ECE 417/598: Homework 1

Max marks: 120

Due on Jan 28, 2021, before class.

You are allowed to use any matrix or linear algebra library (Eigen or xtensor), but no library that implements rotation matrices. You are not allowed to use Eigen/Geometry. You can use the following code for generating random rotation matrices: random_rotation.cpp.

1 Jan 21 Lecture

Problem 1 *Degrees of Freedom of a quantity is the number independent scalar variables needed to represent that quantity. What is degrees of freedom required to*

1. *Position and orientation in 1-D*
2. *Position and orientation in 2-D*
3. *Position and orientation in 3-D*
4. *Position and orientation in 4-D*

(8 marks. Estimated time: 15 min) Justify your answer.

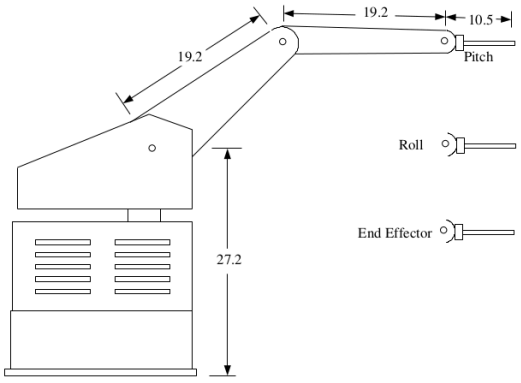
Problem 2 *Write a program in C++ that checks if a given 3x3 matrix is a valid Rotation matrix is a valid Rotation matrix (check for orthonormality i.e. orthogonality and determinant = 1). You may use Eigen's matrix multiplication and determinant() function. (10 marks. Used in the following problems. Estimated time: 15 min).*

Problem 3 *Write a pair of functions in C++ that converts rotation matrix from XYZ Euler angles (roll, pitch, yaw) and vice versa. Test the pair of functions with randomly generated Euler angles. And check if the converted rotation matrix is orthonormal. What happens when pitch = $\pi/2$, are you able to convert from rotation matrix to Euler angle? Why or why not? (50 marks. Estimated time: 30 min)*

Problem 4 *Write a pair of functions in C++ that converts rotation matrix from axis-angle format and vice-versa. Test the pair of functions with randomly generated axis-angle representation. And check if the converted rotation matrix is orthonormal. (50 marks. Estimated time: 30 min)*

Problem 5 *Write a function in C++ that generates a 4x4 transformation given axis-angle representation and translation. (20 marks. Estimated time: 15 min).*

Problem 6 *For the given robot write down the axis-angle rotation from joint to joint assuming the joint angles to be θ_1, θ_2, θ_3 respectively.*



2 Jan 24 Lecture

Problem 7 *The following is known about a smooth trajectory: p(0) = 0, v(0) = 0, p(3) = 2, p(7) = 0, v(7) = 0 and velocity and acceleration are continuous everywhere.*

1. *What is the lowest degree single polynomial which could be used?*
2. *Give two advantages to using a spline curve instead.*

3. *What degree polynomials would you suggest for the splines?*

4. *Write the polynomials.*

5. *Write the set of equations which would be used to solve for the coefficients. Do not use normalized time.*

6. *Solve for the coefficients.*

7. *Write the set of equations which would be used to solve for the coefficients. Use normalized time.*

8. *Solve for the coefficients.*

References