:
        self.fig, self.ax = plt.subplots()

def plot_maze(self):
    self.init_plots()
    replace = { ' ' : 1, '+': 0}
    maze_mat = np.array([[replace[c] for c in line]
                          for line in self.maze_lines])
    return [self.ax.imshow(maze_mat, cmap='gray')]

def plot_step(self, i_node):
    i, (r, c) = i_node
    return [self.ax.text(c, r, '%d' % (i+1))]

def plot_path(self, path):
    self.plot_maze()
    return [self.plot_step((i, (r,c)))
            for i, (r, c) in enumerate(path)]

def animate_search_path(maze, search_path, node2dist):
    maze.init_plots()
    return animation.FuncAnimation(maze.fig, maze.plot_step, frames=[(node2c
                                                                      for n
                                  init_func=maze.plot_maze, blit=True, repea
```

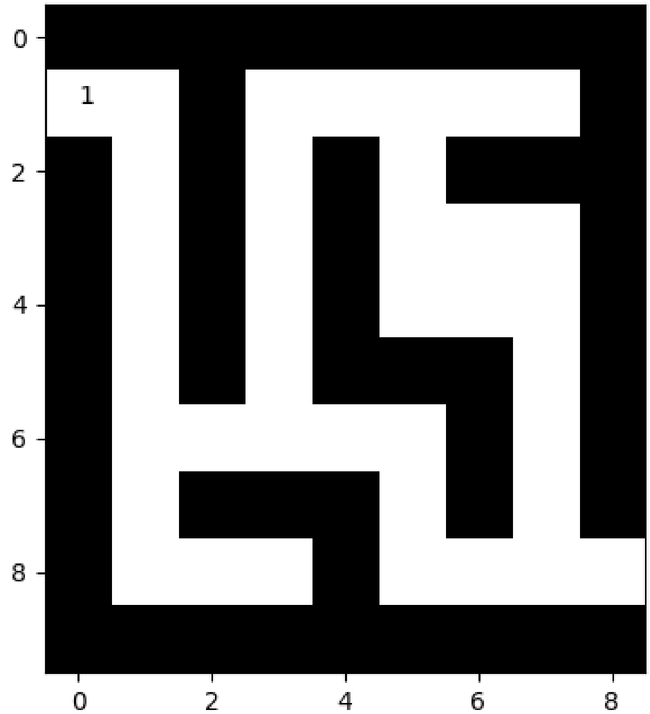
In [9]:
```
import matplotlib.pyplot as plt
import numpy as np
maze_str = \
"""
+++++++++
  +     +
+ + + +++
+ + +   +
+ + +   +
+ + +++ +
```

```
+      + +
+ +++ + +
+    +
+++++++++
"""

class Maze:
    def __init__(self, maze_str, freepath=' '):
        self.maze_lines = [l for l in maze_str.split("\n")
                                   if len(l)]
        self.FREEPATH = freepath
        self.fig = None

    def get(self, node, default):
        (r, c) = node
        m_row = self.maze_lines[r]
        nbrs = []
        if c-1 >= 0 and m_row[c-1] == self.FREEPATH:
            nbrs.append((r, c-1))
        if c+1 < len(m_row) and m_row[c+1] == self.FREEPATH:
            nbrs.append((r, c+1))
        if r-1 >= 0 and self.maze_lines[r-1][c] == self.FREEPATH:
            nbrs.append((r-1, c))
        if r+1 < len(self.maze_lines) and self.maze_lines[r+1][c] == self.FR
            nbrs.append((r+1, c))
        return nbrs if len(nbrs) else default
    init_plots = init_plots
    plot_maze = plot_maze
    plot_step = plot_step
    plot_path = plot_path
    animate_search_path = animate_search_path
```

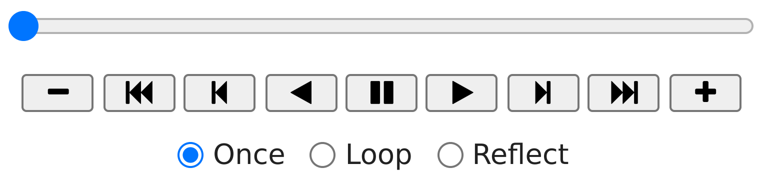
In [10]:

```
import matplotlib.pyplot as plt
import matplotlib.animation as animation
import matplotlib as mpl
%matplotlib inline
mpl.rc('animation', html='jshtml')


maze = Maze(maze_str)
search_path, node2dist = bfs(maze, (1, 0)) # prints the order of search all
maze.plot_maze()

maze.animate_search_path(search_path, node2dist)
```

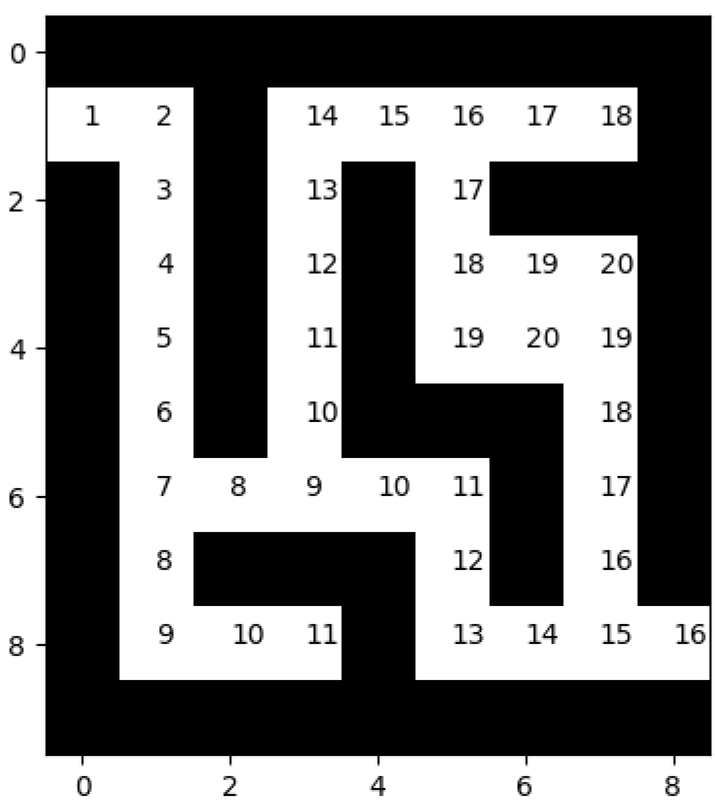
Out[10]:



0
1
2
4
6
8
0 2 4 6 8



Once
Loop
Reflect



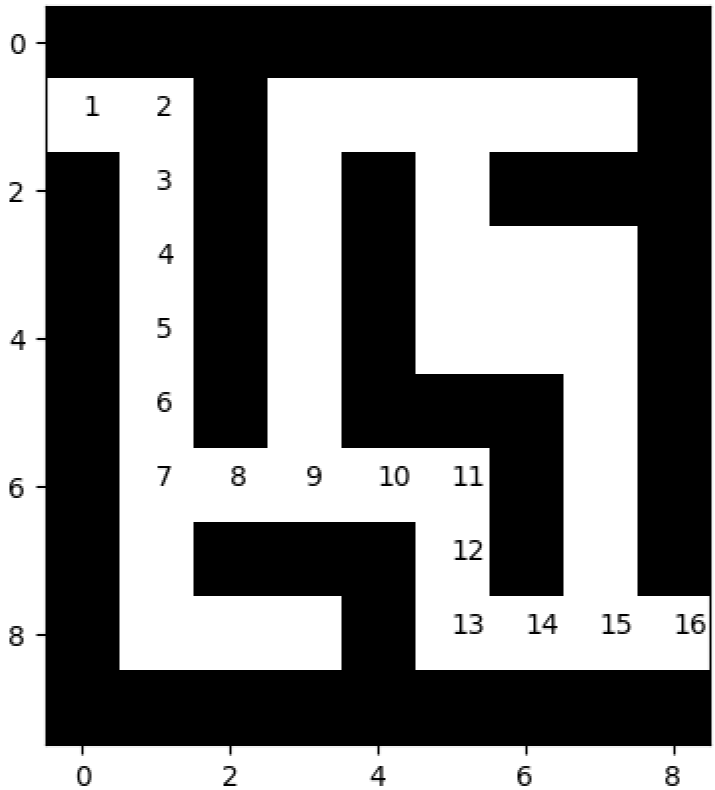
0
1 2 14 15 16 17 18
2
3 13 17
4 12 18 19 20
4
5 11 19 20 19
6 10 18
6
7 8 9 10 11 17
8 12 16
8
9 10 11 13 14 15 16
0 2 4 6 8

In [11]:

```python
def bfs_path(graph, start, goal):
    """
    Returns success and node2parent

    success: True if goal is found otherwise False
    node2parent: A dictionary that contains the nearest parent for node
    """
    seen = [start] # List for seen nodes.
    # Frontier is the boundary between seen and unseen
    frontier = Queue() # Frontier of unvisited nodes as FIFO
    node2parent = dict() # Keep track of nearest parent for each node (requi
    frontier.put(start)

    while not frontier.empty():          # Creating loop to visit each node
        m = frontier.get() # Get the oldest addition to frontier
        if m == goal:
            return True, node2parent

        for neighbor in graph.get(m, []):
            if neighbor not in seen:
                seen.append(neighbor)
                frontier.put(neighbor)
                node2parent[neighbor] = m
    return False, []
```

In [12]:

```python
def backtrace_path(node2parent, start, goal):
    c = goal
    r_path = [c]
    parent = node2parent.get(c, None)
    while parent != start:
        r_path.append(parent)
        c = parent
        parent = node2parent.get(c, None) # Keep getting the parent until yo
        #print(parent)
    r_path.append(start)
    return reversed(r_path) # Reverses the path

maze = Maze(maze_str)
start = (1, 0)
goal = (8, 8)
success, node2parent = bfs_path(maze, (1, 0), (8, 8))
path = backtrace_path(node2parent, (1, 0), (8, 8))
#print(list(path))
maze.plot_path(path) # Draws all the searched nodes
plt.show()
#node2parent
```



Dijkstra algorithm

dijkstra-step-by-step

PriorityQueue

PriorityQueue returns the smallest (or the largest) item in the queue faster than other data structures

In [13]:
```
#from queue import PriorityQueue
from hw2_solution import PriorityQueueUpdatable
from dataclasses import dataclass, field
from typing import Any


# https://docs.python.org/3/library/queue.html#queue.PriorityQueue
@dataclass(order=True)
class PItem:
    dist: int
    node: Any=field(compare=False)

    # Make the PItem hashable
    # https://docs.python.org/3/glossary.html#term-hashable
    def __hash__(self):
        return hash(self.node)

graph = {
    's' : [('x', 5), ('u', 10)],
```

comparable 1 < 2

(~~x~~, 5) < (~~u~~, 10)

node hashable

node dist

BFS : FIFO

DFS : LIFO

Dijkstra : Priority Queue

{u, v, x, y}

priority

Q = [(u,-8), (v,-1), (x,-4)
(y,-3)]

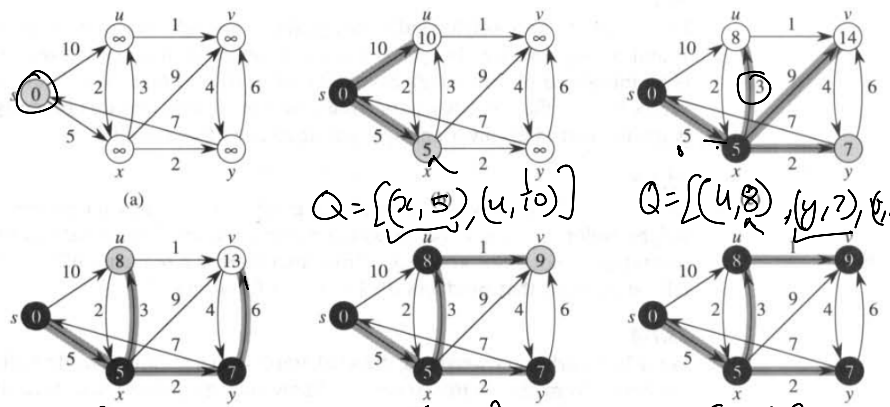
node2parent[v] = x

node2parent[v] = parent

Q = [(s,0)]

Q = [(x,5), (u,10)]

Q = [(u,8), (y,7), (v,14)]



Q = [(u,8), (v,13)]

node2parent[v] = y

Q = [(v,9)] n2p[v]=u

Q = [ ]

Figure 25.5 The execution of Dijkstra's algorithm. The source is the leftmost vertex. The shortest-path estimates are shown within the vertices, and shaded edges indicate predecessor values: if edge (u, v) is shaded, then $\pi[v] = u$. Black vertices are in the set S, and white vertices are in the priority queue $Q = V - S$. **(a)** The situation just before the first iteration of the **while** loop of lines 4–8. The shaded vertex has the minimum d value and is chosen as vertex u in line 5. **(b)–(f)** The situation after each successive iteration of the **while** loop. The shaded vertex in each part is chosen as vertex u in line 5 of the next iteration. The d and π values shown in part (f) are the final values.

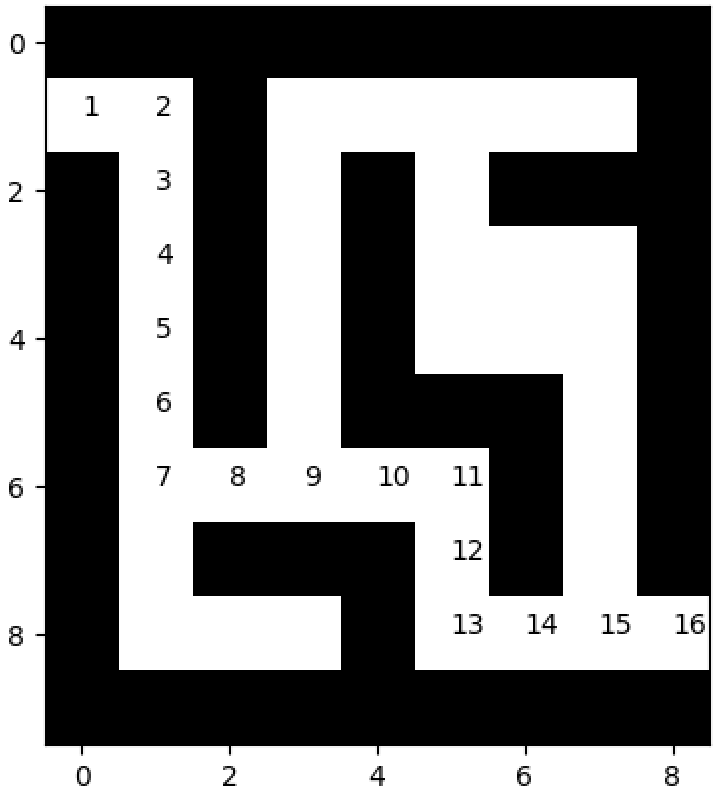
```python
    'u' : [('v', 1), ('x', 2)],
    'x' : [('u', 3), ('v', 9), ('y', 2)],
    'y' : [('v', 6), ('s', 7)],
    'v' : [('y', 4)]
}


def dijkstra(graph, start, goal, debug=False):
    """
    edgecost: cost of traversing each edge

    Returns success and node2parent

    success: True if goal is found otherwise False
    node2parent: A dictionary that contains the nearest parent for node
    """
    seen = set([start]) # Set for seen nodes.
    # Frontier is the boundary between seen and unseen
    frontier = PriorityQueueUpdatable() # Frontier of unvisited nodes as a P
    node2parent = {start : None} # Keep track of nearest parent for each noc
    node2dist = {start: 0} # Keep track of cost to arrive at each node
    search_order = []
    frontier.put(PItem(0, start))
    i = 0
    while not frontier.empty():          # Creating loop to visit each node
        dist_m = frontier.get() # Get the smallest addition to the frontier
        if debug: print("%d) Q = " % i, list(frontier.queue), end='; ')
        if debug: print("dists = " , [node2dist[n.node] for n in frontier.qu
        m = dist_m.node
        m_dist = node2dist[m]
        search_order.append(m)
        if goal is not None and m == goal:
            return True, search_order, node2parent, node2dist

        for neighbor, edge_cost in graph.get(m, []):
            old_dist = node2dist.get(neighbor, float("inf"))
            new_dist = edge_cost + m_dist
            if neighbor not in seen:
                seen.add(neighbor)
                frontier.put(PItem(new_dist, neighbor))
                node2parent[neighbor] = m
                node2dist[neighbor] = new_dist
            elif new_dist < old_dist:
                node2parent[neighbor] = m
                node2dist[neighbor] = new_dist
                # ideally you would update the dist of this item in the pric
                # as well. But python priority queue does not support fast u
                old_item = PItem(old_dist, neighbor)
                if old_item in frontier:
                    frontier.replace(old_item, PItem(new_dist, neighbor))
        i += 1
    if goal is not None:
        return False, [], {}, node2dist
    else:
        return True, search_order, node2parent, node2dist
```

In [14]:
```
success, search_path, node2parent, node2dist = dijkstra(graph, 's', None, de
print(success, node2parent, node2dist)
```

```
0) Q =  []; dists =  []
1) Q =  [PItem(dist=10, node='u')]; dists =  [10]
2) Q =  [PItem(dist=8, node='u'), PItem(dist=14, node='v')]; dists =  [8, 1
4]
3) Q =  [PItem(dist=13, node='v')]; dists =  [13]
4) Q =  []; dists =  []
True {'s': None, 'x': 's', 'u': 'x', 'v': 'u', 'y': 'x'} {'s': 0, 'x': 5,
'u': 8, 'v': 9, 'y': 7}
```

In [15]:
```
import itertools

class MazeD(Maze):
    def get(self, node, default):
        nbrs = Maze.get(self, node, default)
        return zip(nbrs, itertools.repeat(1))

maze = MazeD(maze_str)
success, search_path, node2parent, node2dist = dijkstra(maze, (1, 0), (8, 8)
print(success, node2parent)
if success:
    path = backtrace_path(node2parent, (1, 0), (8, 8))
    maze.plot_path(path) # Draws all the searched nodes
```

```
True {(1, 0): None, (1, 1): (1, 0), (2, 1): (1, 1), (3, 1): (2, 1), (4, 1):
(3, 1), (5, 1): (4, 1), (6, 1): (5, 1), (6, 2): (6, 1), (7, 1): (6, 1), (6,
3): (6, 2), (8, 1): (7, 1), (6, 4): (6, 3), (5, 3): (6, 3), (8, 2): (8, 1),
(6, 5): (6, 4), (4, 3): (5, 3), (8, 3): (8, 2), (7, 5): (6, 5), (3, 3): (4,
3), (8, 5): (7, 5), (2, 3): (3, 3), (8, 6): (8, 5), (1, 3): (2, 3), (8, 7):
(8, 6), (1, 4): (1, 3), (8, 8): (8, 7), (7, 7): (8, 7), (1, 5): (1, 4)}
```



In [16]:
```
maze_str = \
"""
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```

```
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+                  +                                      +
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
"""
start_pos, goal_pos = (35, 9), (5, 50)
```
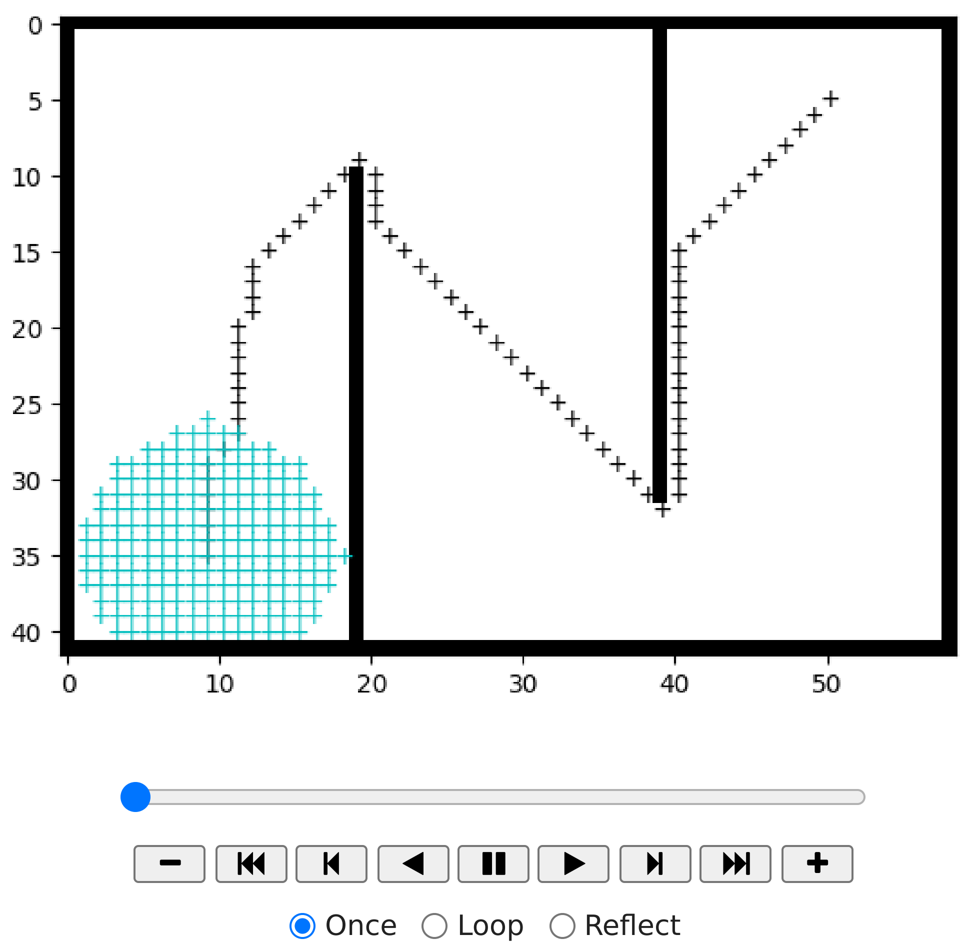

In [17]:
```
import math
from itertools import islice

class Maze8(MazeD):
    def get(self, node, default):
        (r, c) = node
        rmax = len(self.maze_lines)
        cmax = len(self.maze_lines[0])
        m_row = self.maze_lines[r]
        possible_nbrs = [
            ((r, c-1), 1),
            ((r, c+1), 1),
            ((r-1, c), 1),
            ((r+1, c), 1),
            ((r-1, c-1), math.sqrt(2)),
            ((r-1, c+1), math.sqrt(2)),
            ((r+1, c-1), math.sqrt(2)),
            ((r+1, c+1), math.sqrt(2))
        ]
        free_nbrs = []
        for (ri, ci), dist in possible_nbrs:
            if (ri >= 0 and ci >= 0 and ri < rmax and ci < cmax
                   and self.maze_lines[ri][ci] == self.FREEPATH):
                free_nbrs.append(((ri, ci), dist))
        return free_nbrs if len(free_nbrs) else default

    def _plot_path(self, path, char='+', color='c'):
        return [self.ax.text(c-0.5, r+0.5, char, color=color)
               for (r, c) in path]

    def plot_path(self, path, **kw):
        self.plot_maze()
        return self._plot_path(path, **kw)

    def animate(self, path, batch_size=200):
        self.init_plots()
        anim = animation.FuncAnimation(self.fig, self._plot_path,
                                       frames=batched(search_path, batch_siz
```

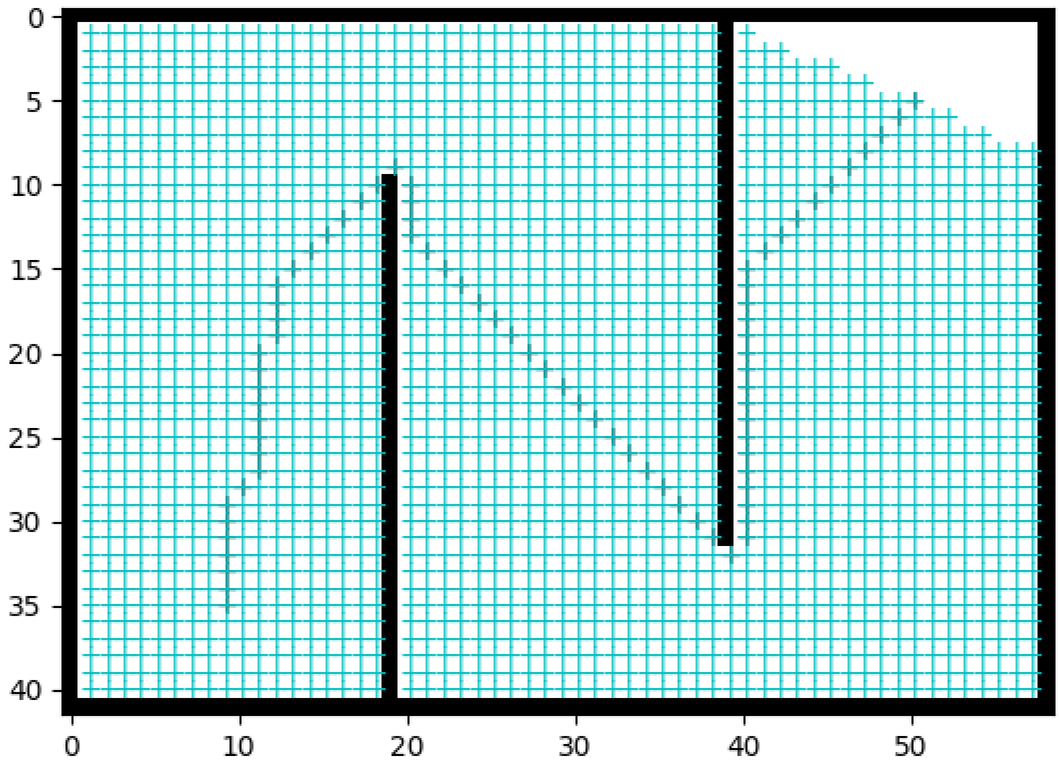
```
                                                  init_func=self.plot_maze, blit=True, r
        return anim
```

In [18]:
```
maze = Maze8(maze_str)
success, search_path, node2parent, node2dist = dijkstra(maze, start_pos, goa
#print(success, search_path)
assert success
anim = maze.animate(search_path)
path = backtrace_path(node2parent, start_pos, goal_pos)
#maze.init_plots(reinit=True)
path_plot = maze.plot_path(path, color='k') # Draws the traced shortest path
anim
```

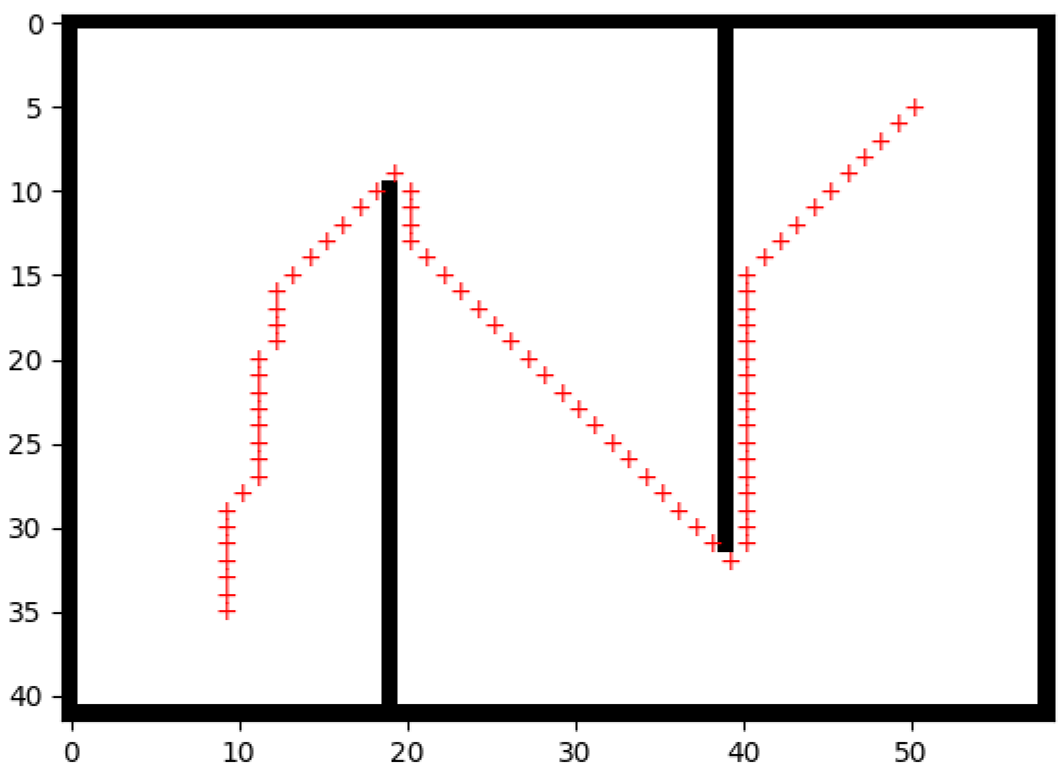
```
/tmp/ipykernel_26974/955263672.py:37: UserWarning: frames=<generator object
batched at 0x7f5c0c3632e0> which we can infer the length of, did not pass an
explicit *save_count* and passed cache_frame_data=True.  To avoid a possibly
unbounded cache, frame data caching has been disabled. To suppress this warn
ing either pass `cache_frame_data=False` or `save_count=MAX_FRAMES`.
  anim = animation.FuncAnimation(self.fig, self._plot_path,
```

Out[18]:



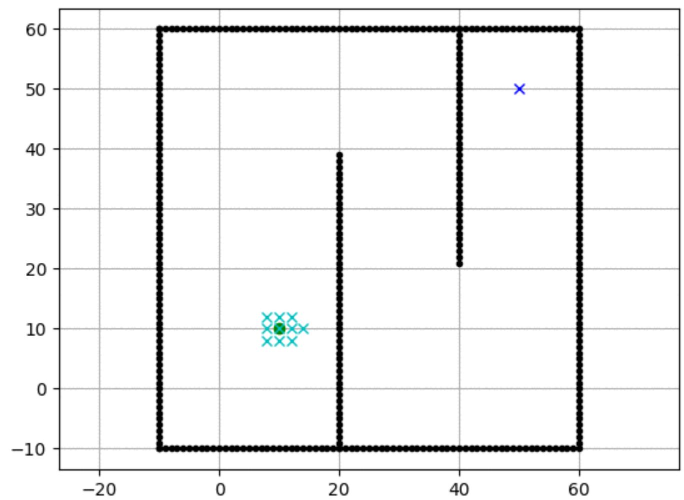


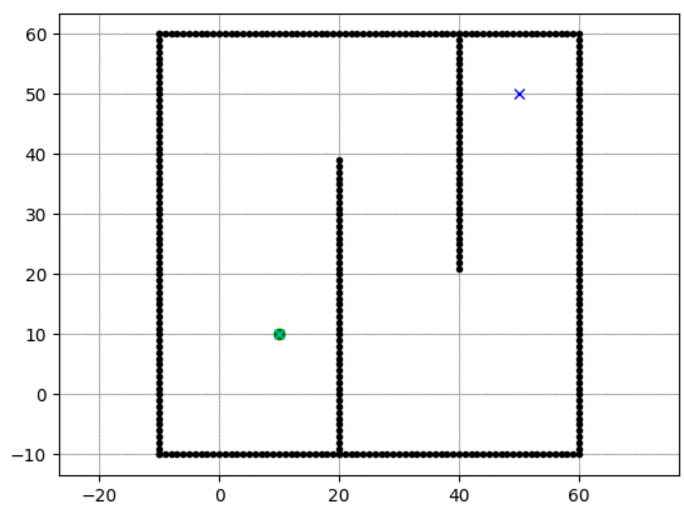
In [19]:
```
path = backtrace_path(node2parent, (35, 9), (5, 50))
maze.init_plots(reinit=True)
maze.plot_path(path, color='r') # Draws the traced shortest path
plt.show()
```



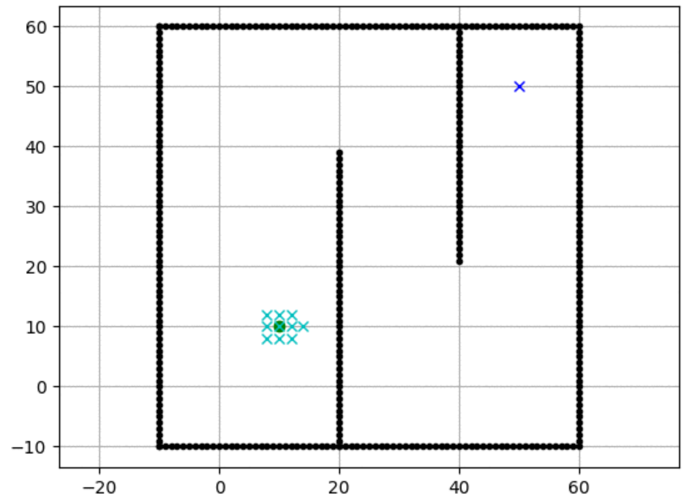
Search order in BFS vs DFS vs Dijkstra

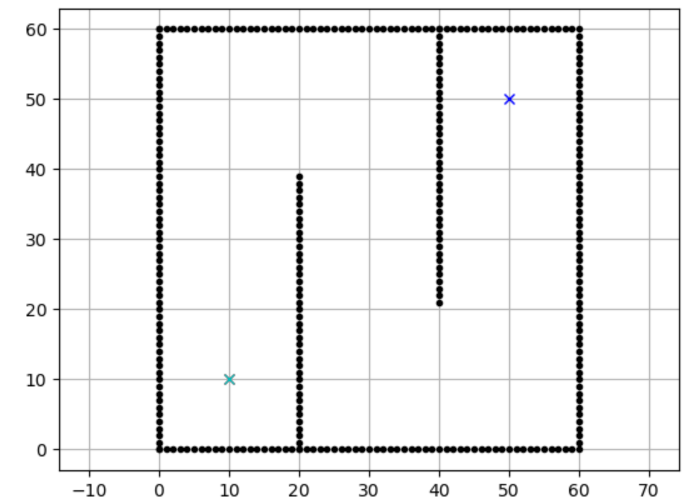
Breadth first search vs Depth first search





Breadth first search vs Dijkstra

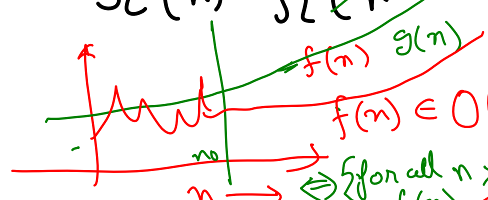




$\Theta(n)$ $\Theta(n^2)$

$\boxed{O(n) \quad O(n^2)}$ Asymptotic Upper bound

$\Omega(n)$ $\Omega(n^2)$



$f(n) \in O(g(n))$

$\Leftrightarrow \{$for all $n > n_0$ $0 < f(n) < g(n)\}$

Computational complexity of BFS

In [20]:

```
# Write down the computational complexity of each line in big-O notation O()
# Assume the graph has |V| nodes and |E| edges
def bfs_barebones(graph, start):
    seen = {start} # Set for seen nodes (contains both frontier and dead sta
    # Frontier is the boundary between seen and unseen (Also called the aliv
    frontier = Queue() # Frontier of unvisited nodes as FIFO # O(1)
    frontier.put(start) # O(1)

    while not frontier.empty(): # Creating loop to visit each node # O(|V|)
        m = frontier.get() # Get the oldest addition to frontier # O(|V| * 1

        for neighbor in graph.get(m, []): # O(|V| * |E|/|V|) = O(|E|)
            if neighbor not in seen: # O(|E| * 1)
                seen.add(neighbor) # O(|E| * 1)
                frontier.put(neighbor) # O(|E| * 1)
```

```
# The computational complexity of BFS is O(|E|). Some books write it as O(|V
# where O(|V|) is the cost of initializing states of different nodes
```

Computational complexity of Dijkstra

In [21]:

```
# Write down the computational complexity of each line in big-O notation O()
# Assume the graph has |V| nodes and |E| edges
def dijkstra_barebones(graph, start):
    seen = {start} # Set for seen nodes (contains both frontier and dead sta
    # Frontier is the boundary between seen and unseen (Also called the aliv
    frontier = PriorityQueue() # Frontier of unvisited nodes as PriorityQueu
    frontier.put(PItem(0, start)) # O(1)
    node2dist = {start: 0} # Keep track of cost to arrive at each node # O(1

    while not frontier.empty(): # Creating loop to visit each node
        dist_and_node = frontier.get() # Get the smallest dist node # O(|V|
        m_dist = dist_and_node.dist
        m = dist_and_node.node

        for neighbor, edge_dist in graph.get(m, []): # O(|V| * |E|/|V|) = O(
            if neighbor not in seen: # O(|E| * 1)
                seen.add(neighbor) # O(|E| * 1)
                frontier.put(neighbor) # # O(|E| * log(1)) # for fibonacci h
                node2dist[neighbor] = m_dist + edge_dist # O(1)
            elif node2dist[neighbor] > m_dist + edge_dist: # O(1)
                node2dist[neighbor] = m_dist + edge_dist # O(1)

# The computational complexity of Dijkstra is O(|V|log(|V|) + |E|) when impl
# using a Fibonacci heap based PriorityQueue
```

min([2, 3, 10, 7, 8, 5, 1]) = O(|V|)

PriorityQueue (Heaps Chapter 7 of Carmen's intro to algorithms)

heap

Heap property

1. $H[\text{Parent(i)}] \geq H[i]$
2. Parent(i) = ceil(i/2)
3. LeftChild(i) = 2i
4. RightRight(i) = 2i+1

Heapify

heapify

Heapify pseudocode

heapify-psuedocode

Total node = $1 + 2 + 2^2 + 2^3 + \ldots + 2^d = \frac{2^{d+1} - 1}{2-1} = 2^{d+1} - 1 = |V|$

$d = \log_2(|V|+1) - 1$

$d \propto \log(|V|)$

Heap property

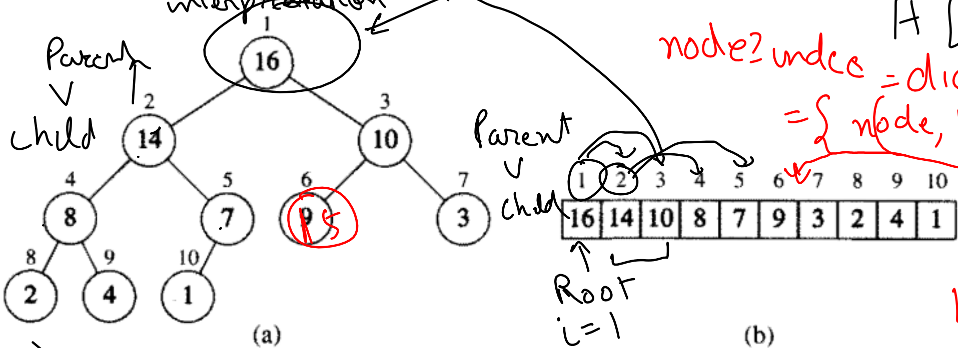
$H[\text{Parent}] > H[\text{Child}]$

Min ⇔ Max

↑ Python

node2index = dict()

= { node, 15 }: 6 }



Heapinsert = $O(\log|V|)$

Figure 7.1 A heap viewed as **(a)** a binary tree and **(b)** an array. The number within the circle at each node in the tree is the value stored at that node. The number next to a node is the corresponding index in the array.

Carmen's book

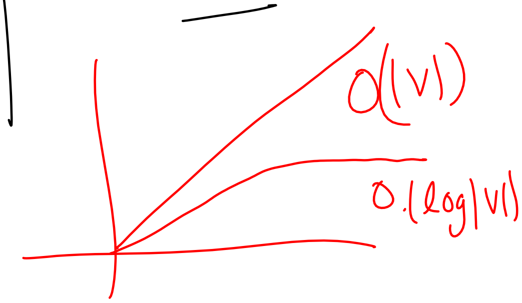
Max

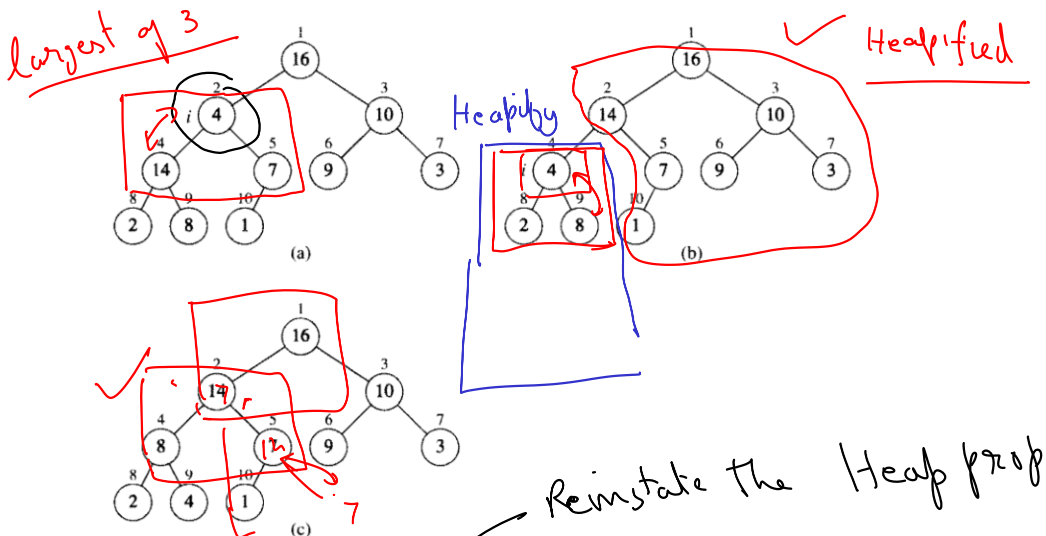
Parent = floor($i/2$)

= $i // 2$

Left Child = $2i$

Right Child = $2i+1$





Reinstate the Heap property

Figure 7.2 The action of HEAPIFY(A, 2), where *heap-size*[A] = 10. **(a)** The initial configuration of the heap, with $A[2]$ at node $i = 2$ violating the heap property since it is not larger than both children. The heap property is restored for node 2 in **(b)** by exchanging $A[2]$ with $A[4]$, which destroys the heap property for node 4. The recursive call HEAPIFY(A, 4) now sets $i = 4$. After swapping $A[4]$ with $A[9]$, as shown in **(c)**, node 4 is fixed up, and the recursive call HEAPIFY(A, 9) yields no further change to the data structure.

Heap Insert

heap-insert

Heap runtimes

heap-runtimes

A-star (A*) algorithm

(Required reading: 3.5.2 of Russel and Norving: Artificial Intelligence)

romania-nodes

In [22]:
```
from hw2_solution import PriorityQueueUpdatable
import sys
def astar(graph, heuristic_dist_fn, start, goal, debug=False, debugf=sys.std
    """
    edgecost: cost of traversing each edge

    Returns success and node2parent

    success: True if goal is found otherwise False
    node2parent: A dictionary that contains the nearest parent for node
    """
    seen = set([start]) # Set for seen nodes.
    # Frontier is the boundary between seen and unseen
    frontier = PriorityQueueUpdatable() # Frontier of unvisited nodes as a P
    node2parent = {start : None} # Keep track of nearest parent for each noc
    hfn = heuristic_dist_fn # make the name shorter
    node2dist = {start: 0  } # Keep track of cost to arrive at each node
    search_order = []
    frontier.put(PItem(0 + hfn(start, goal), start)) #   <------------- Diff

    if debug: debugf.write("goal = "  + str(goal) + '\n')
    i = 0
    while not frontier.empty():          # Creating loop to visit each node
        dist_m = frontier.get() # Get the smallest addition to the frontier
        if debug: debugf.write("%d) Q = " % i + str(list(frontier.queue)) +
        if debug: debugf.write("%d) node = " % i + str(dist_m) + '\n')
        #if debug: print("dists = " , [node2dist[n.node] for n in frontier.q
        m = dist_m.node
        m_dist = node2dist[m]
        search_order.append(m)
        if goal is not None and m == goal:
            return True, search_order, node2parent, node2dist

        for neighbor, edge_cost in graph.get(m, []):
            old_dist = node2dist.get(neighbor, float("inf"))
            new_dist = edge_cost +  m_dist
            if neighbor not in seen:
                seen.add(neighbor)
                frontier.put(PItem(new_dist +  hfn(neighbor, goal), neighbor
```

```
                node2parent[neighbor] = m
                node2dist[neighbor] = new_dist
            elif new_dist < old_dist:
                node2parent[neighbor] = m
                node2dist[neighbor] = new_dist
                # ideally you would update the dist of this item in the prio
                # as well. But python priority queue does not support fast u
                # ------------- Different from dijkstra -------------------
                old_item = PItem(old_dist + hfn(neighbor, goal), neighbor)
                if old_item in frontier:
                    frontier.replace(
                        old_item,
                        PItem(new_dist + hfn(neighbor, goal), neighbor))
        i += 1
    if goal is not None:
        return False, [], {}, node2dist
    else:
        return True, search_order, node2parent, node2dist
```

In [23]:

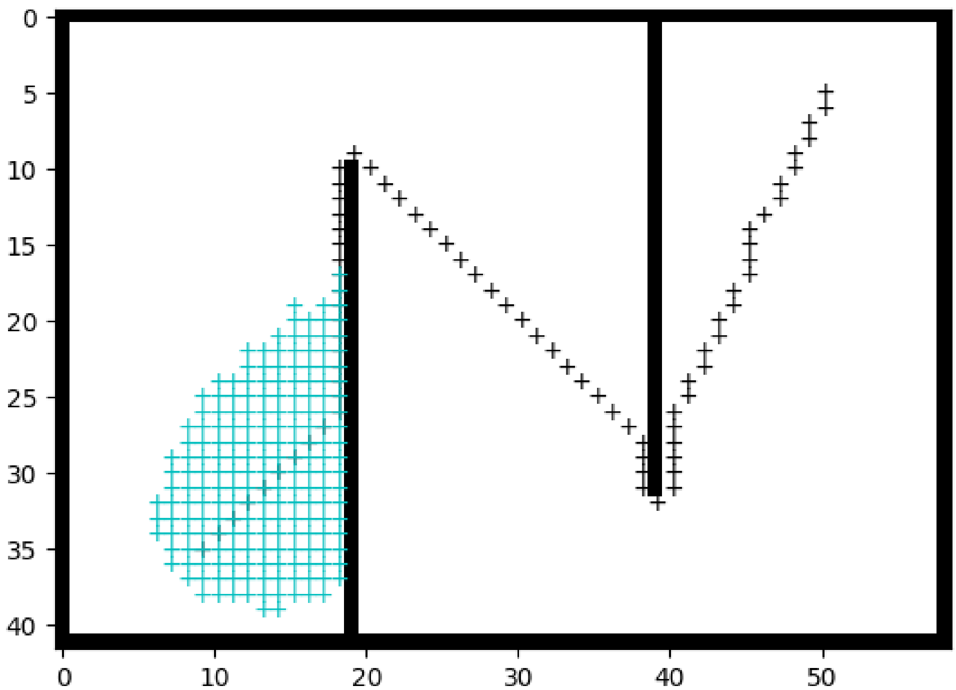
```
import math
from functools import partial

def euclidean_heurist_dist(node, goal, scale=1):
    x_n, y_n = node
    x_g, y_g = goal
    return scale*math.sqrt((x_n-x_g)**2 + (y_n - y_g)**2)
```

In [24]:

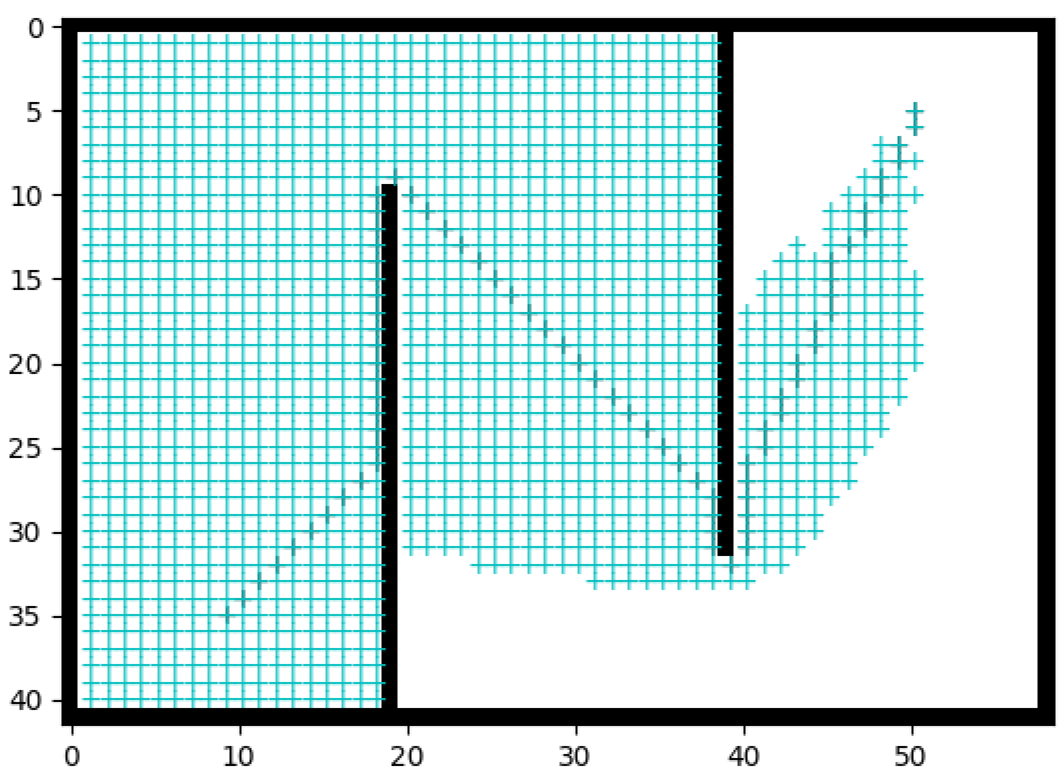
```
maze = Maze8(maze_str)
debugf=open('log.txt', 'w')
success, search_path, node2parent, node2dist = astar(
    maze, partial(euclidean_heurist_dist, scale=1),
    start_pos, goal_pos, debug=True, debugf=debugf)
debugf.close()

#print(success, search_path)
assert success
anim = maze.animate(search_path)
path = backtrace_path(node2parent, start_pos, goal_pos)
#maze.init_plots(reinit=True)
path_plot = maze.plot_path(path, color='k') # Draws the traced shortest path
anim
```

```
/tmp/ipykernel_26974/955263672.py:37: UserWarning: frames=<generator object
batched at 0x7f5c07250430> which we can infer the length of, did not pass an
explicit *save_count* and passed cache_frame_data=True.  To avoid a possibly
unbounded cache, frame data caching has been disabled. To suppress this warn
ing either pass `cache_frame_data=False` or `save_count=MAX_FRAMES`.
  anim = animation.FuncAnimation(self.fig, self._plot_path,
```

Out[24]:



Once Loop Reflect



```python
In [25]: maze_str = \
"""
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+                  +                                      +
+                  +                                      +
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
"""

start_pos, goal_pos = (35, 9), (5, 50)
```
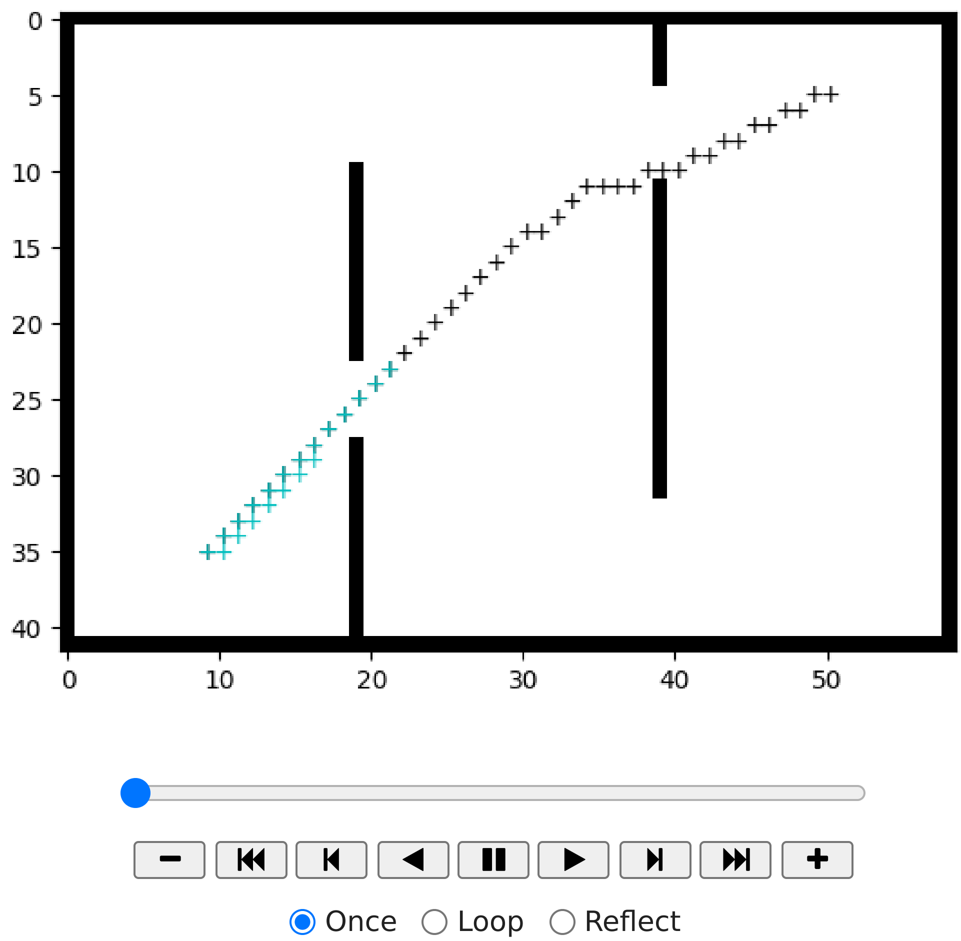
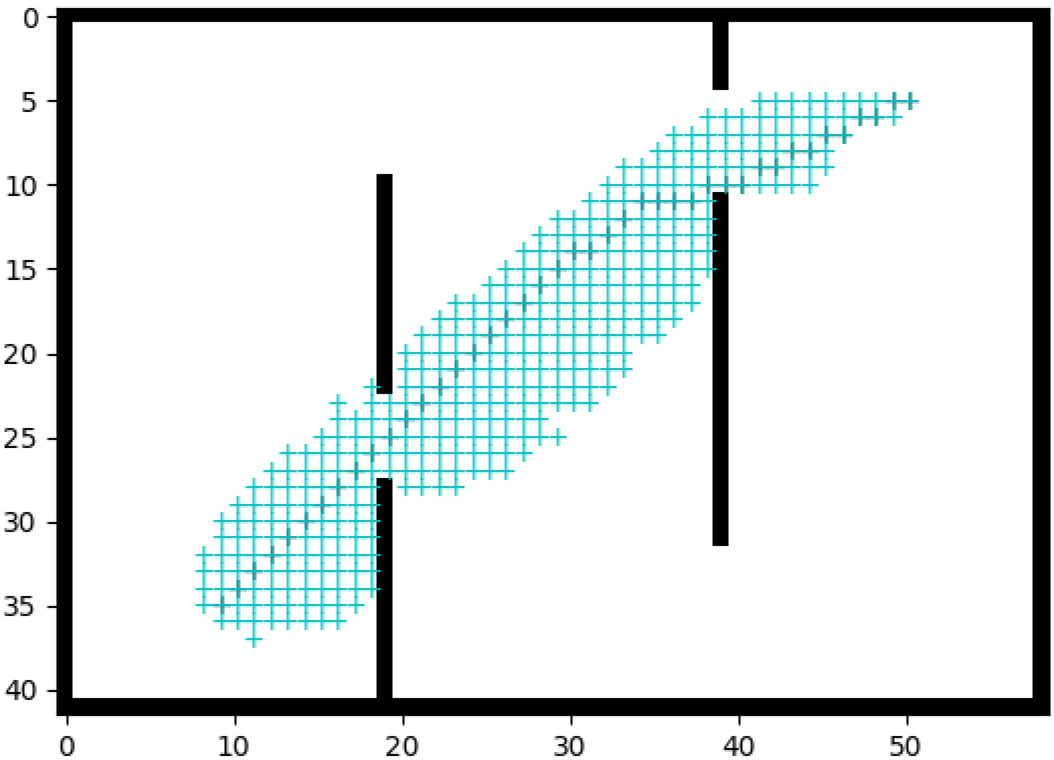

```python
In [26]: maze = Maze8(maze_str)
success, search_path, node2parent, node2dist = astar(
    maze, partial(euclidean_heurist_dist, scale=1),
    start_pos, goal_pos)
#print(success, search_path)
assert success
anim = maze.animate(search_path, batch_size=20)
```

```
path = backtrace_path(node2parent, start_pos, goal_pos)
#maze.init_plots(reinit=True)
path_plot = maze.plot_path(path, color='k') # Draws the traced shortest path
anim
```

```
/tmp/ipykernel_26974/955263672.py:37: UserWarning: frames=<generator object
batched at 0x7f5c0724e2e0> which we can infer the length of, did not pass an
explicit *save_count* and passed cache_frame_data=True.  To avoid a possibly
unbounded cache, frame data caching has been disabled. To suppress this warn
ing either pass `cache_frame_data=False` or `save_count=MAX_FRAMES`.
  anim = animation.FuncAnimation(self.fig, self._plot_path,
```

Out[26]:

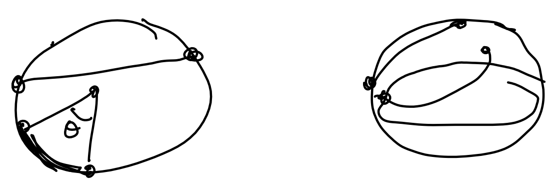


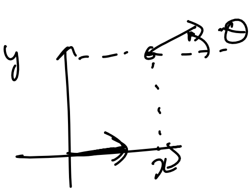


In [27]:
```
maze = Maze8(maze_str)
success, search_path, node2parent, node2dist = astar(
    maze, partial(euclidean_heurist_dist, scale=0),
    start_pos, goal_pos)
#print(success, search_path)
assert success
anim = maze.animate(search_path)
path = backtrace_path(node2parent, start_pos, goal_pos)
#maze.init_plots(reinit=True)
path_plot = maze.plot_path(path, color='k') # Draws the traced shortest path
anim
```

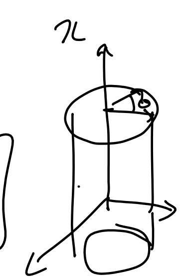
```
/tmp/ipykernel_26974/955263672.py:37: UserWarning: frames=<generator object batched at 0x7f5c060b4e40> which we can infer the length of, did not pass an explicit *save_count* and passed cache_frame_data=True.  To avoid a possibly unbounded cache, frame data caching has been disabled. To suppress this warning either pass `cache_frame_data=False` or `save_count=MAX_FRAMES`.
  anim = animation.FuncAnimation(self.fig, self._plot_path,
```

$x(\theta_s - \theta_G)^2$



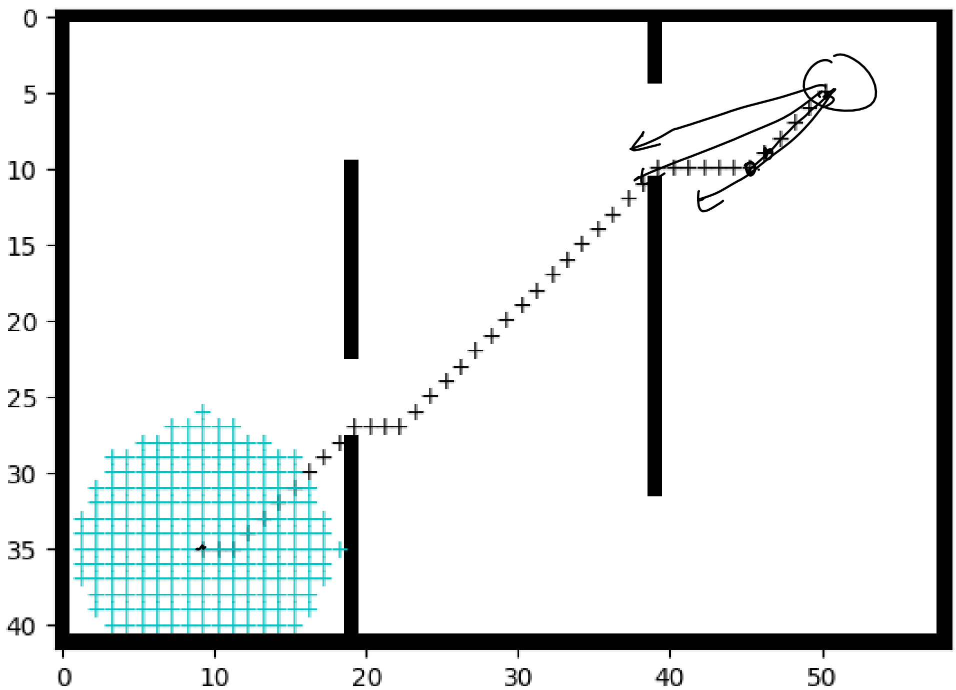


State of a car like robot is described by $\begin{bmatrix} x \\ y \\ r_1 \cos\theta \\ r_2 \sin\theta \end{bmatrix} \to \begin{bmatrix} x \\ \theta \end{bmatrix}$

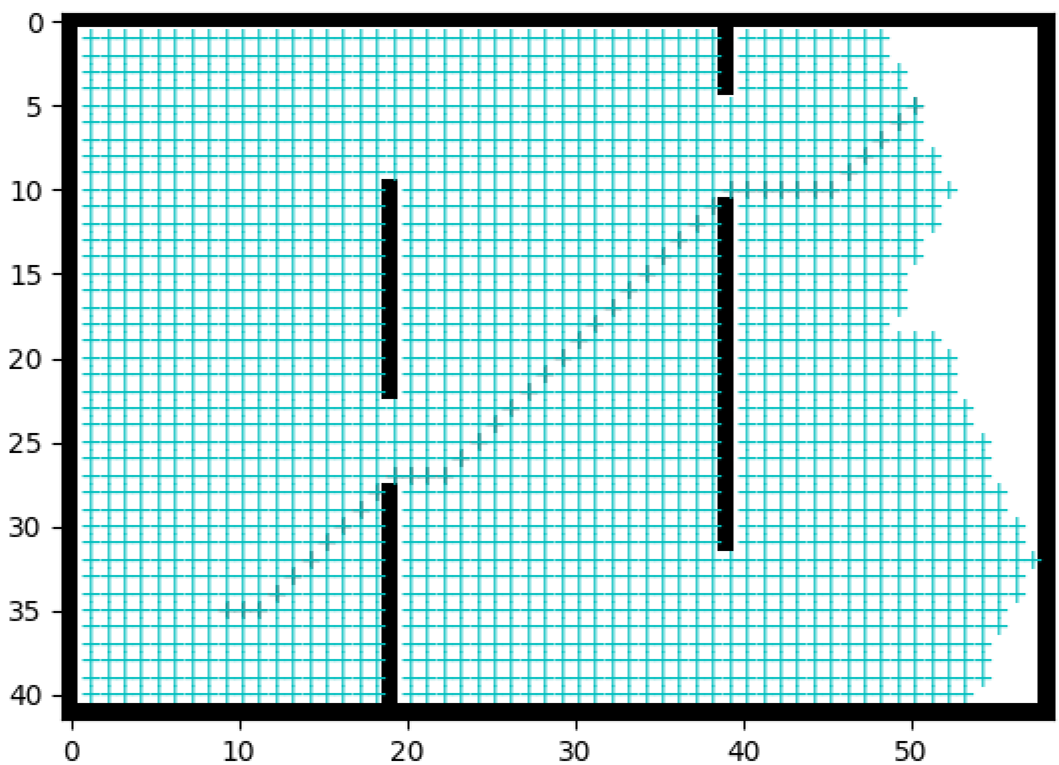


dist $\sqrt{(x_s - x_G)^2 + (y_s - y_G)^2}$

Out[27]:



Once Loop Reflect



```python
In [28]: maze_str = \
"""
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+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
"""
goal_pos = (9-5, 46+5)
start_pos = (9+30, 46-30)
```
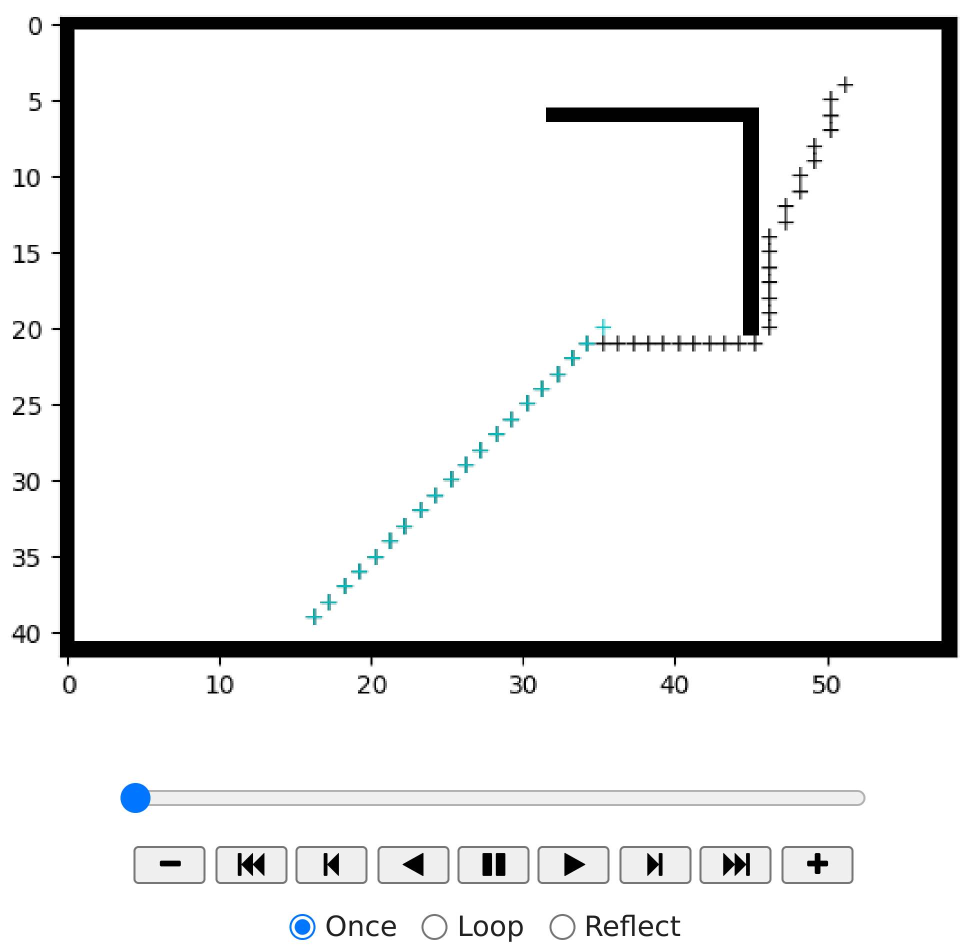
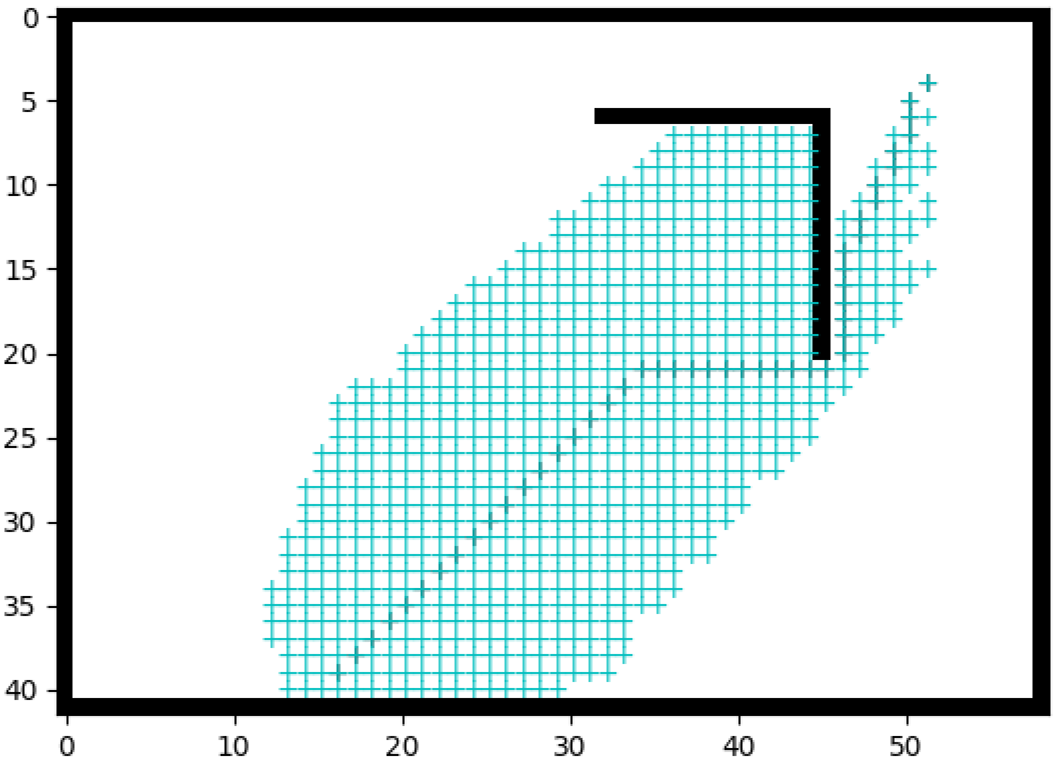

```python
In [29]: maze = Maze8(maze_str)
success, search_path, node2parent, node2dist = astar(
    maze, partial(euclidean_heurist_dist, scale=1),
    start_pos, goal_pos)
#print(success, search_path)
assert success
anim = maze.animate(search_path, batch_size=20)
```

```
path = backtrace_path(node2parent, start_pos, goal_pos)
#maze.init_plots(reinit=True)
path_plot = maze.plot_path(path, color='k') # Draws the traced shortest path
anim
```

```
/tmp/ipykernel_26974/955263672.py:37: UserWarning: frames=<generator object
batched at 0x7f5c05036900> which we can infer the length of, did not pass an
explicit *save_count* and passed cache_frame_data=True.  To avoid a possibly
unbounded cache, frame data caching has been disabled. To suppress this warn
ing either pass `cache_frame_data=False` or `save_count=MAX_FRAMES`.
  anim = animation.FuncAnimation(self.fig, self._plot_path,
```

Out[29]:





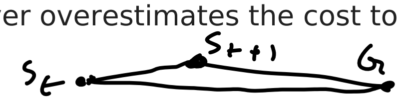
Admisibility and Consistency of heuristic function

$h(s_{t+1}) \leq c(s_t, G)$

$s_{t+1} = G$

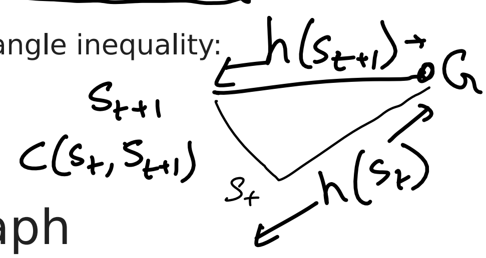
$h(s_{t+1}) = 0$

1. An admissible heuristic is one that never overestimates the cost to reach the goal.

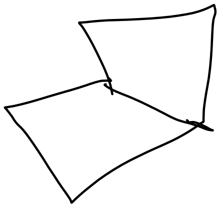


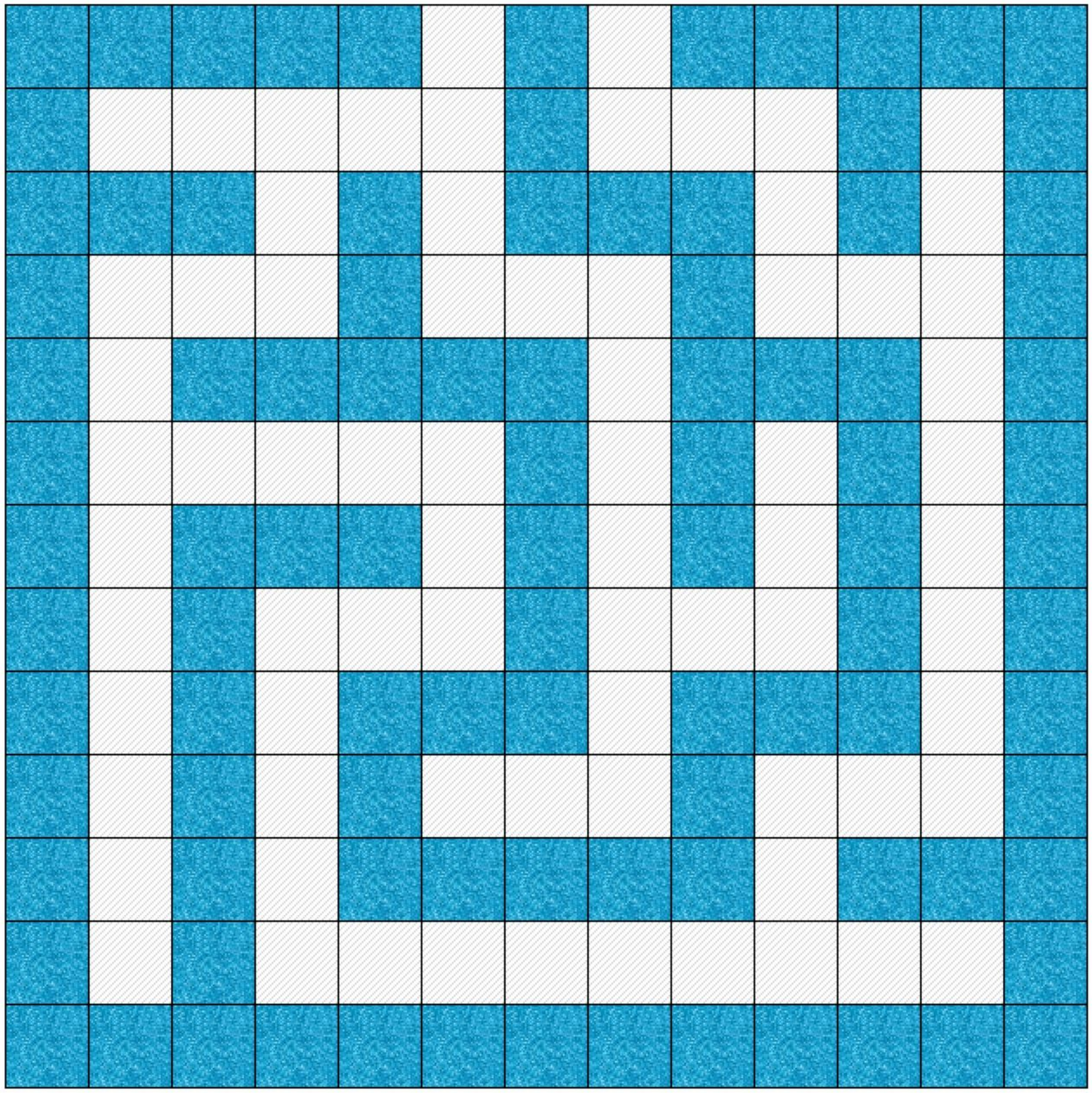
2. A heuristic h(n) is consistent if it satisfies the triangle inequality:

$$h(\mathbf{s}_t) \leq \underbrace{c(\mathbf{s}_t, \mathbf{s}_{t+1})}_{=} + h(\mathbf{s}_{t+1}).$$



Converting maze to grid to graph





Other ways of converting a maze into a graph

Rapidly exploring random trees

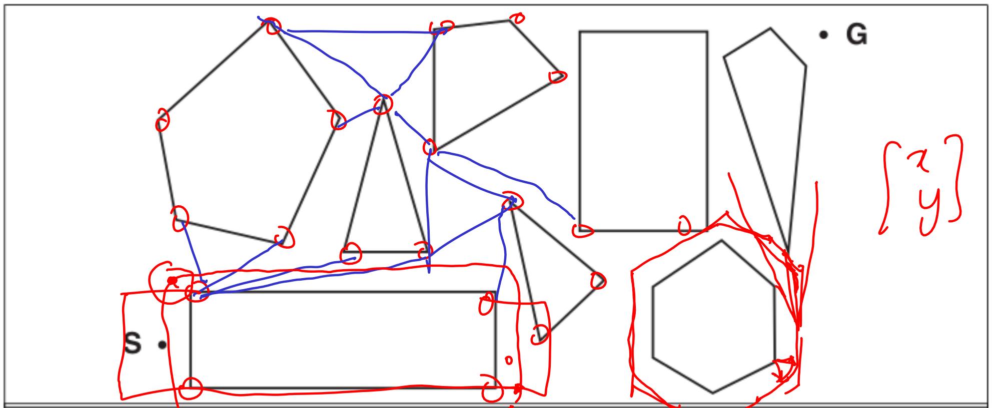
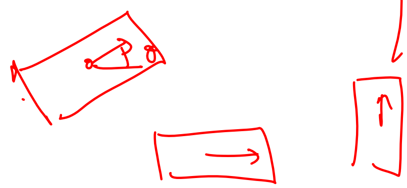


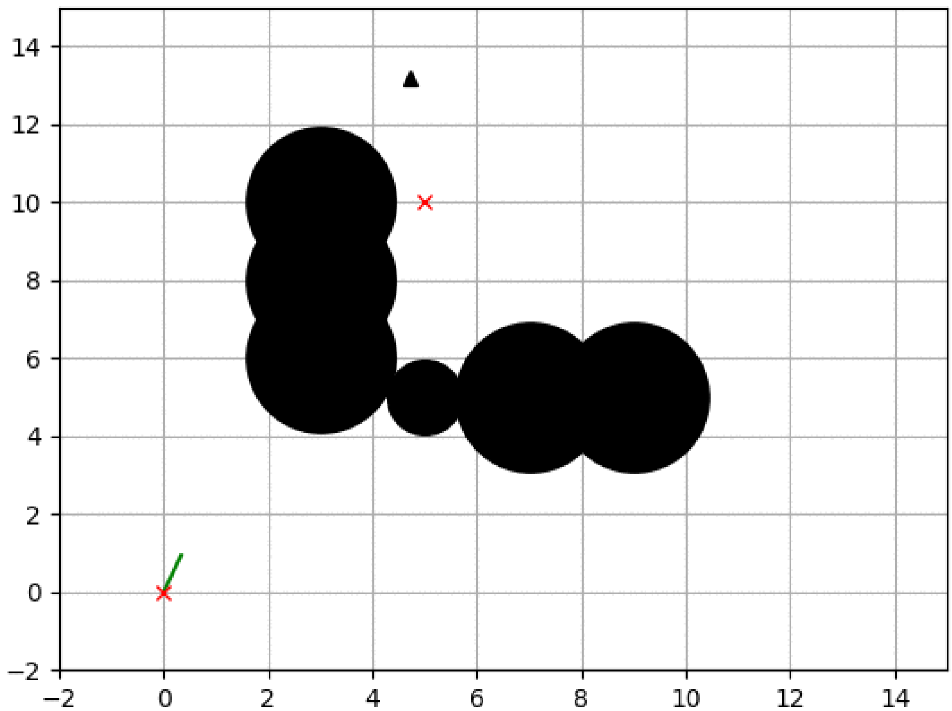
Figure 3.31 A scene with polygonal obstacles. S and G are the start and goal states.

inflated obstacles

$\begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$





In [ ]: