

# ECE 417/598: Camera calibration

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<sup>1</sup>See Hartley and Zisserman's Multiple View Geometry for details.

# Announcements

- ▶ Midterm this Friday.
- ▶ Will cover
  - ▶ Rotation representations: Euler angles, axis-angle representation
  - ▶ Transformations
  - ▶ Denavit-Hartenberg parameters
  - ▶ Camera projection model
  - ▶ Pseudo-inverse
  - ▶ Projective representation for line and point
- ▶ Sample exam today
- ▶ No programming questions. Expect Linear Algebra question though. Only the concepts that we have touched in class.
- ▶ Jetbot update: probably not happening due to global chip shortage

$$u \cong v(\epsilon) \equiv |u - v| \leq \epsilon \min(|u|, |v|) \quad (1)$$

$$u \sim v(\epsilon) \equiv |u - v| \leq \epsilon \max(|u|, |v|) \quad (2)$$

## Homogeneous representation of lines

$$\mathbb{P}^2 = \mathbb{R}^3 - \{(0, 0, 0)^\top\}$$

$$ax + by + 1.c = 0$$

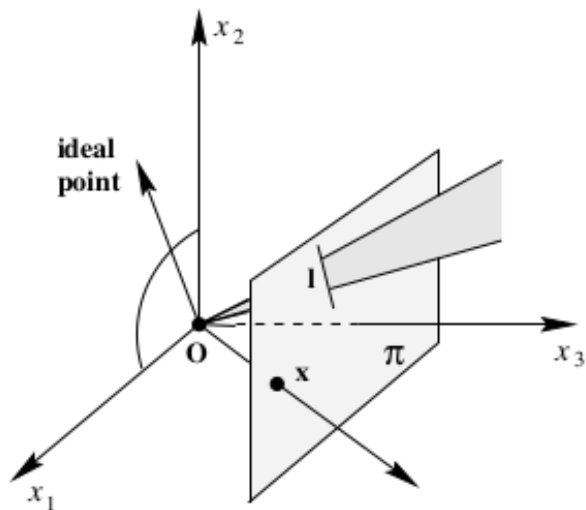
$$\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

The point  $\mathbf{x} \in \mathbb{P}^2$  lies on a line  $\mathbf{l}$  if and only if

$$\mathbf{l}^\top \mathbf{x} = 0$$

Points are rays and lines are planes



## Intersection of lines

Two line  $\mathbf{l}_1$  and  $\mathbf{l}_2$  intersect at  $\mathbf{x} \in \mathbb{P}^2$

$$\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2$$

## Numerical example

Find the intersection of two lines  $x = 1$  and  $y = 1$  using perspective geometry.

## Numerical example

Find the intersection of two lines  $x = y$  and  $x + y = 1$  using perspective geometry.

## Line joining points

Two point  $\mathbf{x}_1$  and  $\mathbf{x}_2$  form a  $\mathbf{l} \in \mathbb{P}^2$

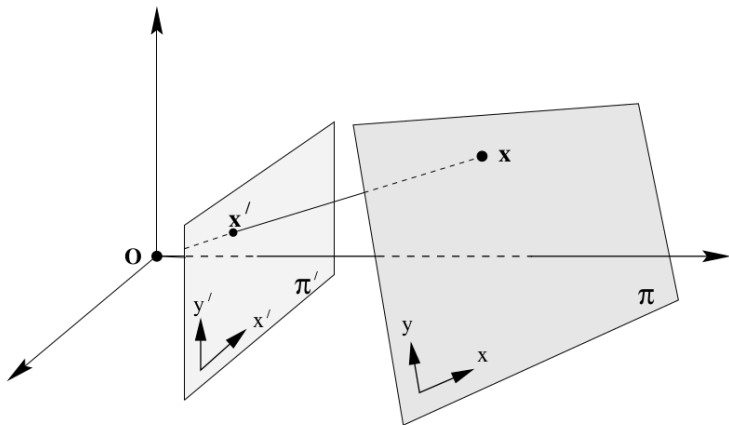
$$\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$



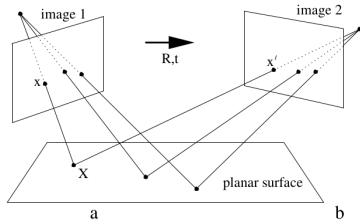
## Numerical example

Find the lines that pass through points  $(1, 1)$  and  $(2, 2)$  using perspective geometry.

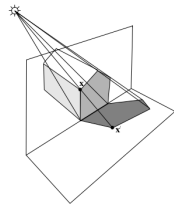
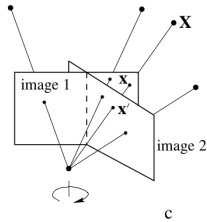
# Homography



# Examples of Homography



b





# Computing Homography



## 2D homography

Given a set of points  $\mathbf{x}_i \in \mathbb{P}^2$  and a corresponding set of points  $\mathbf{x}'_i \in \mathbb{P}^2$ , compute the projective transformation that takes each  $\mathbf{x}_i$  to  $\mathbf{x}'_i$ . In a practical situation, the points  $\mathbf{x}_i$  and  $\mathbf{x}'_i$  are points in two images (or the same image), each image being considered as a projective plane  $\mathbb{P}^2$ .

# Computing Homography



## Solving for Homography derivation



# Direct Linear Transformation (DLT) algorithm

## Objective

Given  $n \geq 4$  2D to 2D point correspondences  $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$ , determine the 2D homography matrix  $\mathbf{H}$  such that  $\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$ .

## Algorithm

- (i) For each correspondence  $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$  compute the matrix  $\mathbf{A}_i$  from (4.1). Only the first two rows need be used in general.
- (ii) Assemble the  $n \times 9$  matrices  $\mathbf{A}_i$  into a single  $2n \times 9$  matrix  $\mathbf{A}$ .
- (iii) Obtain the SVD of  $\mathbf{A}$  (section A4.4(p585)). The unit singular vector corresponding to the smallest singular value is the solution  $\mathbf{h}$ . Specifically, if  $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^T$  with  $\mathbf{D}$  diagonal with positive diagonal entries, arranged in descending order down the diagonal, then  $\mathbf{h}$  is the last column of  $\mathbf{V}$ .
- (iv) The matrix  $\mathbf{H}$  is determined from  $\mathbf{h}$  as in (4.2).

## 3D to 2D camera projection matrix estimation

Given a set of points  $\mathbf{X}_i$  in 3D space, and a set of corresponding points  $\mathbf{x}_i$  in an image, find the 3D to 2D projective  $\mathbf{P}$  mapping that maps  $\mathbf{X}_i$  to  $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$ .

## Eigenvalues and Eigenvectors

For a square matrix  $A$ , the  $\lambda_i$  and  $\mathbf{x}_i$  that satisfy the following equation are called eigenvalues and eigenvectors respectively.

$$A\mathbf{x} = \lambda\mathbf{x} \text{ or } (A - \lambda I)\mathbf{x} = 0 \quad (3)$$

$\lambda$  is chosen to ensure that  $A - \lambda I$  has null space, hence, characteristic equation

$$\det(A - \lambda I) = 0 \quad (4)$$

For symmetrix matrix  $A = A^\top$ , eigenvalues are real, and eigenvectors are orthonormal,

$$A[\mathbf{x}_1, \dots, \mathbf{x}_n] = [\mathbf{x}_1, \dots, \mathbf{x}_n] \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_n \end{bmatrix} \quad (5)$$

$$AS = SA \quad (6)$$

$$\text{if } A = A^\top \text{ then } A = S\Lambda S^\top \quad (7)$$

## Numerical example

## Singular Value Decomposition (SVD)

$$A = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^{\top} \quad (8)$$

$$A^{\top} A = V \Sigma^2 V^{-1} \quad (9)$$

$$A^{\top} A \mathbf{v}_i = \lambda_i \mathbf{v}_i \quad \lambda_i = \sigma_i^2 \quad (10)$$

$$AV = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \quad (11)$$

$$U^+ = \Sigma^{-1} AV^+ \quad (12)$$

## Numerical example