Midterm 1: Obstacle avoidance using ARUCO markers and A-Star/RRT

I will document my process to achieve at this solution.

Thought process

- 1. Get object poses from ARUCO
- 2. Plan path through obstacles using A-star
- 3. Follow path by converting desired movements to cmd_vel

Always test the smallest software component you can test, so that errors are caught early.

Our A-star algorithm from the discrete-planning notebook worked on discrete grids. Let's see if we can extend it to a robot that moves only forward or backward.

Code from previous notebooks

PriorityQueueUpdatable (HW2)

In [1]: #%writefile hw2_solution.py # uncomment and run after you fill in with your ### Fill in with your HW2 solution or ask for the solution from the instruct

A-star algorithm

```
In [2]: %writefile astar.py
from hw2_solution import PriorityQueueUpdatable
import sys
from dataclasses import dataclass, field
from typing import Any

# https://docs.python.org/3/library/queue.html#queue.PriorityQueue
@dataclass(order=True)
class PItem:
    dist: int
    node: Any=field(compare=False)

# Make the PItem hashable
# https://docs.python.org/3/glossary.html#term-hashable
def __hash__(self):
    return hash(self.node)
```

```
def astar(graph, heuristic dist fn, start, goal, debug=False, debugf=sys.std
   edgecost: cost of traversing each edge
   Returns success and node2parent
   success: True if goal is found otherwise False
   node2parent: A dictionary that contains the nearest parent for node
   seen = set([start]) # Set for seen nodes.
   # Frontier is the boundary between seen and unseen
   frontier = PriorityQueueUpdatable() # Frontier of unvisited nodes as a F
   node2parent = {start : None} # Keep track of nearest parent for each nod
   hfn = heuristic dist fn # make the name shorter
   node2dist = {start: 0 } # Keep track of cost to arrive at each node
   search order = []
   frontier.put(PItem(0 + hfn(start, goal), start)) # <----- Diff</pre>
   if debug: debugf.write("goal = " + str(goal) + '\n')
   i = 0
                                       # Creating loop to visit each node
   while not frontier.empty():
        dist m = frontier.get() # Get the smallest addition to the frontier
        if debug: debugf.write("%d) Q = " % i + str(list(frontier.queue)) +
        if debug: debugf.write("%d) node = " % i + str(dist m) + '\n')
       #if debug: print("dists = " , [node2dist[n.node] for n in frontier.q
        m = dist m.node
       m dist = node2dist[m]
        search order.append(m)
        if goal is not None and m == goal:
            return True, search order, node2parent, node2dist
        for neighbor, edge cost in graph.get(m, []):
            old dist = node2dist.get(neighbor, float("inf"))
            new dist = edge cost + m dist
            if neighbor not in seen:
                seen.add(neighbor)
                frontier.put(PItem(new_dist + hfn(neighbor, goal), neighbor
                node2parent[neighbor] = m
                node2dist[neighbor] = new dist
            elif new dist < old dist:</pre>
                node2parent[neighbor] = m
                node2dist[neighbor] = new dist
                # ideally you would update the dist of this item in the pric
                # as well. But python priority queue does not support fast u
                # ----- Different from dijkstra -----
                old item = PItem(old dist + hfn(neighbor, goal), neighbor)
                if old item in frontier:
                    frontier.replace(
                        old item,
                        PItem(new dist + hfn(neighbor, goal), neighbor))
        i += 1
   if goal is not None:
        return False, search order, node2parent, node2dist
   else:
        return True, search order, node2parent, node2dist
```

```
def backtrace_path(node2parent, start, goal):
    c = goal
    r_path = [c]
    parent = node2parent.get(c, None)
    while parent != start:
        r_path.append(parent)
        c = parent
        parent = node2parent.get(c, None) # Keep getting the parent until you #print(parent)
    r_path.append(start)
    r_path.append(start)
    return reversed(r_path) # Reverses the path
```

Overwriting astar.py

Creating a discrete maze

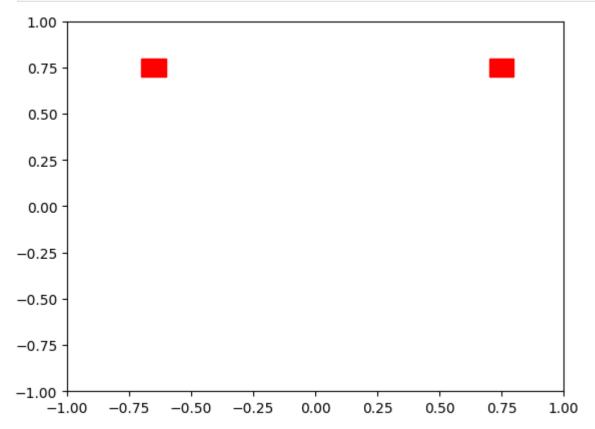
```
In [3]: # Define the bounds of the maze
        import numpy as np
        state min = np.array([
            -1.0, # x min in meters
            -1.0, # y min in meters
            -np.pi # theta min in radians
        ])
        state max = np.array([
            1.0, # x max in meters
            1.0, # y max in meters
            np.pi # theta max in radians
        ])
        state discrete step = np.array([
            0.01, # x min in meters
            0.01, # y min in meters
            np.pi/10 # theta min in radians
        ])
        # Let's put obstacles somewhere (arbitrary for now)
        obstacles = np.array([
            [0.7, 0.7],
            [-0.7, 0.7]
        ])
        # Let's put goal somewhere (arbitrary for now)
        goal = np.array([0, 0.9])
        # The robot must start with a state
        start state = np.array([0.0, 0.0, np.pi/3])
```

Defining numbers is good, but visualizing them is even better. Let's create some visualization code that show the robot and obstacles.

```
import matplotlib.pyplot as plt
import matplotlib.patches as patches

fig, ax = plt.subplots()
ax.set_xlim(state_min[0], state_max[0])
ax.set_ylim(state_min[1], state_max[1])
```

```
# Draw the obstacle as rectangles
# Had to google matplotlib + draw rectangle
# https://stackoverflow.com/questions/37435369/how-to-draw-a-rectangle-on-im
for obs in obstacles:
    rect = patches.Rectangle((obs[0], obs[1]), 0.1, 0.1, linewidth=1, edgecd
    # Add the patch to the Axes
    ax.add_patch(rect)
plt.show()
```



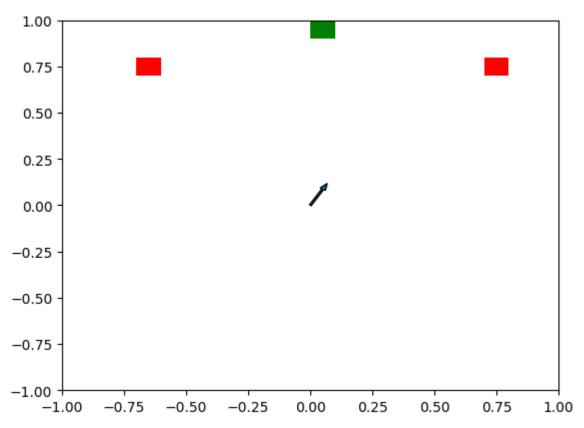
You should test the code often. Usually I would keep modifying the code after editing the rectangles, but to show the intermediate state of the code, I leave the above cell as it is. I modify a copy of the code below. The below code was tested multiple times before

```
In [5]: fig, ax = plt.subplots()
    ax.set_xlim(state_min[0], state_max[0])
    ax.set_ylim(state_min[1], state_max[1])

# Draw the obstacle as rectangles
    # Had to google matplotlib + draw rectangle
    # https://stackoverflow.com/questions/37435369/how-to-draw-a-rectangle-on-in
    for obs in obstacles:
        orect = patches.Rectangle(obs, 0.1, 0.1, facecolor='r')
        # Add the patch to the Axes
        ax.add_patch(orect)

# Draw the goal as a rectangle
    grect = patches.Rectangle(goal, 0.1, 0.1, facecolor='g')
    ax.add_patch(grect)
```

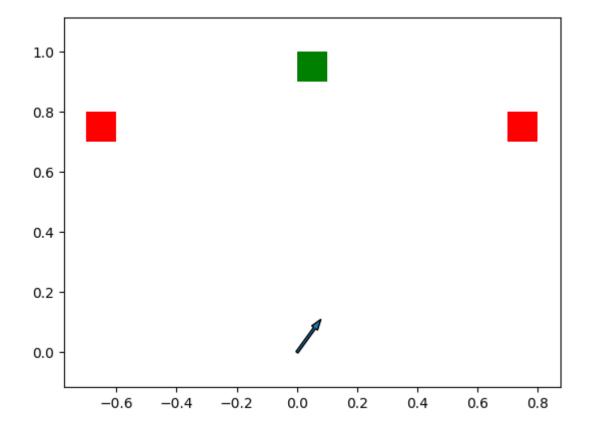
Out[5]: <matplotlib.patches.FancyArrow at 0x7fe44a270760>



Looks like we are wasting a lot of space. Also we will need to visualize this many times. Let's make a function out of this.

```
In [6]: from dataclasses import dataclass
        @dataclass
        class MapProperties:
            state min : np.ndarray
            state max : np.ndarray
            state discrete step : np.ndarray
            obstacles : np.ndarray
        def plot_map(ax, map_properties, goal, current_state,
                    obstaclesize=0.1,
                    goalsize=0.1,
                    robotsize=0.1,
                    obstaclecolor='r',
                    goalcolor='g'):
            state_min = map_properties.state_min
            state max = map properties.state max
            obstacles = map properties.obstacles
            ax.set xlim(state min[0], state max[0])
```

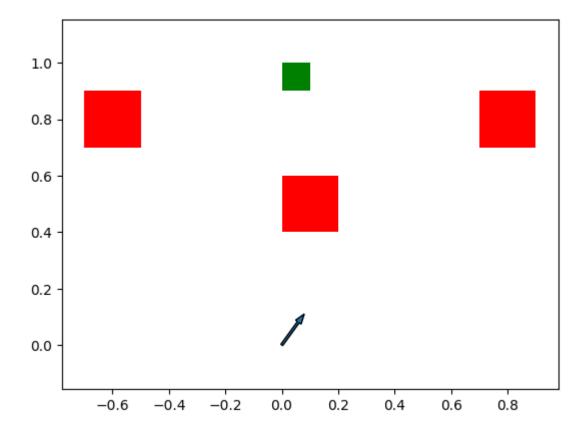
```
ax.set ylim(state min[1], state max[1])
    ax.axis('equal') # keeps square obstacles square
    # Draw the obstacle as rectangles
    # Had to google matplotlib + draw rectangle
    # https://stackoverflow.com/questions/37435369/how-to-draw-a-rectangle-d
    for obs in obstacles:
        orect = patches.Rectangle(obs, obstaclesize, obstaclesize, facecolor
        # Add the patch to the Axes
        ax.add patch(orect)
    # Draw the goal as a rectangle
    grect = patches.Rectangle(goal, goalsize, goalsize, facecolor=goalcolor,
    ax.add patch(grect)
    # Draw the robot as an arrow
    # Google: matplotlib draw arrow
    ax.arrow(current state[0], current state[1],
             robotsize*np.cos(current state[2]), robotsize*np.sin(current st
             width=0.08*robotsize,
             label='robot')
    return ax
map = MapProperties(
    state min = np.array([
        -1.0, # x min in meters
        0.0, # y min in meters
        -np.pi # theta min in radians
    ]),
    state max = np.array([
        1.0, # x max in meters
        1.0, # y max in meters
       np.pi # theta max in radians
    ]),
    state discrete step = np.array([
        0.01, # x min in meters
        0.01, # y min in meters
        np.pi/20 # theta min in radians
    ]),
    # Let's put obstacles somewhere (arbitrary for now)
    obstacles = np.array([
        [0.7, 0.7],
        [-0.7, 0.7]
    ])
# Let's put goal somewhere (arbitrary for now)
goal = np.array([0, 0.9])
# The robot must start with a state
start state = np.array([0.0, 0.0, 3*np.pi/10])
fig, ax = plt.subplots()
plot map(ax, map, goal, start state)
```



It is always a good practice to (as we did above):

- 1. group related variables to classes (or dataclasses)
- 2. change the constants in the code to meaningful keyword arguments or constants.

This problem looks too easy on this visualization. Let's make it a bit harder by increasing the obstacle size and number. We can simplify it back when we move to the real robot.



Now we need to convert this obstacle list as as graph. Recall that the function astar uses only one function on graph:

for neighbor, edge_cost in graph.get(m, [])
So we only need to implement a function that provides the possible neighbors of
the current node (state) along with edge_cost (the cost of taking that action).

```
In [8]: class ObstacleListToGraph:
    def __init__(self, map_properties):
        pass
    def get(self, current_state, default):
        nbrs = []
        return nbrs
```

Now what are neighbors of the robot state in current_state = [x, y, theta].

$$s_t = [x_t, y_t, heta_t]$$

This robot can move only forward, backward with linear velocity v_t and rotate in place with angular velocity ω_t .

$$s_{t+1} = egin{bmatrix} x_{t+1} \ y_{t+1} \ heta_{t+1} \end{bmatrix} = egin{bmatrix} x_t \ y_t \ heta_t \end{bmatrix} + egin{bmatrix} v_t \cos(heta_t) dt \ v_t \sin(heta_t) dt \ \omega_t dt \end{bmatrix}$$
 (1)

This equation represents the state transition model. This particular equation is called the unicycle model.

What should be an ideal dt? Ideally, it should be as small as possible, approaching zero.

What should be good dt if we have already decided on a state_discrete_step $= [\Delta x, \Delta y, \Delta \theta] \text{ size? It should be as small as possible, but at least big enough that when you are moving with minimum absolute linear velocity, you should at least land in the next discrete cell. Otherwise you will stay in the same cell until eternity.$

$$dt = \max\left(rac{\sqrt{(\Delta x)^2 + (\Delta y)^2}}{|v_{min}|}, rac{\Delta heta}{|\omega_{min}|}
ight)$$

Out[9]: 0.5

Creating a discrete graph

The state space is of size 3, so we need to create the graph in 3D. The neighbors of the graph are possible cells where the robot can go in the next time step dt. The full range of neighborhood is determined by min and max linear velocity and angular velocity. We will draw a straight line with the cells that are covered min by min and max linear velocity and all velocities within that range. And for each neighbor reachable by linear displacement we also have the angular velocity possibilities.

```
In [10]:
    class ObstacleListToGraph:
        def __init__(self, map_properties, robot):
            self.map = map_properties
            self.robot = robot

    def __discretize(self, s):
        # divide by the step size, then round to convert the number to the n
        # and then multiply by the step size again.
```

```
return np.round(s / self.map.state discrete step) * self.map.state d
def get(self, current state, default=[]):
    vmin, vmax = self.robot.linvel range
    omegamin, omegamax = self.robot.angvel range
    step size = self.map.state discrete step
    dt = self.robot.dt
    s = np.asarray(current state)
    # The max and min state cells are governed by
    # max and min velocity both linear and angular
    s = self. discretize(s)
    state min = s + np.array([
        vmin * np.cos(s[2]) * dt,
        vmin * np.sin(s[2]) * dt,
        omegamin * dt])
    state min = self. discretize(state min)
    xy min = state min[:2]
    theta min = state min[2]
    state delta = np.array([
        (vmax - vmin) * np.cos(s[2]) * dt,
        (vmax - vmin) * np.sin(s[2]) * dt,
        (omegamax - omegamin) * dt
    1)
    state delta = self. discretize(state delta)
    xy delta = state delta[:2]
    theta delta = state delta[2]
    xy uniq nbrs = self.get lin vel nbrs(s, xy delta, xy min, step size)
    state nbrs = np.empty((len(xy uniq nbrs), len(s)))
    state nbrs[:len(xy uniq nbrs), :2] = xy uniq nbrs
    return state nbrs
def get lin vel nbrs(self, s, xy delta, xy min, step size):
    # all the cells that lie on the straight line between xy delta+xy mi
    # possible neighbors.
    xy dist = np.sqrt((xy delta**2).sum())
    min step = np.min(step size[:2])
    xy_max_steps = xy_dist / min step
    xy dir = min step * np.array([np.cos(s[2]), np.sin(s[2])])
    xy nbrs = np.arange(xy_max_steps)[:, None]*xy_dir + xy_min
    xy uniq nbrs = np.unique(
        np.round(xy nbrs / step size[:2]).astype(dtype=np.int64),
        axis=0
    ) * step size[:2]
    return xy uniq nbrs
```

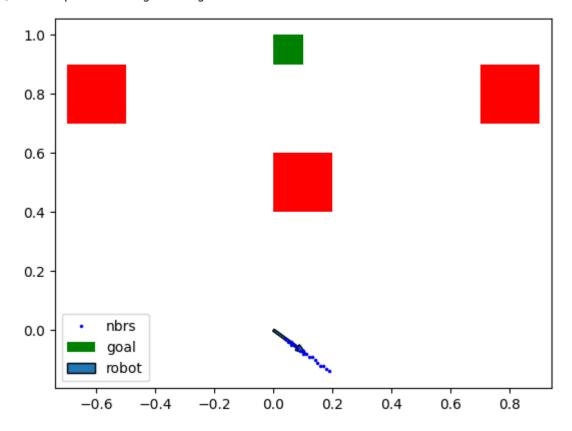
This is a lot of code to be written in one go. Let's test it with different values, visualize it to see if it gives us the possible neighbors that we expect for different robot states.

```
In [11]: graph = ObstacleListToGraph(map, robot)
# start_state = np.array([0., 0., 3*np.pi/10])
```

```
# start_state = np.array([0., 0., 6*np.pi/10])
# start_state = np.array([0., 0., 9*np.pi/10])
# start_state = np.array([0., 0., 12*np.pi/10])
# start_state = np.array([0., 0., 15*np.pi/10])
start_state = np.array([0., 0., 18*np.pi/10])
# start_state = np.array([0., 0., 21*np.pi/10])
# start_state = np.array([0., 0., np.pi/10])
state_nbrs = graph.get(start_state)

fig, ax = plt.subplots()
ax.plot(state_nbrs[:, 0], state_nbrs[:, 1], 'bo', markersize=1.5, label='nbr plot_map(ax, map, goal, start_state, obstaclesize=0.2)
ax.legend()
```

Out[11]: <matplotlib.legend.Legend at 0x7fe44a07d370>



If the parameters do not look good to you about vmin, vmax, etc. Feel free to go back and edit them to make them more realistic.

Finding the set of cells that are covered by a straight line in a discrete grid is a common problem in computer graphics and robotics. The proper way to find these are by Bresenham's line algorithm. The code I wrote is just an inefficient way to do the same. The most inefficient part there is np.unique.

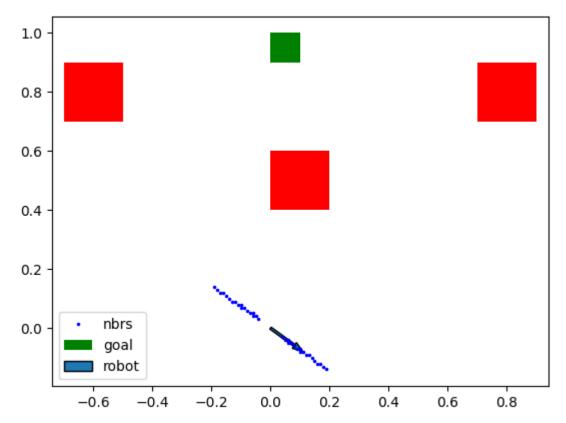
This looks a good set of neighbors in the forward direction. What about the reverse direction? The jetbot can move in reverse. Let's duplicate the set of neighbors in the opposite direction as well.

```
In [12]: class ObstacleListToGraph:
             def init (self, map properties, robot):
                 self.map = map properties
                 self.robot = robot
             def discretize(self, s):
                 # divide by the step size, then round to convert the number to the r
                 # and then multiply by the step size again.
                 return np.round(s / self.map.state discrete step) * self.map.state d
             def get(self, current state, default=[]):
                 vmin, vmax = self.robot.linvel range
                 omegamin, omegamax = self.robot.angvel range
                 step size = self.map.state discrete step
                 dt = self.robot.dt
                 s = np.asarray(current state)
                 # The max and min state cells are governed by
                 # max and min velocity both linear and angular
                 s = self. discretize(s)
                 state_min = s + np.array([
                     vmin * np.cos(s[2]) * dt,
                     vmin * np.sin(s[2]) * dt,
                     omegamin * dt])
                 state_min = self._discretize(state_min)
                 xy min = state min[:2]
                 theta min = state min[2]
                 state delta = np.array([
                      (vmax - vmin) * np.cos(s[2]) * dt,
                      (vmax - vmin) * np.sin(s[2]) * dt,
                      (omegamax - omegamin) * dt
                 ])
                 state delta = self. discretize(state delta)
                 xy delta = state delta[:2]
                 theta delta = state delta[2]
                 xy uniq nbrs = self.get lin vel nbrs(s, xy delta, xy min, step size)
                 xy_uniq_nbrs_opp = s[:2] - (xy_uniq_nbrs - s[:2])
                 xy both side nbrs = np.vstack((xy uniq nbrs,
                                                 xy uniq nbrs opp))
                 state nbrs = np.empty((len(xy both side nbrs), len(s)))
                 state nbrs[:len(xy both side nbrs), :2] = xy both side nbrs
                 return state nbrs
             def get lin vel nbrs(self, s, xy delta, xy min, step size):
                 # all the cells that lie on the straight line between xy delta+xy mi
                 # possible neighbors.
                 xy dist = np.sqrt((xy delta**2).sum())
                 min_step_size = np.min(step_size[:2])
                 xy max steps = xy dist / min step size
                 xy_dir = min_step_size * np.array([np.cos(s[2]), np.sin(s[2])])
```

```
In [13]: graph = ObstacleListToGraph(map, robot)
    start_state = np.array([0., 0., 18*np.pi/10])
    state_nbrs = graph.get(start_state)

fig, ax = plt.subplots()
    ax.plot(state_nbrs[:, 0], state_nbrs[:, 1], 'bo', markersize=1.5, label='nbr
    plot_map(ax, map, goal, start_state, obstaclesize=0.2)
    ax.legend()
```

Out[13]: <matplotlib.legend.Legend at 0x7fe44a02b7f0>



This looks like a set of good linear velocity neighbors but what about rotation? The jetbot can rotate in place and it can rotate while moving as well.

Keeping angles wraparound in sight

We can follow the same process, but there is a catch. Unlike position, theta wraps around 360° . 360° is exactly the same as 0° . So 359.99° is probably a neighbor off 0° . One way to do that is by taking the theta to be the remainder of 2π every time we do a computation on theta. This will keep theta between 0 and 2π , but if you want to keep θ between $[-\pi,\pi)$, then something similar can be done: $((\theta+\pi)\mod 2\pi)-\pi$

What is difference of two angles?

Every pair of angles can have two differences: clockwise difference and anticlockwise difference. When we talk about positive direction of angular velocity, it is in the anticlockwise direction. So all our differences of two angles must be positive in anticlockwise direction, even if the first angle is smaller than the second.

Only angle values needs to be wrapped, not the delta angle values. Sometimes it is better to do this tracking by maintaining different datatypes. But we do not want to deal with subclassing numpy arrays for now. We will leave this exercise (good programming practice) for some other time. For now, we will track this by calling all variable names that represent an angle and need to be wrapped as angle. Everything else can just stay unwrapped.

This kind of difference between type of quantity and its difference also occurs with date and time types. Python has a different datatype for timedelta which is different from datetime type.

The differences of two angles must not be wrapped between 0 and 2π . If you are rotating at 2π radians per sec, it is different from 0 per sec.

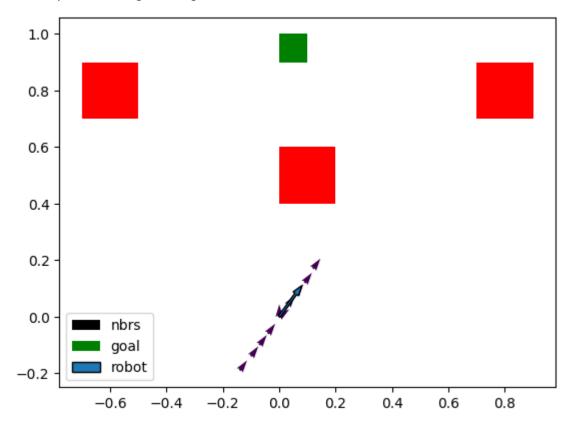
```
In [14]: class Angle:
             @staticmethod
             def wrap(theta):
                 return ((theta + np.pi) % (2*np.pi)) - np.pi
             @staticmethod
             def iswrapped(theta):
                  return (-np.pi <= theta) & (theta < np.pi)</pre>
             @staticmethod
             def diff(a, b):
                 assert Angle.iswrapped(a).all()
                 assert Angle.iswrapped(b).all()
                 # np.where is like a conditional statement in numpy
                 # but it operates on per element level inside the numpy array
                  return np.where(a < b,</pre>
                                  (2*np.pi + a - b),
                                  (a - b))
             @staticmethod
             def dist(a, b):
                 # The distance between two angles is minimum of a - b and b - a.
                 return np.minimum(Angle.diff(a, b), Angle.diff(b, a))
         class ObstacleListToGraph:
             def init (self, map properties, robot):
                 self.map = map properties
                 self.robot = robot
```

```
def discretize(self, s):
    # divide by the step size, then round to convert the number to the n
    # and then multiply by the step size again.
    return np.round(s / self.map.state discrete step) * self.map.state d
def get bounds(self, current state):
    vmin, vmax = self.robot.linvel range
    omegamin, omegamax = self.robot.angvel range
    step size = self.map.state discrete step
    dt = self.robot.dt
    s = np.asarray(current state)
    # The max and min state cells are governed by
    # max and min velocity both linear and angular
    s = self. discretize(s)
    state min = s + np.array([
        vmin * np.cos(s[2]) * dt,
        vmin * np.sin(s[2]) * dt,
        omegamin * dt])
    state_min = self._discretize(state_min)
    xy min = state min[:2]
    theta min angle = Angle.wrap(state min[2]) # Angle type
    state delta = np.array([
        (vmax - vmin) * np.cos(s[2]) * dt,
        (vmax - vmin) * np.sin(s[2]) * dt,
        (omegamax - omegamin) * dt
    ])
    state delta = self. discretize(state delta)
    xy delta = state delta[:2]
    theta delta = state delta[2]
    return s, xy delta, xy min, theta delta, theta min angle, step size
def get(self, current state, default=[]):
    s, xy delta, xy min, theta delta, theta min angle, step size = self.
    xy_uniq_nbrs = self.get_lin_vel_nbrs(s, xy delta, xy min, step size)
    xy uniq nbrs opp = s[:2] - (xy uniq nbrs - s[:2])
    xy both side nbrs = np.vstack((xy uniq nbrs,
                                   xy uniq nbrs opp))
    theta nbrs angle = self.get ang vel nbrs(s, theta delta, theta min a
    theta angle = s[2] # Angle type
    # Note that for theta nbrs angle and theta angle we use diff, but on
    # use normal substraction with theta angle.
    theta_nbrs_opp_angle = Angle.wrap(theta_angle - Angle.diff(theta_nbr
    theta_both_side_nbrs_angle = np.unique(np.hstack((theta_nbrs_angle,
                                           theta nbrs opp angle)))
    state nbrs = np.empty((len(xy both side nbrs) + len(theta both side
    state nbrs[:len(xy both side nbrs), :2] = xy both side nbrs
    state_nbrs[:len(xy_both_side_nbrs), 2] = theta_angle
    state nbrs[len(xy both side nbrs):, :2] = s[:2]
```

```
state nbrs[len(xy both side nbrs):, 2] = theta both side nbrs angle
                 return state nbrs
             def get lin vel nbrs(self, s, xy delta, xy min, step size):
                 # all the cells that lie on the straight line between xy delta+xy mi
                 # possible neighbors.
                 xy dist = np.sqrt(((xy delta)**2).sum())
                 min step = np.min(step size[:2])
                 xy max steps = xy dist / min step
                 xy dir = min step * np.array([np.cos(s[2]), np.sin(s[2])])
                 xy nbrs = np.arange(xy max steps)[:, None]*xy dir + xy min
                 xy uniq nbrs = np.unique(
                     np.round(xy nbrs / step size[:2]).astype(dtype=np.int64),
                     axis=0
                 ) * step size[:2]
                 return xy uniq nbrs
             def get ang vel nbrs(self, s, theta delta, theta min angle, step size):
                 theta steps = theta delta / step size[2]
                 theta nbrs angle = np.unique(Angle.wrap(np.arange(theta steps)*step
                 return theta nbrs angle
In [15]: graph = ObstacleListToGraph(map, robot)
         def rad2deg(theta):
             return theta / np.pi * 180
         for i in range(3, 21, 3):
             start state = np.array([0., 0., Angle.wrap(i*np.pi/10)])
             s, xy delta, xy min, theta delta, theta min, step size = graph. get bour
             theta nbrs = graph.get ang vel nbrs(s, theta delta, theta min, step size
             print(rad2deg(s[2]), rad2deg(theta nbrs))
             print(rad2deg(s[2]), rad2deg(Angle.wrap(s[2] - Angle.diff(theta nbrs, s[
        54.0 [ 63. 72. 81. 90. 99. 108.]
        54.0 [45. 36. 27. 18. 9. 0.]
        108.0 [117. 126. 135. 144. 153. 162.]
        108.0 [99. 90. 81. 72. 63. 54.]
        162.0 [-180. -171. -162. -153. -144. 171.]
        162.0 [144. 135. 126. 117. 108. 153.]
        -144.0 [-135. -126. -117. -108. -99. -90.]
        -144.0 [-153. -162. -171. -180. 171. 162.]
        -90.0 [-81. -72. -63. -54. -45. -36.]
        -90.0 [ -99. -108. -117. -126. -135. -144.]
        -36.0 [-27. -18. -9. 0. 9. 18.]
        -36.0 [-45. -54. -63. -72. -81. -90.]
In [16]: graph = ObstacleListToGraph(map, robot)
         start state = np.array([0., 0., 3*np.pi/10])
         state nbrs = graph.get(start state)
         fig, ax = plt.subplots()
         state nbrs plot = state nbrs[::5, :]
         ax.quiver(state nbrs plot[:, 0], state nbrs plot[:, 1],
                   np.cos(state nbrs plot[:, 2]), np.sin(state nbrs plot[:, 2]),
                   0.7,
                   label='nbrs',
```

```
scale=40)
plot_map(ax, map, goal, start_state, obstaclesize=0.2)
ax.legend()
```

Out[16]: <matplotlib.legend.Legend at 0x7fe449efcaf0>



In [17]: state_nbrs

```
0.04
Out[17]: array([[ 0.03
                                            0.9424778 ],
                [ 0.04
                               0.05
                                            0.9424778 ],
                [ 0.04
                               0.06
                                            0.9424778 ],
                [ 0.05
                               0.06
                                            0.9424778],
                [ 0.05
                               0.07
                                            0.9424778 ],
                               0.08
                [ 0.06
                                            0.9424778],
                             , 0.09
                [ 0.07
                                            0.9424778 ],
                                            0.9424778 ],
                [ 0.07
                               0.1
                [ 0.08
                               0.1
                                            0.9424778],
                               0.11
                                            0.9424778 ],
                [ 0.08
                            , 0.12
                [ 0.09
                                            0.9424778],
                [ 0.09
                             , 0.13
                                            0.9424778 ],
                [0.1]
                               0.14
                                            0.9424778 ],
                            , 0.15
                                            0.9424778],
                [0.11]
                [ 0.12
                               0.16
                                            0.9424778 ],
                             , 0.17
                                         , 0.9424778],
                [ 0.12
                            , 0.18
                                         , 0.9424778],
                [ 0.13
                             , 0.19
                                         , 0.9424778],
                [ 0.14
                             , -0.04
                [-0.03
                                            0.9424778 ],
                             , -0.05
                [-0.04
                                            0.9424778 ],
                                         , 0.9424778],
                [-0.04
                             , -0.06
                [-0.05
                             , -0.06
                                         , 0.9424778],
                             , -0.07
                                         , 0.9424778],
                [-0.05
                             , -0.08
                [-0.06
                                            0.9424778 ],
                             , -0.09
                                            0.9424778],
                [-0.07
                                         , 0.9424778],
                             , -0.1
                [-0.07
                [-0.08
                             , -0.1
                                            0.9424778 ],
                             , -0.11
                                         , 0.9424778],
                [-0.08
                            , -0.12
                [-0.09
                                            0.9424778 ],
                                         , 0.9424778],
                [-0.09]
                             , -0.13
                                         , 0.9424778],
                [-0.1
                             , -0.14
                [-0.11]
                             , -0.15
                                            0.9424778 ],
                             , -0.16
                                         , 0.9424778],
                [-0.12
                                         , 0.9424778],
                             , -0.17
                [-0.12
                                         , 0.9424778 ],
                [-0.13]
                              -0.18
                             , -0.19
                                         , 0.9424778],
                [-0.14]
                [ 0.
                               0.
                                                      ],
                [ 0.
                            , 0.
                                         , 0.15707963],
                [ 0.
                               0.
                                            0.31415927],
                [ 0.
                            , 0.
                                        , 0.4712389],
                            , 0.
                                         , 0.62831853],
                [ 0.
                            , 0.
                                        , 0.78539816],
                [ 0.
                            , 0.
                                         , 1.09955743],
                [ 0.
                [ 0.
                               0.
                                            1.25663706],
                            , 0.
                [ 0.
                                            1.41371669],
                               0.
                                            1.57079633],
                [ 0.
                [ 0.
                               0.
                                            1.72787596],
                [ 0.
                                            1.88495559]])
                               0.
```

I am satisfied with the list of all possible neighbors.

But we also need the neighbors to be checked against obstacles and removed from the neighbor list. The neighbors returned must be only the ones that do not collide with obstacles. Let's add bounds to the obstacle description.

```
In [18]: map = MapProperties(
             state min = np.array([
                 -1.0, # x min in meters
                 -.1, # y min in meters
                 -np.pi # theta min in radians
             ]),
             state max = np.array([
                 1.0, # x max in meters
                 1.0, # y max in meters
                 np.pi # theta max in radians
             ]),
             state discrete step = np.array([
                 0.01, # x min in meters
                 0.01, # y min in meters
                 np.pi/20 # theta min in radians
             # Let's put obstacles somewhere (arbitrary for now)
             obstacles = np.array([
                 [0.7, 0.7, 0.2, 0.1], # x, y, width, height
                 [0., 0.4, 0.2, 0.1], # x, y, width, height
                 [-0.7, 0.7, 0.2, 0.1] # x, y, width, height
             ])
         def plot map(ax, map properties, goal, current state,
                     goalsize=0.1,
                     robotsize=0.1,
                     obstaclecolor='r',
                     goalcolor='g'):
             state min = map properties.state min
             state max = map properties.state max
             obstacles = map properties.obstacles
             ax.set xlim(state min[0], state max[0])
             ax.set ylim(state min[1], state max[1])
             ax.axis('equal') # keeps square obstacles square
             # Draw the obstacle as rectangles
             # Had to google matplotlib + draw rectangle
             # https://stackoverflow.com/questions/37435369/how-to-draw-a-rectangle-d
             for obs in obstacles:
                 xy = obs[:2]
                 width, height = obs[2:]
                 orect = patches.Rectangle(xy, width, height, facecolor=obstaclecolor
                 # Add the patch to the Axes
                 ax.add patch(orect)
             # Draw the goal as a rectangle
             grect = patches.Rectangle(goal, goalsize, goalsize, facecolor=goalcolor,
             ax.add patch(grect)
             # Draw the robot as an arrow
             # Google: matplotlib draw arrow
```

```
In [19]: class ObstacleListToGraph:
             def init (self, map properties, robot):
                 self.map = map properties
                 self.robot = robot
             def discretize(self, s):
                 # divide by the step size, then round to convert the number to the n
                 # and then multiply by the step size again.
                 return np.round(s / self.map.state discrete step) * self.map.state d
             def get bounds(self, current state):
                 vmin, vmax = self.robot.linvel range
                 omegamin, omegamax = self.robot.angvel range
                 step size = self.map.state discrete step
                 dt = self.robot.dt
                 s = np.asarray(current_state)
                 # The max and min state cells are governed by
                 # max and min velocity both linear and angular
                 s = self. discretize(s)
                 state min = s + np.array([
                     vmin * np.cos(s[2]) * dt,
                     vmin * np.sin(s[2]) * dt,
                     omegamin * dt])
                 state min = self. discretize(state min)
                 xy min = state min[:2]
                 theta min angle = wrap theta(state min[2]) # Angle type
                 state delta = np.array([
                     (vmax - vmin) * np.cos(s[2]) * dt,
                     (vmax - vmin) * np.sin(s[2]) * dt,
                     (omegamax - omegamin) * dt
                 ])
                 state delta = self. discretize(state delta)
                 xy delta = state delta[:2]
                 theta delta = state delta[2]
                 return s, xy delta, xy min, theta delta, theta min angle, step size
             def get all nbrs(self, current state, default=[]):
                 s, xy delta, xy min, theta delta, theta min angle, step size = self.
                 xy uniq nbrs = self.get lin vel nbrs(s, xy delta, xy min, step size)
                 xy uniq nbrs opp = s[:2] - (xy uniq nbrs - s[:2])
                 xy both side nbrs = np.vstack((xy uniq nbrs,
                                                xy uniq nbrs opp))
                 theta nbrs angle = self.get ang vel nbrs(s, theta delta, theta min a
                 theta angle = s[2] # Angle type
```

```
# Note that for theta nbrs angle and theta angle we use diff, but on
    # use normal substraction with theta angle.
    theta nbrs opp angle = Angle.wrap(theta angle - Angle.diff(theta nbr
    theta both side nbrs angle = np.unique(np.hstack((theta nbrs angle,
                                            theta nbrs opp angle)))
    state nbrs = np.empty((len(xy both side nbrs) + len(theta both side
    state_nbrs[:len(xy_both_side_nbrs), :2] = xy_both_side_nbrs
    state nbrs[:len(xy both side nbrs), 2] = theta angle
    state nbrs[len(xy both side nbrs):, :2] = s[:2]
    state nbrs[len(xy both side nbrs):, 2] = theta both side nbrs angle
    return state nbrs
def get lin vel nbrs(self, s, xy delta, xy min, step size):
    # all the cells that lie on the straight line between xy delta+xy mi
    # possible neighbors.
    xy dist = np.sqrt(((xy delta)**2).sum())
    min step = np.min(step size[:2])
    xy max steps = xy dist / min step
    xy dir = min step * np.array([np.cos(s[2]), np.sin(s[2])])
    xy nbrs = np.arange(xy max steps)[:, None]*xy dir + xy min
    xy uniq nbrs = np.unique(
        np.round(xy nbrs / step size[:2]).astype(dtype=np.int64),
        axis=0
    ) * step size[:2]
    return xy uniq nbrs
def get ang vel nbrs(self, s, theta delta, theta min angle, step size):
    theta steps = theta delta / step size[2]
    theta nbrs angle = np.unique(Angle.wrap(np.arange(theta steps)*step
    return theta nbrs angle
def get(self, current state, default=[]):
    all nbrs = self.get all nbrs(current state, default)
    nbrs xy = all nbrs[:, :2] # Nx2 with N neighbors and 2 dim x, y. the
    obstacles xy = self.map.obstacles[:, :2] # 0x2
    obstacles wh = self.map.obstacles[:, 2:] # 0x2
    nbr obs diff = nbrs xy[:, None, :] - obstacles xy[None, :, :] # N \times O x
    is nbr in obs = ((0 \le \text{nbr obs diff}) \& (\text{nbr obs diff} < \text{obstacles wh})
    is nbr in any obs = is nbr in obs.any(axis=-1) # N
    return all nbrs[~is nbr in any obs, :] # Return all neighbors that a
```

But we also need the cost of taking action with each neighbor. The costs are very problem dependent. If you are a truck driver, the energy costs matter the most but if you are a F1 drive the time cost is of utmost important. One thing we can do in a principled way (or close enough) is set a relative cost between angular velocity and linear velocity neighbors.

We can do it in a more principled way if we knew the mass to moment of inertia. Anyway, we are not working in acceleration domain, so we will stick to crude approximations. We would take half the distance travelled by each wheel to be a measure of the cost. The cost of moving d distance forward or backward is d.

The separation between two wheels is L=0.12m so the cost of turning by heta radian is L heta/2.

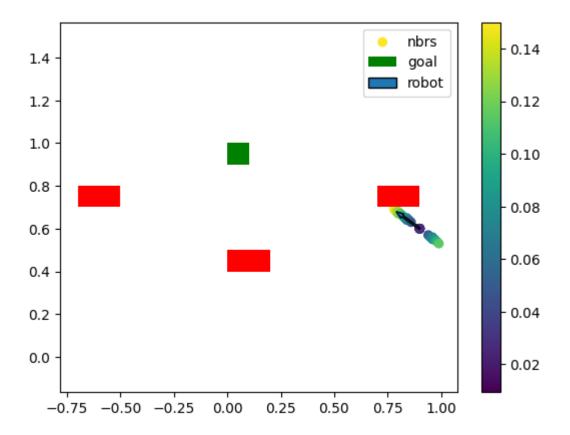
Let's add this to graph class.

```
In [20]: @dataclass
         class Robot:
             linvel range: np.ndarray
             angvel_range: np.ndarray
             dt: float
             wheel base: float
          robot = Robot(linvel range = np.array([0.1, 0.5]),
                        angvel range = np.array([np.pi/10, 3*np.pi/4]),
                        wheel base = 0.12,
                        dt = None
          robot.dt = max(
             np.sqrt((map.state discrete step[:2]**2).sum()) / np.abs(robot.linvel ra
             map.state discrete step[2] / np.abs(robot.angvel range[0]))
         def do points collide(map, xy nbrs):
             # Nx2 with N neighbors and 2 dim x, y. the orientation of robot does not
             in bounds = (
                  (map.state min[:2] <= xy nbrs) &</pre>
                  (xy nbrs < map.state max[:2])).all(axis=-1)</pre>
             obstacles xy = map.obstacles[:, :2] # 0x2
             obstacles wh = map.obstacles[:, 2:] # 0x2
             nbr obs diff = xy nbrs[:, None, :] - obstacles xy[None, :, :] # Nx0x2
             is nbr in obs = ((0 \le \text{nbr obs diff}) \& (\text{nbr obs diff} < \text{obstacles wh})).al
             is nbr in any obs = is nbr in obs.any(axis=-1) # N
              return is nbr in_any_obs | (~in_bounds)
         class ObstacleListToGraph:
             def init (self, map properties, robot):
                  self.map = map properties
                  self.robot = robot
             def discretize(self, s):
                 # divide by the step size, then round to convert the number to the n
                  # and then multiply by the step size again.
                  return np.round(s / self.map.state discrete step) * self.map.state d
             def get bounds(self, current state):
                  vmin, vmax = self.robot.linvel range
                  omegamin, omegamax = self.robot.angvel range
                  step size = self.map.state discrete step
                  dt = self.robot.dt
                  s = np.asarray(current state)
                 # The max and min state cells are governed by
```

```
# max and min velocity both linear and angular
    s = self. discretize(s)
    state min = s + np.array([
        vmin * np.cos(s[2]) * dt,
        vmin * np.sin(s[2]) * dt,
        omegamin * dt])
    state min = self. discretize(state min)
    xy min = state min[:2]
    theta min angle = Angle.wrap(state min[2]) # Angle type
    state delta = np.array([
        (vmax - vmin) * np.cos(s[2]) * dt,
        (vmax - vmin) * np.sin(s[2]) * dt,
        (omegamax - omegamin) * dt
    ])
    state delta = self. discretize(state delta)
    xy delta = state delta[:2]
    theta delta = state delta[2]
    return s, xy delta, xy min, theta delta, theta min angle, step size
def get all nbrs(self, current state, default):
    s, xy delta, xy min, theta delta, theta min angle, step size = self.
    xy uniq nbrs = self.get lin vel nbrs(s, xy delta, xy min, step size)
    xy uniq nbrs opp = self.get lin vel nbrs(s, -xy delta,
                                             s[:2] - (xy min - s[:2]), #
                                             step_size)
    xy both side nbrs = np.vstack((xy uniq nbrs,
                                   xy_uniq_nbrs_opp))
    theta_nbrs_angle = self.get_ang_vel_nbrs(s, theta_delta, theta_min_a
    theta angle = s[2] # Angle type
    # Note that for theta nbrs angle and theta angle we use diff, but on
    # use normal substraction with theta angle.
    theta nbrs opp angle = Angle.wrap(theta angle - Angle.diff(theta nbr
    theta both side nbrs angle = np.unique(np.hstack((theta nbrs angle,
                                           theta nbrs opp angle)))
    state nbrs = np.empty((len(xy both side nbrs) + len(theta both side
    state_nbrs[:len(xy_both_side_nbrs), :2] = xy_both_side_nbrs
    state nbrs[:len(xy both side nbrs), 2] = theta angle
    state nbrs[len(xy both side nbrs):, :2] = s[:2]
    state nbrs[len(xy both side nbrs):, 2] = theta both side nbrs angle
    return state nbrs
def get lin vel nbrs(self, s, xy delta, xy min, step size):
    # all the cells that lie on the straight line between xy delta+xy mi
    # possible neighbors.
    xy dist = np.sqrt((xy delta**2).sum())
    min step = np.min(step size[:2])
    xy max steps = xy dist / min step
    xy_dir = min_step * (xy_min - s[:2]) / np.linalg.norm((xy_min - s[:2])
```

```
xy nbrs = np.arange(xy max steps+1)[:, None]*xy dir + xy min
                 # Check neighbors for collision or being out of bound
                 collisions = do points collide(self.map, xy nbrs)
                 if collisions[0]:
                     return np.empty((0, 2)) # No nbr that does not collide
                 indices, = np.nonzero(collisions)
                 xy non colliding = xy nbrs[:indices[0]-1] if len(indices) else xy nb
                 xy uniq nbrs = np.unique(
                     np.round(xy non colliding / step size[:2]).astype(dtype=np.int64
                     axis=0
                 ) * step size[:2]
                 return xy uniq nbrs
             def get ang vel nbrs(self, s, theta delta, theta min angle, step size):
                 theta steps = theta delta / step size[2]
                 theta nbrs angle = np.unique(Angle.wrap(np.arange(theta steps+1)*ste
                 return theta nbrs angle
             def get nbrs np(self, current state, default):
                 L = self.robot.wheel base
                 nbrs = self.get all nbrs(current state, default)
                 s = np.asarray(current state)
                 nbrs diff = nbrs[:, :2] - s[:2]
                 edge cost = np.sqrt((nbrs diff**2).sum(axis=-1)) + L*Angle.dist(nbrs
                 return nbrs, edge cost
             def get(self, current state, default=[]):
                 nbrs, edge cost = self.get nbrs np(current state, default)
                 return [(tuple(nbr), ecost)
                         for nbr, ecost in zip(nbrs.tolist(), edge cost.tolist())]
In [21]: graph = ObstacleListToGraph(map, robot)
         start state = np.array([0.9, 0.6, Angle.wrap(8*np.pi/10)])
         state nbrs, edge costs = graph.get nbrs np(start state, [])
         fig, ax = plt.subplots()
         a = ax.scatter(state nbrs[:, 0], state nbrs[:, 1], c=edge costs, label='nbr
         plot map(ax, map, goal, start state)
         fig.colorbar(a)
         ax.legend()
```

Out[21]: <matplotlib.legend.Legend at 0x7fe44a0d69a0>



```
In [22]: def euclidean_heurist_dist(node, goal):
    x_n, y_n, theta_n = node
    x_g, y_g = goal
    return np.sqrt((x_n-x_g)**2 + (y_n - y_g)**2)
```

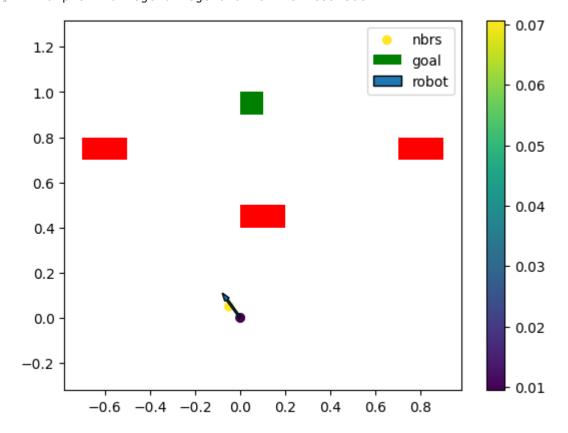
In [23]: from astar import astar
#success, search_path, node2parent, node2dist = astar(graph, euclidean_heuri

The above code takes forever to run. We need to lower the graph complexity.

```
In [24]:
         @dataclass
         class Robot:
             linvel range: np.ndarray
             angvel_range: np.ndarray
             dt: float
             wheel base: float
         robot = Robot(linvel range = np.array([0.1, 0.1]),
                       angvel range = np.array([np.pi/10, np.pi/10]),
                       wheel base = 0.12,
                       dt = None
         robot.dt = max(
             np.sqrt((map.state_discrete_step[:2]**2).sum()) / np.abs(robot.linvel_ra
             map.state discrete step[2] / np.abs(robot.angvel range[0]))
         map = MapProperties(
             state min = np.array([
                 -1.0, # x min in meters
                 0.0, # y min in meters
                 -np.pi # theta min in radians
```

```
]),
    state max = np.array([
        1.0, # x max in meters
        1.0, # y max in meters
        np.pi # theta max in radians
    ]),
    state discrete step = np.array([
        0.05, # x min in meters
        0.05, # y min in meters
        np.pi/20 # theta min in radians
    ]),
    # Let's put obstacles somewhere (arbitrary for now)
    obstacles = np.array([
        [0.7, 0.7, 0.2, 0.1], #x, y, width, height
        [0., 0.4, 0.2, 0.1], # x, y, width, height
        [-0.7, 0.7, 0.2, 0.1] # x, y, width, height
    ])
)
graph = ObstacleListToGraph(map, robot)
start state = np.array([0., 0., Angle.wrap(7*np.pi/10)])
state_nbrs, edge_costs = graph.get_nbrs_np(start_state, [])
fig, ax = plt.subplots()
a = ax.scatter(state nbrs[:, 0], state nbrs[:, 1], c=edge costs, label='nbr
plot map(ax, map, goal, start state)
fig.colorbar(a)
ax.legend()
```

Out[24]: <matplotlib.legend.Legend at 0x7fe44c8a43a0>



It looks like our search never reaches the goal. We need to make the goal check function a region check instead of node check.

```
In [27]: %writefile astar2.py
         from hw2 solution import PriorityQueueUpdatable
         import sys
         from dataclasses import dataclass, field
         from typing import Any
         # https://docs.python.org/3/library/queue.html#queue.PriorityQueue
         @dataclass(order=True)
         class PItem:
             dist: int
             node: Any=field(compare=False)
             # Make the PItem hashable
             # https://docs.python.org/3/glossary.html#term-hashable
             def hash (self):
                 return hash(self.node)
         def default goal check(m, goal):
             return m == goal
         def astar(graph, heuristic dist fn, start, goal,
                   goal check=default goal check,
                   debug=False,
                   debugf=sys.stdout):
             edgecost: cost of traversing each edge
             Returns success and node2parent
             success: True if goal is found otherwise False
             node2parent: A dictionary that contains the nearest parent for node
             seen = set([start]) # Set for seen nodes.
             # Frontier is the boundary between seen and unseen
             frontier = PriorityQueueUpdatable() # Frontier of unvisited nodes as a F
             node2parent = {start : None} # Keep track of nearest parent for each nod
             hfn = heuristic dist fn # make the name shorter
             node2dist = {start: 0 } # Keep track of cost to arrive at each node
```

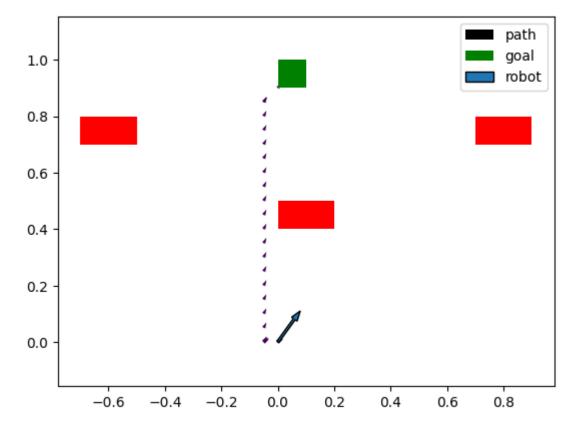
```
search order = []
   frontier.put(PItem(0 + hfn(start, goal), start)) # <----- Diff</pre>
   if debug: debugf.write("goal = " + str(goal) + '\n')
   i = 0
                                       # Creating loop to visit each node
   while not frontier.empty():
       dist m = frontier.get() # Get the smallest addition to the frontier
       if debug: debugf.write("%d) Q = " % i + str(list(frontier.queue)) +
       if debug: debugf.write("%d) node = " % i + str(dist m) + '\n')
       #if debug: print("dists = " , [node2dist[n.node] for n in frontier.q
       m = dist m.node
       m dist = node2dist[m]
       search order.append(m)
       if goal is not None and goal_check(m, goal):
           return True, search order, node2parent, node2dist
       for neighbor, edge cost in graph.get(m, []):
           old dist = node2dist.get(neighbor, float("inf"))
           new dist = edge cost + m dist
           if neighbor not in seen:
               seen.add(neighbor)
               frontier.put(PItem(new dist + hfn(neighbor, goal), neighbor
               node2parent[neighbor] = m
               node2dist[neighbor] = new dist
           elif new dist < old dist:</pre>
               node2parent[neighbor] = m
               node2dist[neighbor] = new dist
               # ideally you would update the dist of this item in the pric
               # as well. But python priority queue does not support fast u
               # ----- Different from dijkstra -----
               old item = PItem(old dist + hfn(neighbor, goal), neighbor)
               if old item in frontier:
                   frontier.replace(
                       old item,
                        PItem(new dist + hfn(neighbor, goal), neighbor))
       i += 1
   if goal is not None:
        return False, search_order, node2parent, node2dist
   else:
       return True, search order, node2parent, node2dist
def backtrace path(node2parent, start, goal):
   c = goal
   r path = [c]
   parent = node2parent.get(c, None)
   while parent != start:
       r path.append(parent)
       c = parent
       parent = node2parent.get(c, None) # Keep getting the parent until yd
       #print(parent)
   r path.append(start)
   return reversed(r path) # Reverses the path
```

```
In [28]: from astar2 import astar, backtrace path
         def goal check region(m, goal, goal wh=(0.1, 0.1)):
             m = np.asarray(m)
             goal = np.asarray(goal)
             goal wh = np.asarray(goal wh)
             mdiff = (m[:2] - goal)
             return ((0 <= mdiff) & (mdiff <= goal wh)).all()</pre>
         start state = np.array([0., 0., 3*np.pi/10])
         start node tuple = tuple(start state.tolist())
         success, search path, node2parent, node2dist = astar(
             graph,
             euclidean heurist dist,
             start node tuple, goal,
             goal check=goal check region,
             debug=True,
             debugf=open('log.txt', 'w'))
```

In [29]: path = np.array(list(backtrace_path(node2parent, start_node_tuple, search_path)

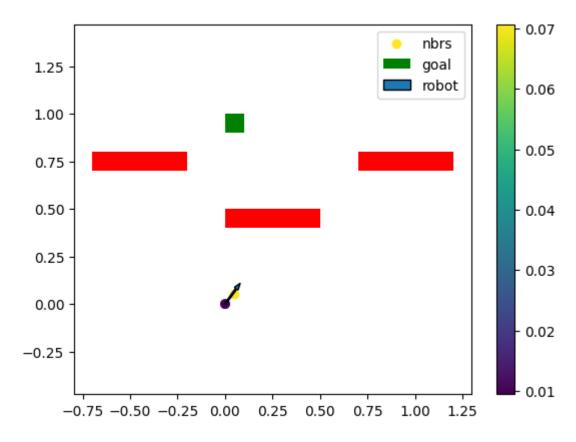
```
In [30]: fig, ax = plt.subplots()
   ax.quiver(path[:, 0], path[:, 1], np.cos(path[:, 2]), np.sin(path[:, 2]), 1.
   plot_map(ax, map, goal, start_state)
   ax.legend()
```

Out[30]: <matplotlib.legend.Legend at 0x7fe449c1c040>

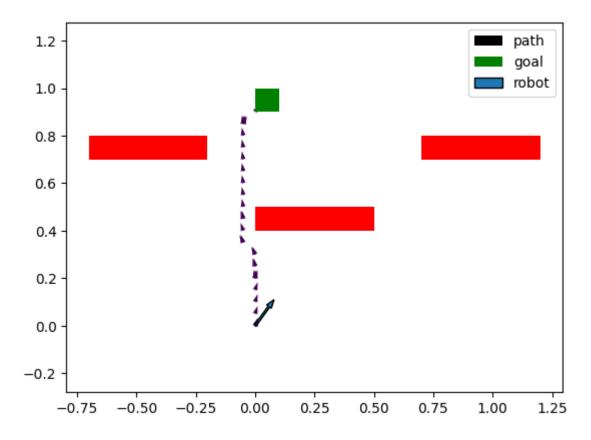


```
dt = None
robot.dt = max(
    np.sqrt((map.state_discrete_step[:2]**2).sum()) / np.abs(robot.linvel ra
    map.state discrete step[2] / np.abs(robot.angvel range[0]))
map = MapProperties(
   state min = np.array([
        -1.0, # x min in meters
        0.0, # y min in meters
        -np.pi # theta min in radians
    ]),
    state max = np.array([
        1.0, # x max in meters
        1.0, # y max in meters
        np.pi # theta max in radians
    ]),
    state discrete step = np.array([
        0.05, # x min in meters
        0.05, # y min in meters
        np.pi/20 # theta min in radians
    ]),
    # Let's put obstacles somewhere (arbitrary for now)
    obstacles = np.array([
        [0.7, 0.7, 0.5, 0.1], # x, y, width, height
        [0., 0.4, 0.5, 0.1], # x, y, width, height
        [-0.7, 0.7, 0.5, 0.1] # x, y, width, height
   ])
)
graph = ObstacleListToGraph(map, robot)
start_state = np.array([0., 0., Angle.wrap(3*np.pi/10)])
state nbrs, edge costs = graph.get nbrs np(start state, [])
fig, ax = plt.subplots()
a = ax.scatter(state nbrs[:, 0], state nbrs[:, 1], c=edge costs, label='nbr
plot_map(ax, map, goal, start_state)
fig.colorbar(a)
ax.legend()
```

Out[31]: <matplotlib.legend.Legend at 0x7fe449cdbf70>



Out[32]: <matplotlib.legend.Legend at 0x7fe449b03580>



This could also have been done with RRT. Once the RRT is implemented the graph construction is much easier with RRT.

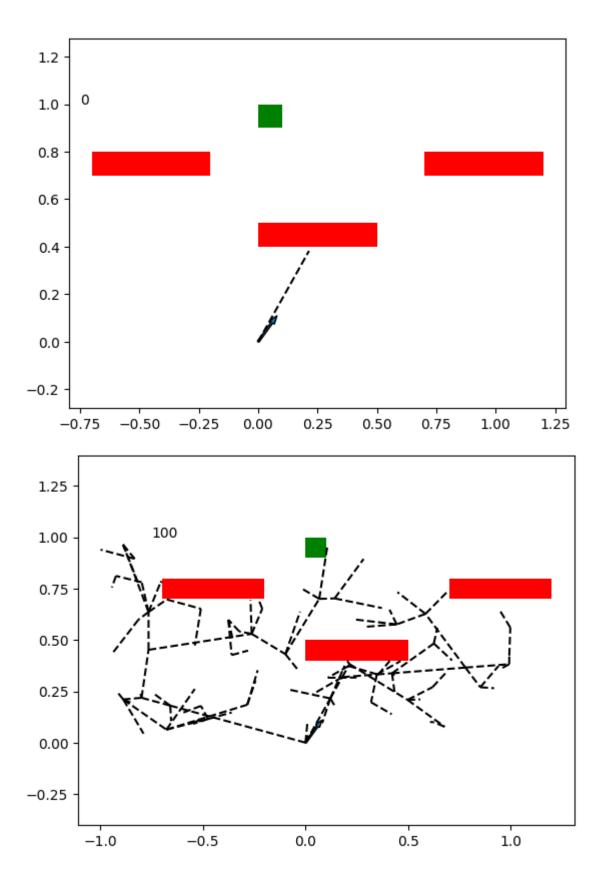
```
In [50]: np.random.seed(1010)
         def closest point on line segs(edges, pt, dist fn rrt=lambda x, y: np.linalg
             assert edges.shape[-2] == 2
             *N, D = edges.shape
             vs, ve = edges[:, 0, :], edges[:, 1, :]
             edge_vec = ve - vs \# *N \times D
             edge mag = dist fn rrt(vs, ve)[..., None] # *N
             edge unit = edge vec / edge mag # *NxD
             \# pt on edge = x = vs + t * edge unit
             \# (x - pt) @ edge unit = 0
             # ((vs + t * edge_unit) - pt) @ edge_unit = 0
             \# t = (pt - vs) @ edge unit
             t = ((pt - vs) * edge unit).sum(axis=-1, keepdims=True) # N x 1
             x = vs + t * edge unit # *N x D
             dist_e = dist_fn_rrt(pt , x)#, axis=-1)
             dist_vs = dist_fn_rrt(vs , pt)#, axis=-1)
             dist_ve = dist_fn_rrt(ve , pt)#, axis=-1)
             is pt on edge = ((0 \le t) \& (t \le edge mag))[..., 0]
             dist v = np.minimum(dist vs, dist ve)
             dist = np.where( is pt on edge,
                              dist e,
                              dist v)
             min idx = np.argmin(dist)
             closest point = (x[min idx]
                                if is pt on edge[min idx] else vs[min idx]
                                if (dist vs[min idx] < dist ve[min idx]) else ve[min i</pre>
```

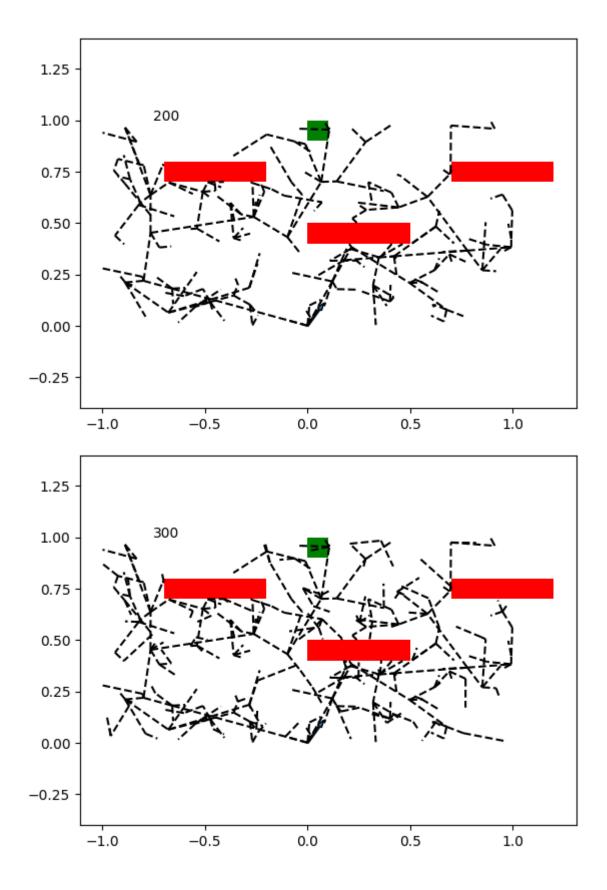
```
return closest point, dist[min idx], ([(vs[min idx], ve[min idx])]
                                          if is pt on edge[min idx] else [])
def closest point on graph(G adjacency list, pt, dist fn rrt):
    vertices = list(G adjacency list.keys())
    edge list = sum(
        [[(v, n)]]
        for n, e in nbrs]
         for v, nbrs in G adjacency list.items()], [])
    if len(edge list):
        return closest point on line segs(np.array(edge list), pt, dist fn n
    else:
        verticesnp = np.array(vertices)
        dists v = dist fn rrt(verticesnp, pt)#, axis=-1)
        min idx = np.argmin(dists v)
        closest point v = verticesnp[min idx]
        return closest point v, dists v[min idx], []
def rrt one step(map, G adjacency list, do points collide rrt, closest point
                 random pt, dist fn rrt, stepsize):
    # 2.B Connect the sampled point to the nearest point (vertex or edge)
    # on the graph, as long as the connecting line does not pass through the
    nearest pt, dist, edges to delete = closest point on graph rrt(
        G adjacency list, random pt, dist fn rrt)
    dist_norm = np.linalg.norm(random_pt - nearest pt) # not using dist fn r
    steps = int(np.floor(dist norm / stepsize))
    if steps <= 0:</pre>
        return False, [], []
    direction = (random pt - nearest pt) / dist norm
    all points = np.arange(1, steps + 1)[:, None]*stepsize*direction+ neares
    collisions = do points collide rrt(map, all points)
    if collisions[0]:
        return False, [], []
    indices, = np.nonzero(collisions)
    first non colliding = (all points[indices[0]-1] if len(indices) else re
    dist fnc = dist fn rrt(first non colliding, nearest pt)
    nearest pt tuple = tuple(nearest pt.tolist())
    first non colliding tuple = tuple(first non colliding.tolist())
    edges to add = [(nearest pt tuple, (first non colliding tuple, dist fnc)
    edges to delete tupled = []
    for vs, ve in edges to delete:
        vs dist = dist fn rrt(nearest pt, vs)
        vs tuple = tuple(vs.tolist())
        edges to add.append((vs tuple, (nearest pt tuple, vs dist)))
        ve dist = dist fn rrt(ve, nearest pt)
        ve tuple = tuple(ve.tolist())
        edges to add.append((nearest pt tuple, (ve tuple, ve dist)))
        edges to delete tupled.append((vs tuple, ve tuple))
    return True, edges to add, edges to delete tupled
def rrt explore(
        map, # map details with obstacles
        start, # start node as a tuple
```

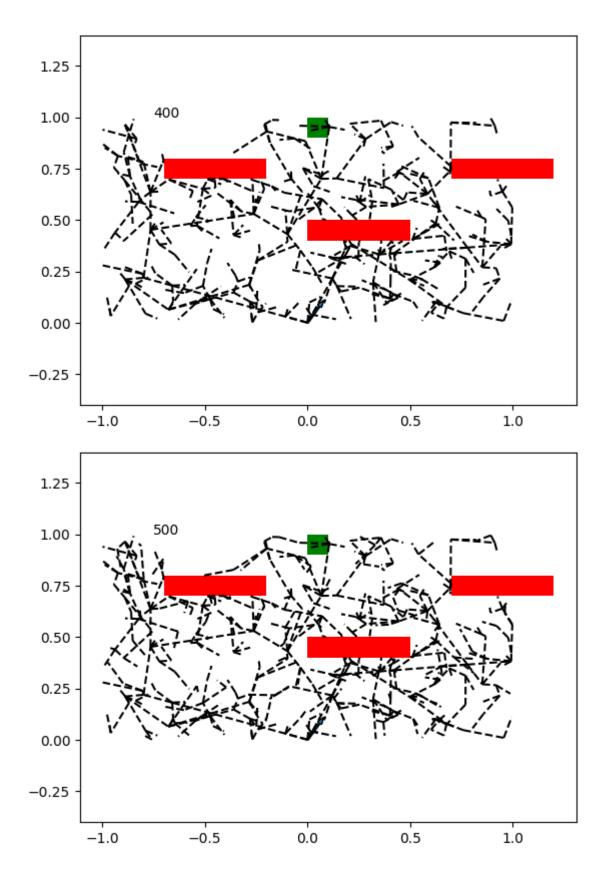
```
closest point on graph rrt,
        do points collide rrt,
        dist fn rrt,
        goal node,
        Npts=100, # we are going to sample 100 points, but start with 1 poin
        stepsize=0.01
    ):
   # Specify the bounds of the map
   pt min = map.state min
   pt max = map.state max
   D = len(map.state min)
   # 1. Initialize an empty graph with the start point
   G adjacency list = { start : [] }
   # 2. While not done
   for i in range(Npts):
       # 2.a Sample points on the chosen area.
       if i == Npts - 1:
            random pt = np.asarray(goal node)
       else:
            random pt = np.random.rand(D) * (pt max - pt min) + pt min
        # 2.B Connect the sampled point to the nearest point (vertex or edge
        # on the graph, as long as the connecting line does not pass through
        success, edges to add, edges to delete = rrt one step(
            map, G adjacency list, do points collide rrt, closest point on g
            random pt, dist_fn_rrt, stepsize)
        if success:
            for nearest_pt_tuple, (first_non_colliding_tuple, dist) in edges
                G adjacency list.setdefault(
                    nearest pt tuple, []
                                        ) append (
                    (first non colliding tuple, dist))
            for nearest pt tuple, pt2 tuple in edges to delete:
                nbrs = G adjacency list.get(nearest pt tuple, [])
                if len(nbrs):
                    idx = [n for n, e in nbrs].index(pt2 tuple)
                    nbrs.pop(idx)
        if i % 100 == 0:
            fig, ax = plt.subplots()
            ax.text(-0.75, 1,'%d' % i)
            plot map(ax, map, goal, start)
            plot graph(ax, G adjacency list)
            plt.show()
    return G adjacency list
def plot_graph(ax, G_adjacency_list):
   ax.axis('equal')
   for v, nbrs in G adjacency list.items():
        for n, e in nbrs:
            ax.plot([v[0], n[0]], [v[1], n[1]], 'k--')
```

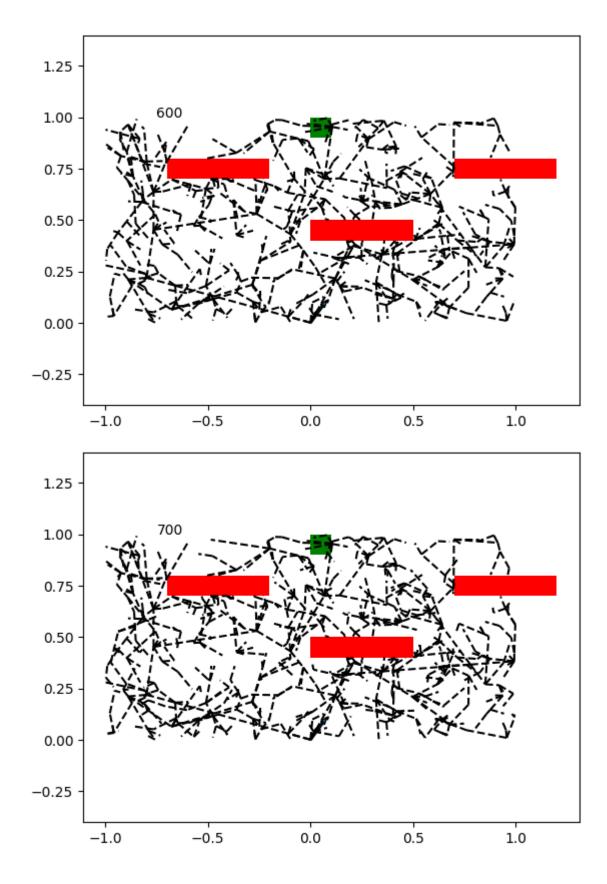
```
In [52]: start_state = np.array([0., 0., 3*np.pi/10])
    start_node_tuple = tuple(start_state.tolist())
```

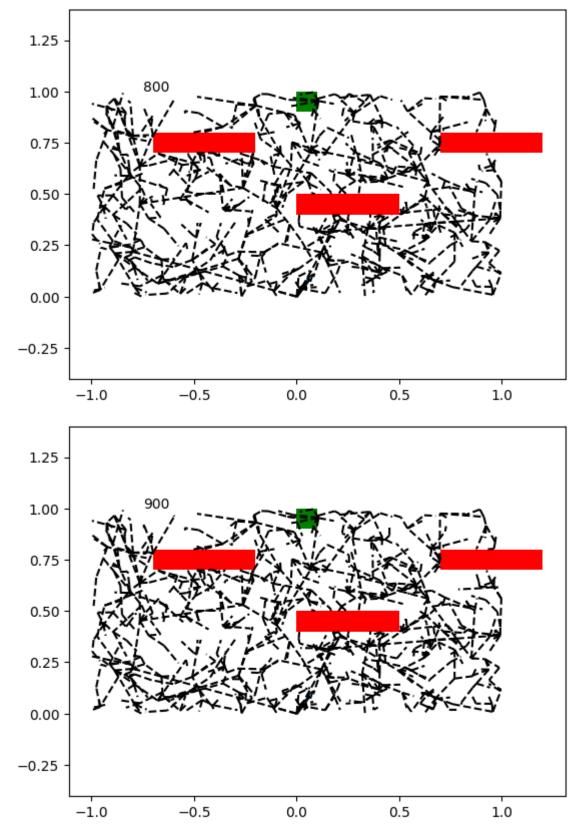
```
def do points collide rrt(map, all points):
    return do points collide(map, all points[:, :2])
def dist fn rrt(robot, end, start):
    L = robot.wheel base
    nbrs diff = end[..., :2] - start[..., :2]
    nbrs dir = nbrs diff / np.linalg.norm(nbrs diff, axis=-1, keepdims=True)
    move angle = np.arctan2(nbrs dir[..., 1], nbrs dir[..., 0])
    return (L*Angle.dist(Angle.wrap(start[..., 2]), move angle)/2 # Turn tow
            + np.sqrt((nbrs diff**2).sum(axis=-1)) # Move
            + L*Angle.dist(move angle, Angle.wrap(end[..., 2]))/2) # Turn to
G rrt = rrt explore(map, start node tuple,
                   closest point on graph rrt=closest point on graph,
                   do points collide rrt=do points collide rrt,
                   dist fn rrt=lambda x, y: dist fn rrt(robot, x, y),
                   goal node=(goal[0], goal[1], 0.0),
                   Npts=1000, stepsize=0.05)
success, search path, node2parent, node2dist = astar(
    G rrt,
    euclidean heurist dist,
    start_node_tuple, goal,
    goal check=goal check region)
path = np.array(list(backtrace path(node2parent, start node tuple, search pa
fig, ax = plt.subplots()
plot_map(ax, map, goal, start_node_tuple)
plot graph(ax, G rrt)
#ax.quiver(path[:, 0], path[:, 1], np.cos(path[:, 2]), np.sin(path[:, 2]), @
ax.plot(path[:, 0], path[:, 1], 'b-', label='path')
```











Out[52]: [<matplotlib.lines.Line2D at 0x7fe449737e50>]

