Before you turn this problem in, make sure everything runs as expected. First, **restart the kernel** (in the menubar, select Kernel→Restart) and then **run all cells** (in the menubar, select Cell→Run All).

Make sure you fill in any place that says `YOUR CODE HERE` or "YOUR ANSWER HERE", as well as your name and collaborators below:

In [ ]:
```
NAME = ""
COLLABORATORS = ""
```

In [ ]:
```
try:
    import torch as t
    import torch.nn as tnn
except ImportError:
    print("Colab users: pytorch comes preinstalled. Select Change Ru")
    print("Local users: Please install pytorch for your hardware using instr
    print("ACG users: Please follow instructions here: https://vikasdhiman.i

    raise
```

In [ ]:
```
def wget(url, filename):
    """
    Download files using requests package.
    Better than wget command line because this is cross platform.
    """
    try:
        import requests
    except ImportError:
        import subprocess
        subprocess.call("pip install --user requests".split())
        import requests
    r = requests.get(url)
    with open(filename, 'wb') as fd:
        for chunk in r.iter_content():
            fd.write(chunk)
```

In [ ]:
```
# Get training features from MNIST dataset.
wget("https://vikasdhiman.info/ECE490-Neural-Networks/notebooks/05-mlp/zero_
     "zero_one_train_features.npz")
```

In [ ]:
```
def draw_features(ax, zero_features, one_features):
    zf = ax.scatter(zero_features[:, 0], zero_features[:, 1], marker='.', la
    of = ax.scatter(one_features[:, 0], one_features[:, 1], marker='+', labe
    ax.legend()
    ax.set_xlabel('Feature 1: count of pixels')
    ax.set_ylabel('Feature 2: Variance along x-axis')
    return [zf, of] # return list of artists
```

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

```
zero_one_train_features = np.load('zero_one_train_features.npz')
FEATURE_MEAN = zero_one_train_features['mean']
FEATURE_STD = zero_one_train_features['std']
features = zero_one_train_features['normed_features']
labels = zero_one_train_features['labels']

fig, ax = plt.subplots()
draw_features(ax, features[labels > 0, :], features[labels < 0, :])
```

In [ ]:

```
if t.cuda.is_available():
    DEVICE="cuda"
elif t.mps.is_available():
    DEVICE="mps"
else:
    DEVICE="cpu"

DTYPE = t.get_default_dtype()

def loss(predicted_labels, true_labels):
    # Make sure predicted_labels and true_labels have same shape
    y = true_labels[..., None]
    yhat = predicted_labels
    assert y.shape == yhat.shape
    return t.maximum(- y * yhat, t.Tensor([0.]).to(device=DEVICE)).sum() / y

# TODO:
# Define model = ?
model = tnn.Sequential(
    tnn.Linear(2, 5),
    tnn.ReLU(),
    tnn.Linear(5, 1))


def train_by_gradient_descent(model, loss, train_features, train_labels, lr=
    predicted_labels = model(train_features)
    #print(predicted_labels)

    loss_t = loss(predicted_labels, train_labels)
    loss_t.backward()
    loss_t_minus_1 = 2*loss_t  # Fake  value to make the while test pass onc
    niter = 0
    while t.abs(loss_t - loss_t_minus_1) / loss_t > 0.01: # Stopping criteri
        with t.no_grad(): # parameter update needs no gradients
            for param in model.parameters():
                assert param.grad is not None
                param.add_( - lr * param.grad)  # Gradient descent

        model.zero_grad()
        # Recompute the gradients
        predicted_labels = model(train_features)
        loss_t_minus_1 = loss_t
        loss_t = loss(predicted_labels, train_labels)
        loss_t.backward() # Compute gradients for next iteration

        # If loss increased, decrease lr. Works for gradient descent, not fo
        if loss_t > loss_t_minus_1:
```

Gradient Descent

Data: $D_{train} = \{(\underline{x}_1, y_1) \dots (\underline{x}_n, y_n)\}$

Model: $\hat{y}_i = f(\underline{x}_i, \theta)$ ← parameters

Loss: $l(y_i, \hat{y}_i)$ ← predicted label, ← true label

Empirical Risk (loss)

$$\theta^* = \arg\min_{\theta} \sum_{i=1}^{n} l(y_i, f(\underline{x}_i, \theta)) \rightarrow L(D_{train}, \theta)$$

$$\theta_{t+1} = \theta_t - \alpha_t \nabla_\theta L(D_{train}, \theta)$$ Gradient Descent

Expected Risk (loss) minimization

$$\theta^* = \arg\min_{\theta} \mathbb{E}_{\underline{x},y}\left[l(Y, f(\underline{X}, \theta)) \right]$$

$$\theta_{t+1} = \theta_t - \alpha_t \nabla_\theta \mathbb{E}_{\underline{x},y}\left[l(Y, f(\underline{X}, \theta)) \right]$$

$$\nabla_\theta L(D_{train}, \theta) \approx \nabla_\theta \mathbb{E}_{x,y}\left[l(Y, f(\underline{X}, \theta)) \right]$$

$$= \nabla_\theta \Sigma\, l(y_i, f(\underline{x}_i, \theta))$$

Stochastic Gradient Descent (SGD)

for t in range(nepochs)

① while not converged {

Batch Size = 1 or 9 or 16 or 64 or 128

shuffle D_{train}

for batches from D_{train} {

$D_{batch} = \{(x_1, y), \ldots (x_B, y_B)\}$

uniformly randomly sampling D_{Batch}

$$\theta_{t+1} = \theta_t - \alpha_t \nabla_\theta \underbrace{\sum_{i=1}^{B} l(y_i, f(x_i, \theta))}_{D_{batch}}$$

}

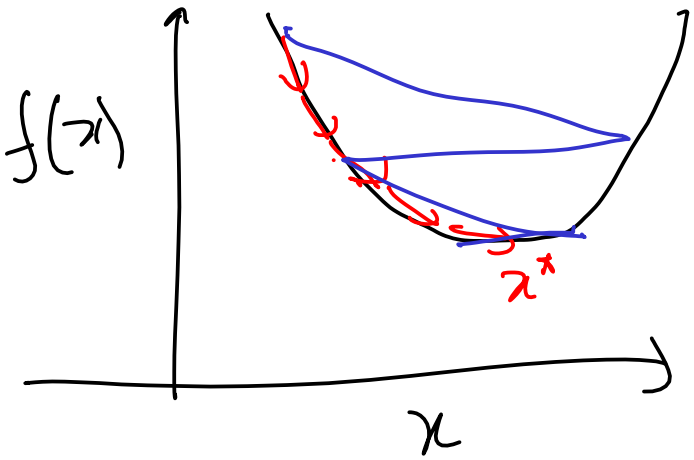
Epoch

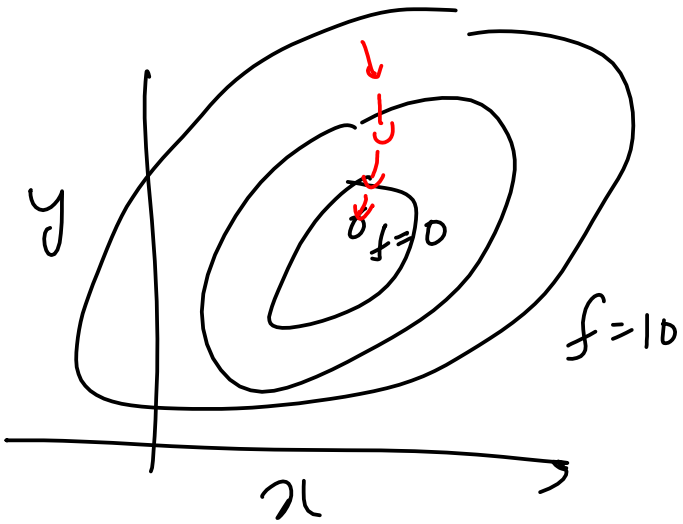
}

$$\sum_{b=1}^{N//B} \nabla_\theta \underbrace{\sum_{i=1}^{B} l(y_i, \hat{y}_i)}_{} \overset{\checkmark}{=} \nabla_\theta \sum_{i=1}^{N} l(y_i, \hat{y}_i)$$

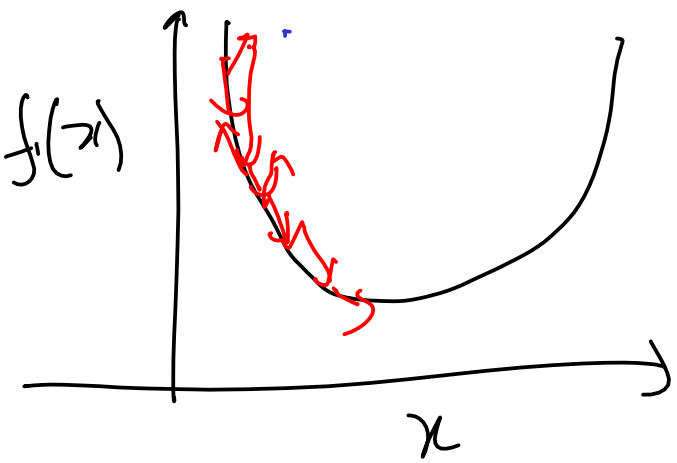
because $\nabla_\theta \sum f(\theta) = \sum \nabla_\theta f(\theta)$

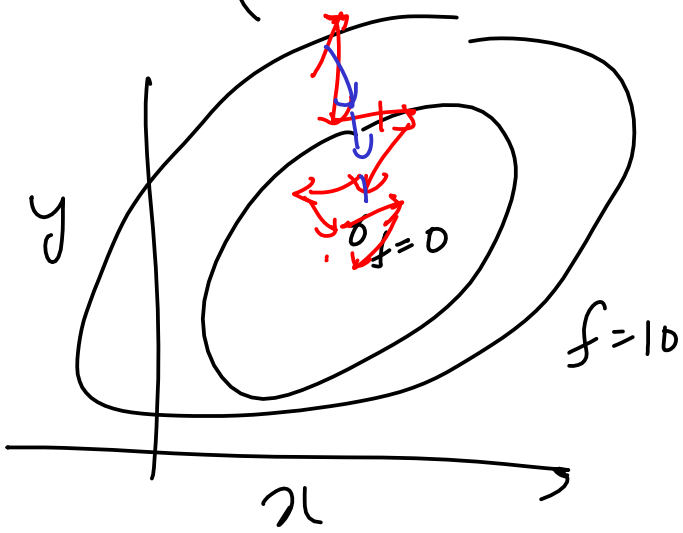
Gradient descent





Stochastic Gradient Descent (SGD)





(3) SGD with momentum

$$\nabla_\theta L(D_{batch}, \theta_t)$$

$$v_{t+1} = \mu v_t + \nabla_\theta \sum_{i=1}^{B} l(y_i, \hat{y}_i)$$

v_{t+1}: velocity

μ: momentum factor

$\mu \approx 0.9$

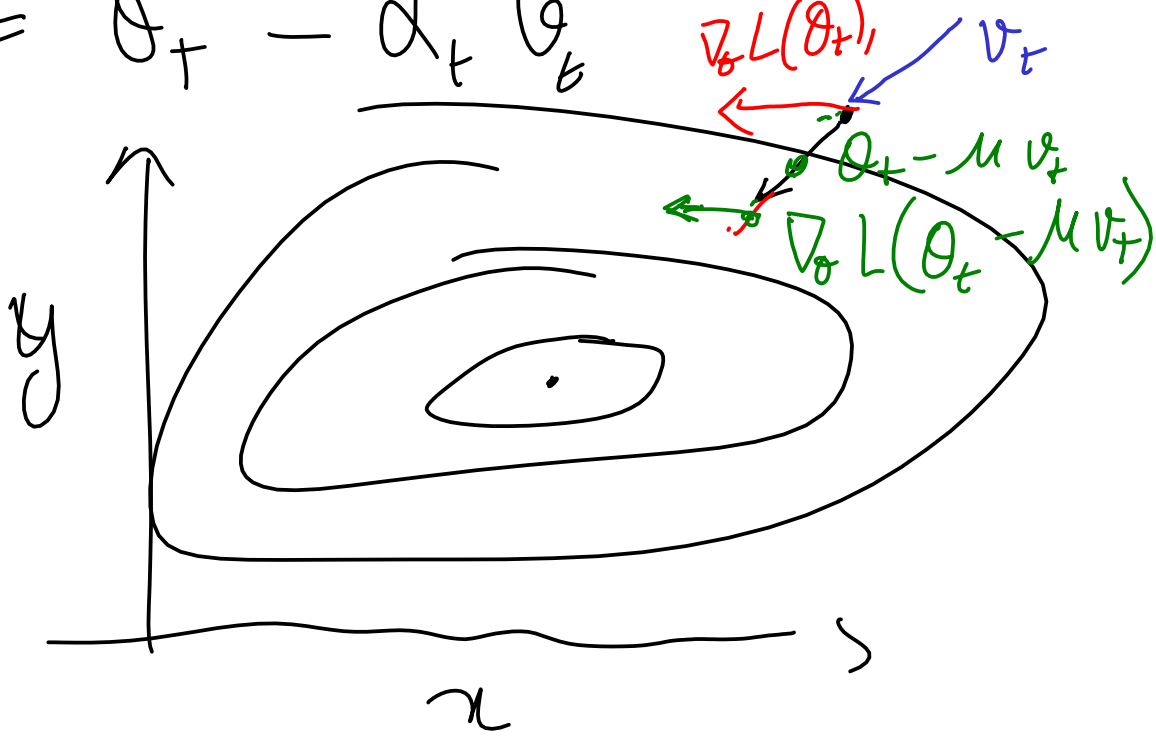
$$\theta_{t+1} = \theta_t - \alpha_t v_t$$

α_t: Learning rate

(4) Nestrov Accelerated Gradient

$$v_{t+1} = \mu v_t + \nabla_\theta L(D_{batch}, \theta_t - \mu v_t)$$

$$\theta_{t+1} = \theta_t - \alpha_t v_t$$



(5) ADAM

Entropy : measure of randomness

: opposite of information

How unlikely an event is a good measure of information

$$\text{Info}(X=x) = \underbrace{-\log_2 P(X=x)}_{\text{bits}} \in [0, \infty)$$

Shanon's entropy

$$H(X) = \mathbb{E}_X\left[-\log_e P(X)\right]$$

Discrete RV

$$= \sum_{x \in \Omega} -x \log P(X=x) \quad \text{for}$$

Cont. RV

$$= \int_{x \in \Omega} -x \log f(x)\, dx \quad \text{for}$$

Cross entropy

$$H(Y, \hat{Y}) = \mathbb{E}_Y\left[-\log_e P(\hat{Y})\right]$$

Y: true, $\hat{Y}$: predicted

Multi-class classification problem

So far

Two

$$\hat{y}_i = f(\underline{x}_i, \theta) \begin{cases} \to +1 & \text{for digit 1} \\ \to -1 & \text{for digit 0} \end{cases}$$

$\hat{y}_i \in \mathbb{R}^{10}$

$\hat{y}_i = f(x_i, \theta) \in \mathbb{R}^{10}$

$\exp(-\hat{y}_i[0])$ = Probability that x_i is digit 0

$\exp(-\hat{y}_i[1])$ = " " x_i is digit 1

$\exp(-\hat{y}_i[9])$ = " " x_i is digit 9

$y_i = 1$ → $P(y_i = 0) = 0$

↳ $P(y_i = 1) = 1$

⋮

↳ $P(y_i = 9) = 0$

$$H(y_i, \hat{y}_i) = -\sum_{j=1}^{n} P(y_i) \log \left(\frac{\exp(-\hat{y}_i[j])}{\sum \exp(-\hat{y}[j])} \right)$$

$P(\hat{y}[j])$

```
            lr = lr / 2

        ### DEBUGing information
        iswrong = (train_labels * predicted_labels.ravel()) < 0
        misclassified = (iswrong).sum() / iswrong.shape[0]
        print(f"loss: {loss_t:04.04f}, delta loss: {loss_t - loss_t_minus_1:
              f"train misclassified: {misclassified:04.04f}")
        if niter % 20 == 0: # plot every 20th iteration
            train_features_cpu = train_features.cpu()
            predicted_labels_cpu = predicted_labels.cpu()
            fig, ax = plt.subplots(1,1)
            draw_features(ax,
                          train_features_cpu[predicted_labels_cpu.ravel() >
                          train_features_cpu[predicted_labels_cpu.ravel() <


        niter += 1
    return model


trained_model = train_by_gradient_descent(model.to(device=DEVICE),
                                          loss,
                                          t.from_numpy(features).to(device=D
                                          t.from_numpy(labels).to(device=DEV


fig, axes = plt.subplots(1,2)
draw_features(axes[0], features[labels > 0, :], features[labels < 0, :])
axes[0].set_title('Train labels')

predicted_labels = trained_model(t.from_numpy(features).to(device=DEVICE, dt
predicted_labels_cpu = predicted_labels.cpu()
draw_features(axes[1], features[predicted_labels_cpu.ravel() > 0, :],
                  features[predicted_labels_cpu.ravel() < 0, :])
axes[1].set_title('Predicted labels');
```

In [ ]:

```
## Doing it the Pytorch way without using our custom feature extraction

import torch
import torch.nn
import torch.optim
import torchvision
from torchvision.transforms import ToTensor
from torch.utils.data import DataLoader

torch.manual_seed(17)

# Getting the dataset, the Pytorch way
all_training_data = torchvision.datasets.MNIST(
    root="data",
    train=True,
    download=True,
    transform=ToTensor()
)

test_data = torchvision.datasets.MNIST(
```

```
    root="data",
    train=False,
    download=True,
    transform=ToTensor()
)
```

In [ ]:
```
training_data, validation_data = torch.utils.data.random_split(all_training_
```

In [ ]:
```
# Hyper parameters
learning_rate = 1e-3 # controls how fast the
batch_size = 64
epochs = 5
momentum = 0.9

training_dataloader = DataLoader(training_data, shuffle=True, batch_size=bat
validation_dataloader = DataLoader(validation_data,  batch_size=batch_size)
test_dataloader = DataLoader(test_data,  batch_size=batch_size)

loss = torch.nn.CrossEntropyLoss()
# TODO:
# Define model = ?

model = tnn.Sequential(
    torch.nn.Flatten(),
    tnn.Linear(28*28, 10),
    tnn.ReLU(),
    tnn.Linear(10, 10))


# Define optimizer
optimizer = torch.optim.SGD(model.parameters(), lr=learning_rate, momentum=m

def loss_and_accuracy(model, loss, validation_dataloader, device=DEVICE):
        # Validation loop
        validation_size = len(validation_dataloader.dataset)
        num_batches = len(validation_dataloader)
        test_loss, correct = 0, 0

        with torch.no_grad():
            for X, y in validation_dataloader:
                X = X.to(device)
                y = y.to(device)
                pred = model(X)
                test_loss += loss(pred, y).item()
                correct += (pred.argmax(dim=-1) == y).type(DTYPE).sum().item

        test_loss /= num_batches
        correct /= validation_size
        return test_loss, correct

def train(model, loss, training_dataloader, validation_dataloader, device=DE
    model.to(device)
    for t in range(epochs):
        # Train loop
        training_size = len(training_dataloader.dataset)
```

```
        for batch, (X, y) in enumerate(training_dataloader):
            X = X.to(device)
            y = y.to(device)
            # Compute prediction and loss
            pred = model(X)
            loss_t = loss(pred, y)

            # Backpropagation
            optimizer.zero_grad()
            loss_t.backward()
            optimizer.step()

            if batch % 100 == 0:
                loss_t, current = loss_t.item(), (batch + 1) * len(X)
                print(f"loss: {loss_t:>7f}  [{current:>5d}/{training_size:>5
        valid_loss, correct = loss_and_accuracy(model, loss, validation_data
        print(f"Validation Error: \n Accuracy: {(100*correct):>0.1f}%, Avg l
    return model

trained_model = train(model, loss, training_dataloader, validation_dataloade

test_loss, correct = loss_and_accuracy(model, loss, test_dataloader)
print(f"Test Error: \n Accuracy: {(100*correct):>0.1f}%, Avg loss: {test_los
```

In [ ]:
```
X, _ = next(iter(test_dataloader))
X.shape
```

In [ ]:
```
import matplotlib.pyplot as plt
plt.imshow(X[0, 0])
```

In [ ]:
```
print("The predicted image label is ", model(X.to(DEVICE)).argmax(dim=-1)[0]
```