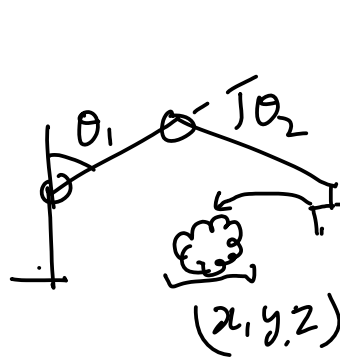


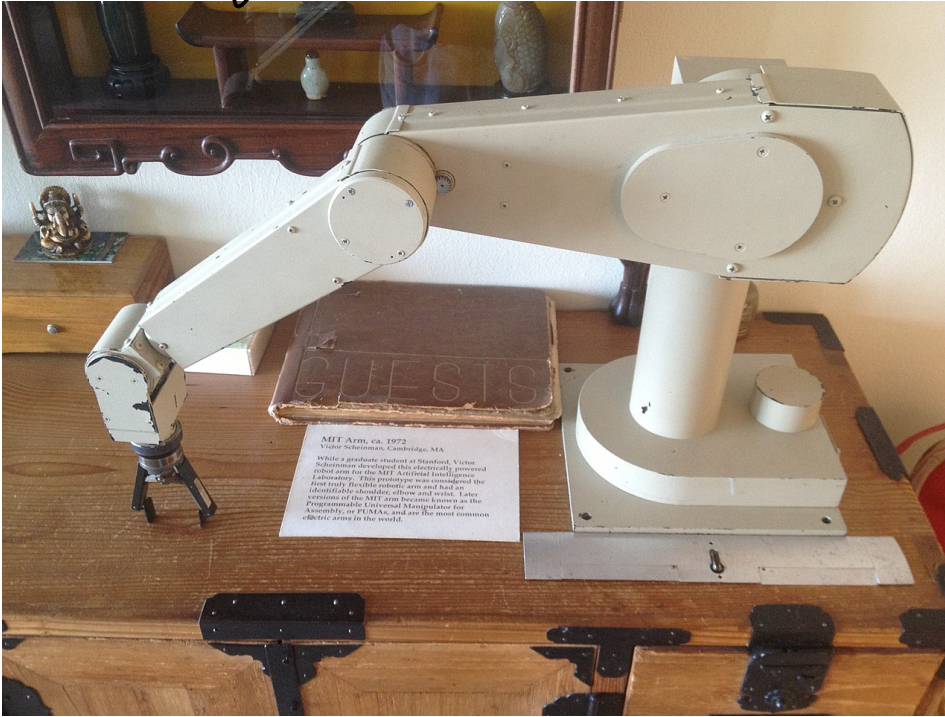
## Forward and inverse kinematics

What should the joint ~~angles~~ <sup>state/conf.</sup> of the robot be so that the end-effector

reaches a desired pose?



How to move the end-effector to a desired pose [position + orientation]  
gripper or suction cup



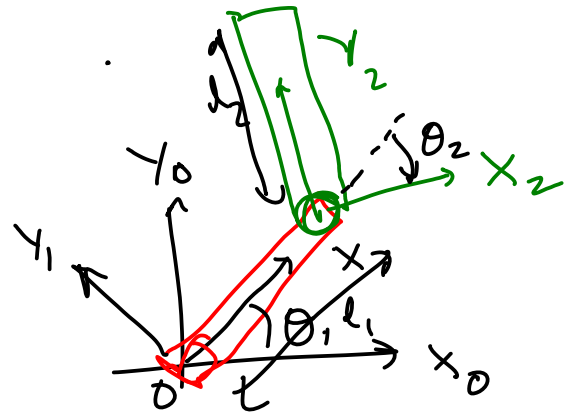
## Forward kine.

If my joint ~~angles~~ <sup>state/conf.</sup> are given what would the pose of end-effector be?

## Forward kinematics

$${}^0T_2 = {}^0T_1(\theta_1, l_1)T_2(\theta_2, l_2)$$

in terms of  $\theta_1$  and  $\theta_2$   
Given

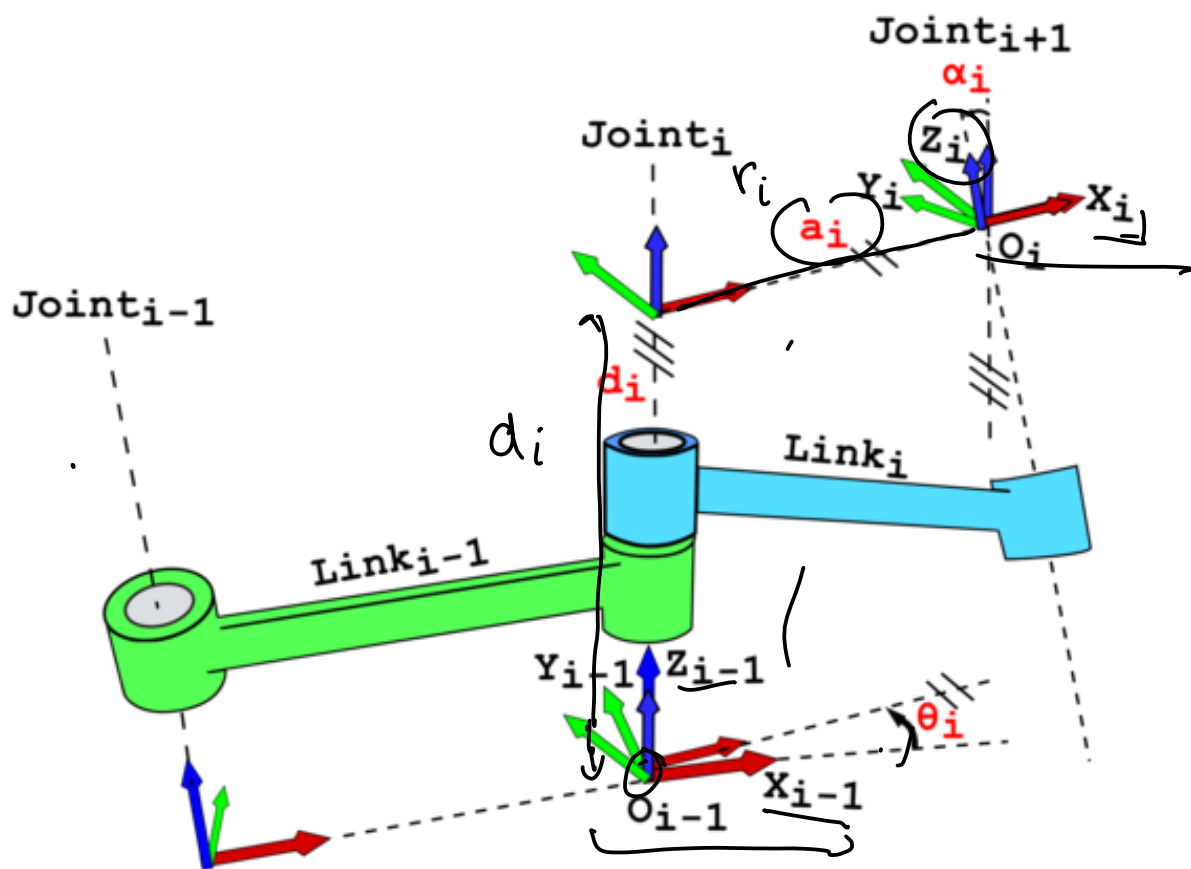


(Denavit Hartenberg)  
Parameters/convention



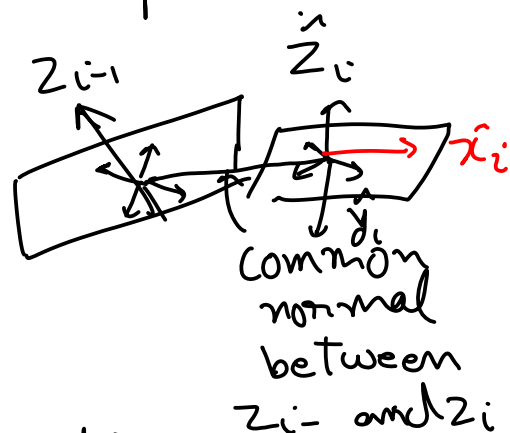
# Denavit Hartenberg parameters

<https://www.youtube.com/watch?v=rA9tm0gTln8>



①  $\hat{Z}_{i-1}, \hat{Z}_i$  aligned along the axis of rotation

② Choose  $\hat{x}_i$  along the common normal between  $\hat{Z}_{i-1}, \hat{Z}_i$



③  $\hat{y}_i = \hat{Z}_i \times \hat{x}_i$

- a)  $\theta_i =$  Rotation along  $Z_{i-1}$  (to align  $x_{i-1}$  with  $x_i$ )
- b)  $d_i =$  translation along  $Z_{i-1}$  (to align the origins)
- c)  $\alpha_i =$  Rotation along  $x_i$  (to align  $Z_{i-1}$  with  $Z_i$ )
- d)  $r_i/a_i =$  translation along  $x_i$  (to align the origin)

(a) and (b) can be swapped  
(c) and (d)

Bwt Transformation along z goes first  
followed by " " " "

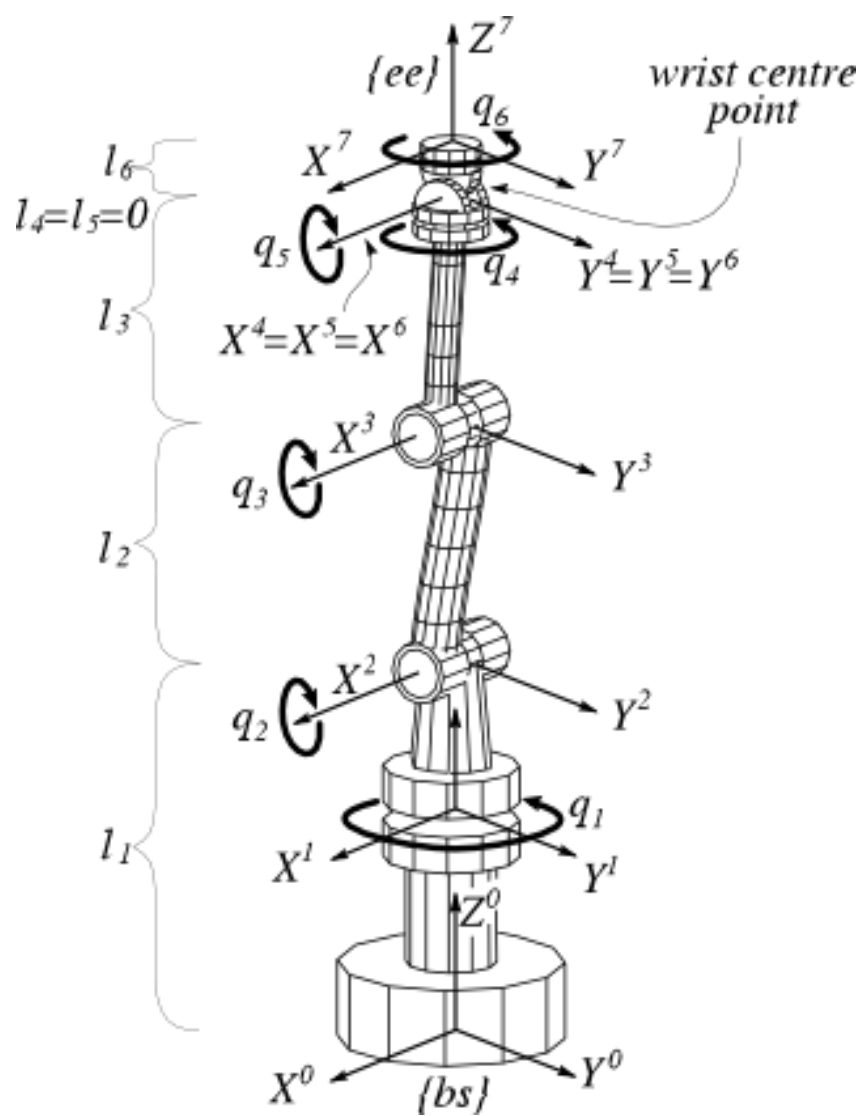
$${}^{i-1}T_i = {}^{i-1}T_{z_i} {}^{i-1}T_{x_i}$$
 target  $\uparrow$  source  $\uparrow$   
 Transformations are applied right to left

$${}^{i-1}T_{x_i} = \left[ \begin{array}{ccc|c} 1 & 0 & 0 & r_i \\ 0 & \cos \alpha_i & -\sin \alpha_i & 0 \\ 0 & \sin \alpha_i & \cos \alpha_i & 0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]_{4 \times 1}$$

$\begin{matrix} 3 \times 3 & 3 \times 1 \end{matrix}$

$${}^{i-1}T_{z_i} = \left[ \begin{array}{ccc|c} \cos \theta_i & -\sin \theta_i & 0 & 0 \\ \sin \theta_i & \cos \theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ \hline 0 & 0 & 0 & 1 \end{array} \right]_{4 \times 1}$$

$\theta_i, d_i$



## Numerical solutions to IK problems: Jacobian inverse technique