Perceptron

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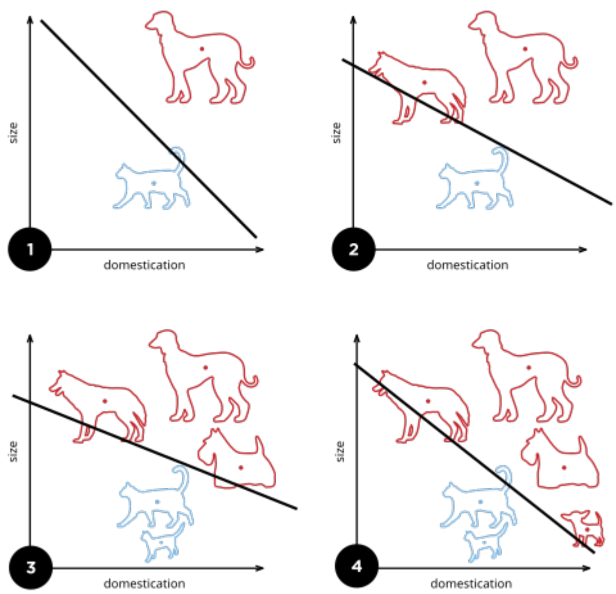
```
!pip install otter -grader
```

```
URL = "https://raw.githubusercontent.com/wecacuee/ECE490 -F25 -Neural -Networks/refs/heads/m
fname = "PerceptronTests.zip"
import urllib
from  zipfile import ZipFile
urllib.request.urlretrieve(URL, fname)
ZipFile(fname).extractall()
import otter
grader = otter.Notebook(tests_dir="./tests")
```

1 Perceptron



1.1 Optimization for classification

1.1.1 Dataset

Let the dataset $\mathcal{D} = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)\}$, where $\mathbf{x}_i \in R^d$ is the feature vector and $y_i \in \{-1, +1\}$ is the binary class label.

1.1.2 Model

We encode the prediction model as

$$\hat{y}_i = f(\mathbf{x}_i; \mathbf{m}, c) = \mathbf{m}^\top \mathbf{x}_i + c, \tag{1}$$

where $\mathbf{m} \in R^d$ and $c \in R$.

We say that the prediction is of class -1, if $\hat{y}_i < 0$ and +1 if $\hat{y}_i > 0$.

1.1.3 Loss function

$$l(y_i, \hat{y}_i; \mathbf{m}, c) = \begin{cases} 0 & \text{if } y_i\hat{y}_i > 0 \text{ or the sign of } y_i \text{ and } \hat{y}_i \text{ is the same} \\ |\hat{y}_i| & \text{if } y_i\hat{y}_i \leq 0 \text{ or the sign of } y_i \text{ and } \hat{y}_i \text{ is different} \end{cases} \tag{2}$$

$$l(y_i, \hat{y}_i; \mathbf{m}, c) = \begin{cases} 0 & \text{if } y_i\hat{y}_i > 0 \\ -y_i\hat{y}_i & \text{if } y_i\hat{y}_i \leq 0 \end{cases} \tag{3}$$

Let

$$\mathbf{w} = \begin{bmatrix} \mathbf{m} \\ c \end{bmatrix} \in R^{d+1}. \tag{4}$$

Then

$$\hat{y}_i = f(\mathbf{x}_i; \mathbf{m}, c) = \mathbf{m}^\top \mathbf{x}_i + c = \begin{bmatrix} \mathbf{x}_i^\top & 1 \end{bmatrix} \begin{bmatrix} \mathbf{m} \\ c \end{bmatrix} = \begin{bmatrix} \mathbf{x}_i^\top & 1 \end{bmatrix} \mathbf{w} \tag{5}$$

We can rewrite loss interms of $\mathbf{w}$,

$$l(y_i, \hat{y}_i; \mathbf{w}) = \begin{cases} 0 & \text{if } y_i\hat{y}_i > 0 \\ -y_i \mathbf{w}^\top \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} & \text{if } y_i\hat{y}_i \leq 0 \end{cases} \tag{6}$$

Optimization We want to find the parameters $\mathbf{w} \in R^{d+1}$ that minimize the loss over the entire dataset,

$$\mathbf{w}^* = \arg\min_{\mathbf{w}} \frac{1}{n} \sum_{(\mathbf{x}_i, y_i) \in \mathcal{D}} l(y_i, \hat{y}_i; \mathbf{w}) \tag{7}$$

To perform gradient descent on the loss function we need the gradient,

$$\nabla_{\mathbf{w}} \frac{1}{n} \sum_{(\mathbf{x}_i, y_i) \in \mathcal{D}} l(y_i, \hat{y}_i; \mathbf{w}) = \frac{1}{n} \sum_{(\mathbf{x}_i, y_i) \in \mathcal{D}} \nabla_{\mathbf{w}} l(y_i, \hat{y}_i; \mathbf{w}) = \frac{1}{n} \sum_{(\mathbf{x}_i, y_i) \in \mathcal{D}} \begin{cases} 0 & \text{if } y_i\hat{y}_i > 0 \\ -y_i \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} & \text{if } y_i\hat{y}_i \leq 0 \end{cases} \tag{8}$$

```
# Download MNIST dataset
!mkdir -p data
!F=train -images -idx3 -ubyte && cd data && \
    [ ! -f $F ] && \
    wget https://raw.githubusercontent.com/wecacuee/mnist/refs/heads/master/dataset/$F.gz  &
    gunzip $F.gz
!F=train -labels -idx1 -ubyte && cd data && \
    [ ! -f $F ] && \
    wget https://raw.githubusercontent.com/wecacuee/mnist/refs/heads/master/dataset/$F.gz  &
    gunzip $F.gz

# Load MNIST dataset from uint8 byte files
import struct
import numpy as np

# Ref:https://github.com/sorki/python -mnist/blob/master/mnist/loader.py
def mnist_read_labels(fname='data/train -labels -idx1 -ubyte'):
    with open(fname, 'rb') as file:
        # The file starts with 4 byte 2 unsigned ints
        magic, size = struct.unpack('>II', file.read(8))
        assert magic == 2049
        labels = np.frombuffer(file.read(), dtype='u1')
        return labels

# Ref:https://github.com/sorki/python -mnist/blob/master/mnist/loader.py
def mnist_read_images(fname='data/train -images -idx3 -ubyte'):
    with open(fname, 'rb') as file:
        # The file starts with 4 byte 4 unsigned ints
        magic, size, rows, cols = struct.unpack('>IIII', file.read(16))
        assert magic == 2051
        image_data = np.frombuffer(file.read(), dtype='u1')
        images = image_data.reshape(size, rows, cols)
        return images

# Visualize the dataset
import matplotlib.pyplot as plt
import matplotlib.animation as animation
import matplotlib as mpl
mpl.rc('animation', html='jshtml')
train_images = mnist_read_images('data/train -images -idx3 -ubyte')
labels = mnist_read_labels('data/train -labels -idx1 -ubyte')
zero_images = train_images[labels==0, ...] # Filter by label == 0
one_images = train_images[labels==1, ...] # Filter by label == 1

fig, ax = plt.subplots()
```

```
# ims is a list of lists, each row is a list of artists to draw in the
# current frame; here we are just animating one artist, the image, in
# each frame
ims = [[ax.imshow(np.hstack((zero_images[i], one_images[i])), animated=True, cmap='gray', vm
    for i in range(60)]
zero_images_anim = animation.ArtistAnimation(fig, ims, interval=50, blit=True,
                                repeat_delay=1000, repeat=False)
zero_images_anim
```

1.2 Images as arrays

```
train_images.shape
```

```
img1 = train_images[0]
img1
```

```
## Visualizing matrices
fig, ax = plt.subplots()
ax.axis('off')
ax.imshow([[1, 0],
           [0, 1]], cmap='gray')
# ax.imshow(np.random.rand(28, 28), cmap='gray')
```

```
zero_images_anim
```

2 What is a feature

Any property of data sample that helps with the task.

```
def feature_n_pxls(imgs):
    n, *shape = imgs.shape
    return np.sum(imgs[:, :, :].reshape(n, -1) > 128, axis=1)


n_pxls_zero_images = feature_n_pxls(zero_images)
n_pxls_one_images = feature_n_pxls(one_images)
fig, ax = plt.subplots()
ax.plot(n_pxls_zero_images, '.')
ax.plot(n_pxls_one_images, '+')
```

```
fig, ax = plt.subplots()
for i in range(5):
    ax.plot(zero_images[i].mean(axis=0), 'b -', label='0')
for i in range(5):
    ax.plot(one_images[i].mean(axis=0), 'r -', label='1')
ax.legend()
ax.set_xlabel('x')
ax.set_ylabel('Averge intensity')
```

We will compute the intensity weighted variance of image along the image rows so that we can take the variance along the rows as a measure fo the spread along the x axis.

Let the `img` be denoted as an array $I \in R^{H \times W}$ where H is the height and W is the width of the array.

Let's first compute the average intensity along the columns and call that `wts` in the python program and $\mathbf{w}$ in maths,

$$w(c) = \frac{1}{H} \sum_{r=1}^{H} I(r, c) \tag{9}$$

$$\mathbf{w} = \begin{bmatrix} w(c=1) & w(c=2) & \dots & w(c=W) \end{bmatrix} \in R^{1 \times W} \tag{10}$$

For the first image in `zero_images`, computing the `wts` vector is one line of code in numpy,

```
wts = zero_images[0].mean(axis=0)
wts
```

Now we want to compute the mean and variance of the column variable c weighted by the weight vector $\mathbf{w}$

$$\mu_c(\mathbf{w}) = \frac{\sum_{c=1}^{W} c w(c)}{\sum_{c=1}^{W} w(c)} \tag{11}$$

```
cs = np.arange(wts.shape[0]) # cs = [1, ..., W]
mean = (cs * wts).sum() / wts.sum()
mean
```

Now we do the same with variance,

$$\sigma_c^2(\mathbf{w}) = \sum_{c=1}^{W} \frac{(c - \mu_c)^2 w(c)}{\sum_{x=1}^{W} w(c)} \tag{12}$$

```
var = ((cs - mean)**2 * wts).sum() / wts.sum()
var
```

Put it all together in a function so that we can repeatedly run in on all the images,

```
def feature_y_var(img):
    wts = img.mean(axis=0)
    mean = (np.arange(wts.shape[0]) * wts).sum() / np.sum(wts)
    var = ((np.arange(wts.shape[0]) - mean)**2 * wts).sum() / np.sum(wts)
    return var

feature_y_var(zero_images[0]), feature_y_var(one_images[0])
```

```
def feature_y_var(imgs):
    wts = imgs.mean(axis= -2)
    arange = np.arange(wts.shape[ -1])
    mean = (arange * wts).sum(axis= -1) / wts.sum(axis= -1)
    mean = mean[:, np.newaxis]
    var = ((arange - mean)**2 * wts).sum(axis= -1) / wts.sum(axis= -1)
    return var

fig, ax = plt.subplots()
ax.plot(feature_y_var(zero_images), '.')
ax.plot(feature_y_var(one_images), '+')

def features_extract(images):
    return np.vstack((feature_n_pxls(images),
                      feature_y_var(images))).T
zero_features = features_extract(zero_images)
one_features = features_extract(one_images)


def draw_features(ax, zero_features, one_features):
    zf = ax.scatter(zero_features[:, 0], zero_features[:, 1], marker='.', label='0', alpha=(
    of = ax.scatter(one_features[:, 0], one_features[:, 1], marker='+', label='1', alpha=0.3
    ax.legend()
    ax.set_xlabel('Feature 1: count of pixels')
    ax.set_ylabel('Feature 2: Variance along x -axis')
    return [zf, of] # return list of artists

fig, ax = plt.subplots()
draw_features(ax, zero_features, one_features)
plt.show()

bfw = np.ones(3)
fig, ax = plt.subplots()
draw_features(ax, zero_features, one_features)
x = np.linspace( -1, 1, 21)
ax.plot(x, (x*bfw[0] + bfw[2])/bfw[1], 'r -')

bfw = np.ones(3)

Y = np.hstack((np.ones(zero_features.shape[0]), np.full(one_features.shape[0], -1.0)))
features = np.vstack((zero_features, one_features))
FEATURES_MEAN = features.mean(axis=0, keepdims=1)
FEATURES_STD = features.std(axis=0, keepdims=1)

def norm_features(features):
    return (features - FEATURES_MEAN) / FEATURES_STD
```

```
X = norm_features(features)

np.savez('zero_one_train_features.npz',
         mean=FEATURES_MEAN, std=FEATURES_STD,
         normed_features=X,
         labels=Y)

fig, ax = plt.subplots()
draw_features(ax, X[Y > 0, :], X[Y < 0, :])
x = np.linspace( -1, 1, 21)
ax.plot(x, -(x*bfw[0] + bfw[2])/bfw[1], 'r -')

X.shape, Y.shape
```

Concatenate 1 to all X

$$\bar{X} = \begin{bmatrix} X_{n\times 2}, & \mathbf{1}_{n\times 1} \end{bmatrix} \in R^{n\times 3} \tag{13}$$

```
X_and_1 = np.hstack((X, np.ones((X.shape[0], 1))))
X_and_1.shape
```

2.0.1 Homework (Perceptron): Problem 1 (10 marks)

Implement a function `model` that takes the dataset inputs $\bar{X} \in R^{n\times 3}$, the dataset labels, and the weight vector $\mathbf{w} \in R^3$ and returns the $\hat{y}_i$ for all elements in the dataset,

$$\hat{y}_i = f(\mathbf{x}_i; \mathbf{w}) = \mathbf{w}^\top \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} \tag{14}$$

```
def model(X_and_1, bfw):
    """
    X_and_1.shape = (n, 3)
    bfw.shape = (3,) or (p, q, 3) for contour plotting

    Return
    Yhat.shape = (n,) when bfw.shape is (3,)
    Yhat.shape = (p, q, n) when bfw.shape is (p, q, 3)
    """
    ...
    return Yhat # Yhat.shape = (p, q, n,)

def test_model(model, env=globals()):
    np = env['np']
    n = 100
    X = np.random.rand(n, 2)
    X_and_1 = np.hstack((X, np.ones((n, 1))))
    bfw1 = np.random.rand(200, 300, 3)
```

```
    Yhat1 = model(X_and_1, bfw1)
    bfw2 = np.random.rand(200, 300, 3)
    Yhat2 = model(X_and_1, bfw2)
    assert np.allclose(model(X_and_1, 13 * bfw1 + 17 * bfw2),
                       13*Yhat1 + 17*Yhat2)
test_model(model)
```

```
grader.check("p1")
```

You are given the implementation of the function `loss` that takes the dataset inputs $\bar{X} \in R^{n\times 3}$, the dataset labels, $Y \in \{0, 1\}^n$ and the weight vector $\mathbf{w} \in R^3$ and returns the total loss for all elements in the dataset,

$$\frac{1}{n} \sum_{(x_i, y_i)\in\mathcal{D}} l(y_i, \hat{y}_i; \mathbf{w}) = \frac{1}{n} \sum_{(x_i, y_i)\in\mathcal{D}} \begin{cases} 0 & \text{if } y_i\hat{y}_i > 0 \\ -y_i \mathbf{w}^\top \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} & \text{if } y_i\hat{y}_i \leq 0 \end{cases} \tag{15}$$

```
def loss(X_and_1, Y, bfw):
    """
    X_and_1.shape = (n, 3)
    Y.shape = (n,)
    bfw.shape = (3,)
    """
    Yhat = model(X_and_1, bfw)
    YYhat = Y * Yhat  # YYhat.shape = (n,)
    l_per_sample = np.where(YYhat > 0, 0, -YYhat) # l_per_sample.shape = (n,)
    l_mean = l_per_sample.mean(axis= -1) # l_per_sample.shape = (n,)
    return l_mean # l_mean.shape = (1,)
```

Homework (Perceptron): Problem 2 (10 marks) Implement a function `grad_loss` that takes the dataset inputs `X_and_1` $\bar{X} \in R^{n\times 3}$, the dataset labels, $Y \in \{0, 1\}^n$ and the weight vector $\mathbf{w} \in R^3$ and returns the total loss for all elements in the dataset,

$$\nabla_{\mathbf{w}} \left(\frac{1}{n} \sum_{(\mathbf{x}_i, y_i)\in\mathcal{D}} l(y_i, \hat{y}_i; \mathbf{w}) \right) = \frac{1}{n} \sum_{(\mathbf{x}_i, y_i)\in\mathcal{D}} \begin{cases} 0 & \text{if } y_i\hat{y}_i > 0 \\ -y_i \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} & \text{if } y_i\hat{y}_i \leq 0 \end{cases} \tag{16}$$

```
def grad_loss(X_and_1, Y, bfw):
    """
    Compute the mean loss gradient

    X_and_1.shape = (n, 3)
    Y.shape = (n,)
    bfw.shape = (3,)
```

```
    """
    ...
    return grad_L_mean # grad_L_mean.shape = (3,)

from functools import partial

def numerical_jacobian(f, x, h=1e -10):
    n = x.shape[ -1]
    eye = np.eye(n)
    x_plus_dx = x + h * eye # n x n
    num_jac = (f(x_plus_dx) - f(x)) / h # limit definition of the formula # n x m
    if num_jac.ndim >= 2:
        num_jac = num_jac.swapaxes( -1, -2) # m x n
    return num_jac

# Compare our grad_f with numerical gradient
def check_numerical_jacobian(f, jac_f,  nD=2, **kwargs):
    x = np.random.rand(nD)
    num_jac = numerical_jacobian(f, x, **kwargs)
    assert np.allclose(num_jac, jac_f(x), atol=1e -06, rtol=1e -4) # m x n

def test_grad_loss(grad_loss, env=globals()):
    np = env['np']
    partial = env['partial']
    numerical_jacobian = env['numerical_jacobian']
    check_numerical_jacobian = env['check_numerical_jacobian']
    loss = env['loss']
    n = 100
    X = np.random.rand(n, 2)
    X_and_1 = np.hstack((X, np.ones((n, 1))))
    Y = np.random.randint(0, 1, size=n)*2 - 1
    check_numerical_jacobian(partial(loss, X_and_1, Y),
                             partial(grad_loss, X_and_1, Y),
                             nD=3)

test_grad_loss(grad_loss, env=globals())

grader.check("p2")

def lr_scheduler(t, lr0=1, lr_end=0.001, t0=0, t_end=500, curv=4):
    normalized = (np.exp( -curv*(t -t0)/(t_end -t0)) -1) / (np.exp( -curv) - 1)
    return (lr_end - lr0) * normalized + lr0

def lr_scheduler_hm(t, lr0=1, lr_end=0.001):
    normralized = 1/(t+1)
    return (lr_end - lr0) * normalized + lr0
```

```
fig, ax = plt.subplots()
ax.plot(lr_scheduler(np.arange(500)))
lr_scheduler(500)
```

Homework (Perceptron): Problem 3a (10 marks) Implement gradient descent to find $\mathbf{w}$ that minimizes $\nabla_{\mathbf{w}} \frac{1}{n} \sum_{(\mathbf{x}_i, y_i) \in \mathcal{D}} l(y_i, \hat{y}_i; \mathbf{w})$

Algorithm 9.3 Gradient descent method.

given a starting point $\mathbf{x} \in \text{dom} f$.

repeat

1. Choose step size α_t
2. Update. $\mathbf{x} := \mathbf{x} - \alpha_t \nabla f(\mathbf{x})$

until stopping criterion is satisfied.

Homework (Perceptron): Problem 3b (10 marks) Create a copy of the Perceptron algorithm. Disable the 2D plotting and the Animation because it works only with 2D/3D parameters. Change the number of features or types of features. You are allowed to use all pixels as features, but you are only allowed to use linear Perceptron model for classification and Hinge Loss. Try to achieve best accuracy within these constraints.

```
def train(X_and_1, Y,
          lr_scheduler = lr_scheduler,
          MAX_ITER=50, # keep this small for autograder
          SMALLEST_GRAD_L_NORM=0.01 # keep this larger for autograder
         ):
    """
    X_and_1.shape is (n, 3)
    Y.shape is (n, )

    Write a function that returns

    bfw : the solution for bfw that you get after gradient descent
    list_of_bfws : the list of bfw at each iteration of gradient descent
    list_of_losses : the list of loss() values that you get at each iteration of gradient de
    """
    bfw = np.random.rand(3)*4 -2
    list_of_bfws = [bfw]
    list_of_losses = []

    grad_l = grad_loss(X_and_1, Y, bfw)
    list_of_losses.append(loss(X_and_1, Y, bfw))
    for t in range(MAX_ITER):
        if np.linalg.norm(grad_l) < SMALLEST_GRAD_L_NORM:
```

```
            break
        learning_rate = lr_scheduler(t)

        # 1. Compute grad_loss for current bfw
        # 2. Update bfw using gradient descent
        ...
        list_of_bfws.append(bfw)
        list_of_losses.append(loss(X_and_1, Y, bfw))
    return bfw, list_of_bfws, list_of_losses

OPTIMAL_BFW, list_of_bfws, list_of_losses = train(X_and_1, Y)
OPTIMAL_BFW /= np.linalg.norm(OPTIMAL_BFW)
print("optimal loss", list_of_losses[ -2:])
print("optimal bfw", OPTIMAL_BFW)
fig, ax = plt.subplots()
ax.plot(list_of_losses)
ax.set_xlabel('t')
ax.set_ylabel('loss')
plt.show()

def project_bfw_to_mc(bfw):
    """
    bfw = [w1, w2, w3]

    Converts equation of line
        w1 x + w2 y + w3 = 0
    to
        - m x + y - c = 0

    return [m, c]
    """
    bfw_normalized = bfw / np.linalg.norm(bfw)
    assert np.abs(bfw_normalized[..., 1]) > 1e -4
    m = -bfw_normalized[0] / bfw_normalized[1]
    c = -bfw_normalized[2] / bfw_normalized[1]
    return m, c

def lift_mc_to_bfw(m, c):
    """
    Converts equation of line
        - m x + y - c = 0
    to
        w1 x + w2 y + w3 = 0

    returns bfw = [w1, w2, w3]
    """
```

```
    bfw_norm_sq = 1 +  m**2 + c**2
    bfw_norm = np.sqrt(bfw_norm_sq)
    w1 = - m / bfw_norm
    w2 = np.sqrt(1 - (m**2 + c**2)/bfw_norm_sq)
    w3 = - c / bfw_norm
    return np.concatenate((w1[..., None], w2[..., None], w3[..., None]),
                          axis= -1)

fig, axes = plt.subplots(2, 1, figsize=(5, 7.5))
class Anim:
    def __init__(self, fig, axes, X_and_1, Y):
        self.fig = fig
        self.ax = axes[0]
        self.ax1 = axes[1]
        self.fts = draw_features(self.ax, X_and_1[Y > 0, :2], X_and_1[Y < 0, :2])
        self.line, = self.ax.plot([], [], 'r -')

        m, c = np.meshgrid(np.linspace( -10, 0, 51), np.linspace( -1, 1, 51))
        bfw = lift_mc_to_bfw(m, c)
        totalloss = np.empty_like(m)
        for i in range(m.shape[0]):
            for j in range(m.shape[1]):
                totalloss[i, j] = loss(X_and_1, Y, bfw[i, j, :])

        self.ctr = self.ax1.contour(m, c, totalloss, 30, cmap='Blues_r')
        self.ax1.set_xlabel('m')
        self.ax1.set_ylabel('c')
        self.ax1.clabel(self.ctr, self.ctr.levels, inline=True, fontsize=6)
        self.m_hist = []
        self.c_hist = []
        self.line2, = self.ax1.plot([], [], 'r* -')


    def anim_init(self):
        return (self.line, self.line2)

    def update(self, bfw):
        x = np.linspace( -2, 2, 21)
        m, c = project_bfw_to_mc(bfw)
        self.line.set_data(x, x * m + c)
        self.m_hist.append(m)
        self.c_hist.append(c)
        self.line2.set_data(self.m_hist, self.c_hist)
        return self.line, self.line2

fig, axes = plt.subplots(2, 1, figsize=(5, 7.5))
```

```
a = Anim(fig, axes, X_and_1, Y)
animation.FuncAnimation(fig, a.update, frames=list_of_bfws[::5],
                        init_func=a.anim_init, blit=True, repeat=False)
```

```
# Download MNIST dataset
!mkdir -p data
!F=t10k -images -idx3 -ubyte && cd data && \
    [ ! -f $F ] && \
    wget https://raw.githubusercontent.com/wecacuee/mnist/refs/heads/master/dataset/$F.gz  &
    gunzip $F.gz
!F=t10k -labels -idx1 -ubyte && cd data && \
    [ ! -f $F ] && \
    wget https://raw.githubusercontent.com/wecacuee/mnist/refs/heads/master/dataset/$F.gz  &
    gunzip $F.gz

test_images = mnist_read_images('data/t10k -images -idx3 -ubyte')
test_labels = mnist_read_labels('data/t10k -labels -idx1 -ubyte')
zero_one_filter = (test_labels == 0) | (test_labels == 1)
zero_one_test_images = test_images[zero_one_filter, ...]
zero_one_test_labels = test_labels[zero_one_filter, ...]
```

```
np.savez('zero_one_PERCEPTRON_optmial_bfw.npz',
         optimal_bfw=OPTIMAL_BFW)

def returnclasslabel(test_imgs):
    Xtest = norm_features(features_extract(test_imgs))
    Xtest_and_1 = np.hstack((Xtest, np.ones((*Xtest.shape[: -1], 1))))
    bfw = OPTIMAL_BFW
    return np.where(
        model(Xtest_and_1, bfw) > 0,
        0,
        1)
zero_one_predicted_labels = returnclasslabel(zero_one_test_images)

accuracy = np.sum(zero_one_test_labels == zero_one_predicted_labels) / len(zero_one_test_lab
accuracy

positive_label = 1
negative_label = 0
TP = np.sum((zero_one_test_labels == positive_label) & (zero_one_predicted_labels == positiv
TP

TN =  np.sum((zero_one_test_labels == negative_label) & (zero_one_predicted_labels == negati
TN
```

```
FP = np.sum((zero_one_test_labels != positive_label) & (zero_one_predicted_labels == positiv
FP

FN = np.sum((zero_one_test_labels != negative_label) & (zero_one_predicted_labels == negativ
FN

# Confusion matrix
fig, ax = plt.subplots()
ax.imshow([[TN, FN],
          [FP, TP]])
ax.set_xlabel('predicted')
ax.set_ylabel('true')
ax.set_xticks([0, 1])
ax.set_yticks([0, 1])
ax.text(0, 0, 'TN = %.3f' % TN)
ax.text(1, 0, 'FN = %.3f' % FN, color='w')
ax.text(0, 1, 'FP = %.3f' % FP, color='w')
ax.text(1, 1, 'TP = %.3f' % TP)

PRECISION = TP / (TP + FP)
PRECISION

RECALL = TP / (TP + FN)
RECALL

fig, ax = plt.subplots()
artists = []
for i in range(60):
    artists.append(
        [ax.imshow(zero_one_test_images[i], animated=True, cmap='gray', vmin=0, vmax=255),
        ax.text(0, 2, 'The number is %d' % zero_one_predicted_labels[i], animated=True, colo
animation.ArtistAnimation(fig, artists, interval=50, blit=True,
                                repeat_delay=1000, repeat=False)
```

2.0.2 Homework (Perceptron): Problem 4 : Course feedback (0 marks)

What parts of the course are going well and should be retained?

What parts of the course are NOT going well? How would you like the course to change to make it better?

2.1 Submission

Make sure you have run all cells in your notebook in order before running the cell below, so that all images/graphs appear in the output. The cell below will generate a zip file for you to submit. **Please save before exporting!**

Upload the generated zip file to the gradescope autograder

```
# Save your notebook first, then run this cell to export your submission.
grader.export(run_tests=True)
```