Roll No. 202001402

Winter Semester

Academic Year 2022-23

Modelling and Simulation (CS302)

Second In-Semester Examination

Dhirubhai Ambani Institute of Information and Communication Technology, Gandhinagar

Time: **2 Hours**

Total Marks: **20**

Note: All questions are compulsory. Provide answers with all relevant steps shown clearly. Answer all the sub-parts of a question together. Marks for each question are indicated next to it. All terms and symbols carry their standard textbook meaning. Use of scientific calculators is allowed.

1. With relevant mathematical formulae and plots, write brief notes on: A. Clustering coefficient in small-world networks. B. Degree distributions in random networks and scale-free networks. **[1+1=2]**

2. If $x(t)$ is the war potential of a nation and $y(t)$ is the war potential of its enemy nation, then Richardson's Theory of Conflict gives $\dot{x} = ky + g - \alpha x$ and $\dot{y} = lx + h - \beta y$ (in which $k, l, g, h, \alpha, \beta > 0$).

   A. Write the mathematical conditions for an arms race without any history of animosity. B. Derive the solution of $y \equiv y(x)$ and plot y versus x with clear labels. C. Derive the solutions of both $x \equiv x(t)$ and $y \equiv y(t)$. Show that when $t \longrightarrow \infty$, both $x(t), y(t) \longrightarrow \infty$. **[1+1+2=4]**

3. A nation can arm itself in proportion to the *difference* between its own armaments and that of its enemy nation. The equations of Richardson's Theory of Conflict are then modified as $\dot{x} = k(y - x) + g - \alpha x$ and $\dot{y} = l(x - y) + h - \beta y$ (in which $k, l, g, h, \alpha, \beta > 0$).

   A. Write the mathematical conditions for the absence of grievance. Then give the equilibrium solutions of x and y for mutual disarmament. • B. If initially both $x(0) \neq 0$ and $y(0) \neq 0$, derive a mathematical condition for the equilibrium solutions to be stable. **[1+1=2]**

4. A Red Army and a Blue Army are engaged in conventional-conventional combat. The combat effectiveness of the Red Army is r and of the Blue Army is b. Assume $r = b = 1$. For the Red Army the number of troops is $R(t)$ and for the Blue Army it is $B(t)$. Initially, $R(0) = 10000$ and $B(0) = 8000$. Use Lanchester's square law for the following:

   A. Giving a relevant numerical value, predict which army will win the battle if both fight with full force.
   B. Divide the battle into two stages. In the first stage the entire Blue Army fights half of the Red Army soldiers. Giving a relevant numerical value, predict who will win the first stage of the battle.
   • C. Calculate the number of soldiers of the victorious army who survive at the end of the first stage (when the soldiers of the defeated army are dead). By hand plot the graph for the victorious army, with R along the horizontal axis and B along the vertical axis. Provide clear labels in the plot. **[1+1+2=4]**

5. For a conventional force the number of troops is $x(t)$ and for a guerilla force the troops number is $y(t)$. Their combat is modelled by the coupled equations $\dot{x} = -ay$ and $\dot{y} = -by - cxy$ (in which $a, b, c > 0$) according to a modified system of Lanchester's equations.

   A. By integration derive $y \equiv y(x)$. B. Plot y versus x with clear labels. **[1+1=2]**

6. Use proper mathematical and practical arguments to carry out the following:

   A. Write the coupled dynamical equations for competition between two similar species with populations $x(t)$ and $y(t)$, according to the Principle of Competitive Exclusion. B. Write the coupled dynamical equations for the co-existence of a prey species and a predator species with populations $x(t)$ and $y(t)$, respectively, according to Volterra's predator-prey model. • C. Find the optimal equilibrium solutions for both the cases above and compare them. **[1+1+2=4]**

7. In the Threshold Theorem of Epidemiology, a population, through which an infection spreads, is divided into three classes — the infected class $x(t)$, the susceptible class $y(t)$, and the recovered class $z(t)$. For $x(t)$ and $y(t)$, the coupled dynamics is given by $\dot{x} = Axy - Bx$ and $\dot{y} = -Axy$ (in which $A, B > 0$).

   • A. For $A = 2.18 \times 10^{-3}\,\text{day}^{-1}$ and $B = 0.44\,\text{day}^{-1}$, state with mathematical reasons if an epidemic should break out in a population of 762 members. • B. For recurring epidemics, the $\dot{y}$ equation is modified as $\dot{y} = -Axy + C$ (in which $C > 0$). Find the finite equilibrium values of both x and y. **[1+1=2]**