Differentiation & Integration Basics (Cheat-Sheet)

Differentiation & Integration Basics (Cheat-Sheet)

Differentiation

Definition: Rate of change / slope of a function.

If y = f(x), derivative is: dy/dx or f'(x)

Rules:

1. d/dx(c) = 0
2. d/dx(x^n) = n * x^(n-1)
3. d/dx[c f(x)] = c f'(x)
4. d/dx[f(x)+g(x)] = f'(x)+g'(x)
5. (fg)' = f'g + fg'
6. (f/g)' = (f'g - fg') / g^2
7. Chain Rule: y=f(g(x)) -> dy/dx = f'(g(x)) * g'(x)

Common Derivatives:

- d/dx(x) = 1
- d/dx(x^n) = n * x^(n-1)
- d/dx(e^x) = e^x
- d/dx(ln x) = 1/x
- d/dx(sin x) = cos x
- d/dx(cos x) = -sin x
- d/dx(tan x) = sec^2 x

Applications in Data Science:

- Gradient Descent (optimization)
- Finding maxima/minima

- Sensitivity analysis

Integration

Definition: Reverse of differentiation; accumulated area under a curve.

If F'(x) = f(x), then: integral f(x) dx = F(x) + C

Rules:

1. integral c dx = cx + C
2. integral x^n dx = (x^(n+1))/(n+1) + C, n not equal to -1
3. integral [f(x)+g(x)] dx = integral f(x) dx + integral g(x) dx
4. integral c f(x) dx = c * integral f(x) dx

Common Integrals:

- integral x^n dx = (x^(n+1))/(n+1) + C
- integral e^x dx = e^x + C
- integral 1/x dx = ln|x| + C
- integral sin x dx = -cos x + C
- integral cos x dx = sin x + C
- integral sec^2 x dx = tan x + C

Applications in Data Science:

- Probability density functions (PDFs)
- Cumulative distribution function (CDF)
- Expectation (mean)
- Variance

Relationship (Fundamental Theorem of Calculus)

- Differentiation and Integration are inverse operations:

  d/dx(integral f(x) dx) = f(x)

$\int f'(x)\,dx = f(x) + C$

- Area under curve = change in antiderivative:

$\int_a^b f(x)\,dx = F(b) - F(a)$

Worked Examples

1. Derivative of x^3:

$\frac{d}{dx}(x^3) = 3x^2$

2. Integral of x:

$\int x\,dx = x^2/2 + C$

3. Uniform Distribution PDF:

$f(x) = 1$ for $0 \le x \le 1$

$P(0.2 \le X \le 0.5) = \int_{0.2}^{0.5} 1\,dx = 0.3$

4. Uniform Distribution CDF:

$F(x) = \int_0^x 1\,dt = x$, for $0 \le x \le 1$

$F(0.7) = 0.7$

5. Expectation of Normal(0,1):

$E[X] = \int_{-\infty}^{\infty} x \left(1/\sqrt{2\pi}\right) e^{-x^2/2}\,dx = 0$

Summary:

- Derivative = slope (rate of change)
- Integral = area (accumulation)

Both are essential for Data Science & Probability.