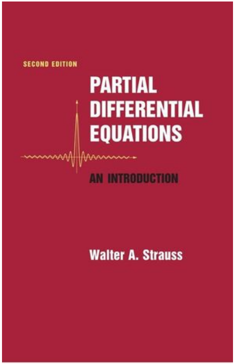
MH4110 Partial Differential Equations

Course Overview

Welcome to MH4110!

Course Information

- Textbook: Partial Differential Equations: An Introduction, *Walter Strauss*, 2nd edition.



- Synopsis: Chapters 1-8 (mainly starred sections).

Course Information

- Instructors: Tong Ping
- Time:
  - Lecture: Mondays 2:30 pm–4:20 pm, SPMS-LT5,
    Thursdays 1:30 pm–2:20 pm, SPMS-LT5.
  - Tutorial: Thursdays 2:30 pm–3:20 pm, SPMS-LT5, Weeks 2-13.
- Office hours
  - Office hours are held on Thursdays from 3:30–5:30 pm at SPMS-MAS 04-17 (Tel: 6513 7457). Zoom or in-person meetings can be arranged by email at tongping@ntu.edu.sg.

Course Evaluation

- 10% Group Presentation: Students should form groups of 3–4 members and prepare a presentation on one of the following topics:
  - Option A: A rigorous and step-by-step derivation of Maxwell's equations.
  - Option B: Paper review – independently identify a scientific paper that applies machine learning to solve a partial differential equation (PDE). The group should explain the problem setting, the methodology, and how machine learning is used to solve the PDE.

  Each group will present during the tutorial session on an assigned Thursday. The presentation duration must not exceed 30 minutes.
- 20% Midterm Test: 9 March 2026 (Monday, Week 8), 90 minutes. Coverage includes lectures up to Week 7 (approximately Chapters 1–4). One double-sided A4 cheatsheet (2 pages) is permitted.
- 10% Quiz: 6 April 2026 (Monday, Week 12), 30 minutes. Open-book.
- 60% Final exam: Chapters 1-7, strictly open (1 double-sided A4 cheatsheat, 2 pages).

About MH4110: Partial Differential Equations

Goals

- Formulate PDEs
- Solve PDEs analytically or numerically
- Analyse the solutions

Prerequisites:

- Calculus, Linear Algebra, Ordinary Differential Equations (ODEs).

About MH4110: Partial Differential Equations

Observations and useful tips from ODEs

- DEs arise from mathematical modeling of real problems, and most of the models are PDEs.
- Only special types of DEs can be solved analytically, so the solutions of most DEs have to resort to numerical means.
- For solvable DEs, it is essential to identify the type and use the right technique to resolve the underlying problem.
  Find the right KEY to open the DOOR!
- Solving PDEs is much more complicated than solving ODEs.
- ODEs can be viewed as a special case of PDEs.

Why MH4110: Partial Differential Equations?

PDEs is a subject at the forefront of research in modern science.

Wide applications

PDEs can be used to describe a wide variety of phenomena such as sound, heat, electrodynamics, fluid dynamics, elasticity, quantum mechanics, or earthquakes.

- Heat equation: $u_t - \alpha\,(u_{xx} + u_{yy} + u_{zz}) = f(t, x, y, z)$

  Natural Convection in a room with a heater

  https://www.youtube.com/watch?v=Zr8WbV8F_5g

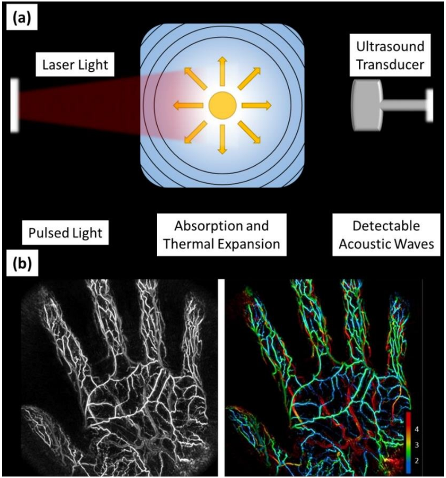
- Wave equation: $u_{tt} - c^2\,(u_{xx} + u_{yy} + u_{zz}) = f(t, x, y, z)$

  Seismic Waves from 2011 Tohoku Japan earthquake

  https://www.youtube.com/watch?v=ZCAOMMjjoN4

Why MH4110: Partial Differential Equations?

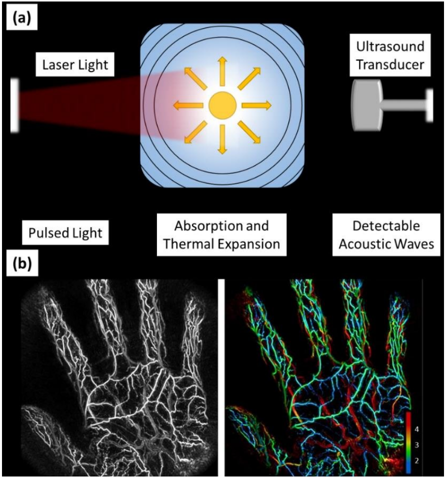
- Biomedical Imaging (Photoacoustic Imaging)



A schematic of photoacoustic imaging with two example images. (a) Shown is how pulsed light is absorbed by a biological tissue, which then causes thermal expansion and the emission of acoustic waves that can be detected by an ultrasound transducer. (b) A maximum intensity projected photoacoustic image of the palm of an individual's hand alongside the same image with vessel depth represented through color coding. Numbering on the bottom right of the figure represents vessel depth in mm. Permission for reuse granted under the Creative Commons Attribution License.

Deegan and Wang, Physics in Medicine and Biology, 2019.

- Biomedical Imaging (Photoacoustic Imaging)



- The general photoacoustic equation

$$\left(\nabla^2 - \frac{1}{v_s^2}\frac{\partial^2}{\partial t^2}\right) p(\mathbf{r},\ \mathbf{t}) = -\frac{\beta}{\kappa \mathbf{v_s^2}}\frac{\partial^2 \mathbf{T}(\mathbf{r},\ \mathbf{t})}{\partial \mathbf{t^2}},$$

where $p(\mathbf{r},\ \mathbf{t})$ denotes the acoustic pressure at location $\mathbf{r}$ and time t and T denotes the temperature rise. The left-hand side of this equation describes the wave propagation, whereas the right-hand side represents the source term.

- The thermal equation

$$\rho C_v \frac{\partial T(\mathbf{r},\ \mathbf{t})}{\partial t} = H(\mathbf{r},\ \mathbf{t}).$$

Here $H(\mathbf{r},\ \mathbf{t})$ is the heating function defined as the thermal energy converted per unit volume and per unit time.

Wang, L. V. and Wu, H.-i.; Biomedical Optics: Principles and Imaging. (Wiley-Interscience, 2007)

Why MH4110: Partial Differential Equations?

Finance: Pricing of Financial Derivatives

A call option is a contract giving the owner the right but not the obligation to buy a specified amount of an underlying security (e.g., stock) at a specified price (strike) with a specified time.

The price $c(t, S(t))$ of a call option on a stock $S(t)$ with strike K at expiration time T satisfies the **Black-Scholes-Merton partial differential equation**

$$c_t(t,x) + rxc_x(t,x) + \frac{1}{2}\sigma^2 x^2 c_{xx}(t,x) = rc(t,x),\ t \in [0, T),\ x \geq 0,$$

which satisfies the **terminal condition**

$$c(T,x) = (x-K)^+.$$

and the following **boundary conditions**

$$c(t,0) = 0, \quad \lim_{x\to\infty} \left(c(t,x) - x + Ke^{-r(T-t)} \right) = 0, \qquad t \in [0, T).$$

$S(t)$ is the price of the underlying stock. σ denotes the volatility of the stock price. r is the interest rate of the money market.