NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2025/26 Semester 2 MH4110 Partial Differential Equations Tutorial 2, 29 January

Problem 1 (Ex. 1 on Page 9) Solve the first-order equation $2u_t + 3u_x = 0$ with the auxiliary condition $u = \sin x$ when $t = 0$.

Problem 2 Solve $3u_x - 5u_y = 0$ satisfying the condition $u(0, y) = \cos y$.

Problem 3 (Ex. 2 on Page 9) Solve the equation $3u_y + u_{xy} = 0$.

Problem 4 (Modified from Ex. 9 on Page 10) Use both the geometric method and the coordinate method to solve the equation

$$u_x + u_y = 1.$$

Problem 5 Solve $au_x + bu_y = c$ using the coordinate method, where a, b, c are constants and $a \neq 0$.

Problem 6 Solve the equation $yu_x + xu_y = 0$ with the condition $u(0, y) = y$. In which region of the xy plane is the solution uniquely determined?

Problem 7 Find the general solution of

$$u_x + e^y\, u_y = 1.$$

Problem 8 (Ex. 6 on Page 10) Solve the equation

$$\sqrt{1 - x^2}\, u_x + u_y = 0, \quad u(0, y) = y.$$

Problem 9 Find the general solution of the equation

$$\text{(a)} \qquad -2u_x + u_y + 3u = e^{x+y};$$

$$\text{(b)} \qquad xu_x - yu_y + y^2 u = y^2, \qquad x, y \neq 0.$$

Problem 10 (Ex. 13 on Page 10) Use the coordinate method to solve the equation

$$u_x + 2u_y + (2x - y)u = 2x^2 + 3xy - 2y^2.$$

Problem 11 Find the solution to general constant coefficient linear first-order equations

$$au_x + bu_y + cu = g(x, y).$$

Problem 12 (Ex. 1 on Page 19) Carefully derive the equation of a string in a medium in which the resistance is proportional to the velocity.

Problem 13 (Ex. 6 on Page 19) Consider heat flow in a long circular cylinder where the temperature depends only on t and on the distance r to the axis of the cylinder. Here $r = \sqrt{x^2 + y^2}$ is the cylindrical coordinate. From the three-dimensional heat equation derive the equation $u_t = k(u_{rr} + u_r/r)$.

Problem 14 Solve the following first order PDE and find where the solution is defined in the x - y plane.

$$u_x + xyu_y = 0, \qquad u(x, -1) = \frac{1}{2}x^2.$$

Problem 15 Solve the following first order PDE

$$x^2u_x + xyu_y = u.$$