Deep Reinforcement Learning

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BF Skinner's Reinforcement Learning for Pigeons

Video

[1]

[1]Image source:bfskinner.org

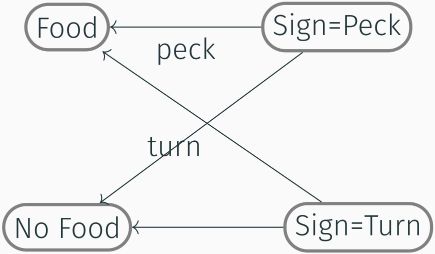
Video

[1]

[1] Image source:bfskinner.org

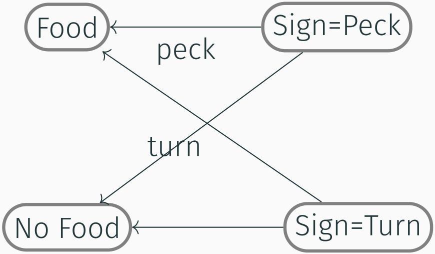
1. BF Skinner demonstrated that pigeons could learn to repeat an action that lead them to a particular reward. [3, p15]

RL terminology



State $(s_t \in \mathcal{S})$

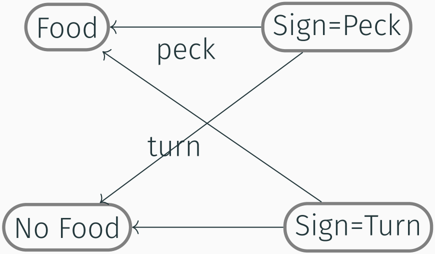
RL terminology



State $(\mathsf{s}_t \in \mathcal{S})$

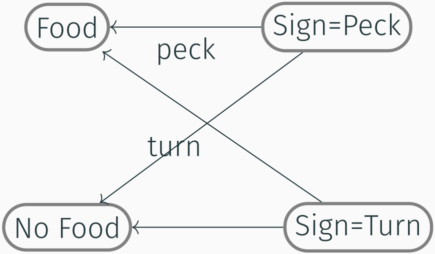
Actions $(\mathsf{a}_t \in \mathcal{A})$

RL terminology



$$\text{State} \quad (s_t \in \mathcal{S})$$
$$\text{Actions} \quad (a_t \in \mathcal{A})$$
$$\text{Transition probabilities} \quad (P(s_{t+1}|s_t, a_t))$$

RL terminology



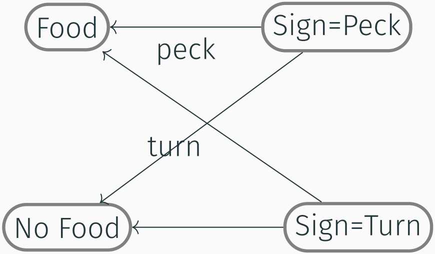
State $(s_t \in \mathcal{S})$

Actions $(a_t \in \mathcal{A})$

Transition probabilities $(P(s_{t+1}|s_t, a_t))$

Rewards $(r_t \sim P(.|s_t, a_t) \in \mathbb{R})$

RL terminology



State $(\mathsf{s}_t \in \mathcal{S})$

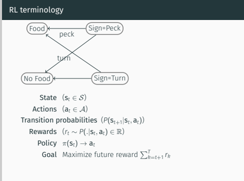
Actions $(\mathsf{a}_t \in \mathcal{A})$

Transition probabilities $(P(\mathsf{s}_{t+1}|\mathsf{s}_t, \mathsf{a}_t))$

Rewards $(r_t \sim P(.|\mathsf{s}_t, \mathsf{a}_t) \in \mathbb{R})$

Policy $\pi(\mathsf{s}_t) \to \mathsf{a}_t$

Goal Maximize future reward $\sum_{k=t+1}^{T} r_k$



State is the full description of the world at time t that captures the entire history. Example: in this example the state can be captured with two bits $\mathbf{s}_t = [f_t; p_t]$, where $f_t \in \{0, 1\}$ describes a food or no food state and $p_t \in \{0, 1\}$ describes the sign showing peck or turn.

Applications of RL

Atari games[1]

Applications of RL

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Alpha Go[2]

Applications of RL

Atari games[1]

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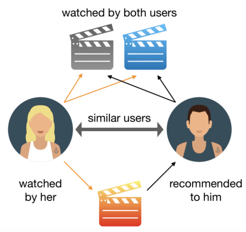


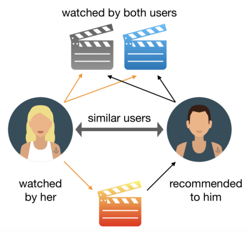
Recommender systems

a

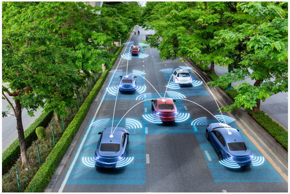
Applications of RL

Atari games[1]

Alpha Go[2]



Recommender systems



Autonomous cars

[a]

[a] Img: towardsdatascience.com

[b] Img:udacity.com

Exercise: Modeling the Breakout game

1. State space?
2. Action space?
3. Rewards?

RL problem

Discount factor $\gamma \in (0,1]$.

$$\pi^*(.) = \arg \max_{\pi} \mathbb{E}\left[\sum_{t=0}^{T} \gamma^t r_t\right]$$

Value Function

$$V_\pi(\mathsf{s}_k) = \mathbb{E}\left[\sum_{t=k}^{T} \gamma^{t-k} r_t \middle| \mathsf{s}_k\right]$$

Action Value Function

$$V_\pi(\mathsf{s}_k) = \mathbb{E}\left[\sum_{t=k}^{T} \gamma^{t-k} r_t \middle| \mathsf{s}_k\right]$$

$$Q_\pi(\mathsf{s}_k, \mathsf{a}_k) = \mathbb{E}\left[\sum_{t=k+1}^{T} \gamma^{t-k-1} r_t \middle| \mathsf{s}_k, \mathsf{a}_k\right]$$

Advantage Function

$$V_\pi(\mathsf{s}_k) = \mathbb{E}\left[\sum_{t=k}^{T} \gamma^{t-k} r_t \middle| \mathsf{s}_k\right]$$

$$Q_\pi(\mathsf{s}_k, \mathsf{a}_k) = \mathbb{E}\left[\sum_{t=k+1}^{T} \gamma^{t-k-1} r_t \middle| \mathsf{s}_k, \mathsf{a}_k\right]$$

$$A_\pi(\mathsf{s}_k, \mathsf{a}_k) = Q_\pi(\mathsf{s}_k, \mathsf{a}_k) - V_\pi(\mathsf{s}_k)$$

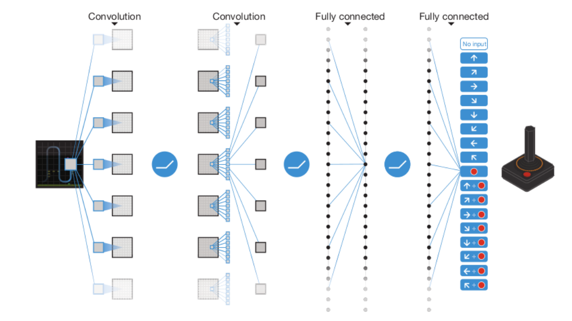
Bellman Equation

$$
\begin{aligned}
V_*(\mathsf{s}_k) &= \mathbb{E}\left[\sum_{t=k}^{T} \gamma^{t-k} r_t \middle| \mathsf{s}_k\right] \\
Q_*(\mathsf{s}_k, \mathsf{a}_k) &= \mathbb{E}\left[\sum_{t=k+1}^{T} \gamma^{t-k-1} r_t \middle| \mathsf{s}_k, \mathsf{a}_k\right] \\
&= \mathbb{E}[r_k + \gamma V_*(\mathsf{s}_{k+1})] \\
&= \mathbb{E}[r_k + \gamma \max_{\mathsf{a}'} Q_*(\mathsf{s}_{k+1}, \mathsf{a}') | \mathsf{s}_k, \mathsf{a}_k]
\end{aligned}
$$

DQN (2013): Bellman equation as a loss function

$$L(\theta) = \mathbb{E}_{(s,a,r,s')\sim U(D)}\left[\left(r + \gamma \max_{a'} Q(s', a'; \theta^-) - Q(s, a; \theta)\right)^2\right]$$

$$\nabla_\theta L(\theta) = \mathbb{E}_{(s,a,r,s')\sim U(D)}\left[-\nabla_\theta Q(s, a; \theta)\left(r + \gamma \max_{a'} Q(s', a'; \theta^-) - Q(s, a; \theta)\right)\right]$$



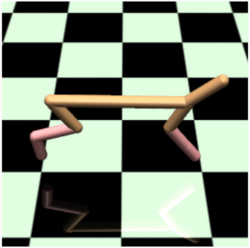
Other design considerations

1. Exploration vs exploitation trade-off
2. Off-policy vs on-policy learning

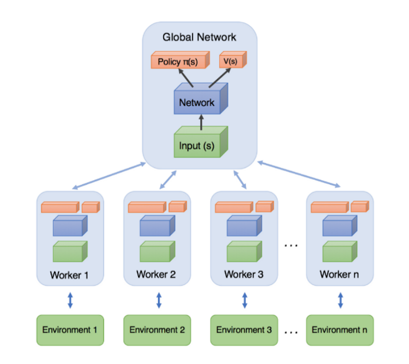
DDPG (2015) : Deep deterministic Policy gradients

$$\nabla_\phi L(\phi) = -\mathbb{E}_{(s,a,r,s')\sim U(D)}[\nabla_a Q(s, a; \theta)\nabla_\phi \pi(s; \phi)]$$

$$\nabla_\theta L(\theta) = \mathbb{E}_{(s,a,r,s')\sim U(D)}\left[-\nabla_\theta Q(s, \pi(s; \theta); \theta)\left(r + \gamma \max_{a'} Q(s', \pi(s'; \phi^-); \theta^-) - Q(s, \pi(s; \phi); \theta)\right)\right]$$



A3C (2016): Async. Advantage Actor Critic



[2]

$$\nabla_\theta L_V(\theta) = \mathbb{E}_{(s,a,r,s')\sim U(D)} \left[-\nabla_\theta V(s;\theta_V)(r + \gamma V(s';\theta_V^-) - V(s;\theta_V)) \right]$$

$$\nabla_\phi L_\pi(\phi) = -\mathbb{E}_{(s,a,r,s')\sim U(D)} \left[\nabla_\phi \log \pi(a|s;\phi) A(s,a;\theta) \right]$$

[2] Img:towardsdatascience.com

My work

References

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