Reconstruction (of micro-objects) based on focus-sets using blind deconvolution

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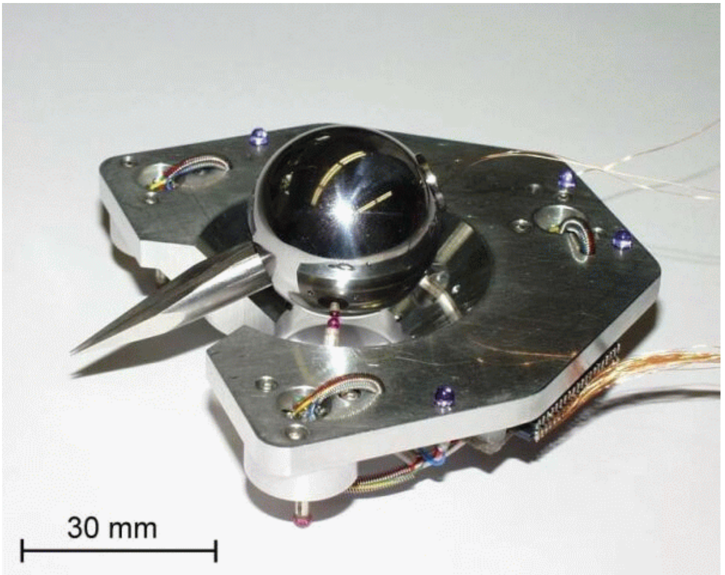
MINIMAN-project

Esprit Project No. 33915

Miniaturised Robot for Micro Manipulation

http://www.miniman-project.com/



Miniman III

people: SHU staff

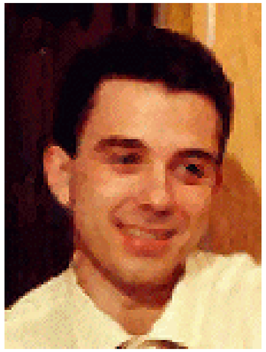
Sheffield Hallam University

microsystems & machine vision lab

http://www.shu.ac.uk/mmvl/

Prof. J. Travis

B. Amavasai



F. Caparrelli

no picture

A. Selvan

people: UKA staff

Universität Karlsruhe (TH)

Institute für Prozeßrechentechnik, Automation und Robotik

http://wwwipr.ira.uka.de/microrobots/

J. Wedekind

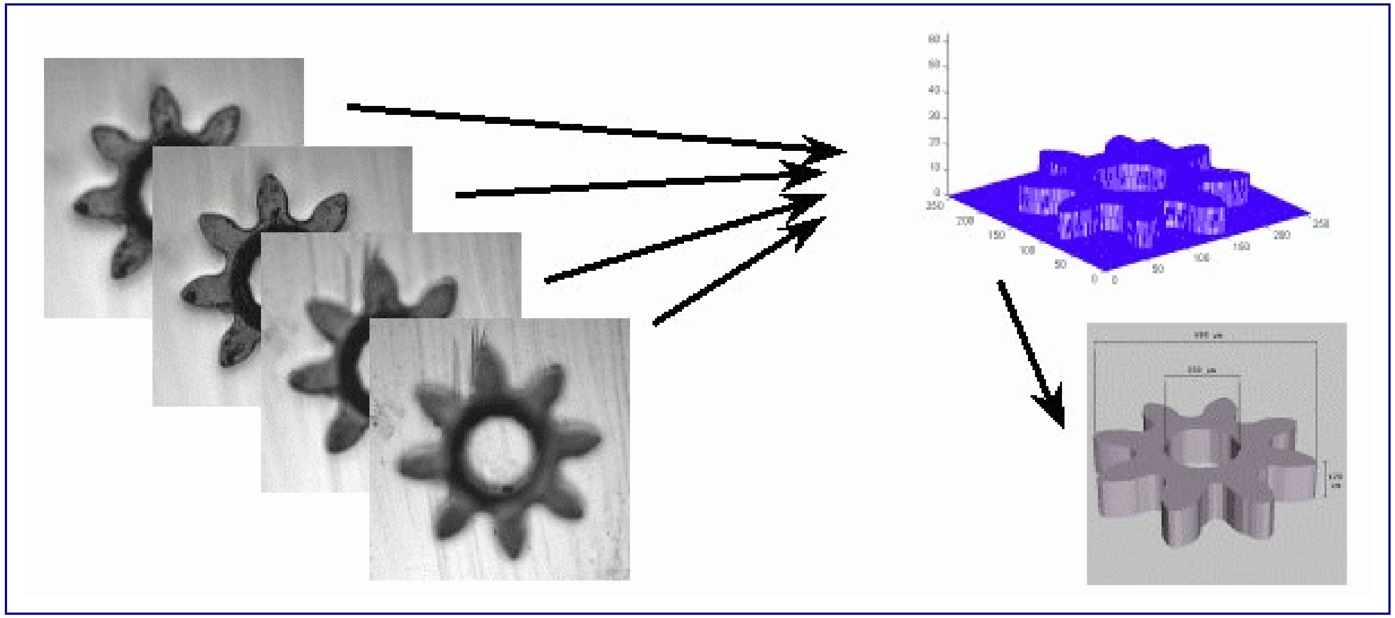
overview: environment

- Leica DM RXA microscope
  - 2 channel illumination
  - motorized z-table (piezo-driven 0.1 μm)
  - filter-module
- motorized xy-table
- Dual Pentium III with 1GHz processors
  - Linux OS
  - C++ and KDE/QT
- CCD camera 768×576

  $\Rightarrow$ resolution up to $0.74 \frac{\mu m}{\text{pixel}}$

Leica DM RXA microscope

overview: objective



- reconstruct from focus-set:
  - surface
  - luminosity and coloring
- identify model-parameters and quality of assembly

basics: discrete fourier transform

- 1D DFT

definition $$F_k := \sum_{x=0}^{N-1} \mathbf{f}_x \, e^{-\frac{2\pi x k i}{N}} \qquad \text{where } i^2 = -1$$

$$\Leftrightarrow \qquad \mathbf{f}_x = \frac{1}{N} \sum_{k=0}^{N-1} F_k \, e^{+\frac{2\pi x k i}{N}}$$

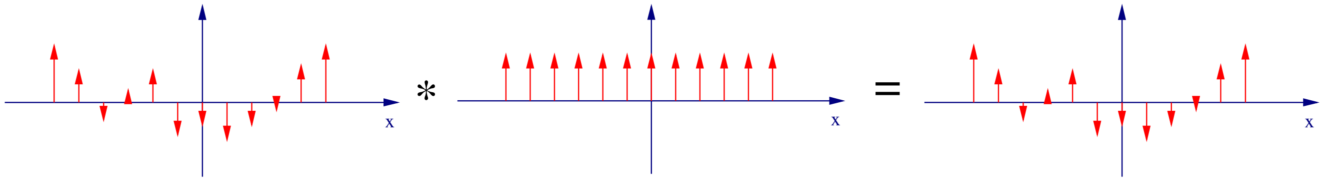
- 2D DFT

analogous $$F_{kl} := \sum_{x,y=0}^{N-1} \mathbf{f}_{xy} \, e^{-\frac{2\pi x k i}{N}} \, e^{-\frac{2\pi y l i}{N}} \qquad , \; i^2 = -1$$

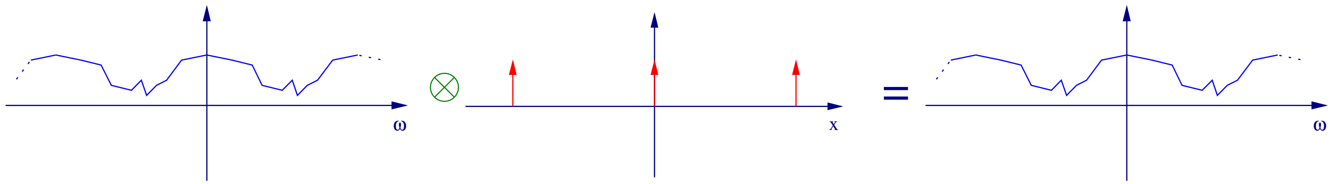
$$\Leftrightarrow \qquad \mathbf{f}_{xy} = \frac{1}{N^2} \sum_{k,l=0}^{N-1} F_{kl} \, e^{+\frac{2\pi x k i}{N}} \, e^{+\frac{2\pi y l i}{N}}$$

basics: 1D discrete fourier transform

image domain



frequency domain

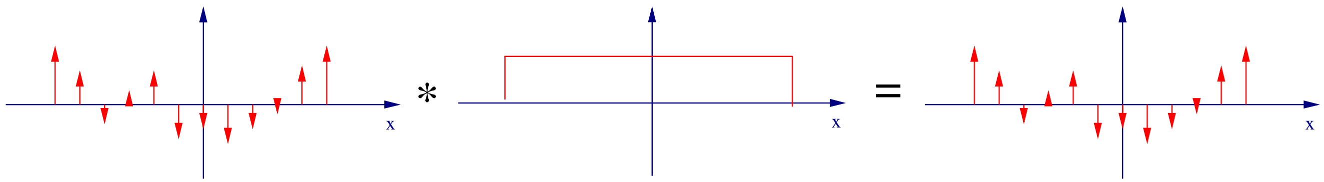


$$\forall x \in \mathbf{R} : f(x) = \int_{-\infty}^{\infty} f(x) \sum_{n=-\infty}^{\infty} \delta(x - Tn)\ \mathrm{d}x$$

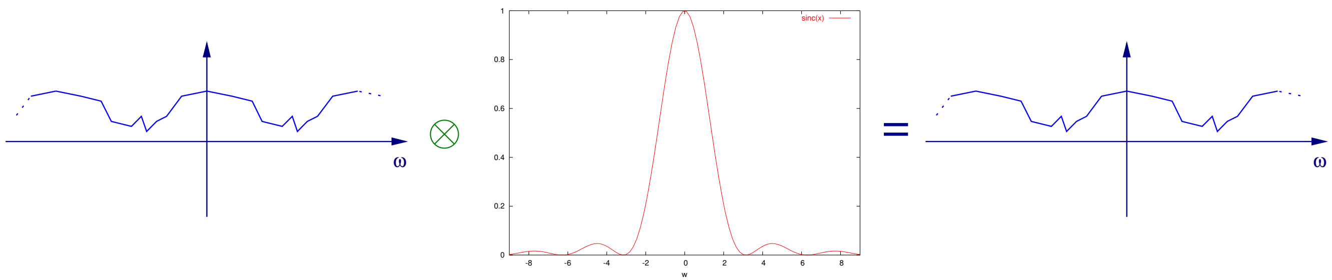
$$\Leftrightarrow \quad \forall \omega \in \mathbf{C} : F(\omega) = (F \otimes \lambda\widetilde{\omega}.[\frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\widetilde{\omega} - n\frac{2\pi}{T})])(\omega)$$

basics: 1D discrete fourier transform

image domain



frequency domain



$$\forall x \in \mathbf{R} : f(x) = f(x)\,\mathrm{rect}(\frac{t}{2T})$$

$$\Leftrightarrow \quad \forall \omega \in \mathbf{C} : F(\omega) = (F \otimes \lambda\tilde{\omega}.[2T\,\frac{\sin(\tilde{\omega}T)}{\tilde{\omega}T}])(\omega)$$

sparse matrices and vectors: images

conservative: $$\mathbf{x} = \begin{pmatrix} x_{00} & \cdots & x_{0(N-1)} \\ \vdots & \ddots & \vdots \\ x_{(N-1)0} & \cdots & x_{(N-1)(N-1)} \end{pmatrix} \in \mathbf{R}^{N \times N}$$

vector repr.: $$\vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \vdots \\ \vec{x}_{N-1} \end{pmatrix} = \begin{pmatrix} x_{00} \\ x_{01} \\ \vdots \\ x_{10} \\ \vdots \\ x_{(N-1)(N-1)} \end{pmatrix} \in \mathbf{R}^{NN}$$

sparse matrices and vectors: convolution (i)

matrix repr. of convolution with a 1D-LSI-filters

$$\underbrace{\begin{pmatrix} \widetilde{g}_0 \\ \widetilde{g}_1 \\ \vdots \\ \widetilde{g}_{K+N-1} \end{pmatrix}}_{\in \mathbf{R}^{(K+N-1)}} = \underbrace{\begin{pmatrix} \widetilde{h}_0 & 0 & \cdots & 0 \\ \widetilde{h}_1 & \widetilde{h}_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \widetilde{h}_{K-1} & & & \widetilde{h}_0 \\ 0 & \ddots & & \widetilde{h}_1 \\ \vdots & \ddots & & \vdots \end{pmatrix}}_{\in \mathbf{R}^{(K+N-1)\times N}} * \underbrace{\begin{pmatrix} \widetilde{f}_0 \\ \widetilde{f}_1 \\ \vdots \\ \widetilde{f}_{N-1} \end{pmatrix}}_{\in \mathbf{R}^{N}}$$

- vectors of different size $\Rightarrow$ not feasible
- matrix is clumsy and baffles mathematical approaches

sparse matrices and vectors: circulant matrices

$$\mathcal{A} = \begin{pmatrix} a(0) & a(N-1) & \cdots & a(1) \\ a(1) & \ddots & \ddots & \vdots \\ \vdots & \ddots & & a(N-1) \\ a(N-1) & \cdots & a(1) & a(0) \end{pmatrix}$$

without proof:

- eigenvalues of $\mathcal{A}$: $\lambda(k) = \sum_{j=0}^{N-1} a(j)\, e^{-\dfrac{2\pi kji}{N}}$
- $\mathcal{A} = \mathcal{F} Q_A \mathcal{F}^{-1}$ with
  - $Q_A = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ and
  - fourier-kernel $\mathcal{F}^{-1}$

sparse matrices and vectors: fourier kernel

definition of fourier-kernel: $\mathcal{F}^{-1} := \left(e^{\dfrac{2\pi u_{kl} i}{N}}\right)$

with $\mathcal{U} := \begin{pmatrix} 0 & 0 & 0 & \cdots \\ 0 & 1 & 2 & \cdots \\ 0 & 2 & 4 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$ and $i^2 = -1$

note:

- $\mathbf{X} = \mathrm{DFT}\{\mathbf{x}\} \mathrel{\widehat{=}} X = \mathcal{F}^{-1}\vec{x}$
- $\mathbf{x} = \mathrm{DFT}^{-1}\{\mathbf{X}\} \mathrel{\widehat{=}} \vec{x} = \mathcal{F}X$ where $\mathcal{F} = \frac{1}{N}(\mathcal{F}^{-1})^*$
- $\mathcal{F} = \mathcal{F}^\top$

sparse matrices and vectors: convolution (ii)

approximated 1D convolution

$$\underbrace{\begin{pmatrix} \tilde{g}_0 \\ \tilde{g}_1 \\ \vdots \\ \tilde{g}_{N-1} \end{pmatrix}}_{=:\vec{\tilde{g}} \in \mathbf{R}^N} \approx \underbrace{\begin{pmatrix} h_0 & h_{N-1} & \cdots & h_1 \\ h_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & & h_{N-1} \\ h_{N-1} & \cdots & h_1 & h_0 \end{pmatrix}}_{=:\tilde{H} \in \mathbf{R}^{N \times N}} * \underbrace{\begin{pmatrix} \tilde{f}_0 \\ \tilde{f}_1 \\ \vdots \\ \tilde{f}_{N-1} \end{pmatrix}}_{=:\vec{\tilde{f}} \in \mathbf{R}^N}$$

$$\vec{\tilde{g}} = \tilde{H} \vec{\tilde{f}}$$

sparse matrices and vectors: convolution (iii)

$$\vec{g}_k = \sum_{l=0}^{N-1} H_{(k-l)\,mod\,N} \vec{f}_l$$

$$\underbrace{\begin{pmatrix} \vec{g}_0 \\ \vec{g}_1 \\ \vdots \\ \vec{g}_{N-1} \end{pmatrix}}_{=:\vec{g}\in\mathbf{R}^{N^2}} \approx \underbrace{\begin{pmatrix} H_0 & H_{N-1} & \cdots & H_1 \\ H_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & & H_{N-1} \\ H_{N-1} & \cdots & H_1 & H_0 \end{pmatrix}}_{=:H\in\mathbf{R}^{N^2\times N^2}} * \underbrace{\begin{pmatrix} \vec{f}_0 \\ \vec{f}_1 \\ \vdots \\ \vec{f}_{N-1} \end{pmatrix}}_{=:\vec{f}\in\mathbf{R}^{N^2}}$$

$\Rightarrow$ $\vec{g} = H\vec{f}$ is 2D convolution!

$\Rightarrow$ H is (block) circulant

EM-algorithm: description

given:

- "incomplete" data-set y
- many-to-one transform $y = g(z_1, z_2, \ldots, z_M) = g(z)$
- pdf $p_z(z;\theta)$ and cond. pdf $p(z|y;\theta)$

EM-algorithm:

1. expectation: Determine expected log-likelihood of complete data
$$U(\theta,\theta_p) := E_z\{\ln p_z(z;\theta)|y;\theta_p\} = \int \ln p_z(z;\theta)\, p(z|y;\theta_p)\mathrm{d}z$$
2. maximisation: Maximize $\theta_{p+1} = \underset{\theta}{\operatorname{argmax}}\, U(\theta,\theta_p)$
3. goto step 1 until θ converges

EM-algorithm: overview of representations

meaning	conversative		sparse matrices	
	real	fourier	real	fourier
sets	$\mathbf{R}^{N\times N}$	$\mathbf{C}^{N\times N}$	$\mathbf{R}^{N^2,N^2\times N^2}$	$\mathbf{C}^{N^2,N^2\times N^2}$
image	$\mathbf{x}$	$\mathbf{X}$	$\vec{x}$	X
PSF	h	Δ	D	Q_D
cov. image	C_x	S_x	Λ_x	Q_x
cov. noise	$\sigma_v^2 I$	(σ_v^2)	Λ_v	Q_v
convolution	$\mathbf{x} \otimes \mathbf{h}$	$\mathbf{X} \circledast \Delta$	$D\vec{x}$	$Q_D X$
DFT	-	$\mathrm{DFT}\{\mathbf{x}\}$	-	$\mathcal{F}^{-1}\vec{x}$
inv. DFT	$\mathrm{DFT}^{-1}\{\mathbf{X}\}$	-	$\mathcal{F}X$	-

EM-Algorithm: derivation (i)

- incomplete/complete data-set: $\vec{y} \in \mathbf{R}^N$ / $(\vec{x}^\top, \vec{y}^\top)^\top \in \mathbf{R}^{2N}$
- many-to-one transform: $\vec{y} =: g\left(\begin{pmatrix} \vec{x} \\ \vec{y} \end{pmatrix}\right) \forall \vec{y}$
- pdf of $\vec{z}$ is zero-mean normal distr.: $p_z(\vec{z}, \theta = \{D, \Lambda_x, \Lambda_v\}) =$

$$\frac{1}{\sqrt{(2\pi)^{2N^2} \begin{vmatrix} \Lambda_x & \Lambda_x D^\top \\ D\Lambda_x & D\Lambda_x D^\top + \Lambda_v \end{vmatrix}}} e^{-\frac{1}{2}\begin{pmatrix} \vec{x} \\ \vec{y} \end{pmatrix}^\top \underbrace{\begin{pmatrix} \Lambda_x & \Lambda_x D^\top \\ D\Lambda_x & D\Lambda_x D^\top + \Lambda_v \end{pmatrix}}_{=:C}^{-1} \begin{pmatrix} \vec{x} \\ \vec{y} \end{pmatrix}}$$

EM-Algorithm: derivation (ii)

- conditional pdf $p(\vec{z}|\vec{y},\theta) = p(\vec{x}|\vec{y},\theta)$ is (conditional) normal distr.:
  - with mean $M_{x|y} = \Lambda_x D^\top (D\Lambda_x D^\top + \Lambda_v)^{-1}\vec{y}$ and
  - covariance $S_{x|y} = \Lambda_x - \Lambda_x D^\top (D\Lambda_x D^\top + \Lambda_v)^{-1} D\Lambda_x$

---

derivation of

$$\ln p_z(z;\theta) = -N^2 \ln(2\pi) - \frac{1}{2}\ln|C| - \frac{1}{2}(\vec{x}^\top, \vec{y}^\top)\, C^{-1} \begin{pmatrix} \vec{x} \\ \vec{y} \end{pmatrix}:$$

$$\begin{vmatrix} \Lambda_x & \Lambda_x D^\top \\ D\Lambda_x & D\Lambda_x D^\top + \Lambda_v \end{vmatrix} = \det(\Lambda_x(D\Lambda_x D^\top + \Lambda_v) - D\Lambda_x\Lambda_x D^\top)$$

$= \det(\Lambda_x\Lambda_v) = |\Lambda_x||\Lambda_v|$, because Λ_x and Λ_v are symmetric and positive definite.

EM-Algorithm: derivation (iii)

- observe that $|\Lambda_v| = (\sigma_v^2)^{N^2}$.
- $|\Lambda_x| = |\mathcal{F} Q_x \mathcal{F}^{-1}| = |Q_x| = \prod_{kl} S_x(k,l)$

using $\mathcal{A} = \begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{pmatrix} =$

$$\begin{pmatrix} (\mathcal{A}_{11} - \mathcal{A}_{12}\mathcal{A}_{22}^{-1}\mathcal{A}_{21})^{-1} & \mathcal{A}_{11}^{-1}\mathcal{A}_{12}(\mathcal{A}_{21}\mathcal{A}_{11}^{-1}\mathcal{A}_{12} - \mathcal{A}_{22})^{-1} \\ (\mathcal{A}_{21}\mathcal{A}_{11}^{-1}\mathcal{A}_{12} - \mathcal{A}_{22})^{-1}\mathcal{A}_{21}\mathcal{A}_{11}^{-1} & (\mathcal{A}_{22} - \mathcal{A}_{21}\mathcal{A}_{11}^{-1}\mathcal{A}_{12})^{-1} \end{pmatrix}$$

we get $C^{-1} = \begin{pmatrix} \Lambda_x & \Lambda_x D^\top \\ D\Lambda_x & D\Lambda_x D^\top + \Lambda_v \end{pmatrix}^{-1} =$

EM-Algorithm: derivation (iv)

$$C^{-1} = \begin{pmatrix} (I - D^\top(D\Lambda_x D^\top + \Lambda_v)^{-1} D\Lambda_x)^{-1}\Lambda_x^{-1} & -D^\top\Lambda_v^{-1} \\ -\Lambda_v^{-1}D & \Lambda_v^{-1} \end{pmatrix}.$$

With $\vec{u}^\top \mathcal{A}\vec{v} = \vec{u}^* \mathcal{F} Q_A \mathcal{F}^{-1}\vec{v} = (\mathcal{F}^*\vec{u})^* Q_A \mathcal{F}^{-1}\vec{v}$

$= (\frac{1}{N^2}\mathcal{F}^{-1}\vec{u})^* Q_A \mathcal{F}^{-1}\vec{v} = \frac{1}{N^2} U^* Q_A V$ we obtain

$$\ln p_z(z;\theta) = -N^2 \ln(2\pi) - \frac{N^2}{2}\ln(\sigma_v^2) - \frac{1}{2}\sum_{kl} \ln S_{kl} - \frac{1}{2N^2} Y^* Q_v^{-1} Y$$

$$+ \frac{1}{N^2}\mathrm{Re}\{Y^* Q_D Q_v^{-1} X\} - \frac{1}{2}\sum_{kl} Q_x (Q_D Q_D^* Q_v^{-1} + Q_x^{-1})$$

$$- \frac{1}{2N^2} X^* (Q_D Q_D^* Q_v^{-1} + Q_x^{-1}) X$$

EM-Algorithm: derivation (v)

Replacing the remaining diagonal matrices with DFT-arrays yields:

$$\ln p_z(z;\theta) = -N^2 \ln(2\pi) - \frac{N^2}{2}\ln(\sigma_v^2) - \frac{1}{2}\sum_{kl} \ln S_{kl} - \frac{1}{2N^2}\mathbf{Y}^*\mathbf{Y}(\sigma_v^2)^{-1}$$
$$+\frac{1}{N^2}\mathrm{Re}\{\mathbf{Y}^*\Delta(\sigma_v^2)^{-1}\mathbf{X}\} - \frac{1}{2}S_x(\Delta\Delta^*(\sigma_v^2)^{-1} + S_x^{-1})$$
$$-\frac{1}{2N^2}\mathbf{X}^*(\Delta\Delta^*(\sigma_v^2)^{-1} + S_x^{-1})\mathbf{X}$$

The last steps are:

- substitute $\mathbf{X}$ and $\mathbf{X}^* \circledast \ldots \circledast \mathbf{X}$ with their expected values.
  $\Rightarrow U(\theta, \theta_k)$
- determine $\underset{\theta}{\mathrm{argmax}}$ by setting derivative $\dfrac{\delta U(\theta, \theta_k)}{\delta \theta}$ to zero.

EM-Algorithm: implementation

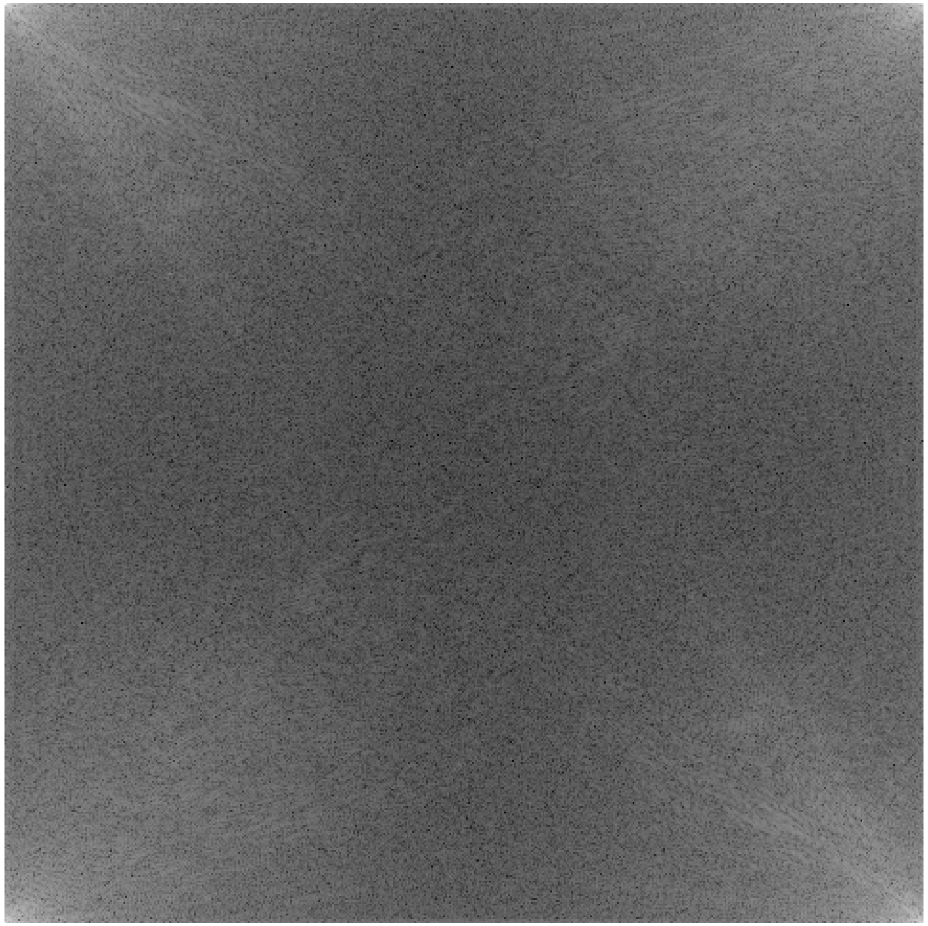
The resulting iteration step:

- $S_x^{(p+1)}(k,l) = S_{x|y}^{(p)}(k,l) + \frac{1}{N^2}|M_{x|y}(k,l)|^2$
- $\Delta^{(p+1)}(k,l) = \frac{1}{N^2}\frac{Y(k,l)M_{x|y}^*(k,l)}{S_{x|y}^{(p)}(k,l) + \frac{1}{N^2}|M_{x|y}(k,l)|^2}$
- $\sigma_v^2 = \frac{1}{N^2}\sum_{kl}\Big\{|\Delta^{(p+1)}(k,l)|^2\big(S_{x|y}^{(p)}(k,l) + \frac{1}{N^2}|M_{x|y}(k,l)|^2\big) + \frac{1}{N^2}\big(|Y(k,l)|^2 - 2\mathrm{Re}\big[Y^*(k,l)\Delta^{(p+1)}(k,l)M_{x|y}(k,l)\big]\big)\Big\}$

results: deblur picture

conditional mean after 10. iteration

results: statistic and psf



log-DFT

result: comparison

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