ECE 417/598: Camera calibration

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[1] See Hartley and Zisserman's Multiple View Geometry for details.

Homogeneous representation of lines

$$ax + by + c = 0$$

Projective space

$$\mathbb{P}^2 = \mathbb{R}^3 - \{(0,0,0)^\top\}$$

Homogeneous representation of points

$$ax + by + c = 0$$

Eq of line in Projective space

The point $\mathbf{x} \in \mathbb{P}^2$ lies on a line $\mathbf{l}$ if and only if $\mathbf{x}^\top \mathbf{l} = 0$.

Intersection of lines

Intersection of parallel lines

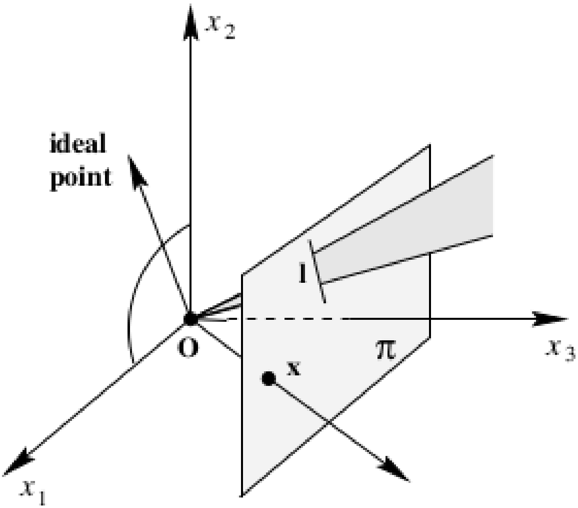
Find the intersection of parallel lines $ax + by + c = 0$ and $ax + by + c' = 0$.

Numerical example

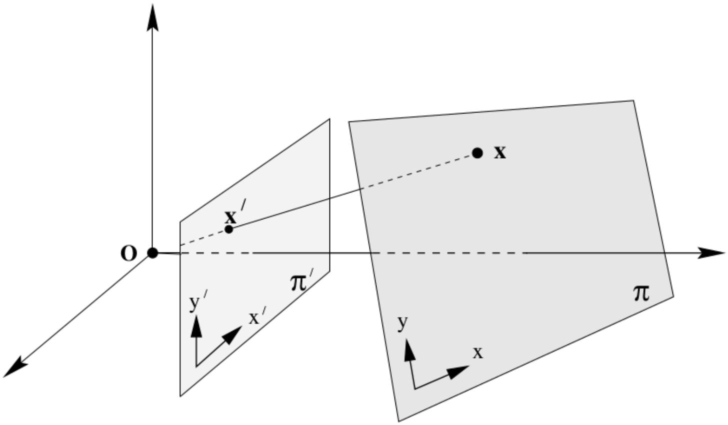
Find the intersection of $x = 1$ and $y = 1$ using perspective geometry.

Line joining points

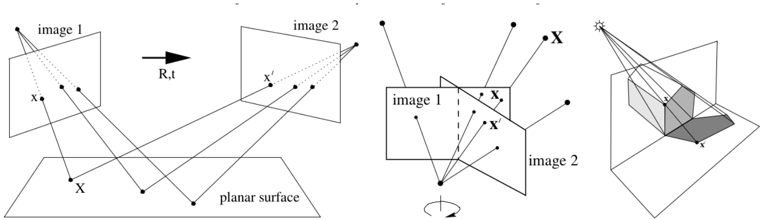
Points are rays and lines are planes



Homography



Examples of Homography



a b c

Computing Homography


Top view
Graphic
Look! Safe to move?
Settings

2D homography

Given a set of points $\mathbf{x}_i \in \mathbb{P}^2$ and a corresponding set of points $\mathbf{x}'_i \in \mathbb{P}^2$, compute the projective transformation that takes each $\mathbf{x}_i$ to $\mathbf{x}'_i$. In a practical situation, the points $\mathbf{x}_i$ and $\mathbf{x}'_i$ are points in two images (or the same image), each image being considered as a projective plane $\mathbb{P}^2$.

Direct Linear Transformation (DLT) algorithm

Objective

Given $n \geq 4$ 2D to 2D point correspondences $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$, determine the 2D homography matrix $\mathtt{H}$ such that $\mathbf{x}'_i = \mathtt{H}\mathbf{x}_i$.

Algorithm

(i) For each correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ compute the matrix $\mathtt{A}_i$ from (4.1). Only the first two rows need be used in general.

(ii) Assemble the n 2×9 matrices $\mathtt{A}_i$ into a single $2n \times 9$ matrix $\mathtt{A}$.

(iii) Obtain the SVD of $\mathtt{A}$ (section A4.4(p585)). The unit singular vector corresponding to the smallest singular value is the solution $\mathbf{h}$. Specifically, if $\mathtt{A} = \mathtt{UDV}^\mathsf{T}$ with $\mathtt{D}$ diagonal with positive diagonal entries, arranged in descending order down the diagonal, then $\mathbf{h}$ is the last column of $\mathtt{V}$.

(iv) The matrix $\mathtt{H}$ is determined from $\mathbf{h}$ as in (4.2).

3D to 2D camera projection matrix estimation

Given a set of points $\mathbf{X}_i$ in 3D space, and a set of corresponding points $\mathbf{x}_i$ in an image, find the 3D to 2D projective $\mathbf{P}$ mapping that maps $\mathbf{X}_i$ to $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$.

Eigenvalues and Eigenvectors

For a square matrix A, the λ_i and $\mathbf{x}_i$ that satisfy the following equation are called eigenvalues and eigenvectors respectively.

$$A\mathbf{x} = \lambda\mathbf{x} \text{ or } (A - \lambda I)\mathbf{x} = 0 \tag{1}$$

λ is chosen to ensure that $A - \lambda I$ has null space, hence, characteristic equation

$$\det(A - \lambda I) = 0 \tag{2}$$

For symmetrix matrix $A = A^\top$, eigenvalues are real, and eigenvectors are orthonormal,

$$A[\mathbf{x}_1, \ldots, \mathbf{x}_n] = [\mathbf{x}_1, \ldots, \mathbf{x}_n] \begin{bmatrix} \lambda_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \lambda_n \end{bmatrix} \tag{3}$$

$$AS = S\Lambda \tag{4}$$

$$\text{if } A = A^\top \text{ then } A = S\Lambda S^\top \tag{5}$$

Singular Value Decomposition (SVD)

$$A = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^\top \tag{6}$$

$$A^\top A = V \Sigma^2 V^{-1} \tag{7}$$

$$A^\top A \mathbf{v}_i = \lambda_i \mathbf{v}_i \qquad \lambda_i = \sigma_i^2 \tag{8}$$

$$AV = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

$$U^+ = \Sigma^{-1} A V^+ \tag{10}$$