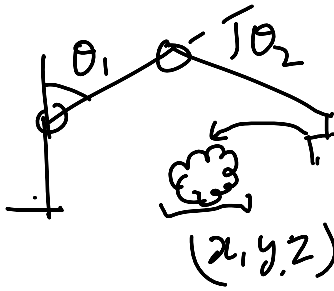
Forward and $\boxed{\text{inverse kinematics}}$

What should the joint ~~angles~~ state/conf. of the robot be so that the end-effector reaches a desired pose?



[How to move the end-effector to a desired pose [position + orientation]]

← end-effector

gripper or suction cup

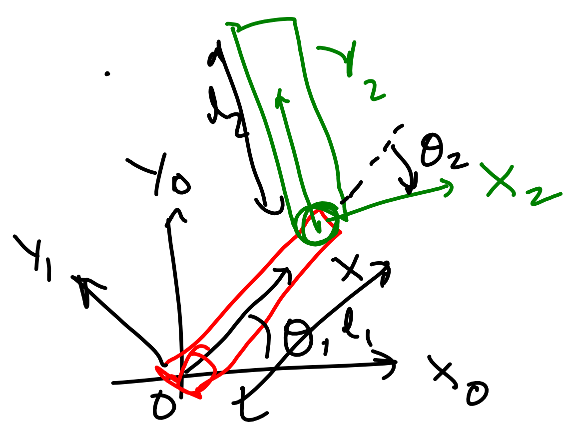
Forward kine.

If my joint ~~angles~~ state/conf. are given what would the pose of end-effector be?

Forward kinematics

$$ {}^{0}T_2 = {}^{0}T_1(\theta_1, l_1)\,{}^{1}T_2(\theta_2, l_2) $$

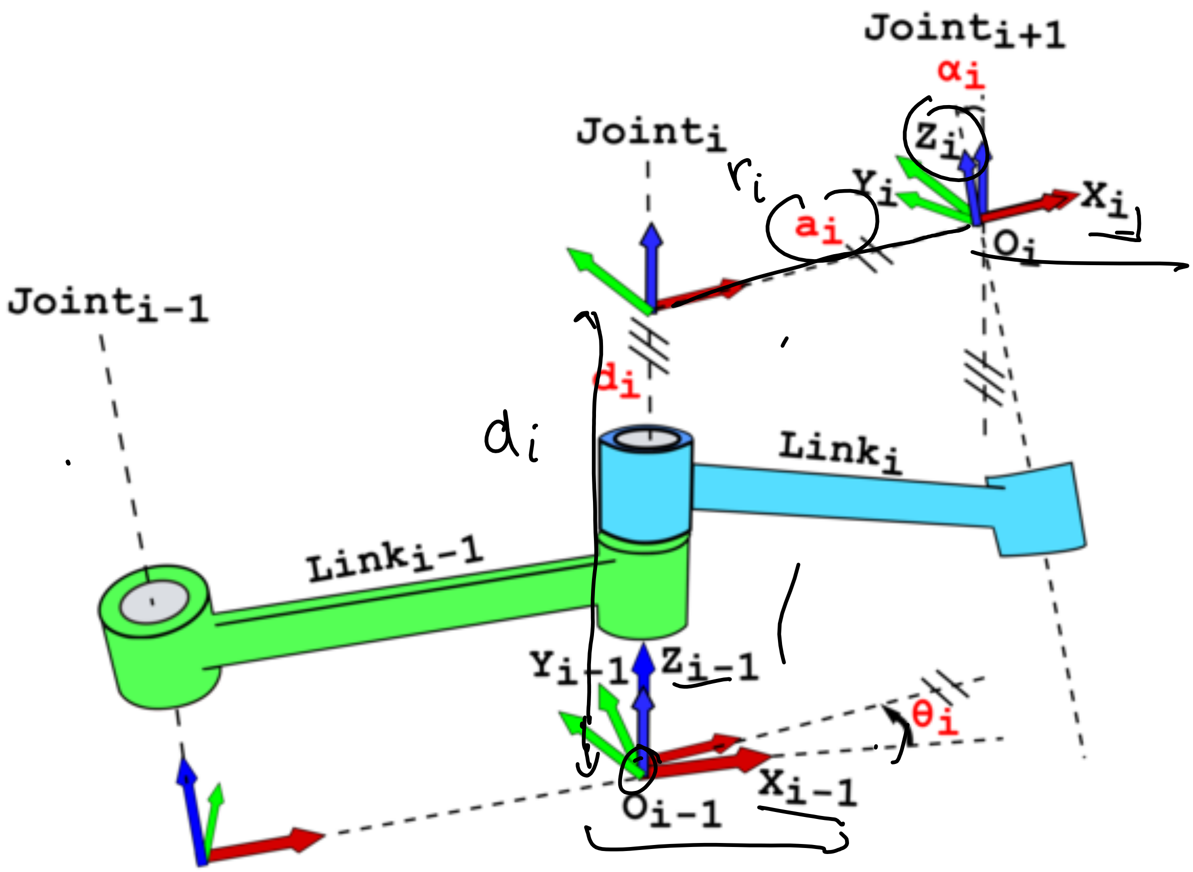
in terms of $\underbrace{\theta_1 \text{ and } \theta_2}_{\text{Given}}$



[Denavit Hartenberg Parameters/Convention]

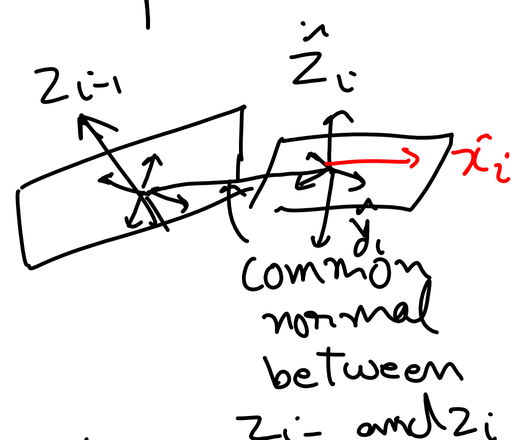
Denavit Hartenberg parameters

https://www.youtube.com/watch?v=rA9tm0gTln8



① $\hat{z}_{i-1}, \hat{z}_i$ aligned along the axis of rotation

② Choose $\hat{x}_i$ along the common normal between $\hat{z}_{i-1}, \hat{z}_i$



③ $\hat{y}_i = \hat{z}_i \times \hat{x}_i$

a) θ_i = Rotation along z_{i-1} (to align x_{i-1} with x_i)

b) d_i = translation along z_{i-1} (to align the origins)

c) α_i = Rotation along x_i (to align z_{i-1} with z_i)

d) r_i/a_i = translation along x_i (to align the origin)

(a) and (b) can be swapped
(c) and (d)

But Tranformation along z goes first
followed by " " x

$$ {}^{i-1}T_i = {}^{i-1}T_{zi} \; {}^{i-1}T_{xi} $$

$i-1$ ↑ target

i ↑ source

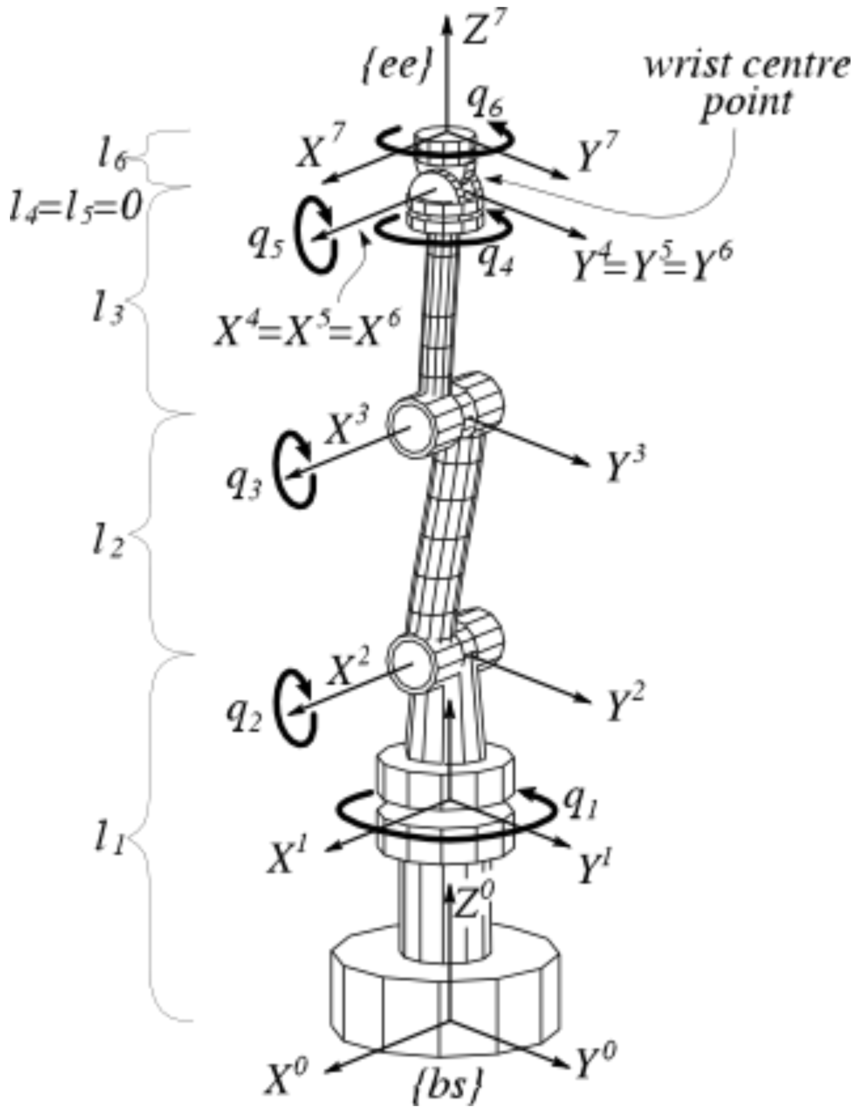
Tranformations are applied right to left

$$ {}^{i-1}T_{xi} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & r_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]_{4\times 1} $$

3×3 $\quad$ 3×1

$$ {}^{i-1}T_{zi} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 1} $$

θ_i, d_i

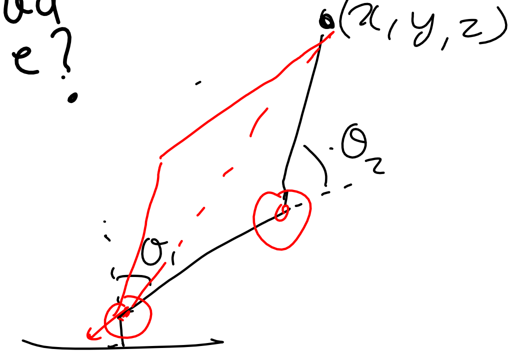


Z7
{ee}
wrist centre point
q6
l6
X7
Y7
l4=l5=0
q5
q4
Y4=Y5=Y6
l3
X4=X5=X6
X3
q3
Y3
l2
X2
q2
Y2
q1
l1
X1
Y1
Z0
X0
Y0
{bs}

Numerical solutions to IK problems: Jacobian inverse technique

Forward and inverse kinematics

What should the joint ~~angles~~ state/conf. of the robot be so that the end-effector reaches a desired pose?



Inverse kinematics

↳ closed form solution

↳ Numerical/Iterative solutions

only polynomials of degree ≤ 5 have closed form solutions

$\cancel{\theta_1 = x_1^2 + y_2 + \cos^{-1}\left(\frac{z}{y}\right)}$

$\cos(\theta_1)$ $\quad$ $\underbrace{\sin(\theta_1)}_{x_1}$

$\sqrt{1 - x_1^2}$

Newton-Raphson method (Gradient descent)
(optimization solution)

Suppose a function $y = f(x)$
to find

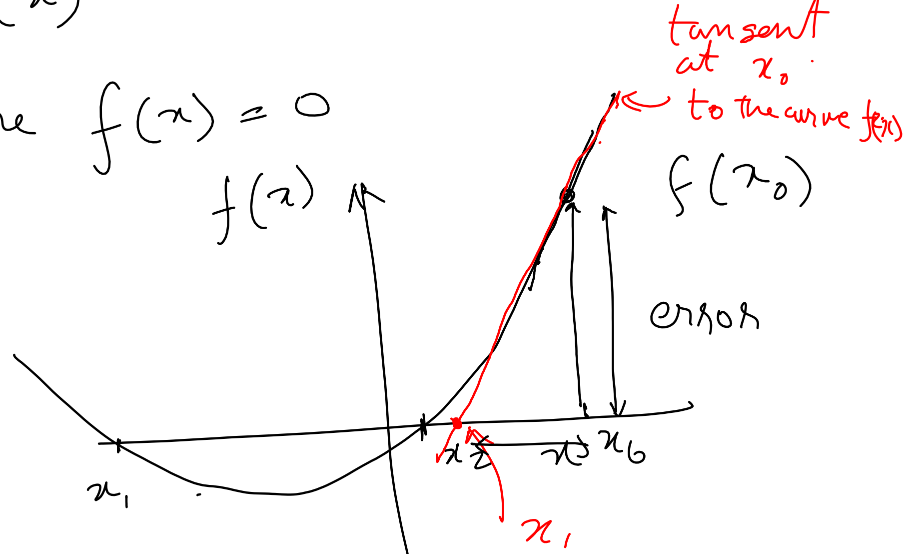
we want ~~all~~ any x where $f(x) = 0$

① Initial guess

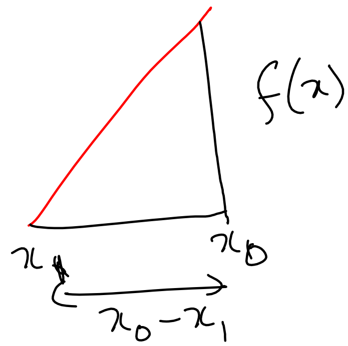
x_0

↑

iteration



(2) Improve the initial guess



$$f'(x)\Big|_{x_0} = \frac{f(x_0)}{x_0 - x_1}$$

$$\Rightarrow \quad x_0 - x_1 = \left[f'(x_0)\right]^{-1} f(x_0)$$

$$x_1 = x_0 - \left[f'(x_0)\right]^{-1} f(x_0)$$

$$\underline{x}_1 = \underline{x}_0 - \left[J_{\underline{x}}\, f(x)\right]^{-1} f(x_0)$$