ECE 498/598 Midterm 2 2023

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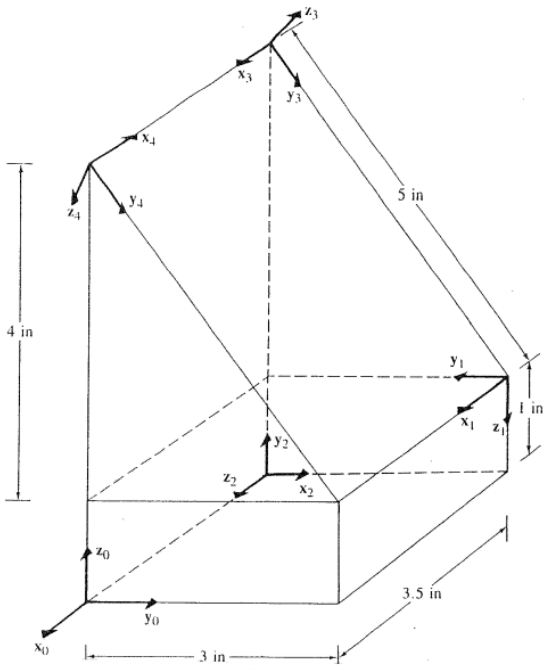
(1) Student name: Student email:

About the exam

1. There are total 4 problems. You must attempt all 4.
2. Maximum marks: 50.
3. Maximum time allotted: 50 min
4. Calculators are allowed.
5. One US Letter size or A4 size cheat sheet (both-sides) is allowed.

Problem 1 *Find the 4x4 transformation matrix ${}^1T_0$ that transforms coordinates from coordinate frame 1 to coordinate frame 0 (5 marks).*



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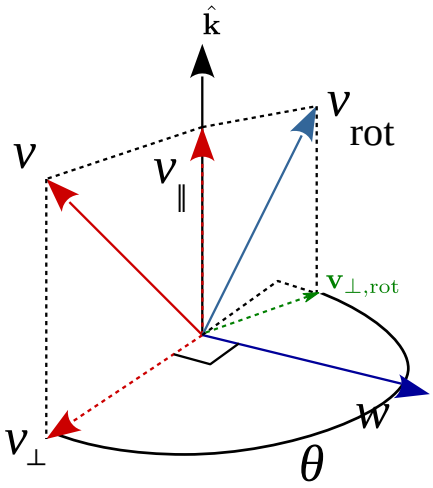
Problem 2 *Consider a coordinate system OUVW whose ordered set of basis vectors given by* $\mathbf{u} = [3/7, 2/7, 6/7]^\top$, $\mathbf{v} = [2/7, 6/7, 3/7]^\top$ *and* $\mathbf{w} = [4, 5, 6]^\top$. *Another coordinate system OXYZ whose order set of basis vectors is,* $\mathbf{x} = [2/7, 6/7, -3/7]^\top$, $\mathbf{y} = [-6/7, 3/7, 2/7]^\top$ *and* $\mathbf{z} = [3/7, 2/7, 6/7]^\top$. *Find the rotation matrix* ${}^{ouvw}R_{oxyz}$ *that converts coordinates from frame OXYZ to frame OUVW. (10 marks)*

Problem 3 *(Rodrigues formula) In the figure below, we are rotating point* $\mathbf{v}$ *around axis unit-vector* $\hat{\mathbf{k}}$ *by an angle* θ*. A unit vector* $\hat{\mathbf{w}}$ *is perpendicular to the both* $\mathbf{v}$ *and* $\hat{\mathbf{k}}$*. Another vector* $\mathbf{v}_\perp$ *is the projection of* $\mathbf{v}$ *onto a plane that is perpendicular to* $\hat{\mathbf{k}}$*. Note that* $\mathbf{v}_\perp$ *is perpendicular to both* $\hat{\mathbf{w}}$ *and* $\hat{\mathbf{k}}$*. First, (a) write the unit-vector* $\hat{\mathbf{w}}$ *in terms of* $\mathbf{v}$ *and* $\hat{\mathbf{k}}$*. (b) Then write the vector (including the correct magnitude)* $\mathbf{v}_\perp$ *in terms of* $\mathbf{v}$ *and* $\hat{\mathbf{k}}$*. (c) A vector* $\mathbf{v}_{\perp,rot}$ *is obtained by rotating* $\mathbf{v}_\perp$ *by an angle* θ*. Write the vector* $\mathbf{v}_{\perp,rot}$ *in terms of* $\mathbf{v}_\perp$*,* $\hat{\mathbf{w}}$ *and* θ*. (15 marks)*



Problem 4 *The Euler angles of rotation YZX are given as θ, ϕ and ψ. Derive the rotation matrix corresponding to the Euler angle representation $R = R_x(\psi)R_z(\phi)R_y(\theta)$. Also derive an expression to convert the rotation matrix back to Euler angles. (20 marks).*