Lecture - 6 - DC, 23-24

We know $G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-j\pi n f/W}$ — ①

& $G(f) = \frac{1}{2W} G_\delta(f)$; $\boxed{-W < f < W}$ — ②

from ① & ②

$$G(f) = \begin{cases} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-j\pi n f/W}, & -W < f < W \\ 0, & \text{elsewhere} \end{cases}$$ ③

$\{g(n/2W)\}$ are the samples of $g(t)$ with $f_s = 2W$

Therefore, if the sample values $g(n/2W)$ of a signal $g(t)$ are specified for 'all n', then FT $G(f)$ of the signal is uniquely deter-mined by using the DTFT in eq ③.

g(t) & G(f) are related through Inverse FT. Hence g(t) is itself uniquely determined by the sample values $\{g(n/2W)\}$ for $-\infty < n < \infty$

#

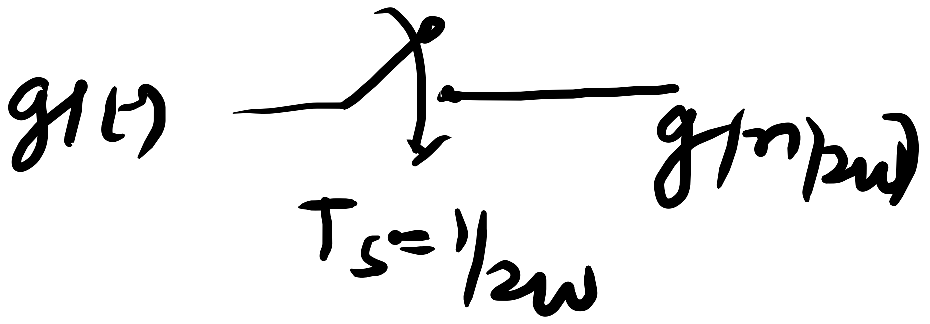
$$g(n/2W) \xrightarrow{\text{DTFT}} G(f) \xrightarrow{\text{IFT}} g(t)$$

(a, b) is a FT pair

$\Rightarrow a \xrightarrow{\text{FT}} b$

$b \xrightarrow{\text{IFT}} a$

$\boxed{\text{Seq } g(n/2W) \text{ has all the information contained in } g(t)}$



Recovery / Reconstruction :-

$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi ft}\, df$

$$g(t) = \int_{-W}^{W} G(f)\, e^{j2\pi ft}\, df \quad (4)$$

$$\frac{\sin(2\pi Wt - n\pi)}{2\pi Wt - n\pi} \leftarrow \text{C.E.} \int_{-W}^{W} e^{j2\pi f\left(t - \frac{n}{2W}\right)}\, df$$

$$\int_{-w}^{w} e^{j2\pi f\left(t-\frac{n}{2w}\right)} df = \left.\frac{e^{j2\pi f\left(t-\frac{n}{2w}\right)}}{j2\pi\left(t-\frac{n}{2w}\right)}\right|_{-w}^{w}$$

$e^{j\theta} - e^{-j\theta}$

$\cos\theta + j\sin\theta$

$-\cos\theta + j\sin\theta$

$$\frac{e^{j2\pi\left(wt-\frac{n}{2}\right)} - e^{-j2\pi\left(wt-\frac{n}{2}\right)}}{j2\pi\left(t-\frac{n}{2w}\right)} = \frac{\cancel{2j}\sin\left(2\pi\left(wt-\frac{n}{2}\right)\right)}{\cancel{2j}\left(\pi t - \frac{n\pi}{2w}\right)}$$

$$\boxed{2W\,\frac{\sin\left[2\pi W t - n\pi\right]}{\left[2\pi W t - n\pi\right]}} \quad -(5)$$

put $G(f)$ in terms of $g(n/2w)$ in eq (4)

$$g(t) = \int_{-W}^{W} \frac{1}{2w}\sum_{n} g\left(\frac{n}{2w}\right) e^{-j\pi n f/W} \times e^{j2\pi f t}\, df$$

rearranging,

$$g(t) = \sum_{n} g\left(\frac{n}{2w}\right)\frac{1}{2w}\int_{-w}^{W} e^{j2\pi f\left(t-\frac{n}{2w}\right)} df,$$ using eq (5) we have

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt-n) \quad ; \quad -\infty < t < \infty$$

$$\operatorname{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

This is called as the interpolation formula for reconstructing the original signal $g(t)$ from 'seq. of sample values' $\{g(\frac{n}{2W})\}$ with sinc function [$\operatorname{sinc}(2Wt)$] as the interpolation function

delay sinc → multiply
add

Sampling theorem:-

For strictly BL signals of finite energy,

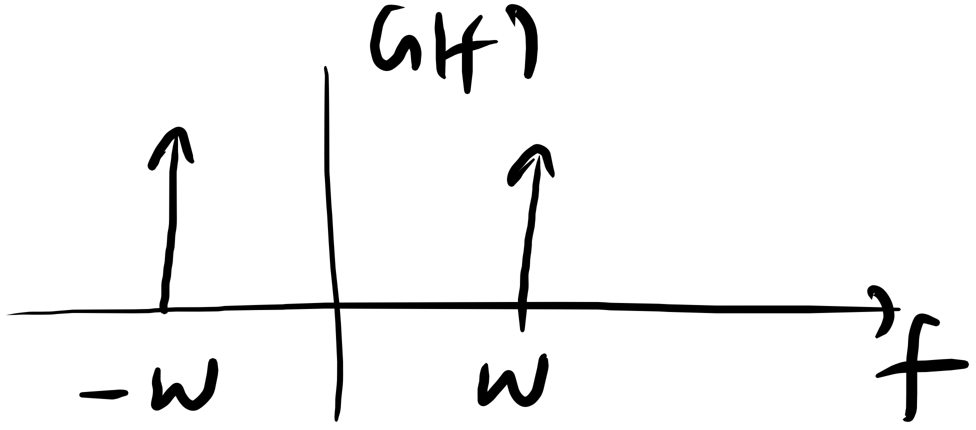
1. such a func^n / signal is completely described by values separated in time by $1/2W$ seconds.

or 2. signal can be recovered from knowledge of its samples taken at the rate of $2W$ samples/sec.

Nyquist rate :- $2W$ samples/sec
" Interval :- $1/2W$ (in sec)

→ for BL signals of BW → W Hz

#2 $f_s = 2W$ or $\boxed{f_s > 2W}$ or $f_s \geq 2W$?

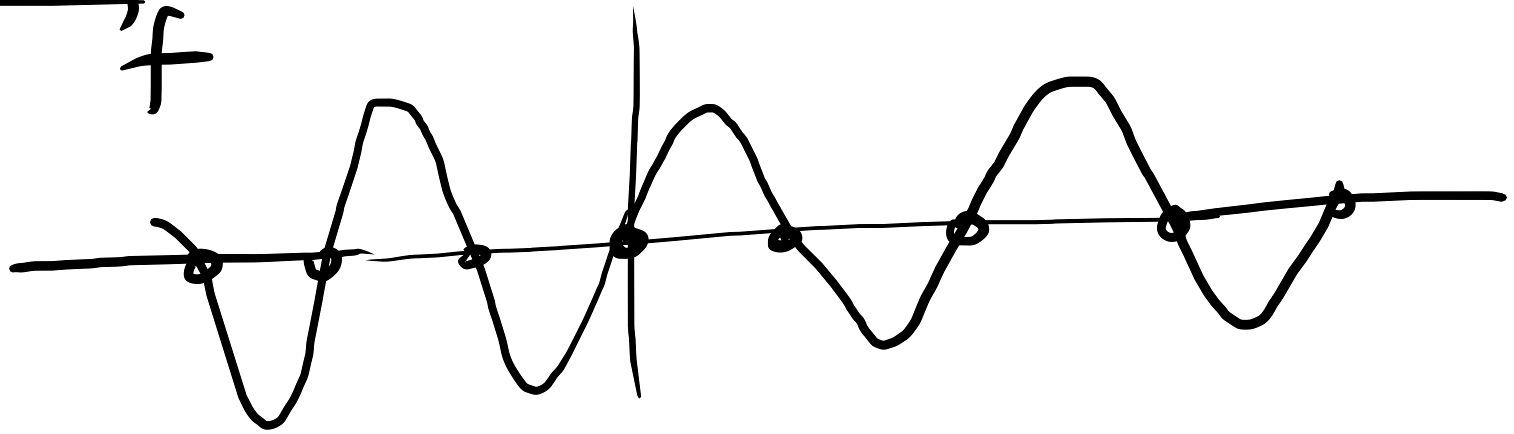


$G(f) = 0, \quad |f| \geq W$

$G(f) = 0$ for $f = W$
& $f = -W$

sin/cos

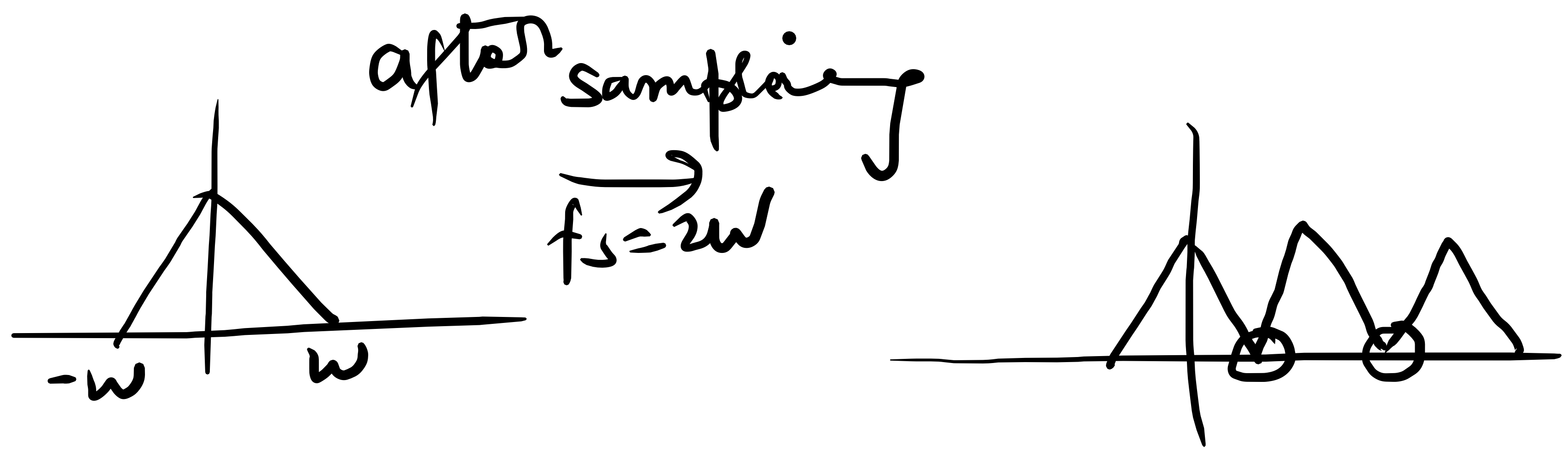
$W + \delta$



$g(t) = \sin(2\pi W t)$

$\{g(n/2W)\}$, $\sin\left(2\pi \frac{W n}{2W}\right) = \sin(n\pi) = 0$

Possibility of $f_s = 2W$: If the spectrum $G(f)$ has no impulse at the highest freq. W, then the overlap is still zero as long as the sampling rate $\geq$ (*) Nyquist rate



On the other hand, if $G(f)$ contains an impulse at the highest freq $\pm W$, then the equality (*) must be removed or else overlap will occur. $f_s > 2W$ Hz