

8.5 张量运算



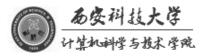
■基本数学运算

□ 加减乘除运算

算术操作	描述
tf.add(x, y)	将x和y逐元素相加
tf.subtract(x, y)	将x和y逐元素相减
tf.multiply(x, y)	将x和y逐元素相乘
tf.divide(x, y)	将x和y逐元素相除
tf.math.mod(x, y)	对x逐元素取模

参数分别是参与运算的两个张量

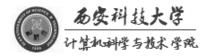
```
In [1]: import tensorflow as tf
In [2]: a = tf.constant([0,1,2])
b = tf.constant([3,4,5])
tf.add(a, b)
Out[2]: <tf.Tensor: id=2, shape=(3,), dtype=int32, numpy=array([3, 5, 7])>
```





□ 幂指对数运算

算术操作	描述	
tf.pow(x, y)	对x求y的幂次方	
tf.square(x)	对x逐元素求计算平方	
tf.sqrt(x)	对x逐元素开平方根	
tf.exp(x)	计算e的x次方	
tf.math.log(x)	计算自然对数,底数为e	



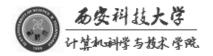


■ 一维张量幂运算

```
In [3]: x = tf.range(4)
x

Out[3]: <tf.Tensor: id=6, shape=(4,), dtype=int32, numpy=array([0, 1, 2, 3])>
In [4]: tf.pow(x, 2)
Out[4]: <tf.Tensor: id=8, shape=(4,), dtype=int32, numpy=array([0, 1, 4, 9])>
```

对x张量中的每个元素计算了2次方



■ 二维张量幂运算

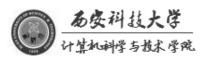
```
In [5]:
        x=tf. constant([[2,
        y=tf. constant([[8, 16], [2, 3]])
        tf.pow(x, y)
Out[5]: <tf. Tensor: id=11, shape=(2, 2), dtype=int32, numpy=
        array([[ 256, 65536],
                          27]])>
                                        张量的元素必须是浮点数类型
In [6]:
        x=tf. constant([1., 4., 9., 16.])
        pow(x, 0.5)
Out[6]: <tf. Tensor: id=14, shape=(4,), dtype=float32, numpy=array([1., 2., 3., 4.], dtype=float32)>
                                                                       平方根
```

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■ 平方和平方根运算

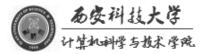
```
In [7]: x=tf. constant([1, 2, 3, 4])
        tf. square(x)
Out[7]: <tf. Tensor: id=16, shape=(4,), dtype=int32, numpy=array([1, 4, 9, 16])>
In [8]: x=tf. constant([1., 4., 9., 16.])
        tf. sqrt(x)
Out[8]: <tf. Tensor: id=18, shape=(4,), dtype=float32, numpy=array([1., 2., 3., 4.], dtype=float32)>
In [9]: f = tf.constant([[1., 9.], [16., 100.]])
        tf. sqrt(f)
Out[9]: <tf. Tensor: id=20, shape=(2, 2), dtype=float32, numpy=
        array([[ 1., 3.],
                [ 4., 10.]]
                             dtype=float32)>
```





■ 自然指数和自然对数运算

TensorFlow中只有以e为底的自然对数,没有提供以其他数值为底的对数运算函数



■ 对数运算

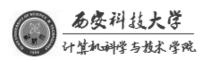
$$\log_a b = \frac{\log_c b}{\log_c a}$$

要计算其他底数的对数,可以利用对数的换底公式,间接的通过 tf.math.log(x)函数来实现。

```
\log_2 256 = \frac{\log_e 256}{256}
In \lceil 12 \rceil: x=tf. constant (256.)
          y=tf. constant(2.)
          tf. math. log(x)/tf. math. log(y)
Out[12]: (tf. Tensor: id=30, shape=(), dtype=float32, numpy=8.0)
In [13]: x = tf. constant([[1., 9.], [16., 100.]]) _ 真数
          y = tf. constant([[2., 3.], [2., 10.]])
                                                          底数
          tf. math. log(x) / tf. math. log(y)
Out[13]: <tf. Tensor: id=35, shape=(2, 2), dtype=float32, numpy=
          array ([[0., 2.].
                  [4., 2.]], dtype=float32)>
```

□ 其他运算

函 数	描述
tf.sign(x)	返回x的符号
tf.abs(x)	对x逐元素求绝对值
tf.negative(x)	对x逐元素求相反数, y = -x
tf.reciprocal(x)	取x的倒数
tf.logical_not(x)	对x逐元素求的逻辑非
tf.ceil(x)	向上取整
tf.floor(x)	向下取整
tf.rint(x)	取最接近的整数
tf.round(x)	对x逐元素求舍入最接近的整数
tf.maximum(x, y)	返回两tensor中的最大值
tf.minimum(x, y)	返回两tensor中的最小值



□ 三角函数和反三角函数运算

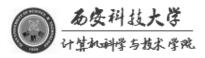
函 数	描述
tf.cos(x)	三角函数cos
tf.sin(x)	三角函数sin
tf.tan(x)	三角函数tan
tf.acos(x)	反三角函数arccos
tf.asin(x)	反三角函数arcsin
tf.atan(x)	反三角函数arctan



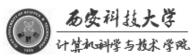


■ 重载运算符

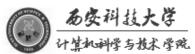
运算符	构造方法	运算符	构造方法
x+y	tf.add()	x&y	tf.logical_and()
x-y	tf.subtract()	x y	tf.logical_or()
x*y	tf.multiply()	x^y	tf.logical_xor()
x/y(python2.0)	tf.divide()	~x	tf.logical_not()
x/y(python3.0)	tf.truediv()	x <y< td=""><td>tf.less()</td></y<>	tf.less()
X//y(python3.0)	tf.floordiv()	x<=y	tf.less_equal()
x%y	tf.math.mod()	x>y	tf.greater()
x**y	tf.pow()	x>=y	tf.greater_equal()
-X	tf.neg()		
abs(x)	tf.abs()		



```
In [14]: a = tf. constant([0, 1, 2, 3])
         b = tf. constant([4, 5, 6, 7])
         a+b
Out[14]: <tf. Tensor: id=38, shape=(4,), dtype=int32, numpy=array([4, 6, 8, 10])>
In [15]: a-b
Out[15]: <tf. Tensor: id=39, shape=(4,), dtype=int32, numpy=array([-4, -4, -4, -4])>
In [16]: a*b
Out[16]: <tf.Tensor: id=40, shape=(4,), dtype=int32, numpy=array([0, 5, 12, 21])>
In [17]: a/b
Out[17]: <tf. Tensor: id=43, shape=(4,), dtype=float64, numpy=array([0.
                                                                               , 0.2
              , 0.33333333, 0.42857143])>
```



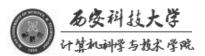
```
In [18]: a=tf. constant([0, 1, 2, 3])
          b = 2
          a%b
Out[18]: <tf. Tensor: id=46, shape=(4,), dtype=int32, numpy=array([0, 1, 0, 1])>
In [19]: a=tf. constant([[0, 1, 2, 3], [0, -1, -2, -3]])
Out[19]: <tf. Tensor: id=49, shape=(2, 4), dtype=int32, numpy=
          array [[ 0, 0, 1, 1],
                 [0, -1, -1, -2]
In [20]: a=tf. constant([0, 1, 2, 3])
          b=2
          a**h
Out[20]: <tf. Tensor: id=52, shape=(4,), dtype=int32, numpy=array([0, 1, 4, 9])>
```





■ 广播机制 (broadcasting)

```
In [21]: a = tf. constant([1, 2, 3])
          a
Out[21]: <tf. Tensor: id=53, shape=(3,), dtype=int32, numpy=array([1, 2, 3])>
In [22]: import numpy as np
          b=tf. constant (np. arange (12). reshape (4, 3))
Out[22]: <tf. Tensor: id=54, shape=(4, 3), dtype=int32, numpy=
          array([[
                   0,
                           5],
```



□ 一维张量+二维张量

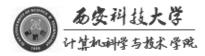
```
a
[1, 2, 3]
[ 0, 1, 2]
[ 3, 4, 5]
[ 7, 8, 9]
[ 10, 11, 12]]
```

两个张量最后一个维度的长度必须相等

□ 一维张量+ 三维张量

□ 数字+N维张量

当张量和一个数字进行运算时,会将这个数字值广播到张量的各个元素

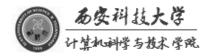




■ 张量和NumPy数组之间的相互转换

NumPy数组转化为张量: tf.constant(); tf.convert_to_tensor

张量转换为NumPy数组: Tensor.numpy()

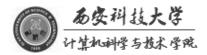


当张量和NumPy数组共同参与运算时:

执行TensorFlow操作,
 TensorFlow将自动的
 把NumPy数组转换为
 张量

■ 执行NumPy操作, NumPy将自动的张量 转换为NumPy数组

```
In [27]:
          nd = np. ones([2, 2])
          t = tf. multiply (nd, 36)
Out[27]:
          <tf.Tensor: id=62, shape=(2, 2), dtype=float64, numpy=</pre>
          array([[36., 36.],
                  [36., 36.]])>
In [28]:
          np. add (nd, t)
Out[28]:
         array([[37., 37.],
                  [37., 37.]]
```

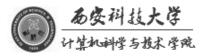




使用运算符操作

只要操作数中**有一个** Tensor对象,就把所有的操作数都转化为张量, 然后再进行运算。

```
In [29]:
         a=tf. constant([1, 2, 3])___
                                 张量
         b=np. array((4, 5, 6))
                 运算符重载为张量加法
Out [29]:
         <tf. Tensor: id=65, shape=(3,), dtype=int32, numpy=array([5, 7, 9])>
In [30]:
         a=np. array((4, 5, 6))
                                2个NumPy数组相加
         b=np. ones(3)
         a+b
Out[30]: array([5., 6., 7.])
In [31]:
         a=1
                   Python整数相加
         b=2
         a+b
Out[31]: 3
```



■ 张量乘法

□ 元素乘法: tf.multiply(), *运算符

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 4 & 12 \end{bmatrix}$$

□ 向量乘法: tf.matmul(), @运算符

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \times \begin{vmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{bmatrix} 10 & 13 \\ 28 & 40 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 13 \\ 28 & 40 \end{bmatrix}$$

```
import tensorflow as tf
         print("TensorFlow version:", tf. version )
         TensorFlow version: 2.0.0
In [2]: import numpy as np
                                                        定义张量a
        a = tf. constant (np. arange (6), shape= (2, 3))
Out[2]: <tf. Tensor: id=2, shape=(2, 3), dtype=int32, numpy=
        array([[0, 1, 2],
                [3, 4, 5]])
In [3]: b = tf. constant (np. arange (6), shape= (3, 2))
                                                              定义张量b
        <tf. Tensor: id=5, shape=(3, 2), dtype=int32, numpy=</pre>
        array([[0, 1],
                [2, 3],
                [4, 5]])>
```

■ 向量乘法

tf.matmul(), @运算符

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 13 \\ 28 & 40 \end{bmatrix}$$



- 多维向量乘法—— 三维张量×二维张量
 - □ 最后两维做向量乘法
 - □ 高维采用广播机制

```
In [6]: a = tf. random. normal([2, 3, 5])
                                          (3,5)×(5,4)→(3,4)<del>广播</del> (2,3,4)
         b = tf. random. normal([5, 4])
         tf. matmul (a, b)
Out[6]: <tf. Tensor: id=20, shape=(2, 3, 4), dtype=float32, numpy=
        array([[[ 0.13672318, -0.3784006 , -0.2231093 , -1.2624099 ],
                 [ 2. 196111 , 1. 2696512 , -4. 518535 , 3. 6673522 ],
                 \begin{bmatrix} 0.62343776, -2.2611952, 1.4009535, 0.22248505 \end{bmatrix}
                [[-0.7526479, -0.5529423, 2.294831, -2.4406729],
                 [-0. 12710276, 1. 5016136, 1. 420251, 0. 81047237],
                 [ 1.014232 , 1.3020521 , -4.1052294 , 1.0766649 ]]],
               dtype=float32)>
```

创建张量

```
In [7]: a = tf. constant (np. arange (12), shape=(2, 2, 3))
Out[7]: <tf. Tensor: id=23, shape=(2, 2, 3), dtype=int32, numpy=
        array [[[ 0, 1,
                 3,
                      4, 5]],
                [[ 6, 7, 8],
                 [ 9, 10, 11]]])
In [8]: b = tf. constant (np. arange (12), shape=(2, 3, 2))
Out[8]: <tf. Tensor: id=26, shape=(2, 3, 2), dtype=int32, numpy=
        array [[[ 0,
                      1],
                      3],
                      5]],
                [ 6,
                      7].
                 [8, 9],
                 [10, 11]])>
```

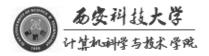


■ 多维向量乘法—— 三维张量×三维张量

张量乘法过程

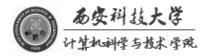
- □ 最后两维做向量乘法
- □ 高维采用广播机制

```
In [7]: a = tf. constant (np. arange (12), shape=(2, 2, 3))
Out[7]: <tf. Tensor: id=23, shape=(2, 2, 3), dtype=int32, numpy=
        array([[[ 0, 1, 2],
                           5]],
                                                         10
                                                             13
                [[ 6, 7, 8],
                                                         28 40
                 [ 9, 10, 11]])>
In [8]: b = tf. constant (np. arange (12), shape=(2, 3, 2))
Out[8]:
        <tf. Tensor: id=26, shape=(2, 3, 2), dtype=int32, numpy=</pre>
         array([[[ 0,
                                                        172
                                                             193
                                                        244 274
                      7],
                [[ 6,
                 [10, 11]])>
```



■ 多维向量乘法—— 三维张量×三维张量

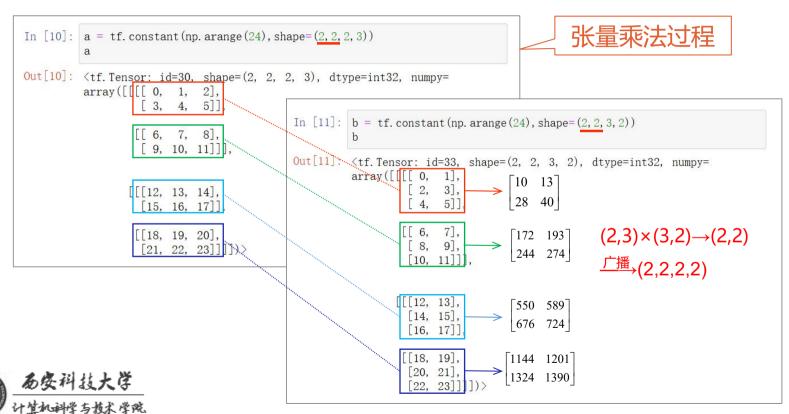
```
运行结果
```





■ 多维向量乘法—— 四维张量×四维张量

```
创建张量
In [10]: a = tf. constant (np. arange (24), shape=(2, 2, 2, 3))
Out[10]: <tf. Tensor: id=30, shape=(2, 2, 2, 3), dtype=int32, numpy=
         array([[[ 0, 1, 2],
                   [ 3, 4, 5]],
                                           In [11]: b = tf. constant (np. arange (24), shape= (2, 2, 3, 2))
                  [[ 6, 7, 8],
                  [ 9, 10, 11]]],
                                           Out[11]: <tf.Tensor: id=33, shape=(2, 2, 3, 2), dtype=int32, numpy=
                                                    array([[[ 0,
                 [[12, 13, 14],
                                                              2,
                  [15, 16, 17]],
                                                            [[6, 7],
                  [[18, 19, 20],
                                                              8, 9],
                   [21, 22, 23]]]))
                                                             [10, 11]]],
                                                           [[12, 13],
                                                             [14, 15],
                                                             [16, 17]],
                                                            [[18, 19],
                                                             [20, 21],
杨安科技大学
                                                             [22, 23]]])>
```



```
运行结果
                    In [12]: a@b
                    Out[12]: <tf. Tensor: id=34, shape=(2, 2, 2, 2), dtype=int32, numpy=
                                        10,
                                              13],
                             array([[[
                                        28,
                                              40]],
                                             193],
                                     [[ 172,
                                     244,
                                             274]]],
                                    [[[ 550, 589],
                                     676,
                                            724]],
                                     [[1144, 1201],
                                      [1324, 1390]]])>
```

□ 数据统计: 求张量在某个维度上、或者全局的统计值

函数	描述
tf.reduce_sum(input_tensor,axis)	求和
tf.reduce_mean(input_tensor,axis)	求平均值
tf.reduce_max(input_tensor,axis)	求最大值
tf.reduce_min(input_tensor,axis)	求最小值

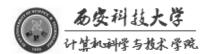
 $\begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$

3 4 5

全局最大值:5

每行的最大值: [2,5]

每列的最大值: [3,4,5]



```
In [13]: a = tf. constant([[1, 5, 3], [4, 2, 6]])
Out[13]: <tf. Tensor: 1id=35, shape=(2, 3), dtype=int32, numpy=
          array [[1, 5, 3],
                                        axis=0
In [14]: tf. reduce_sum(a, axis=0)
Out[14]: <tf. Tensor: id=37, shape=(3,), dtype=int32, numpy=array([5, 7, 9])>
                                        axis=1
In [15]: tf. reduce_sum(a, axis=1)
Out[15]: <tf. Tensor: id=39, shape=(2,), dtype=int32, numpy=array([ 9, 12])>
In [16]: tf. reduce sum(a)
Out[16]: <tf. Tensor: id=41, shape=(), dtype=int32, numpy=21>
```

■ 求均值函数——tf.reduce_mean()

$$\begin{bmatrix}
1 & 5 & 3 \\
4 & 2 & 6
\end{bmatrix}$$
2.5 3.5 4.5

```
张量元素的数据类型是int32,
                                                         因此求得的均值也是int32
In [17]: tf. reduce mean (a, axis=0)
Out[17]: <tf. Tensor: id=43, shape=(3,), dtype=int32, numpy=array([2, 3, 4])>
                                                   张量元素采用浮点数
In [18]: a = tf. constant([[1., 5., 3.], [4., 2., 6.]])
         tf. reduce mean (a, axis=0)
                                                                得到浮点数的均值
Out[18]: <tf. Tensor: id=46, shape=(3,), dtype=float32, numpy=array([2.5, 3.5, 4.5],
          dtype=float32)>
                                               为浮点数,再求均值
In [19]: a = tf. constant([[1, 5, 3], [4, 2, 6]])
         tf.reduce_mean(tf.cast(a, tf.float32), axis=0)
Out[19]: <tf. Tensor: id=50, shape=(3,), dtype=float32, numpy=array([2.5, 3.5, 4.5],
          dtype=float32)>
```

■ 求最大值、最小值函数——tf.max(), tf.min()

$$\begin{bmatrix}
1 & 5 & 3 \\
4 & 2 & 6 \\
4 & 5 & 6
\end{bmatrix}$$

```
In [20]: tf.reduce_max(a, axis=0)
Out[20]: <tf.Tensor: id=52, shape=(3,), dtype=int32, numpy=array([4, 5, 6])>
In [21]: tf.reduce_max(a, axis=1)
Out[21]: <tf.Tensor: id=54, shape=(2,), dtype=int32, numpy=array([5, 6])>
In [22]: tf.reduce_max(a)
Out[22]: <tf.Tensor: id=56, shape=(), dtype=int32, numpy=6>
```

■ 求最值的索引——tf.argmax(), tf.argmin()

$$\begin{bmatrix}
1 & 5 & 3 \\
4 & 2 & 6 \\
0 & 1 & 2
\end{bmatrix}$$

```
In [23]: a = tf.constant([[1,5,3], [4,2,6]])
    tf.argmax(a,axis=0)

Out[23]: <tf.Tensor: id=59, shape=(3,), dtype=int64, numpy=array([1, 0, 1], dtype=int64)>
In [24]: tf.argmax(a,axis=1)
Out[24]: <tf.Tensor: id=61, shape=(2,), dtype=int64, numpy=array([1, 2], dtype=int64)>
In [25]: tf.argmax(a)
Out[25]: <tf.Tensor: id=63, shape=(3,), dtype=int64, numpy=array([1, 0, 1], dtype=int64)>
```

没有指定axis参数时,默认axis=0

