NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2025/26 Semester 2 MH4110 Partial Differential Equations Tutorial 1, 22 January

Problem 1 Solve the equations

(a)
$$2\frac{dy}{dx} + (\tan x)y = \frac{(4x+5)^2}{\cos x}y^3$$

(b)
$$\frac{dy}{dx} + \frac{2}{x}y = (-x^2\cos x)y^2$$

Problem 2 (Ex. 2 on Page 5) Which of the following operators are linear?

(a) $\mathscr{L}u = u_x + xu_y$

(b) $\mathscr{L}u = u_x + uu_y$

(c) $\mathscr{L}u = u_x + u_y^2$

(d) $\mathscr{L}u = u_x + u_y + 1$

(e) $\mathscr{L}u = \sqrt{1+x^2}(\cos y)u_x + u_{yxy} - [\arctan(x/y)]u$

Problem 3 Prove that the first-order equation is linear.

$$a(x,y)u_x(x,y) + b(x,y)u_y(x,y) + c(x,y)u(x,y) = f(x,y)$$

Problem 4 (Ex. 3 on Page 5) For each of the following equations, state the order and whether it is nonlinear, linear inhomogeneous, or linear homogeneous; provide reasons.

(a) $u_t - u_{xx} + 1 = 0$

(b) $u_t - u_{xx} + xu = 0$

(c) $u_t - u_{xxt} + uu_x = 0$

(d) $u_{tt} - u_{xx} + x^2 = 0$

(e) $iu_t - u_{xx} + u/x = 0$

(f) $u_x\left(1+u_x^2\right)^{-1/2} + u_y\left(1+u_y^2\right)^{-1/2} = 0$

(g) $u_x + e^y u_y = 0$

(h) $u_t + u_{xxxx} + \sqrt{1+u} = 0$

Problem 5 (Ex. 4 on Page 6) Show that the difference of two solutions of an inhomogeneous linear equation $\mathscr{L}u = g$ with the same g is a solution of the homogeneous equation $\mathscr{L}u = 0$.

Problem 6 (Ex. 12 on Page 6) Verify by direct substitution that

$$u_n(x, y) = \sin(nx)\sinh(ny)$$

is a solution of $u_{xx} + u_{yy} = 0$ for every $n > 0$.