📘Differentiation & Integration Basics (Cheat-Sheet)

This sheet covers the **core rules, formulas, and applications** of differentiation and integration, especially for **Data Science & Probability**.

🕰️Differentiation

👉**Definition:** Rate of change / slope of a function.
If $y = f(x)$, derivative is:

$$\frac{dy}{dx} \quad \text{or} \quad f'(x)$$

Rules

1. Constant: $\frac{d}{dx}(c) = 0$
2. Power: $\frac{d}{dx}(x^n) = nx^{n-1}$
3. Constant multiple: $\frac{d}{dx}[cf(x)] = cf'(x)$
4. Sum: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
5. Product: $(fg)' = f'g + fg'$
6. Quotient: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
7. Chain: If $y = f(g(x))$, then $dy/dx = f'(g(x))g'(x)$

Common Derivatives

- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\ln x) = 1/x$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$

Applications in Data Science

- Gradient Descent (optimization)
- Finding maxima/minima (critical points)
- Sensitivity analysis

🕰️Integration

👉**Definition:** Reverse of differentiation; gives accumulated area under a curve.
If $F'(x) = f(x)$, then:

$$\int f(x)dx = F(x) + C$$

Rules

1. Constant: $\int cdx = cx + C$
2. Power: $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
3. Sum: $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
4. Constant multiple: $\int cf(x)dx = c \int f(x)dx$

Common Integrals

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
- $\int e^x dx = e^x + C$
- $\int 1/x dx = \ln|x| + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sec^2 x dx = \tan x + C$

Applications in Data Science

- Probability density functions (PDFs):
  $P(a \leq X \leq b) = \int_a^b f(x)dx$
- Cumulative distribution function (CDF):
  $F(x) = \int_{-\infty}^x f(t)dt$
- Expectation (mean):
  $E[X] = \int_{-\infty}^{\infty} xf(x)dx$
- Variance:
  $Var(X) = E[X^2] - (E[X])^2$

🔗Relationship (Fundamental Theorem of Calculus)

- Differentiation and Integration are **inverse operations**:
- $\frac{d}{dx}\left(\int f(x)dx\right) = f(x)$
- $\int f'(x)dx = f(x) + C$
- Area under curve = change in antiderivative:
  $\int_a^b f(x)dx = F(b) - F(a)$

📊Worked Examples

Example 1: Basic Derivative

Find $\frac{d}{dx}(x^3)$:

$$\frac{d}{dx}(x^3) = 3x^2$$

Example 2: Basic Integral

Find $\int x dx$:

$$\int x dx = \frac{x^2}{2} + C$$

Example 3: Probability Density Function (PDF)

For Uniform distribution on [0,1], PDF is:

$$f(x) = 1, \quad 0 \leq x \leq 1$$

Probability that X lies between 0.2 and 0.5:

$$P(0.2 \leq X \leq 0.5) = \int_{0.2}^{0.5} 1 dx = (0.5 - 0.2) = 0.3$$

Example 4: Cumulative Distribution Function (CDF)

For the same Uniform(0,1),

$$F(x) = \int_0^x 1 dt = x, \quad 0 \leq x \leq 1$$

So, $F(0.7) = 0.7$.

Example 5: Expectation of Normal Distribution

For standard Normal(0,1):

$$E[X] = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 0$$

(Symmetric function → mean is 0)

👉🏽Use this sheet for **exam prep & intuition**: - Derivative = *slope* - Integral = *area*
Both are essential for **Data Science & Probability**.