Before you turn this problem in, make sure everything runs as expected. First, **restart the kernel** (in the menubar, select Kernel→Restart) and then **run all cells** (in the menubar, select Cell→Run All).

Make sure you fill in any place that says `YOUR CODE HERE` or "YOUR ANSWER HERE", as well as your name and collaborators below:

In [ ]:
```
NAME = ""
COLLABORATORS = ""
```

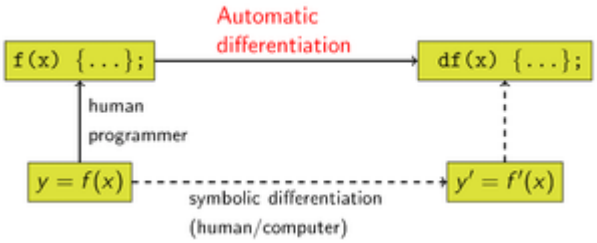
Differentiation options

1. Numerical differentiation
2. Symbolic differentiation
3. Automatic differentiation
    A. Forward mode differentiation
    B. Reverse mode differentiation

1. Numerical differentiation

2. Symbolic differentiation

3. Automatic differentiation



3.A Forward mode

Example:

$$z = f(x_1, x_2) = x_1 x_2 + \sin(x_1)$$

3.B Reverse mode

Example:

$$z = f(x_1, x_2) = x_1 x_2 + \sin(x_1)$$

In [ ]:
```
import numpy as np
class ForwardDiff:
    def __init__(self, value, grad=None):
        self.value = value
        self.grad = np.zeros_like(value) if grad is None else grad


    def __add__(self, other):
        cls = type(self)
        other = other if isinstance(other, cls) else cls(other)
        out = cls(self.value + other.value,
                    self.grad + other.grad)
        return out
    __radd__ = __add__

    def __repr__(self):
        return f"{self.__class__.__name__}(data={self.value}, grad={self.gra
x = ForwardDiff(2, 1)
y = ForwardDiff(3, 0)

f = x + y
f
```

In [ ]:
```
oldFD = ForwardDiff # Bad practice: do not do it
class ForwardDiff(oldFD):
    def __mul__(self, other):
        cls = type(self)
        other = other if isinstance(other, cls) else cls(other)
        out = cls(self.value * other.value,
                    other.value * self.grad+
                     self.value * other.grad)
        return out

    __rmul__ = __mul__

x = ForwardDiff(2, 0)
y = ForwardDiff(3, 1)

f1 = x * y
f2 = 2*x + 3*y + x*y
f1, f2
```

In [ ]:
```
oldFD = ForwardDiff # Bad practice: do not do it
class ForwardDiff(oldFD):
    def log(self):
        cls = type(self)
        return cls(np.log(self.value),
                     1/self.value * self.grad)
    def exp(self):
        cls = type(self)
```

```
        out_val = np.exp(self.value)
        return cls(out_val,
                   out_val * self.grad)

    def sin(self):
        cls = type(self)
        return cls(np.sin(self.value),
                   np.cos(self.value) * self.grad)

    def cos(self):
        cls = type(self)
        return cls(np.cos(self.value),
                  -np.sin(self.value) * self.grad)

    def __pow__(self, other):
        cls = type(self)
        other = other if isinstance(other, cls) else cls(other)
        return (self.log() * other).exp()

    def __neg__(self): # -self
        return self * -1

    def __sub__(self, other): # self - other
        return self + (-other)

    def __truediv__(self, other): # self / other
        return self * other**-1

    def __rtruediv__(self, other): # other / self
        return other * self**-1



x = ForwardDiff(2, 1)
y = ForwardDiff(3, 0)

f = x**y
f
```

In [ ]:

```
import numpy as np
def add_vjp(a, b, grad):
    return grad, grad


def no_parents_vjp(grad):
    return (grad,)

class ReverseDiff:
    def __init__(self, value, parents=(), op='', vjp=no_parents_vjp):
        self.value = value
        self.parents = parents
        self.op = op
        self.vjp = vjp
        self.grad = None

    def backward(self, grad):
```

```
        self.grad = grad
        op_args = [p.value for p in self.parents]
        grads = self.vjp(*op_args, grad)
        for g, p in zip(grads, self.parents):
            p.backward(g)

    def __add__(self, other):
        cls = type(self)
        other = other if isinstance(other, cls) else cls(other)
        out = cls(self.value + other.value,
                  parents=(self, other),
                  op='+',
                  vjp=add_vjp)
        return out

    __radd__ = __add__

    def __repr__(self):
        cls = type(self)
        return f"{cls.__name__}(value={self.value}, parents={self.parents},

x = ReverseDiff(2)
y = ReverseDiff(3)

f = x + y + 3
f.backward(1)
f
x.grad, y.grad
```

In [ ]:
```
oldRD = ReverseDiff # Bad practice: do not do it

def mul_vjp(a, b, grad):
    return grad * b, grad * a

class ReverseDiff(oldRD):
    def __mul__(self, other):
        cls = type(self)
        other = other if isinstance(other, cls) else cls(other)
        out = cls(self.value * other.value,
                  parents=(self, other),
                  op='*',
                  vjp=mul_vjp)
        return out

    __rmul__ = __mul__

x = ReverseDiff(2)
y = ReverseDiff(3)

f1 = 5*x + 7* y
f1.backward(1)
x.grad, y.grad
```

In [ ]:
```
f2 = x*y
f2.backward(1)
```

```
x.grad, y.grad
```