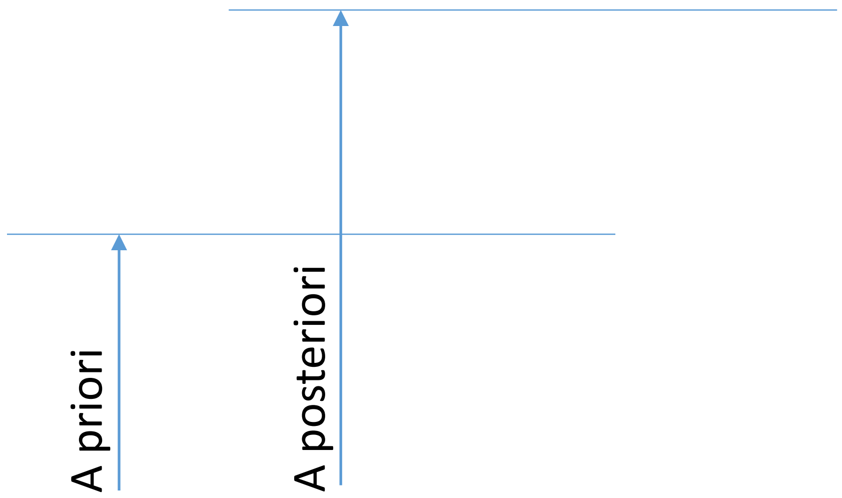
Assignments: wrapping things up

- Class Monday in 820 WBB, first half of spring ends Tuesday
- Online/take home final for undergrads/grads will be posted soon
- Undergrads could use MLIB next Wednesday IF you don't have a class in this time slot during the 2nd half
- It's ok to discuss coding issues of exam with others; not the answers to any questions, work independently on those

Setting the statistical bar

- a priori ("from the earlier") vs. a posteriori ("from the later")
  - a priori- can be verified independently: any conclusions drawn do not depend on a prior analysis of empirical evidence
  - a posteriori- involves some level of subjective exposure to the data from which a conclusion will be drawn



ESSAY

AWARENESS OF BOTH TYPE 1 AND 2 ERRORS IN CLIMATE SCIENCE AND ASSESSMENT

BY WILLIAM R. L. ANDEREGG, ELIZABETH S. CALLAWAY, MAXWELL T. BOYKOFF, GARY YOHE, AND TERRY L. ROOT

Climate science and assessment sometimes focus too strongly on avoiding false-positive errors, when false-negative errors may be just as important.

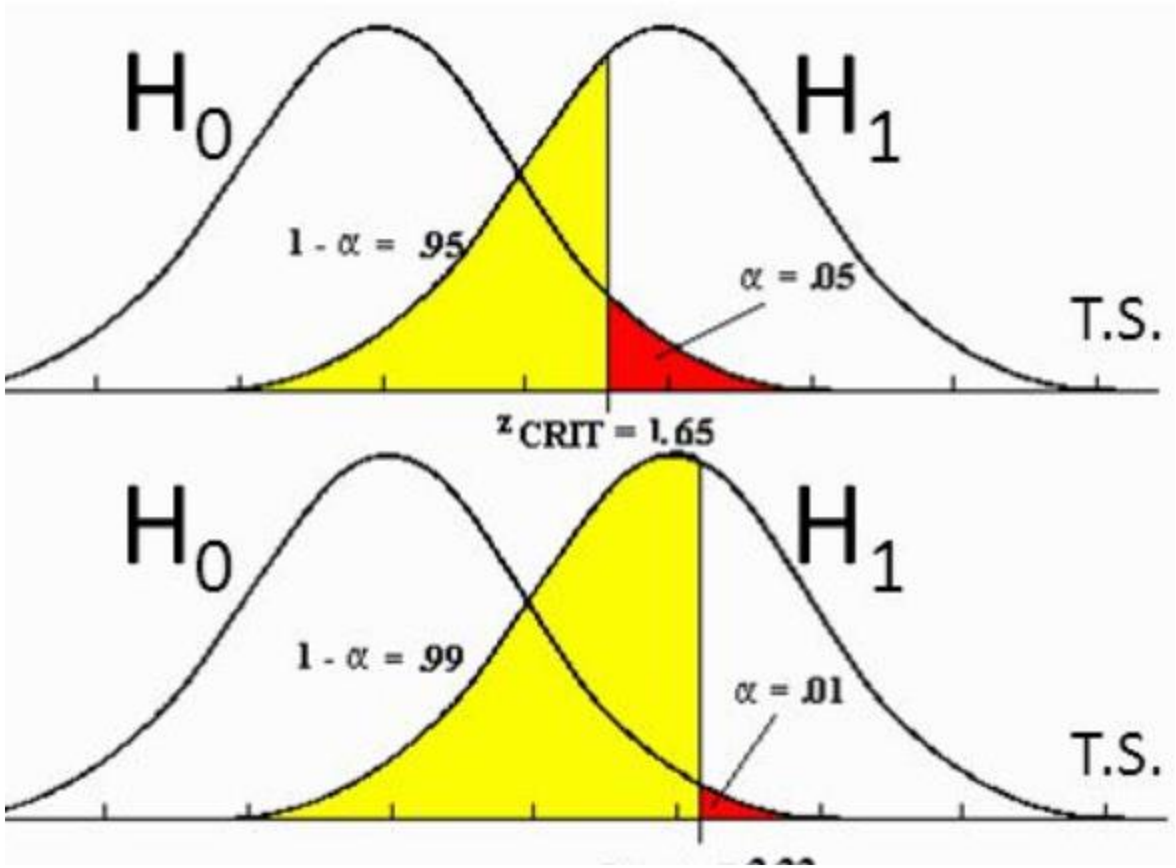
Situation / Decision	Null hypothesis is TRUE	Null hypothesis is FALSE
Reject null hypothesis	Type I Error (False positive)	Correct outcome! (True positive)
Fail to reject null hypothesis	Correct outcome! (True negative)	Type II Error (False negative)

FIG. 1. Graphical representation of type 1 and type 2 errors.

Type 1 errors are a false positive: a researcher states that a specific relationship exists when in fact it does not. Type 1 errors are typically avoided in hypothesis testing by determining whether a p value, roughly the probability that a result could be obtained by chance alone, falls below a predetermined threshold.

Type 2 errors are the reverse: a null hypothesis would not be rejected despite being false- A scientist says no relationship exists when, in fact, one exists

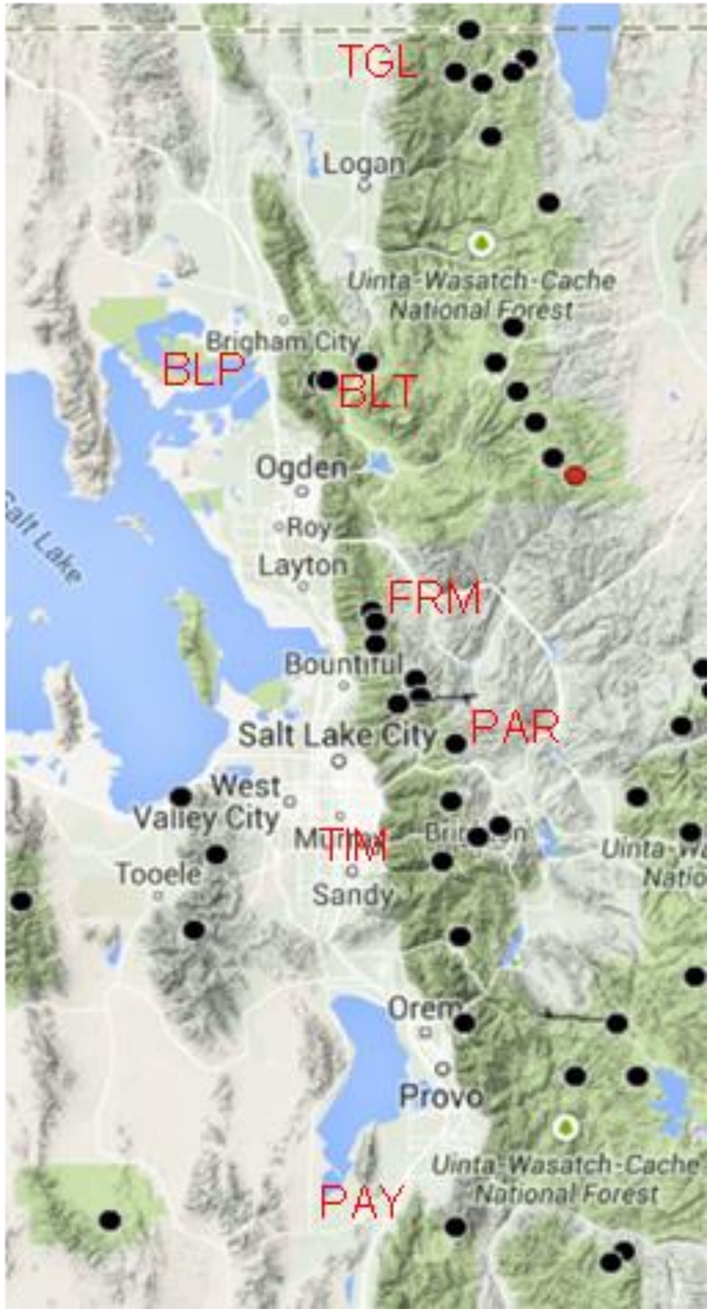
Type I vs Type II errors



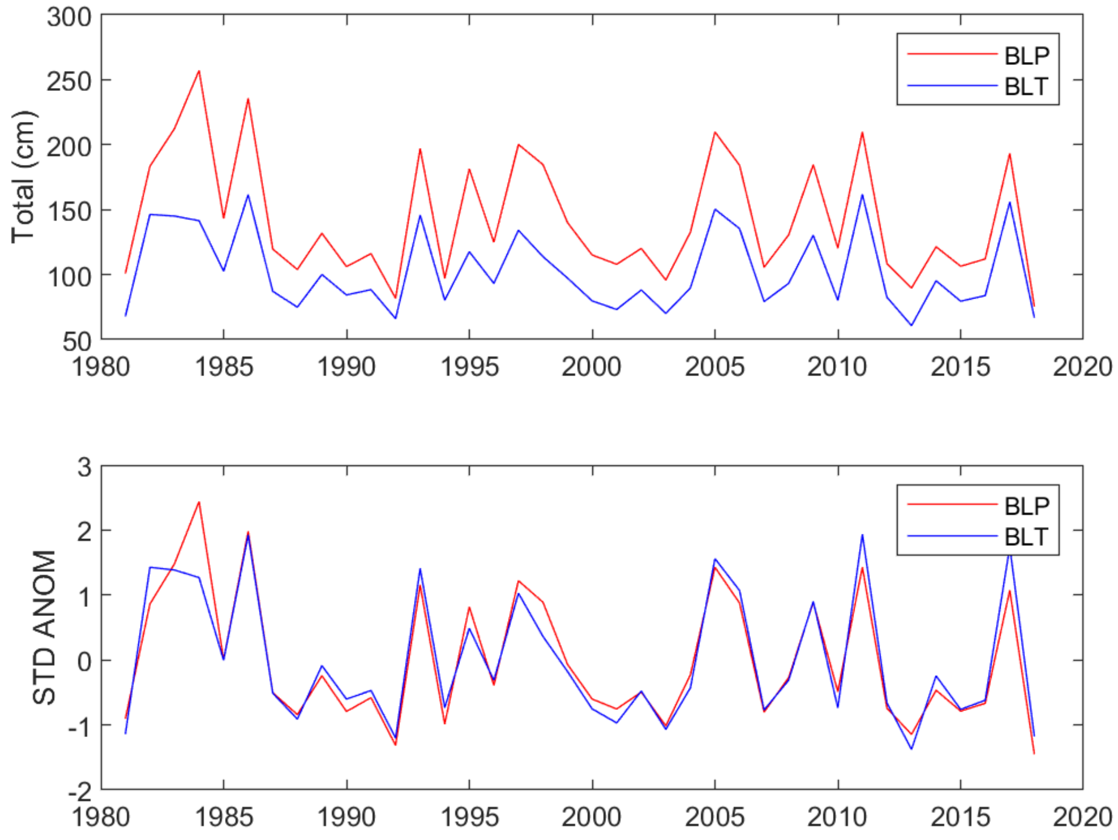
- H_0 - the null distribution
- α : level of the test (or level of risk for a Type I error)
- *Reject the null hypothesis as improbable if test statistic is in the red portion (accepting 5% or 1% risk of Type 1 error)*

- H_1 – (H_A) distribution of the alternative distribution (usually don't know this as many alternative hypotheses)
- β = 1 – α : power of the test; level of risk for a Type II error
- To reduce the risk of a Type II error need to reduce the yellow area, but that increases the risk of a Type I error
- *Want the distribution of an alternative hypothesis to be far from that of the null distribution to have both small Type I and Type II errors*

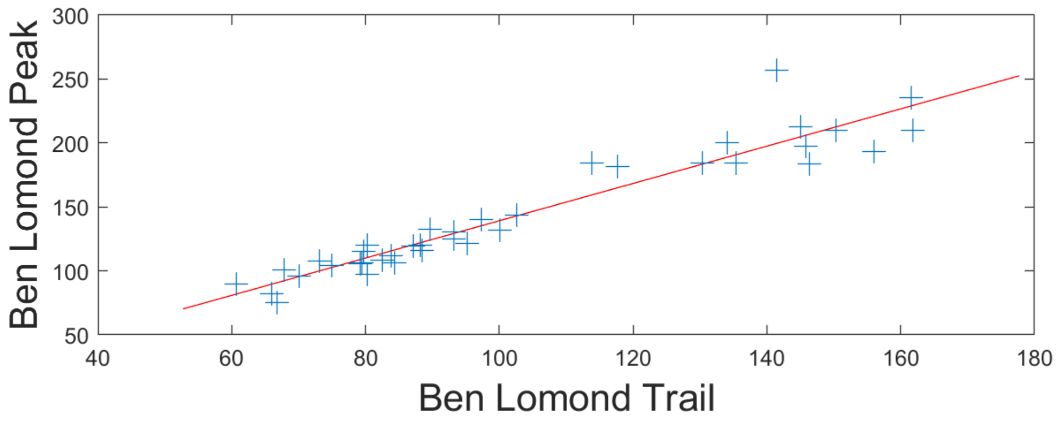
SNOTEL Sites

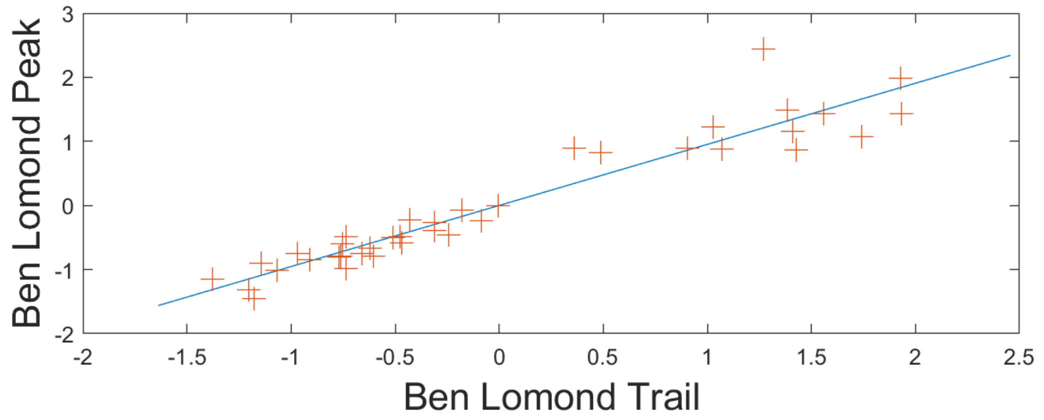


Accum Precip



Scatter plots





Estimating Values of One Variable From Another

- X- Ben Lomond Trail
- Y- Ben Lomond Peak
- Want to estimate Peak from Trail
- Use pairs of observations from sample
- Need to determine coefficient b or r
- b- slope of linear estimate
- r- linear correlation

$$\hat{y}_i = \bar{y} + b(\hat{x}_i - \bar{x})$$

$$\hat{y}^*{}_i = r\hat{x}^*{}_i$$

Definitions

- Estimate $\hat{y}_i = \bar{y} + b(\hat{x}_i - \bar{x})$
- Error of estimate $e_i = y_i' - \hat{y}_i'$
- Want $\sum_{i=1}^{n} e_i^2$ to be a minimum
- Need to find the value of b that minimizes that sum

$$\frac{\partial}{\partial b}\sum_{i=1}^{n} e_i^2 = 0$$

$$b = \overline{x_i' y_i'} / \overline{(x_i')^2} = \overline{x_i' y_i'} / s_x^2$$

- The value of b that minimizes the total error in the sample

Covariance

- Relates how departures of x and y from respective means are related
- Units are the product of the units of the two variables x and y
- Large and positive if sample tendency for:
  - large + anomalies of x occurring when large + anomalies of y AND
  - large - anomalies of x occurring when large - anomalies of y
- Large and negative if sample tendency for:
  - large + anomalies of x occurring when large - anomalies of y AND
  - large - anomalies of x occurring when large + anomalies of y
- Near zero when tendency for cancellation
  - large + anomalies of x occurring when both large – and + anomalies of y AND
  - large - anomalies of x occurring when both large – and + anomalies of y

Linear Correlation

$$r^2 = b^2 s_x{}^2 / s_y{}^2 = (\overline{x_i' y_i'})^2 / (s_x{}^2 s_y{}^2) \qquad r = (\overline{x_i' y_i'}) / \sqrt{\overline{x_i'^2}\;\overline{y_i'^2}}$$

$$x_i{}^* = x_i' / s_x, y_i{}^* = y_i' / s_y, r = (\overline{x_i^* y_i^*})$$

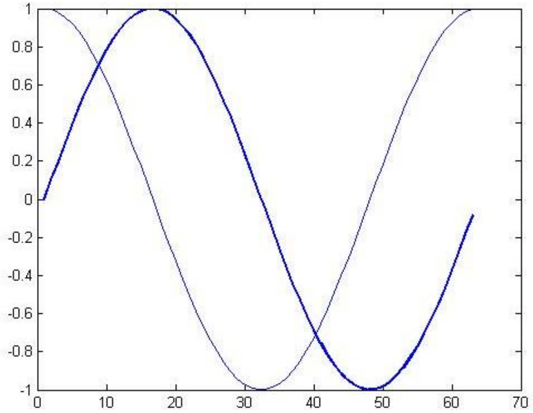
$$1 = r^2 + \frac{\overline{e_i^2}}{s_y{}^2}$$

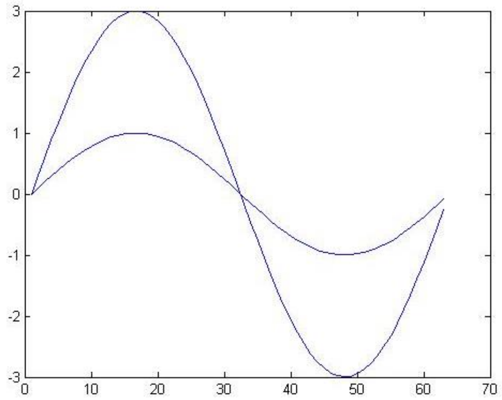
y's total sample variance = fraction of variance estimated by x + fraction of variance NOT explained by x

- Dimensionless number relates how departures of x and y from respective means are related taking into account variance of x and y
- r = 1. Linear fits estimates ALL of the variability of the y anomalies and x and y vary identically
- r = -1 perfect linear estimation but when x is positive, y is negative and vice versa
- r = 0. linear fit explains none of the variability of the y anomalies in the sample. Best estimate of y is the mean value

Stop and think before blindly computing correlations

- tendency to use correlation coefficients of 0.5 - 0.6 to indicate “useful” association.
  – 75%-64% of the total variance is NOT explained by a linear relationship if the correlation is in that range
- linear correlations can be made large by leaving in signals that may be irrelevant to the analysis. Annual and diurnal cyles may need to be removed
- large linear correlations may occur simply at random, especially if we try to correlate one variate with many, many others
- relationships in the data that are inherently nonlinear will not be handled well
- when two time series are in quadrature with one another then the linear correlation is 0
- Linear correlation provides no information on the relative amplitudes of two time series





Getting things mucked up

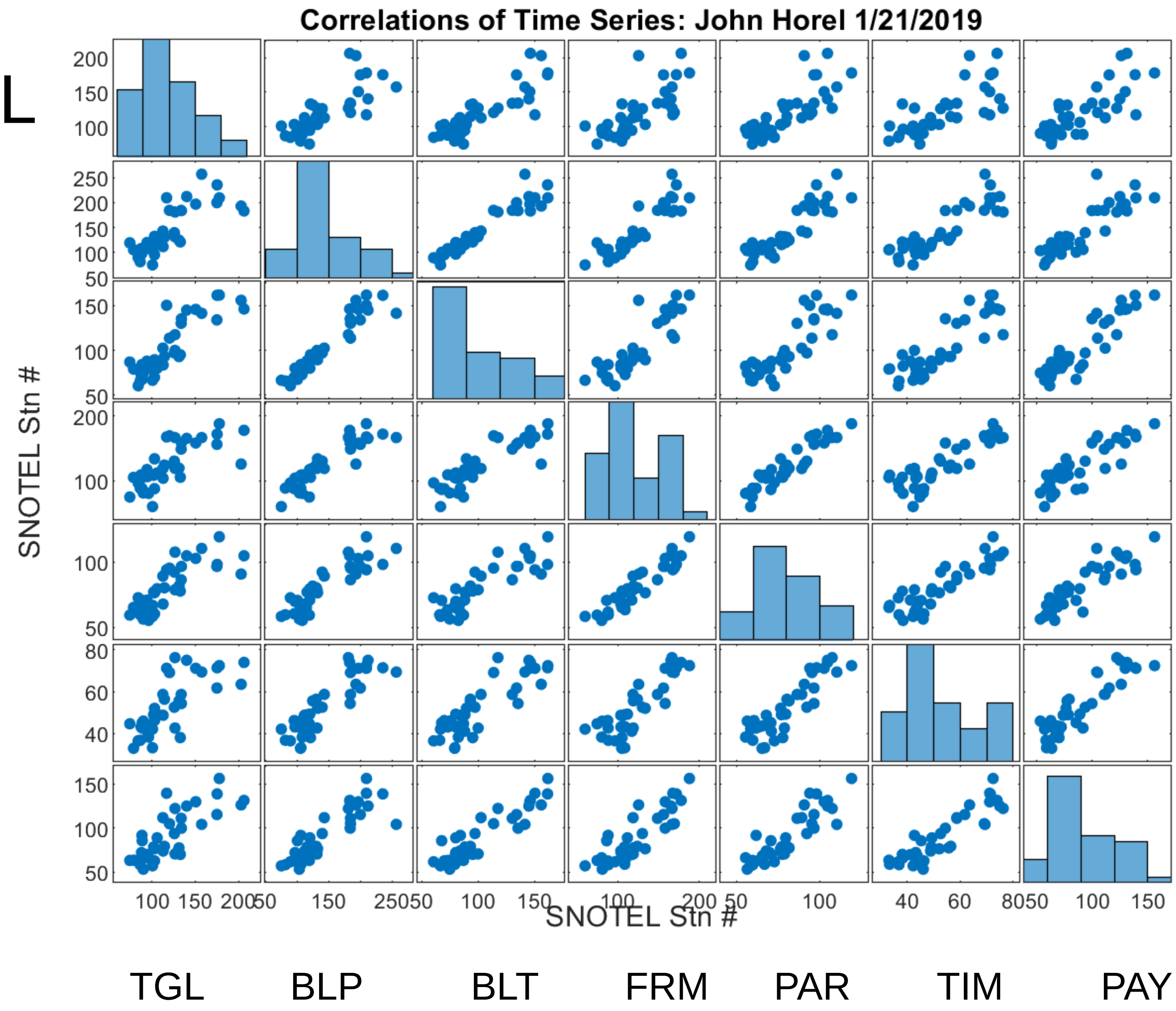
- Incorrectly assuming causation from correlation
  - Having it backwards: wet ground causes rain
  - Something else is the underlying cause: decreased arctic ice will cause temperatures in Utah to increase (global warming?)

Multivariate Linear Correlations

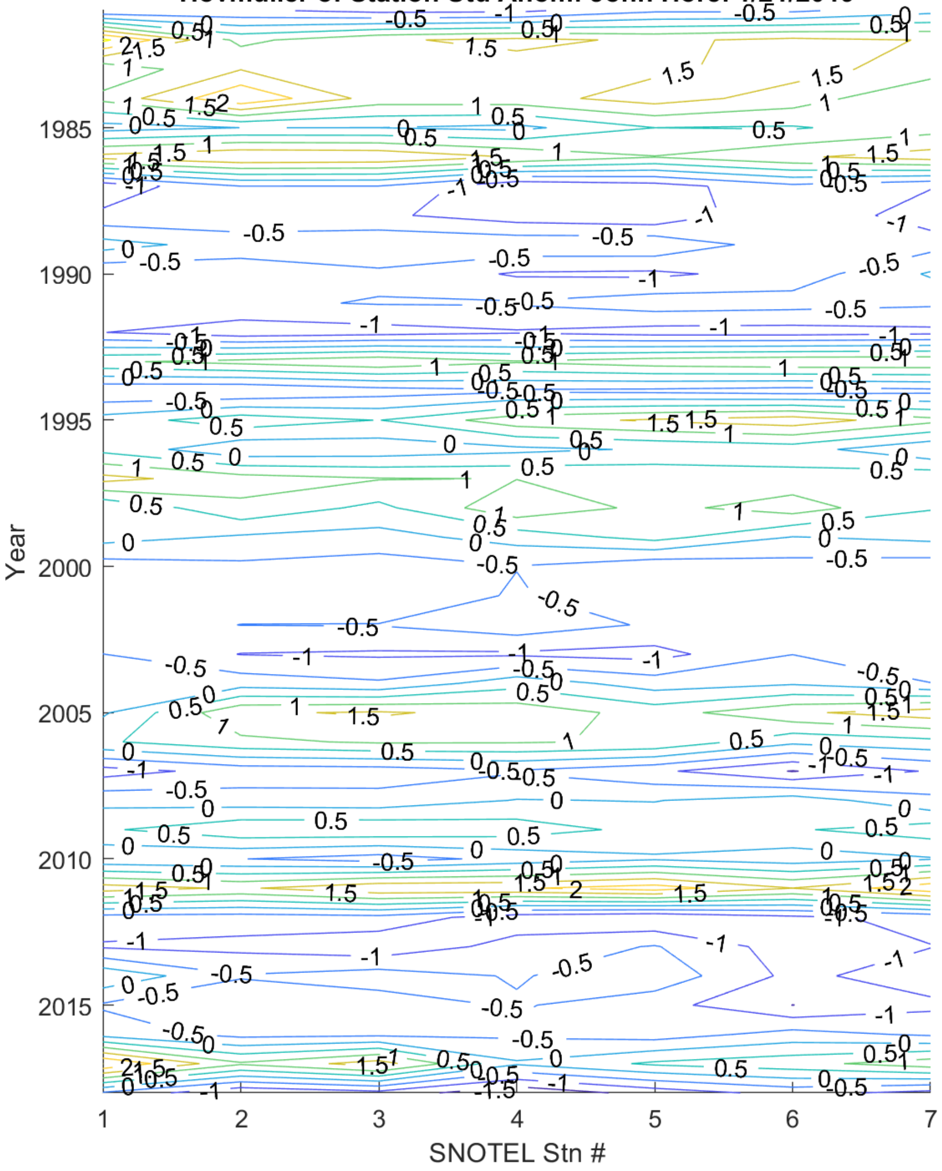
$$\vec{X}* = \begin{bmatrix} x*_{11} & x*_{12} & \ldots & x*_{17} \\ x*_{21} & x*_{22} & \ldots & x*_{27} \\ \ldots & \ldots & \ldots & \ldots \\ x*_{n1} & x*_{n2} & \ldots & x*_{n7} \end{bmatrix}$$

- 7 stations and n=38 years
- Standardized anomalies

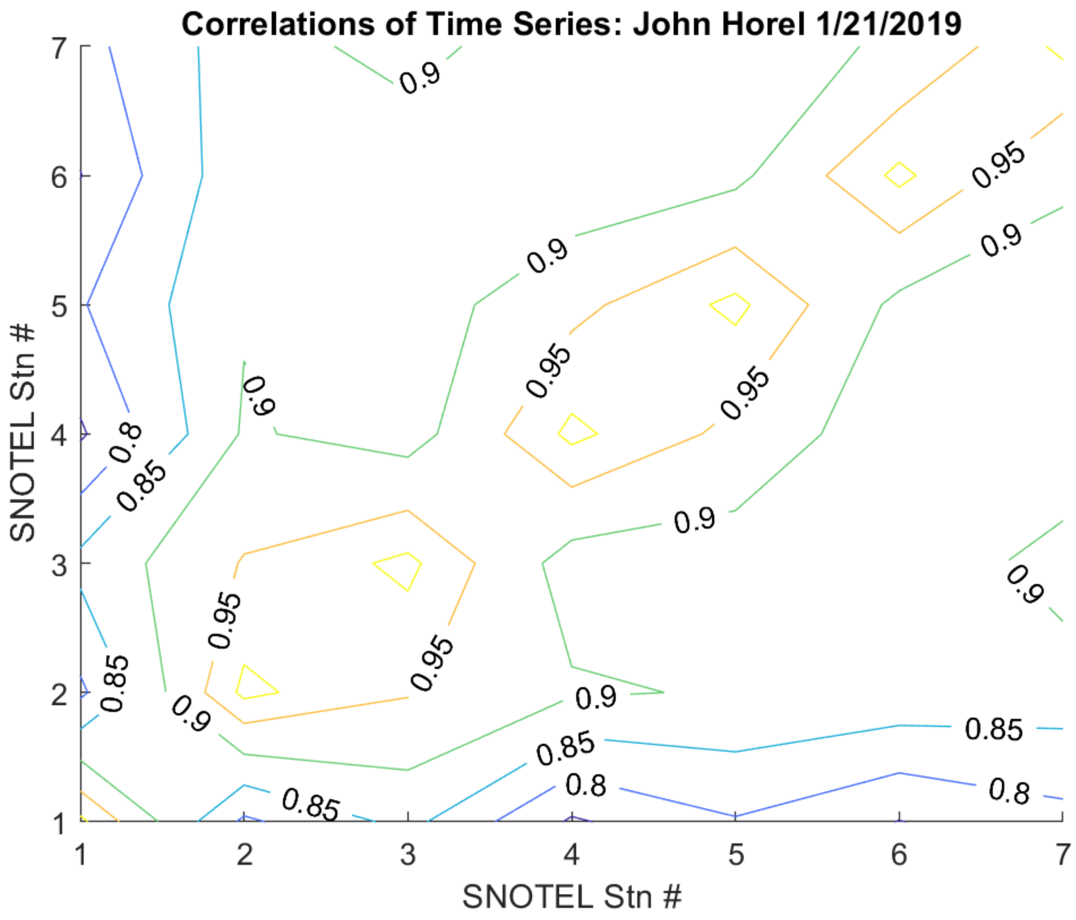
- TGL


Correlations of Time Series: John Horel 1/21/2019
SNOTEL Stn #
SNOTEL Stn #
TGL
BLP
BLT
FRM
PAR
TIM
PAY

Hovmuller of Station Std Anom: John Horel 1/21/2019



Multivariate Linear Correlations

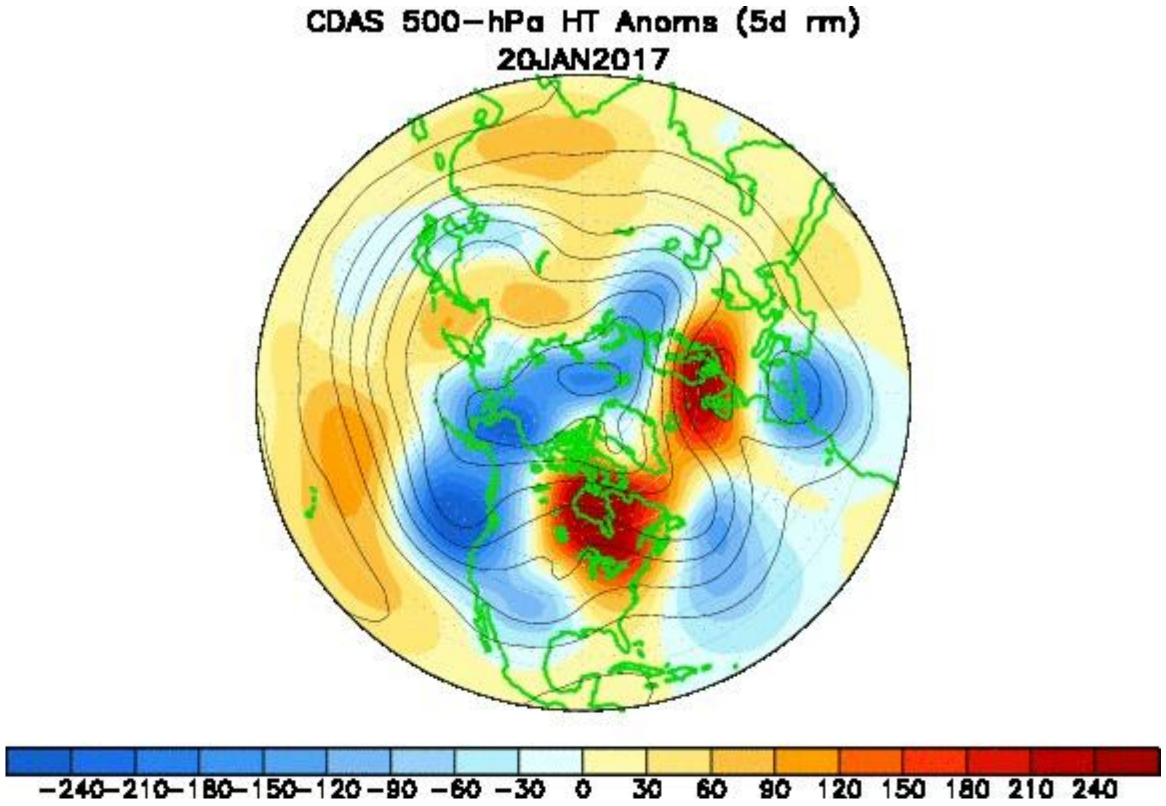


$$\vec{X}* = \begin{bmatrix} x*_{11} & x*_{12} & \dots & x*_{17} \\ x*_{21} & x*_{22} & \dots & x*_{27} \\ \dots & \dots & \dots & \dots \\ x*_{n1} & x*_{n2} & \dots & x*_{n7} \end{bmatrix}$$

$$\vec{R} = \vec{X}*^{T} \ \vec{X}*/n$$

Daily Anomaly Maps

http://www.cpc.ncep.noaa.gov/products/intraseasonal/z500_nh_anim.shtml



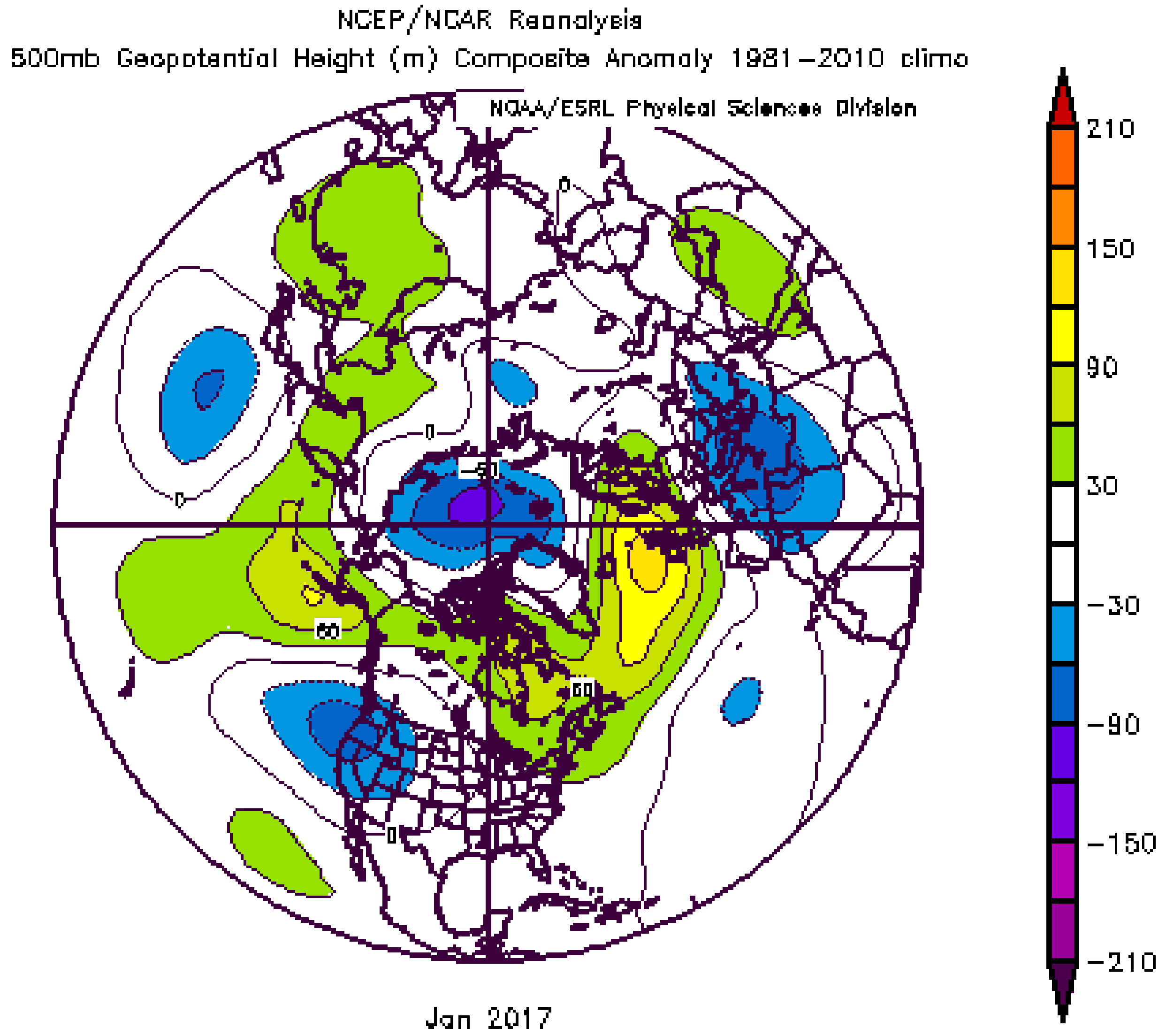
Teleconnection

a causal connection or correlation between meteorological or other environmental phenomena that occur a long distance apart

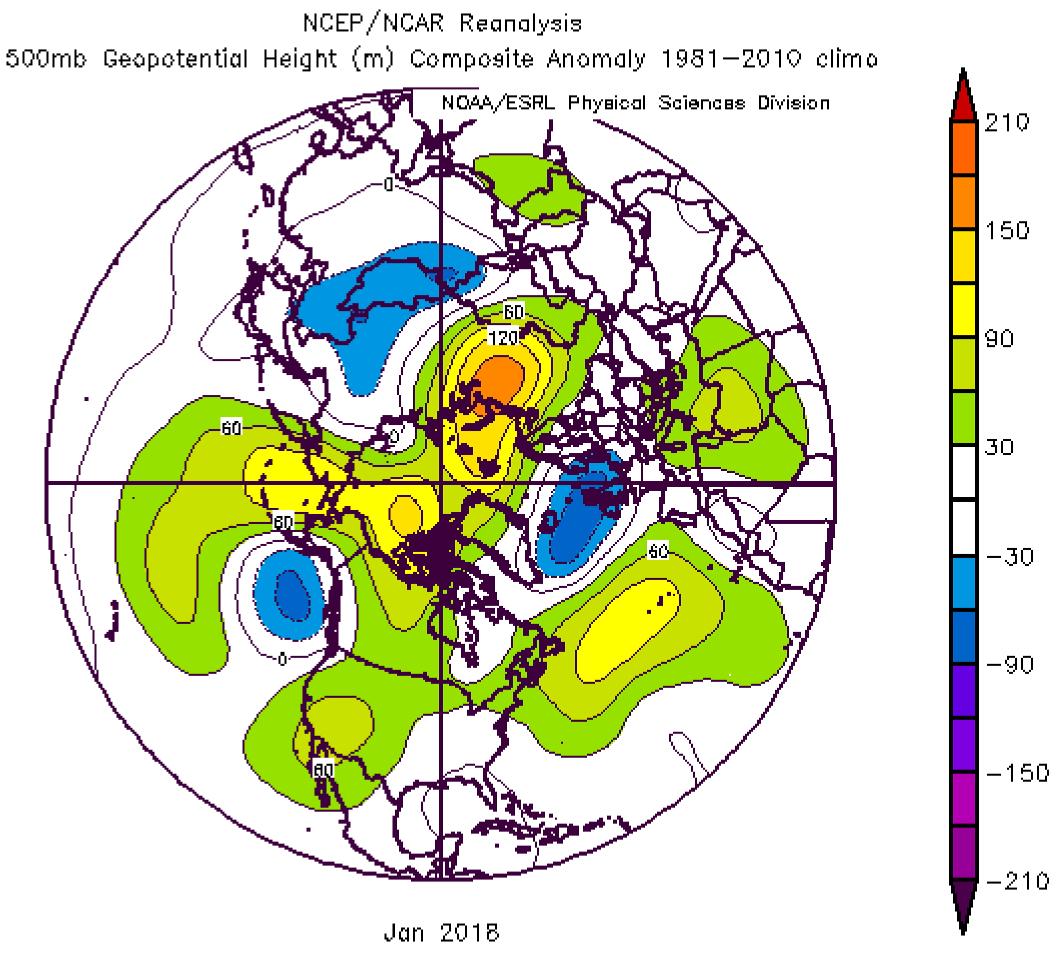
http://www.cpc.ncep.noaa.gov/data/teledoc/teleintro.shtml

Teleconnections: “the study of blobs”

500 hPa Height Anomalies
Jan 2017

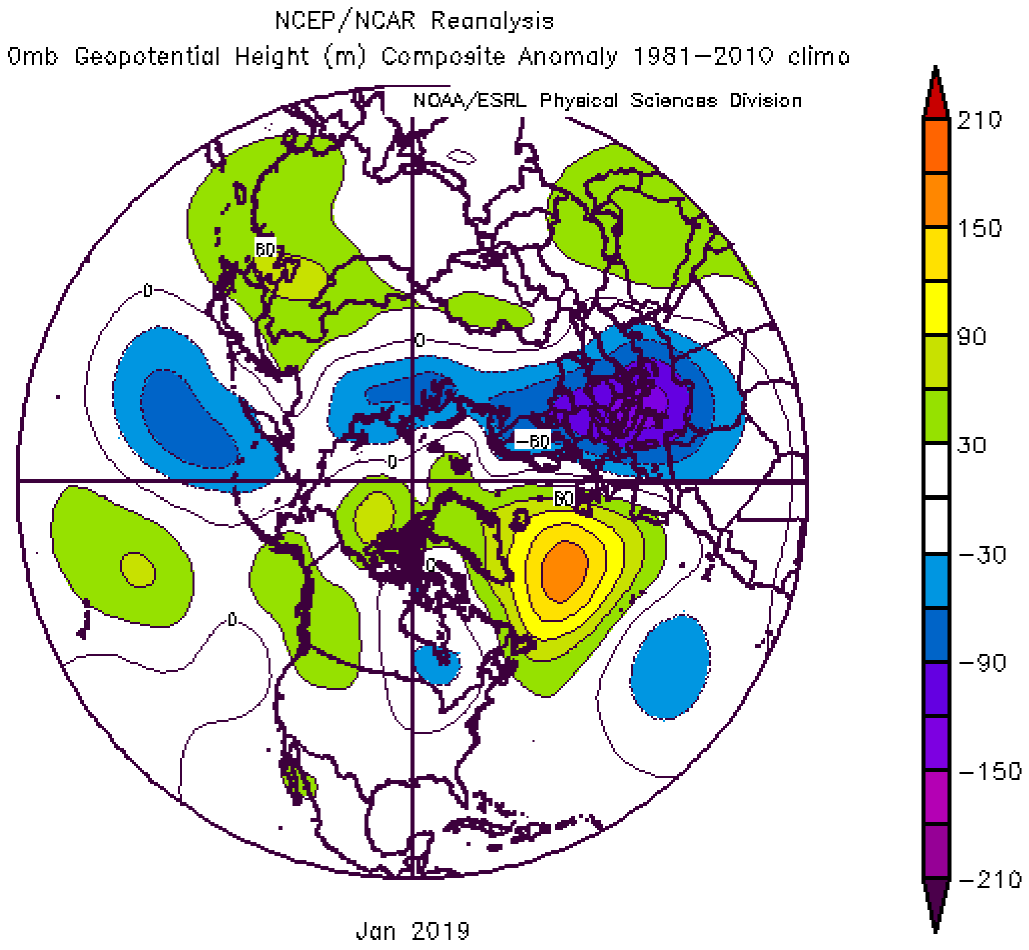


500 hPa Height Anomalies
Jan 2018



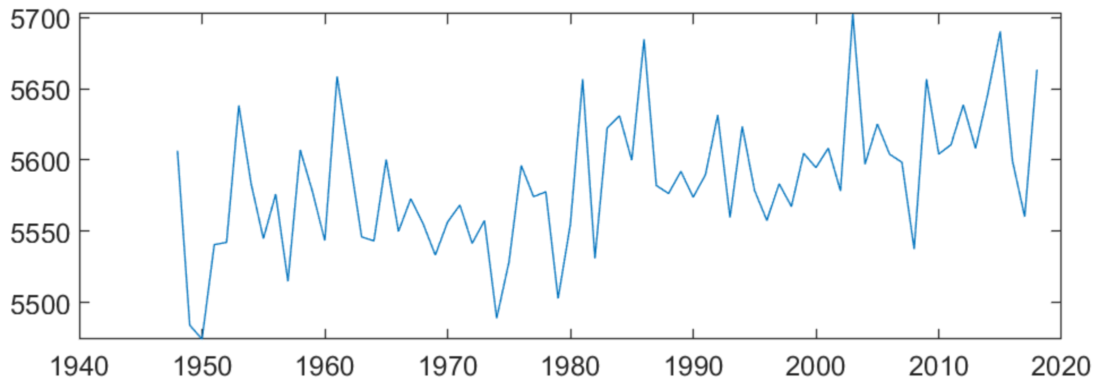
500 hPa Height Anomalies
Jan 2019

https://www.esrl.noaa.gov/psd/cgi-bin/data/composites/printpage.pl

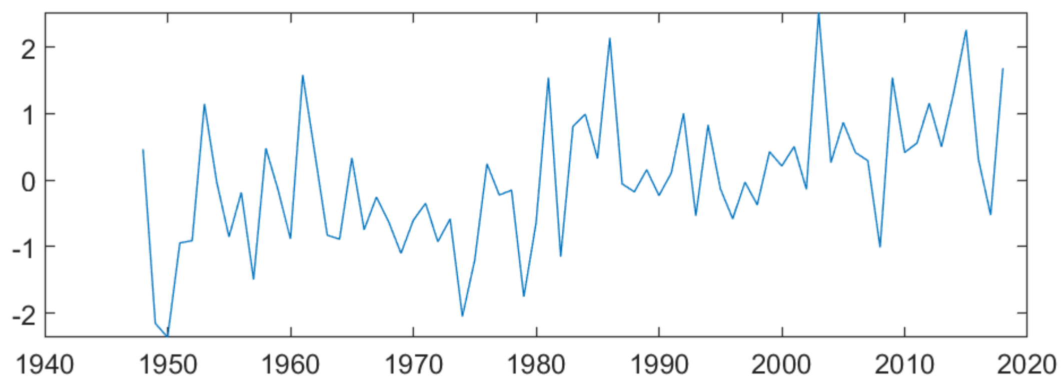


SLC 500 hPa time series

hPa



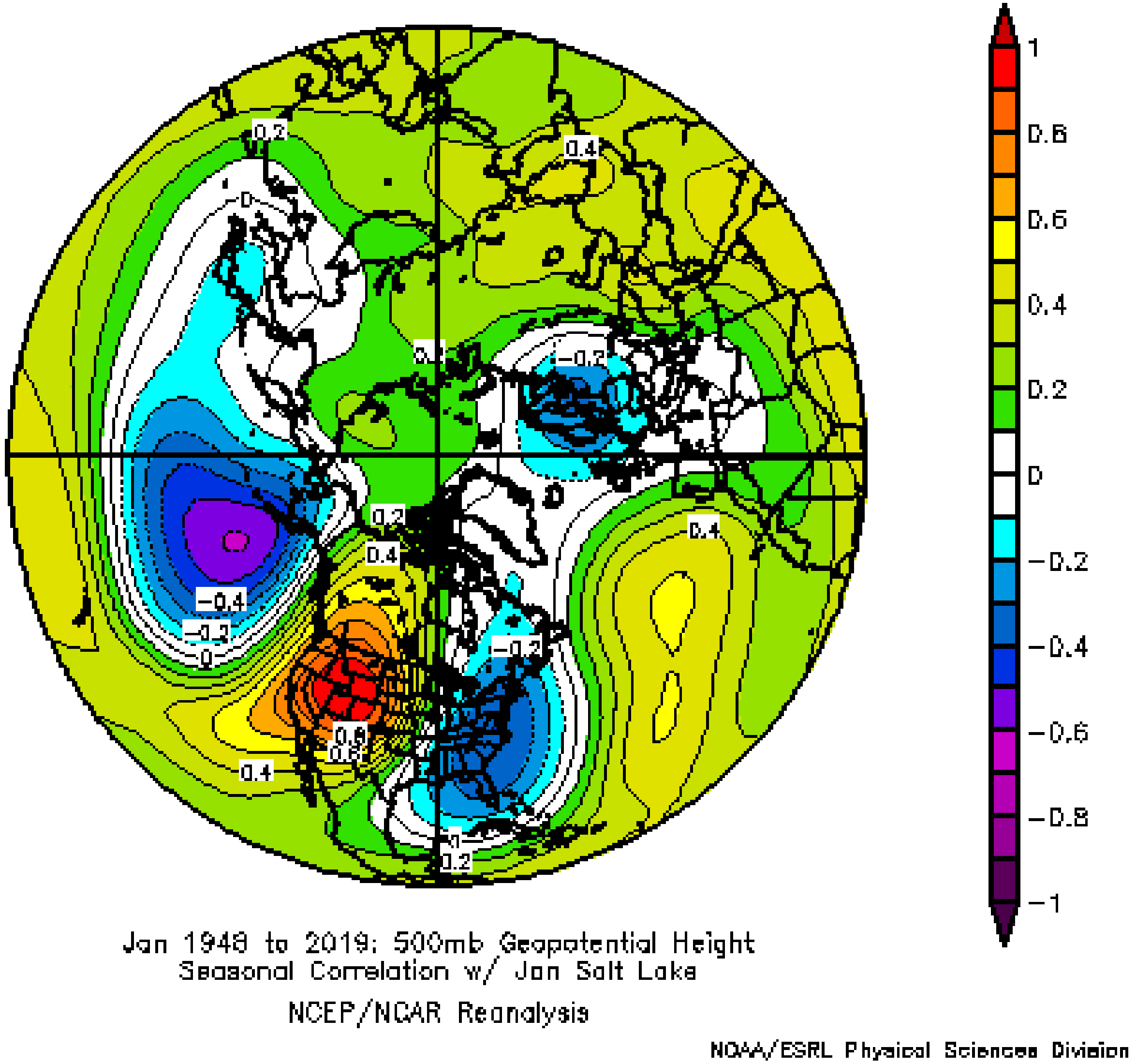
Std
anom



Correlate 500 hPa heights over SLC with every other point in NH

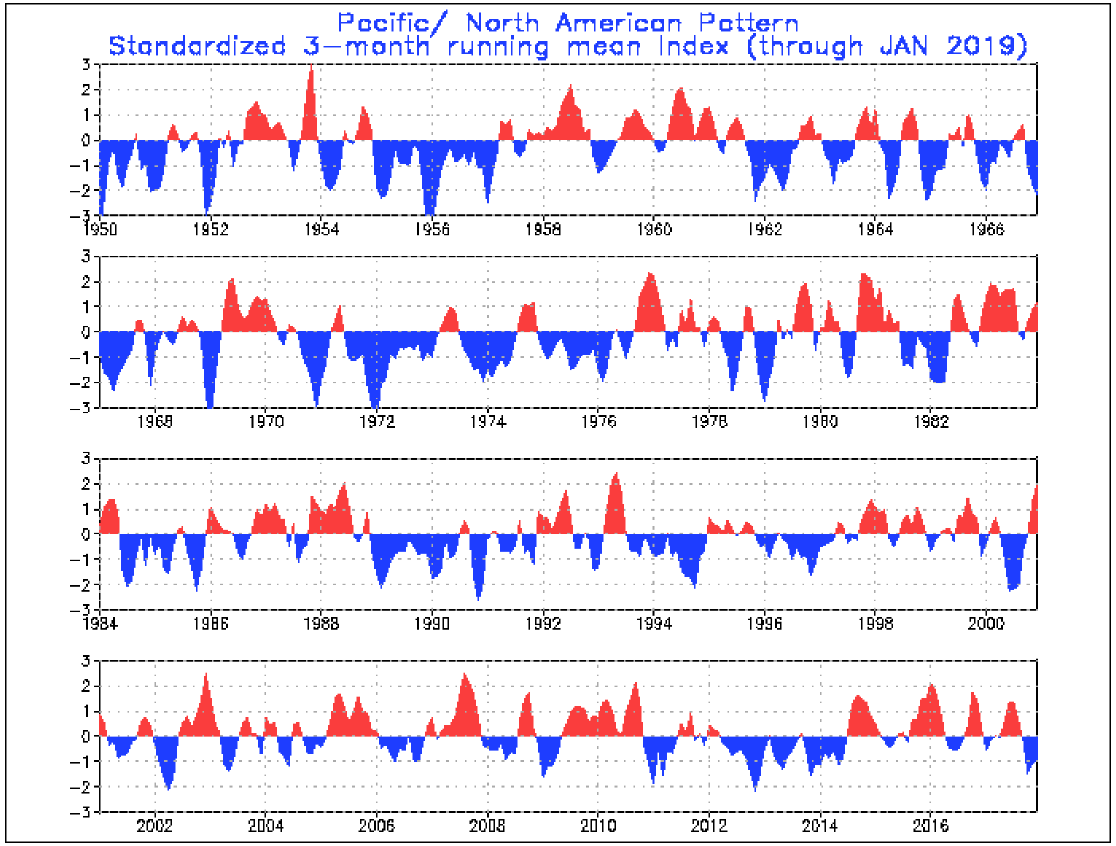
- https://www.esrl.noaa.gov/psd/data/correlation//Public/incoming/timeseries/jdh_slc_500ht.txt
- Create plot

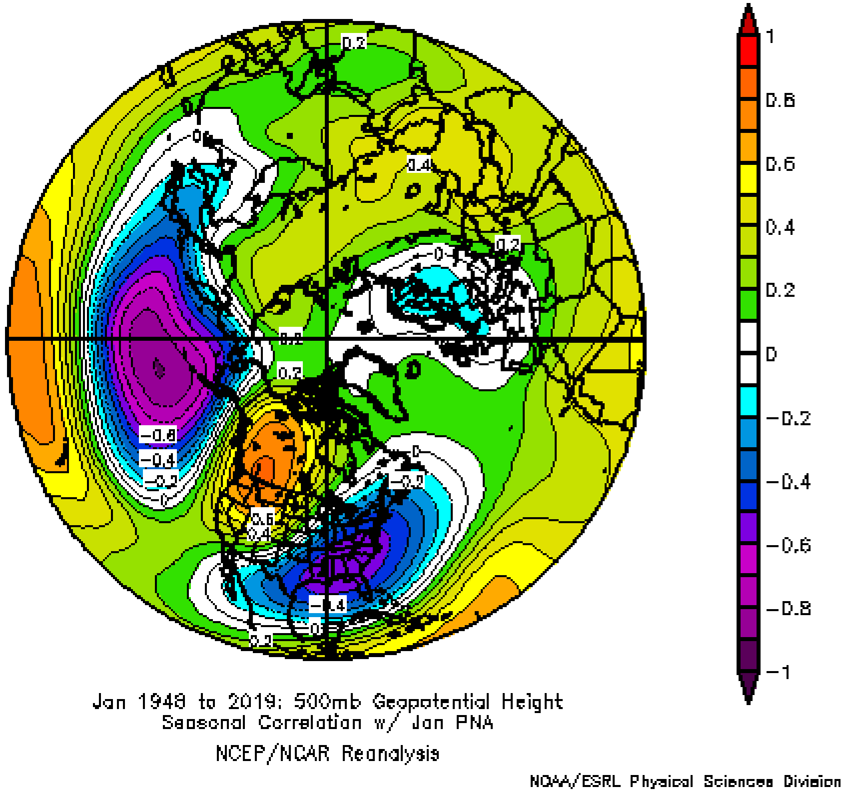
Correlation between SLC time series and all other 500 hPa timeseries during January
A Teleconnection map



PNA Teleconnection Pattern

PNA: https://www.esrl.noaa.gov/psd/data/correlation/

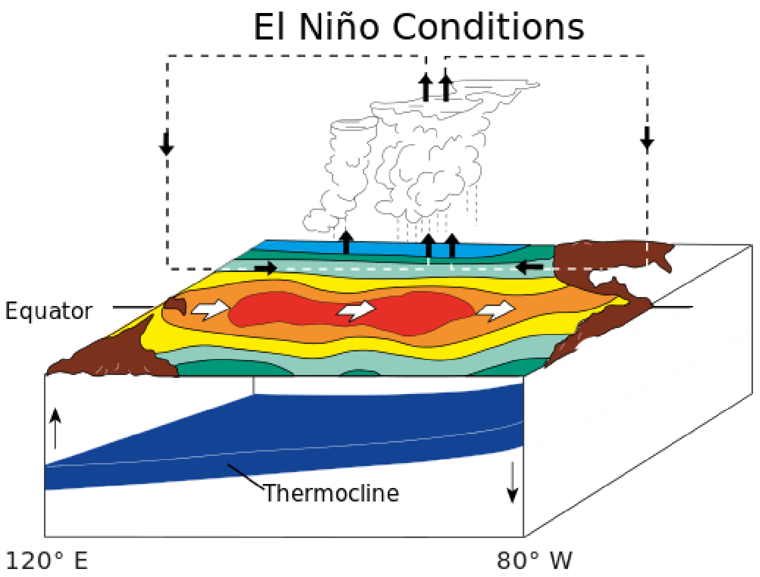


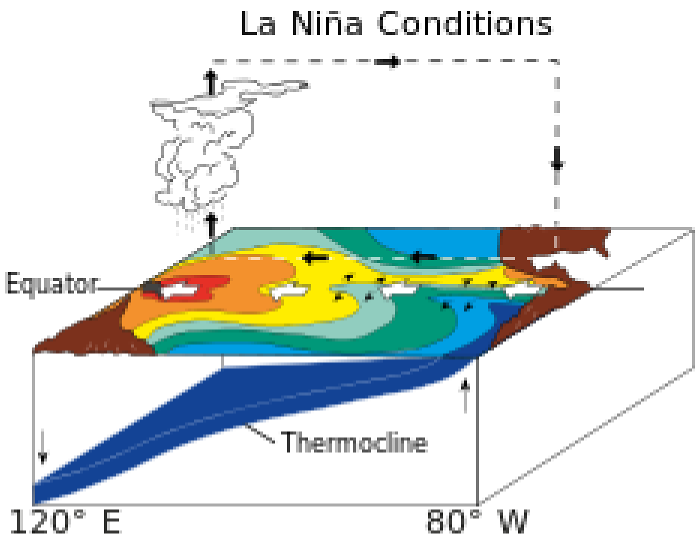


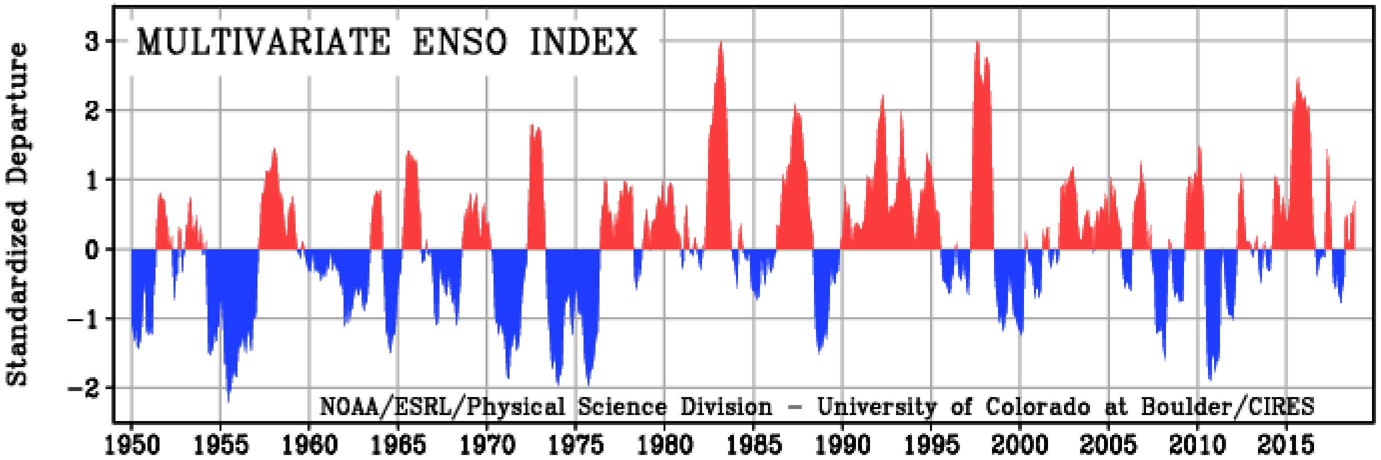
Pacific North American teleconnection

http://www.cpc.ncep.noaa.gov/data/teledoc/pna.shtml

ENSO Teleconnections







ENSO Related Linear Correlation Coefficients

TABLE 1. Matrix of contemporaneous correlation coefficients (×100) between the time series shown in Figs. 2, 3, 4, 7 and 10. The number of winter seasons used in each correlation, if less than the complete 28 seasons (1951–78), is indicated in parentheses.

	SST Index	SLP Index	200 mb Index	PNA Index	WP Index	Tarawa rainfall	Canton rainfall	Christmas rainfall	Fanning rainfall
SST Index									
SLP Index	−83								
200 mb Index	80	−68							
PNA Index	46	−31	57						
WP Index	67	−57	44	−00					
Tarawa rainfall	78	−71	63	32	65				
Canton rainfall	82 (23)	−65 (23)	61 (23)	40 (23)	65 (23)	60 (23)	(23)		
Christmas rainfall	64 (24)	−49 (24)	57 (24)	40 (24)	48 (24)	55 (24)	82 (19)	(24)	
Fanning rainfall	79 (23)	−60 (23)	69 (23)	38 (23)	70 (23)	72 (23)	85 (18)	84 (23)	(23)

Horel and Wallace (1981)

ENSO Teleconnections

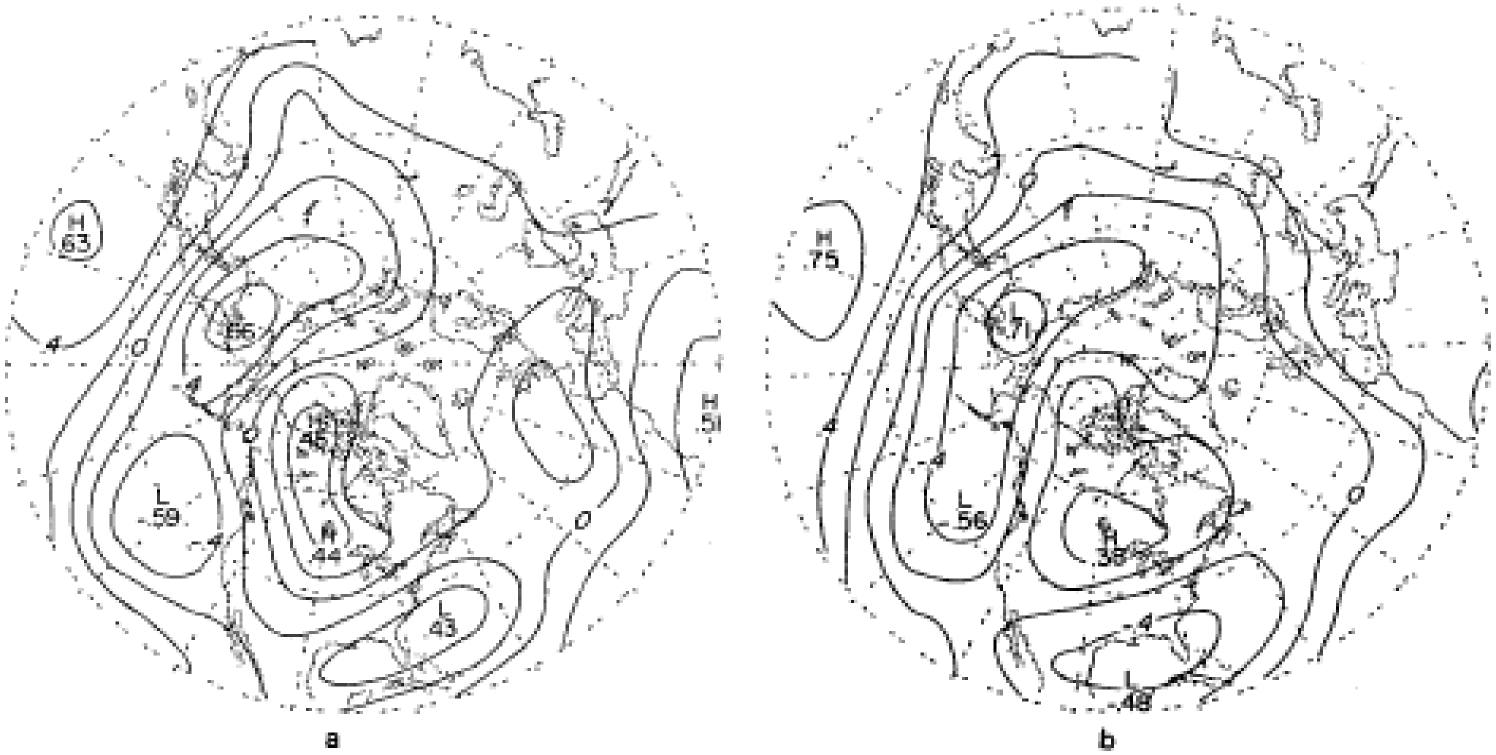
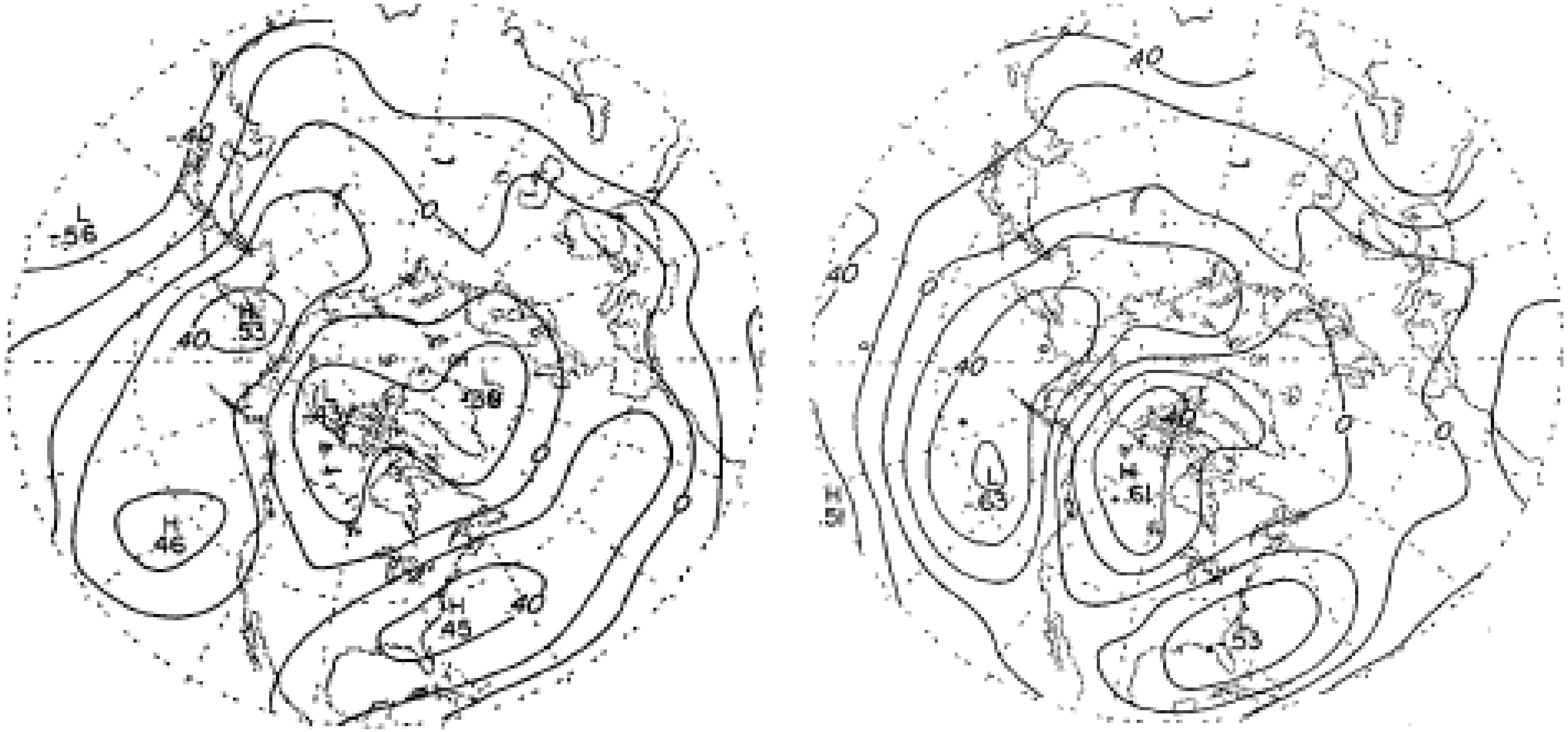


FIG. 9. Correlation coefficients between 700 mb geopotential height at gridpoints poleward of 20°N and (a) the Sea Surface Temperature Index, (b) December–February rainfall at Fanning, (c) our Southern Oscillation Index, and (d) the tropical 200 mb Index. Contour interval 0.2. The locations of the centers of action of the Pacific/North American and West Pacific patterns are denoted, respectively, by dots and open circles in Fig. 9d.

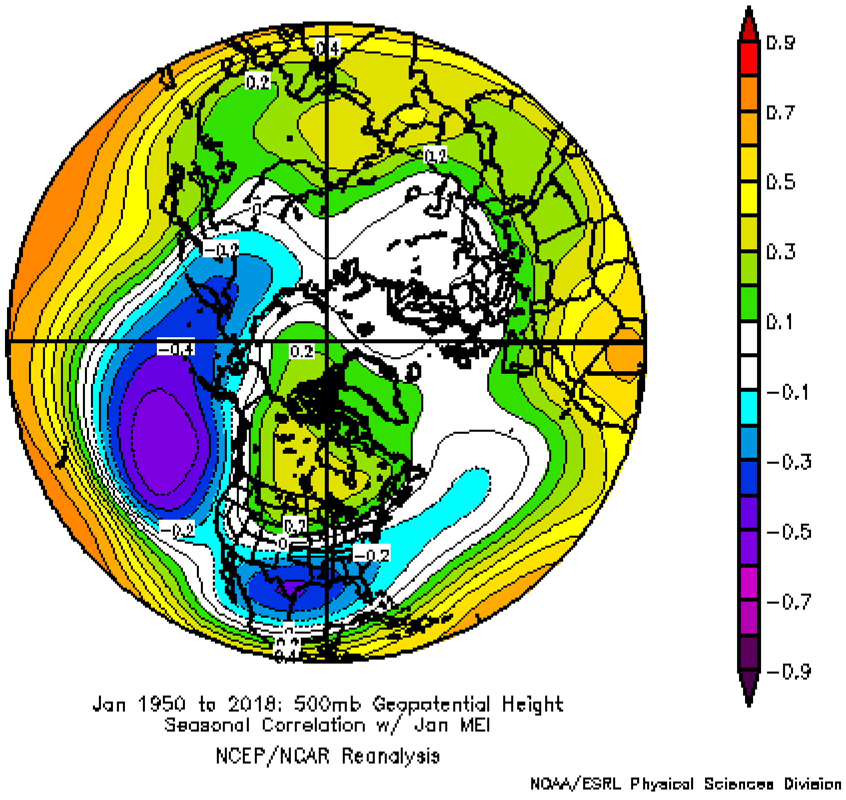
Teleconnections

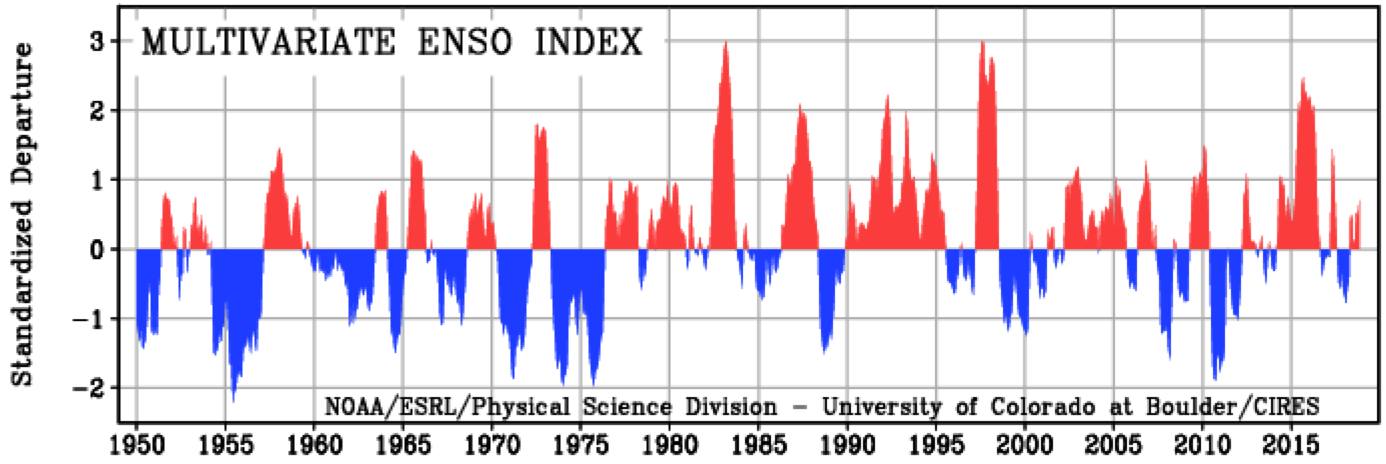


c   d

FIG. 9. Correlation coefficients between 700 mb geopotential height at gridpoints poleward of 20°N and (a) the Sea Surface Temperature Index, (b) December–February rainfall at Fanning, (c) our Southern Oscillation Index, and (d) the tropical 200 mb Index. Contour interval 0.2. The locations of the centers of action of the Pacific/North American and West Pacific patterns are denoted, respectively, by dots and open circles in Fig. 9d.

Correlate MEI Index with 500 mb heights





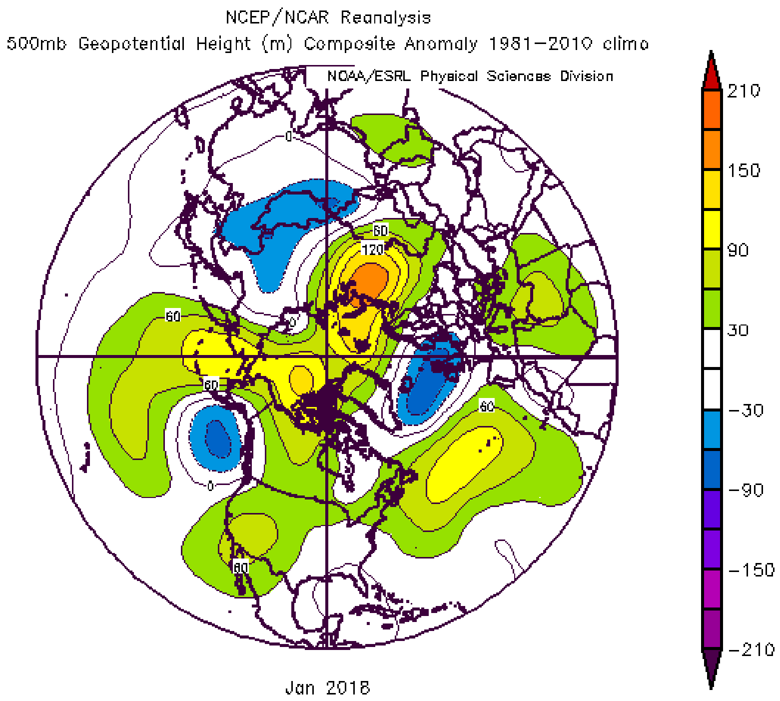
Correlating Maps Rather Than Time Series

$$\hat{\vec{X}} = \begin{bmatrix} \hat{x}_{1,1} & \hat{x}_{1,2} & \cdots & \hat{x}_{1,n} \\ \hat{x}_{2,1} & \hat{x}_{2,2} & \cdots & \hat{x}_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ \hat{x}_{m,1} & \hat{x}_{m,2} & \cdots & \hat{x}_{m,n} \end{bmatrix}$$

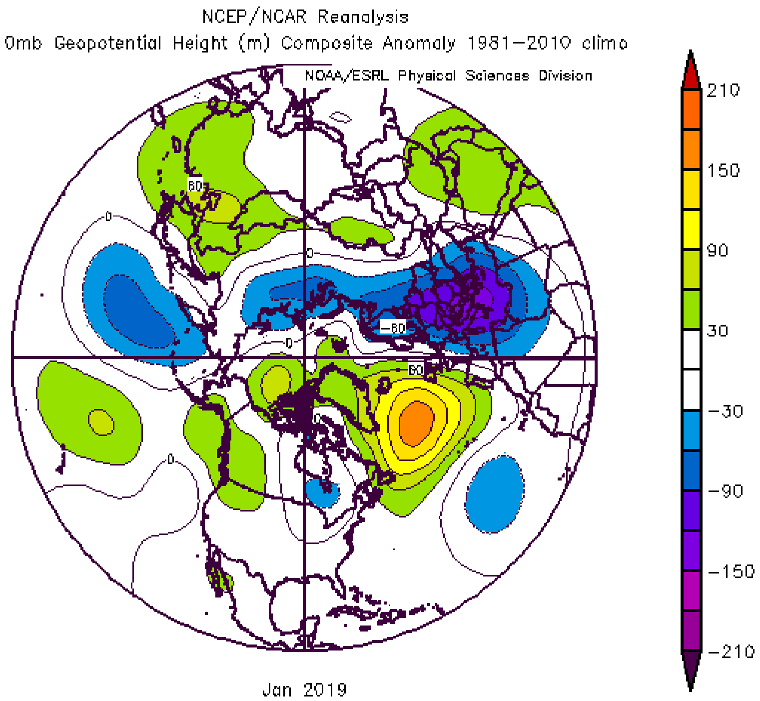
- Comparing variability over m locations at one time to the variability in all of the other n times

$$\vec{S} = \hat{\vec{X}} *^{T} \hat{\vec{X}} * \ / m$$

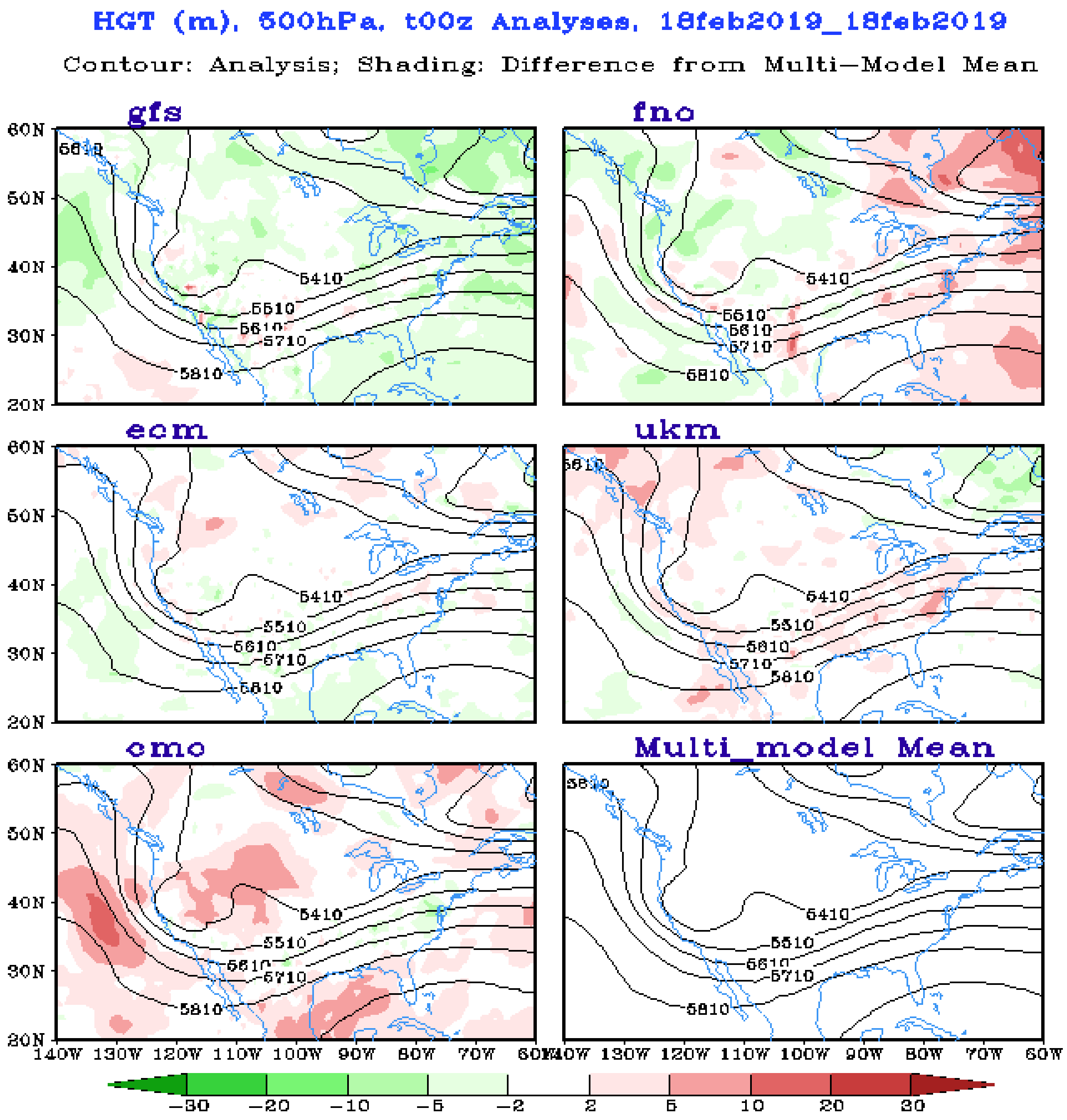
How similar are these two anomaly maps?



JAN 2018

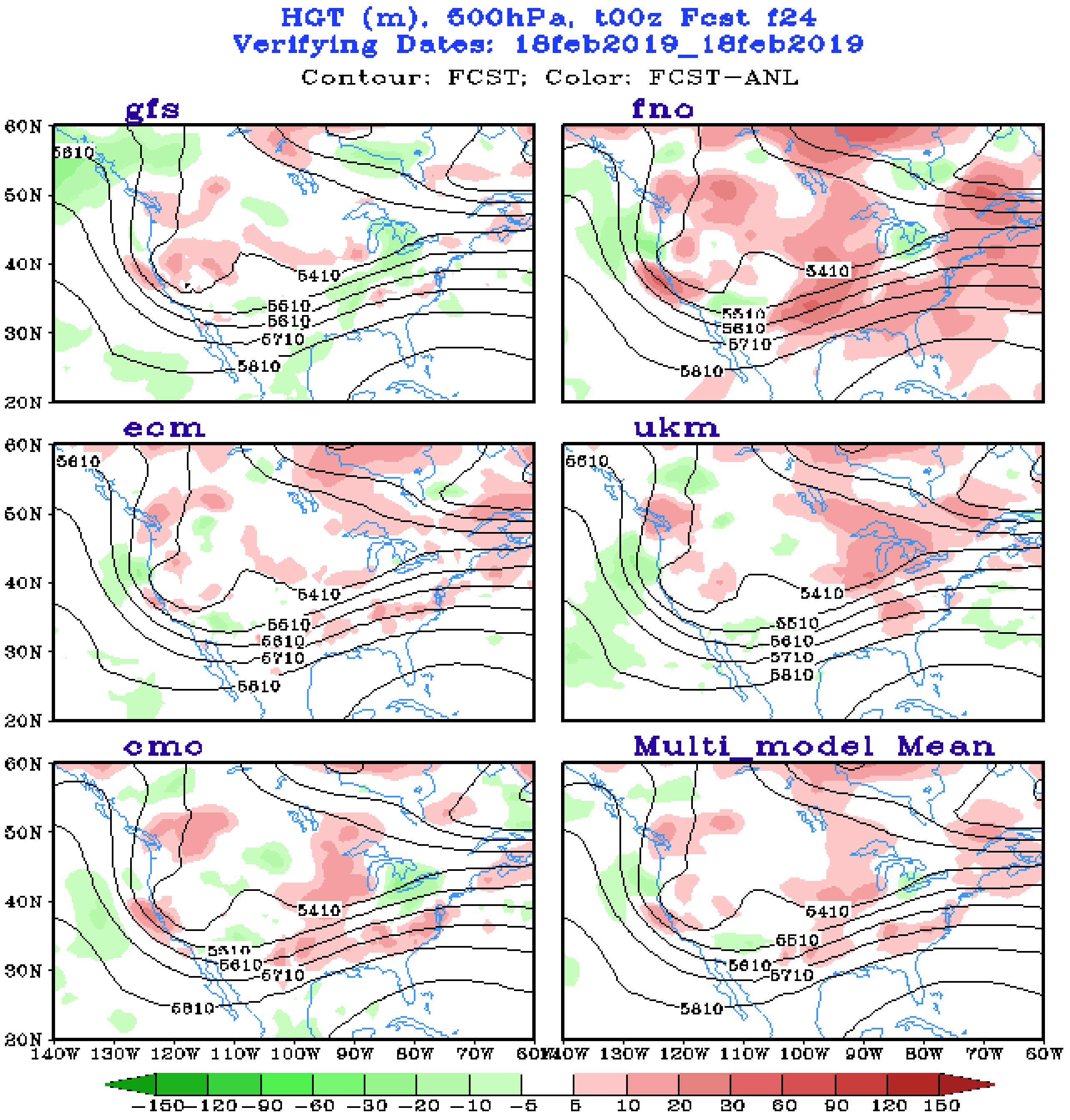


Jan 2019



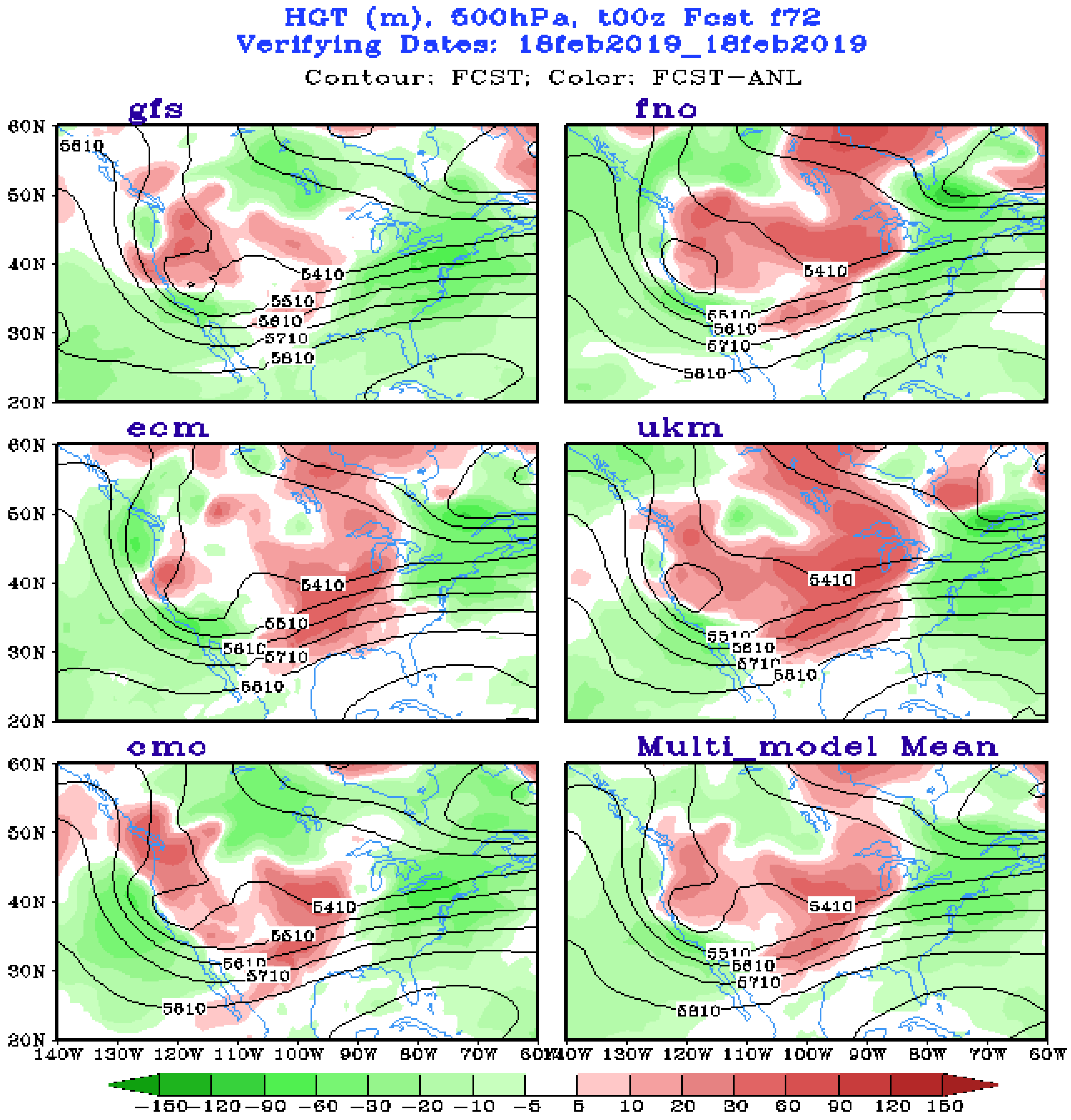
Compare analyses

http://www.emc.ncep.noaa.gov/gmb/STATS_vsdb/



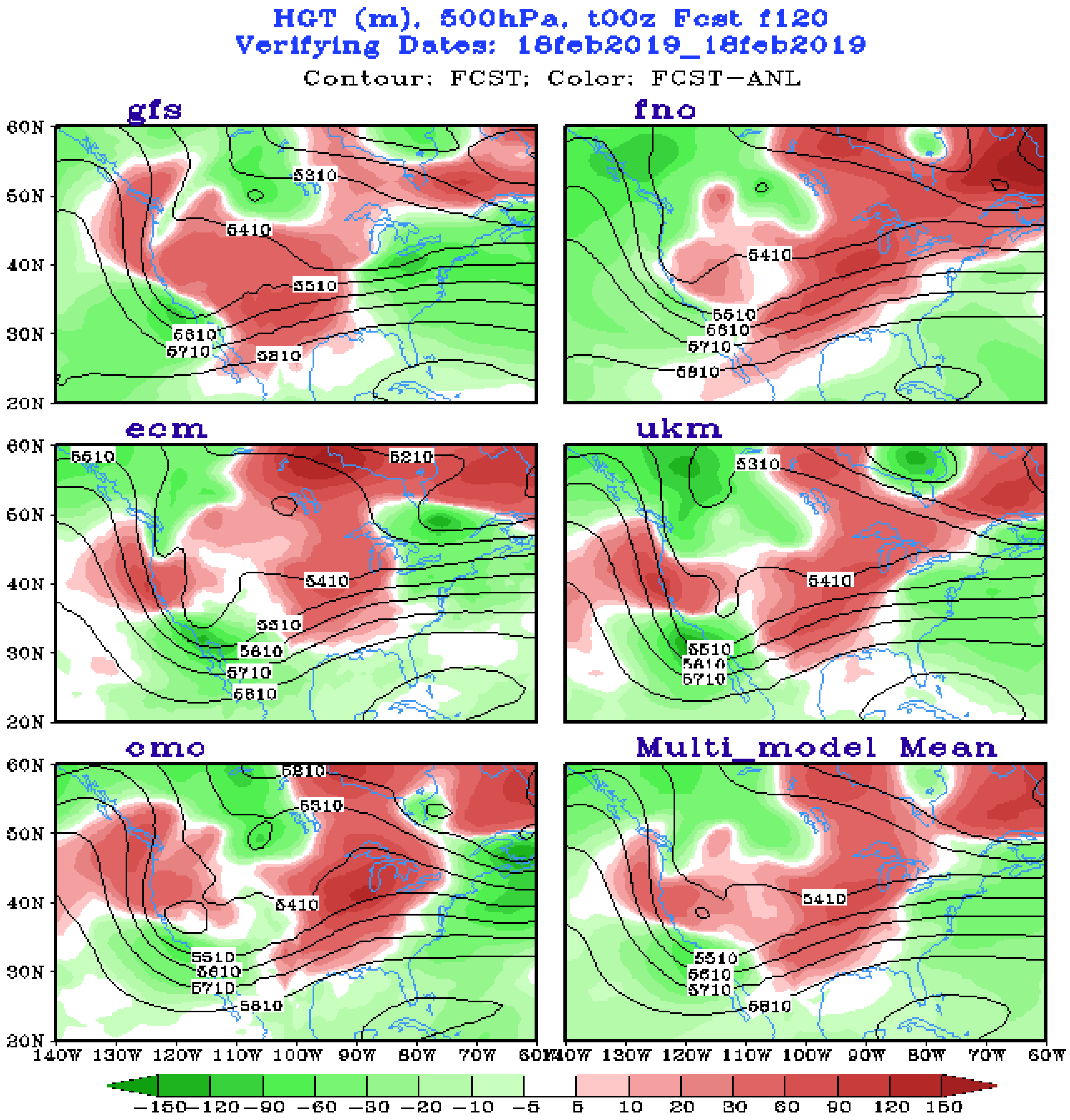
Verify 24 h Forecasts

http://www.emc.ncep.noaa.gov/gmb/STATS_vsdb/



Verify 72 h Forecasts

http://www.emc.ncep.noaa.gov/gmb/STATS_vsdb/

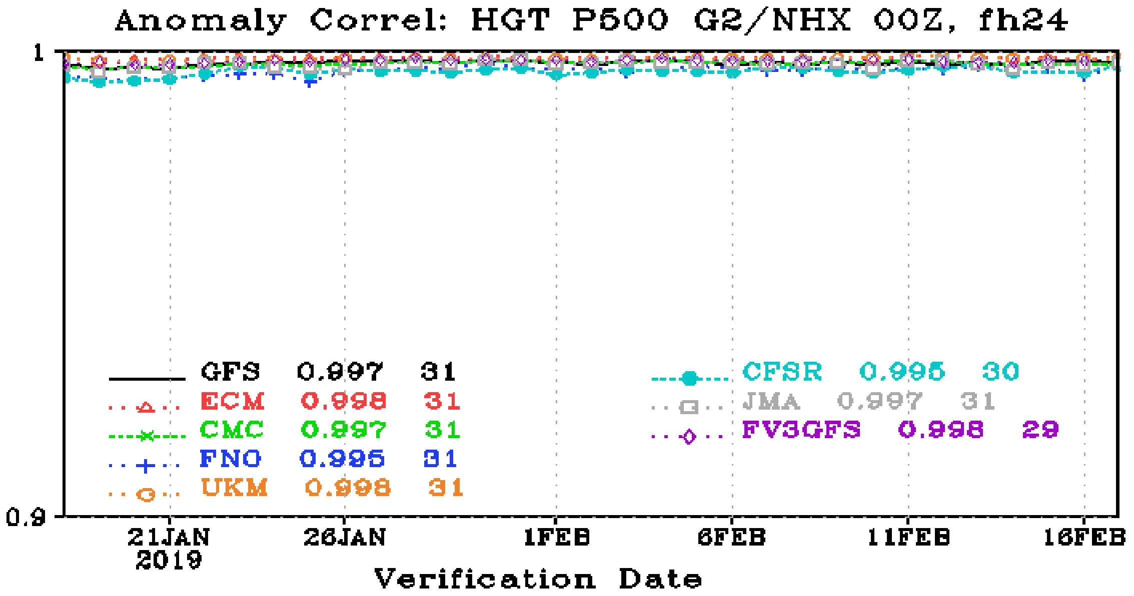


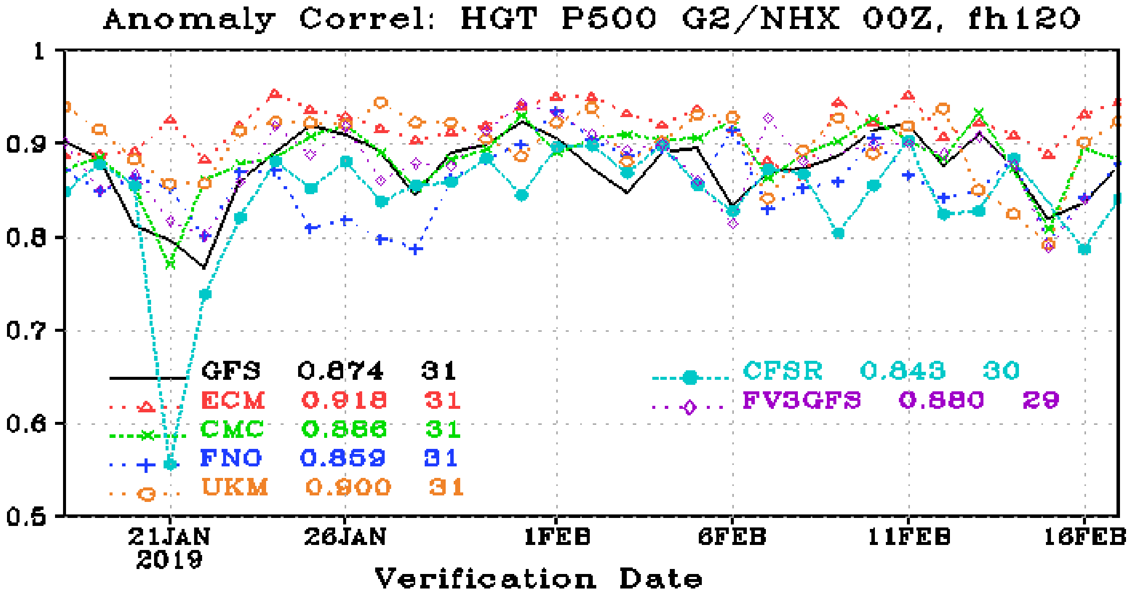
Verify 120 h Forecasts

http://www.emc.ncep.noaa.gov/gmb/STATS_vsdb/

Verifying Forecast Anomaly Maps

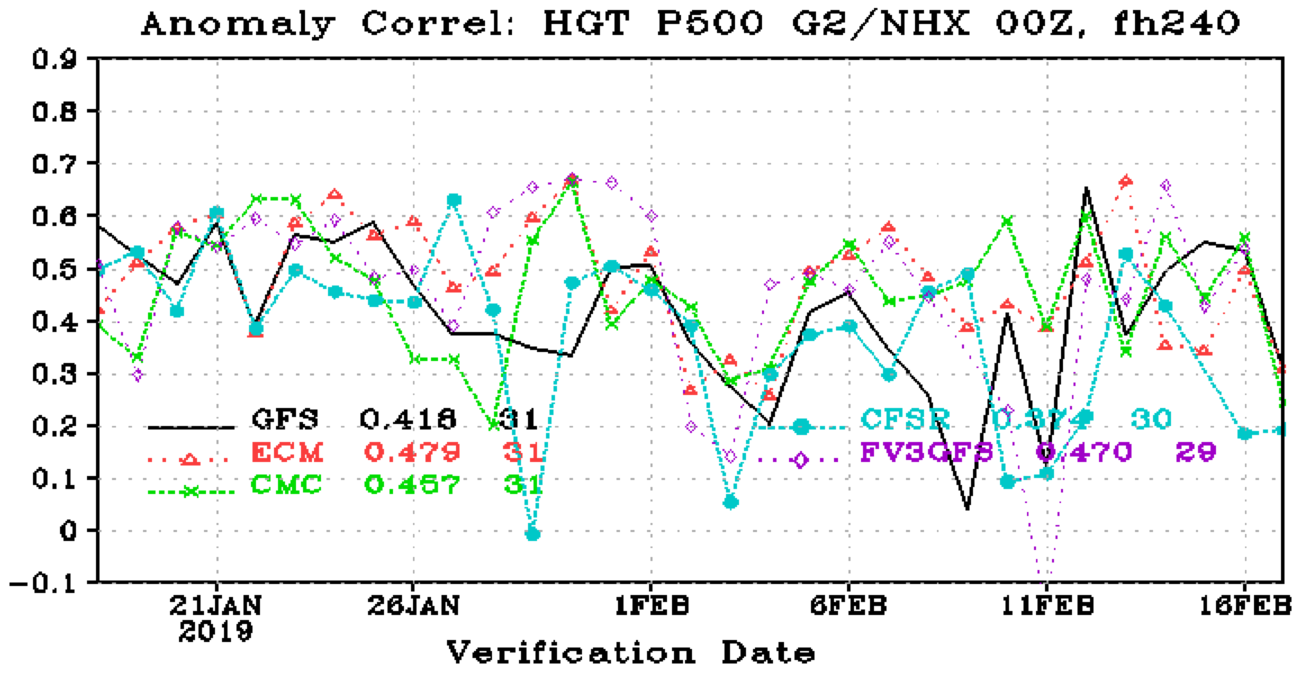
http://www.emc.ncep.noaa.gov/gmb/STATS_vsdb/



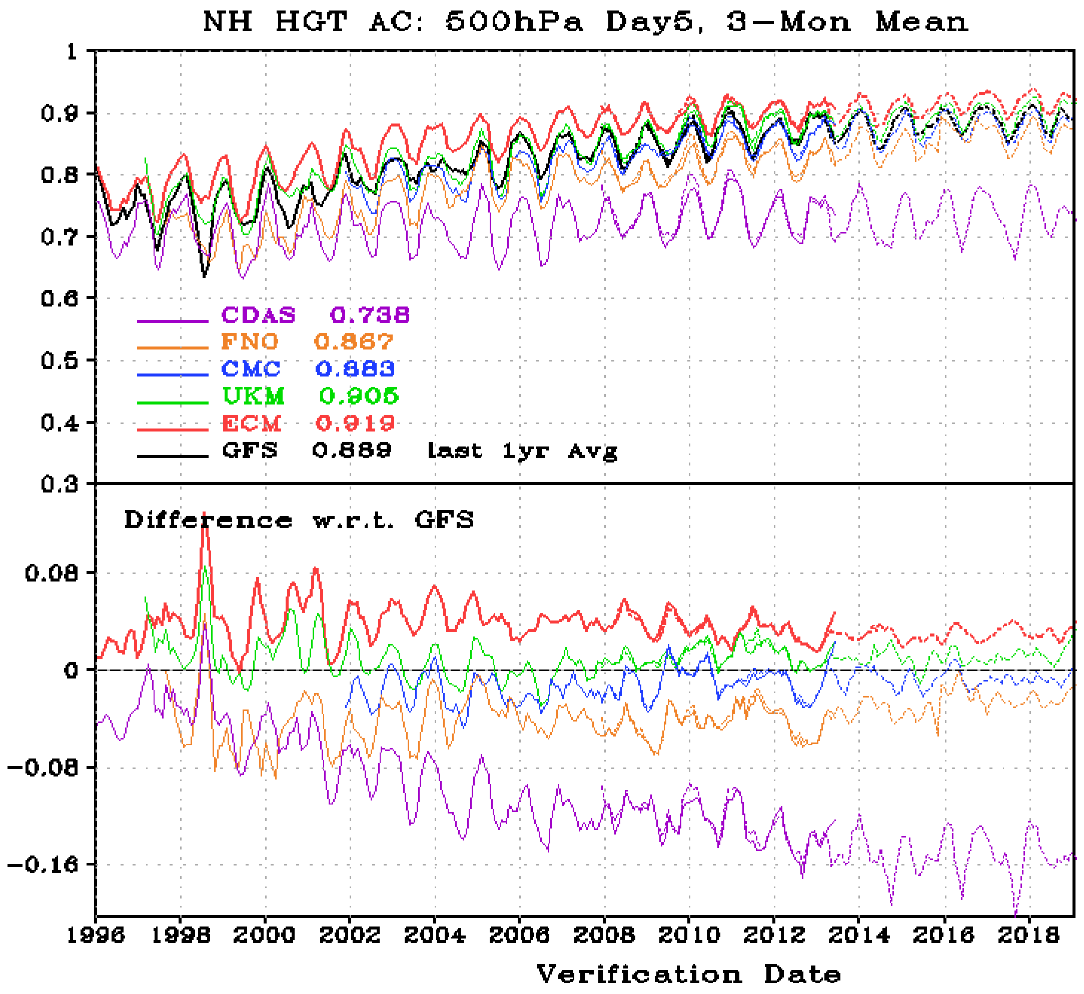


Verifying Forecast Anomaly Maps

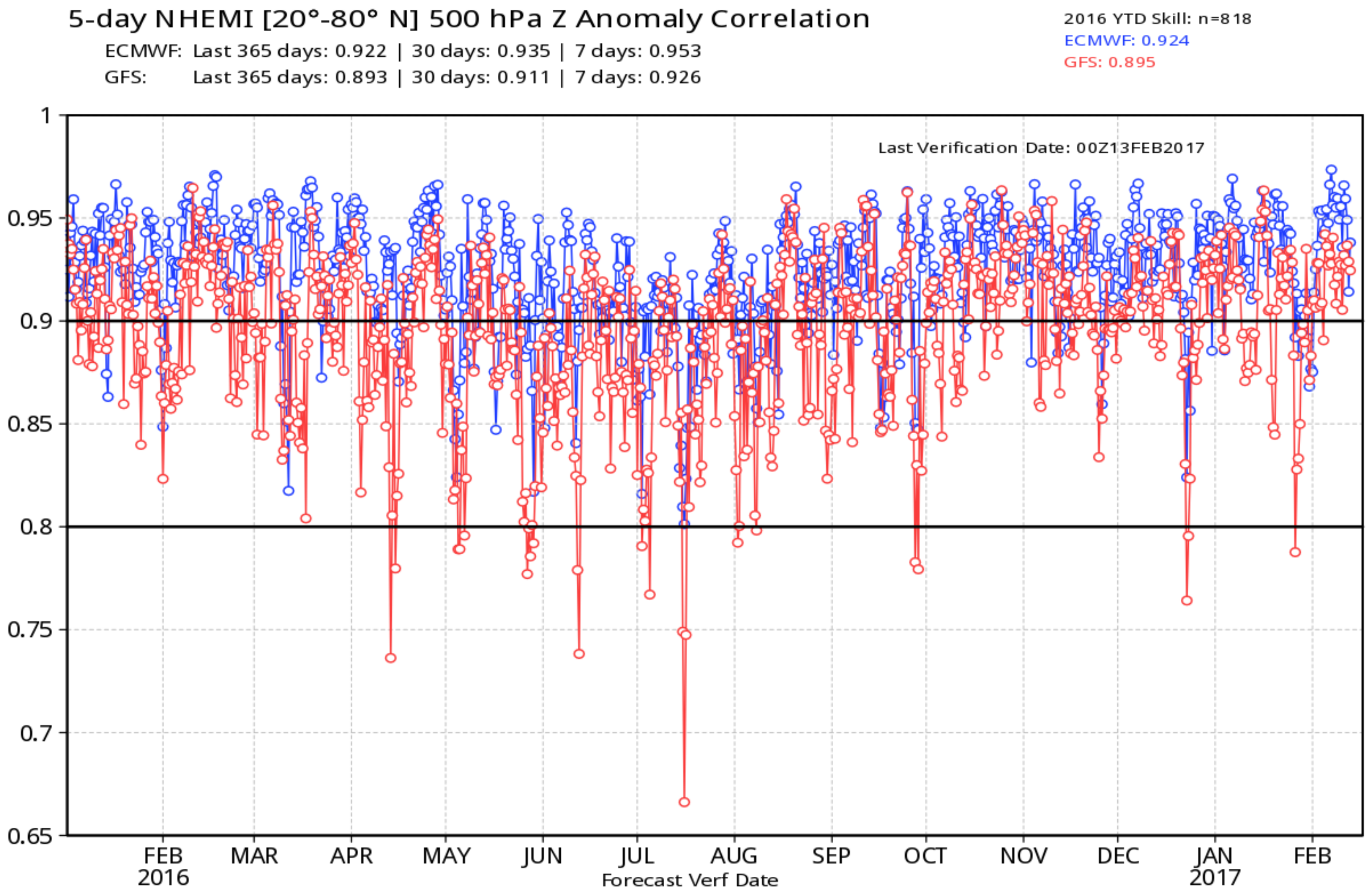
http://www.emc.ncep.noaa.gov/gmb/STATS_vsdb/



http://www.emc.ncep.noaa.gov/gmb/STATS/STATS.html



Daily Showing Drop Offs

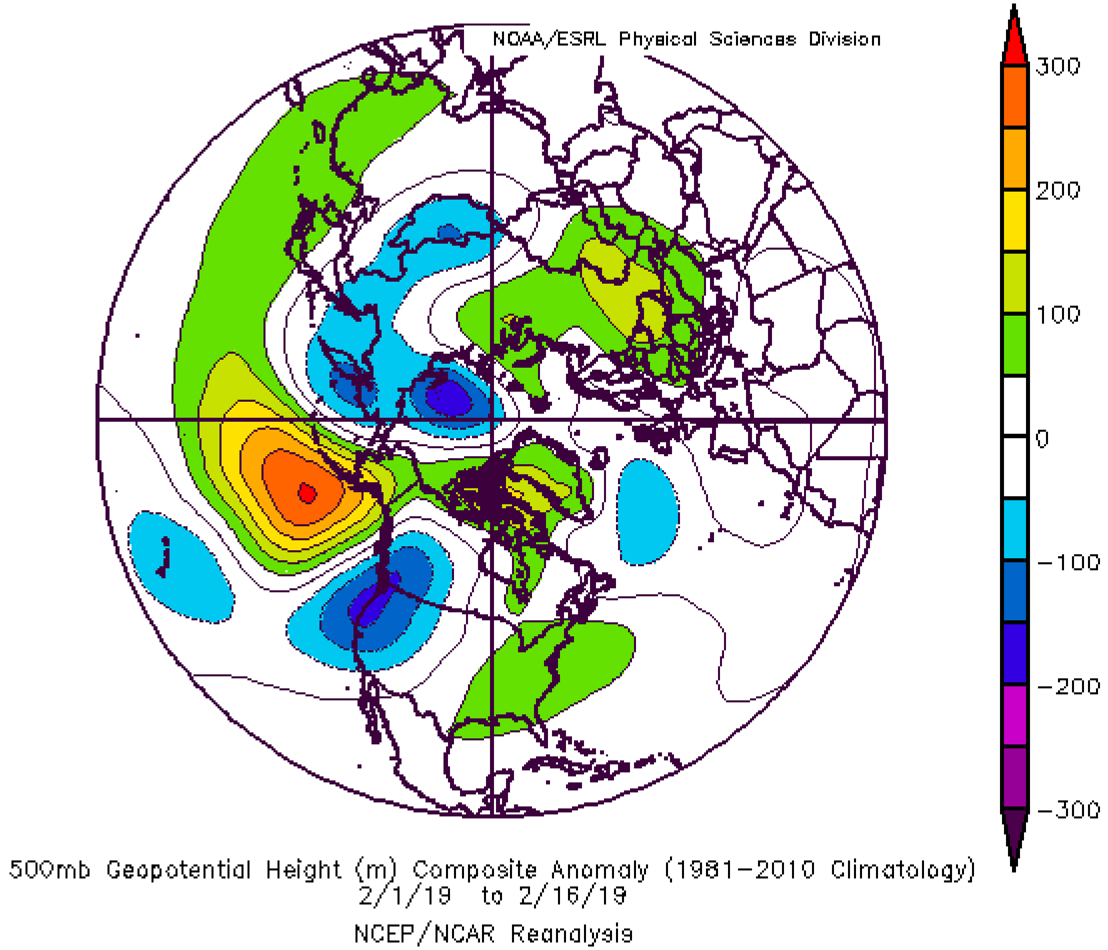


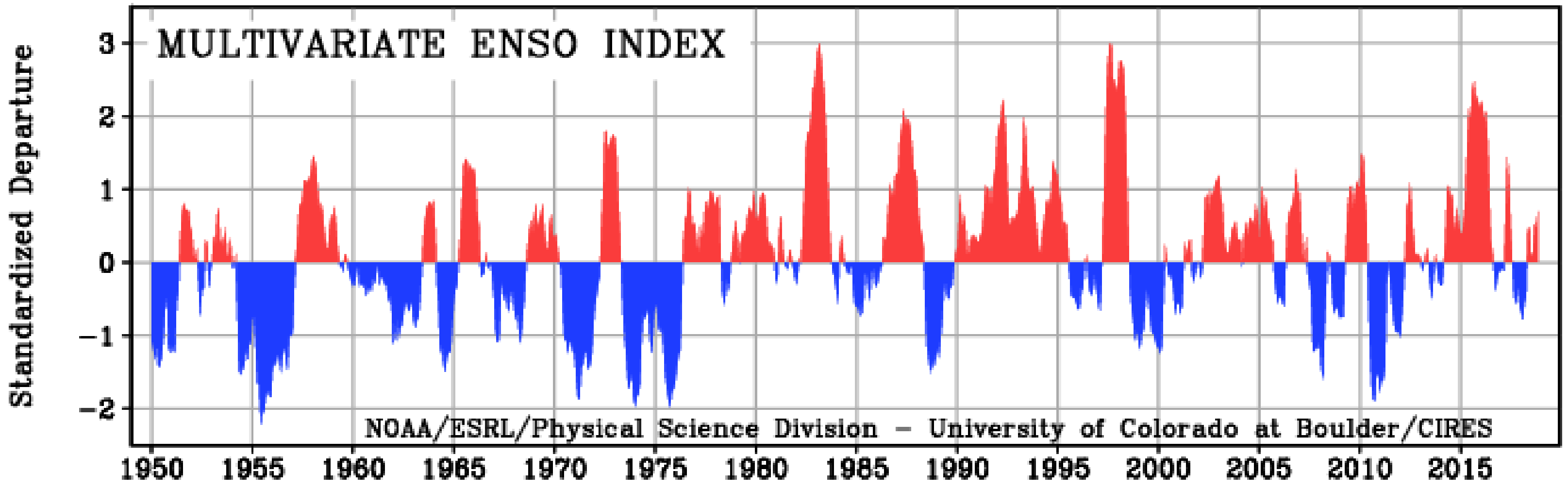
Compositing (Superposed Epoch)

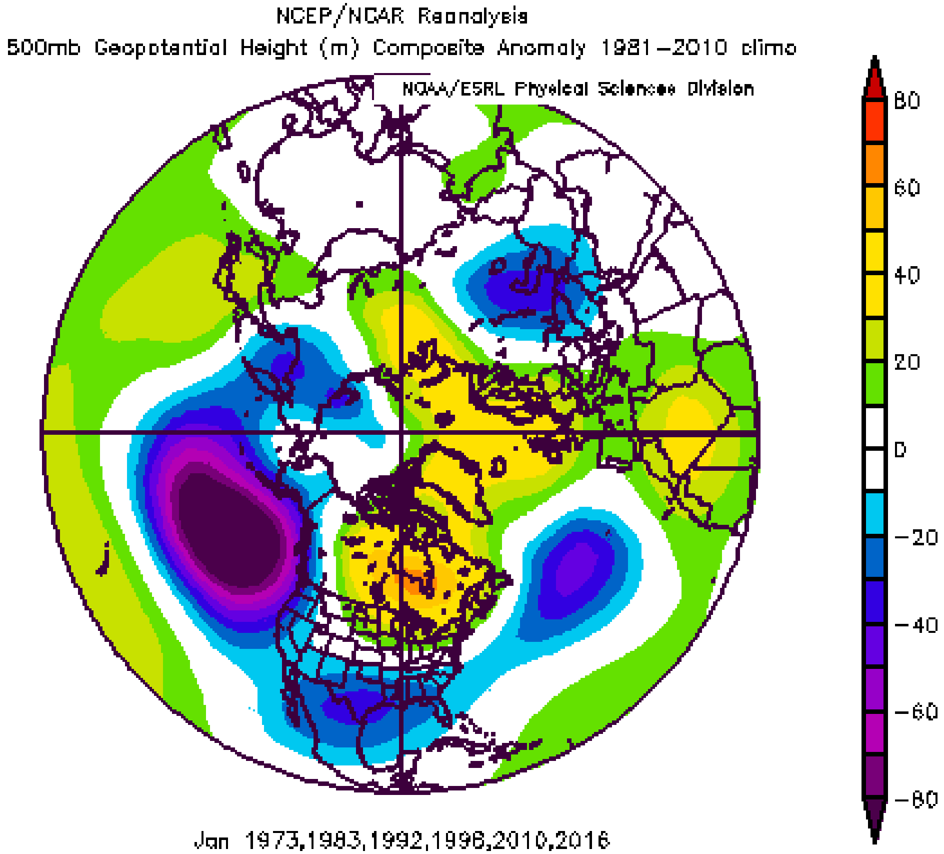
- Identify common characteristics of a sample of events
- Simplest- average conditions before, during, and after some “rare” event
- Has an advantage over linear correlation since no linear assumption necessary
- Limitation- to what extent does sample mean used in composite differ from population?
- Day composites: https://www.esrl.noaa.gov/psd/data/composites/day
- Monthly/seasonal composites:
- https://www.esrl.noaa.gov/psd/cgi-bin/data/composites/printpage.pl

Composite 500 hPa height anomaly over the first 16 days

Plot link




MULTIVARIATE ENSO INDEX
Standardized Departure
3
2
1
0
−1
−2
NOAA/ESRL/Physical Science Division − University of Colorado at Boulder/CIRES
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015

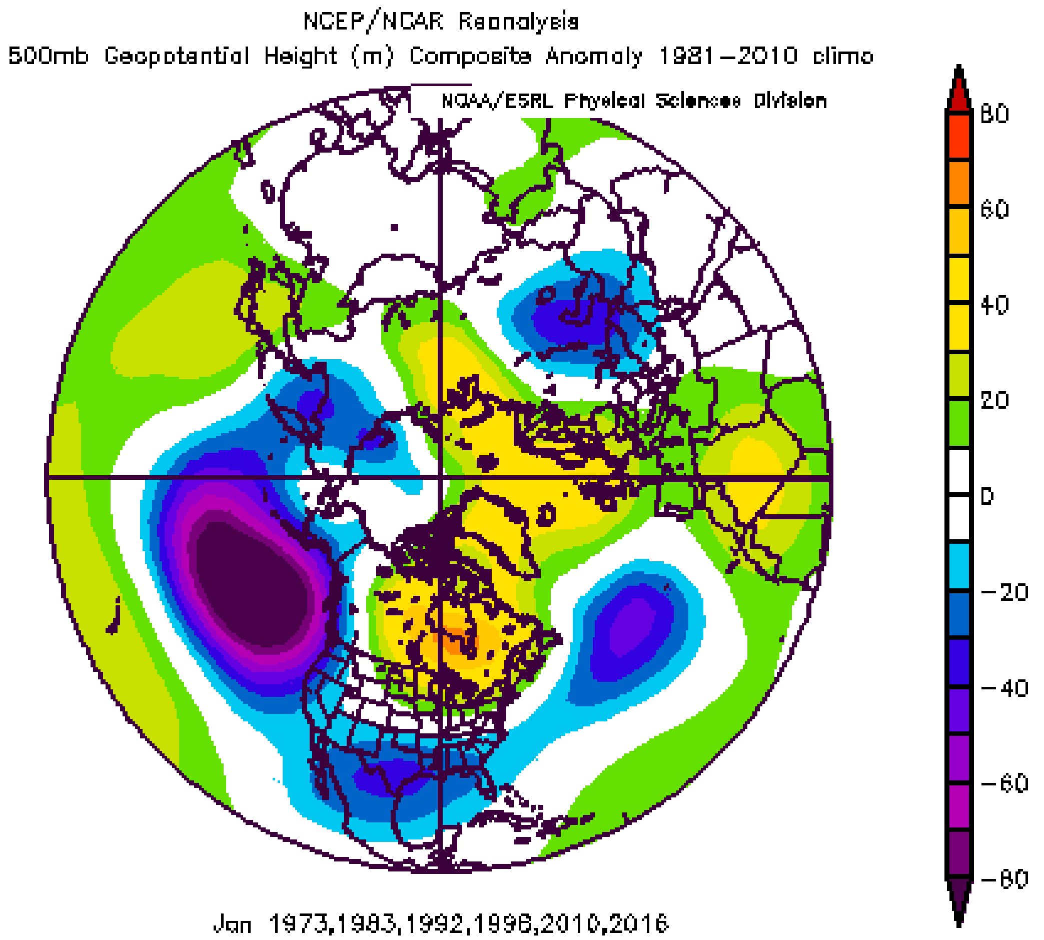

NCEP/NCAR Reanalysis
500mb Geopotential Height (m) Composite Anomaly 1981−2010 climo
NOAA/ESRL Physical Sciences Division
80
60
40
20
0
−20
−40
−60
−80
Jan 1973,1983,1992,1998,2010,2016

Compositing Steps

- Select the basis for compositing: why are you doing it? Physical reasoning hopefully?
- Define the categories on which you define the events: above, below normal? Or …?
- Compute the means and statistics for each category (minimum is standard deviation)
- Organize and display the results
- Validate the results:
  - Significance test? t test is the bare minimum to do
  - Reproduce in an independent sample?
  - Are the results sensible in space and time?
  - Is it consistent with theory?

How many spatial degrees of freedom?

- Count the anomaly blobs



Don’t overdo it…

- Was there a reason a priori to expect the relationship?
- How arbitrary was the choice for defining the composites?
- How subjective and biased was your analysis? Did you tweak your approach to get better results?
- Do the results make sense?
- Are there simpler explanations possible?

Assignments: wrapping things up

- Class Monday in 820 WBB, first half of spring ends Tuesday
- Online/take home final for undergrads/grads will be posted soon
- Undergrads could use MLIB next Wednesday IF you don't have a class in this time slot during the 2nd half
- It's ok to discuss coding issues of exam with others; not the answers to any questions, work independently on those