Bonn-Aachen International Center for Information Technology
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Brain Tractography Registration with Nonrigid ICP

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ABSTRACT

The registration problem or sometimes called alignment or absolute orientation is one of fundamental problem in computer vision. If we have two objects and we need to align them that means we should reduce the distance between them by making one object fix and move the other one to the closest distance, this simple form of alignment called rigid transformation, if we add scaling then it call non-rigid transformation and it will extend the size of the object as well. Due to its fundamental importance in computer vision, it is necessary step in many different applications, for instance: object recognition, tracking, range data fusion, graphics, medical image alignment, robotics and structural bioinformatics etc [**?**].

As we mention, the registration is important in medical image alignment, we are going to apply it on Brain fiber pathways, which is 3D images data. This type of data is not easy to align due to slightly different shape of the same bundle in different brains, and sometimes alignment occur between left side and right side of the same bundle.

One of the famous methods to solve this problem is Iterative Closest Point (ICP), wich we apply in this thesis. The main idea is in each point on the moving object we find the closest point on the fixed object, with respect to threshold, and try to minimize the overall distance between them, the minimal overall distance can be reached by solving the cost function that we build, which we will discuss later in this thesis. ICP can be applied in different algorithms, in our case we use LSQR algorithm to solve our problem and we have sufficient results.

Then we compare our result with registration method from dipy library.

1

INTRODUCTION

The human brain, which is the focus of our work in this thesis, is the central organ of the human nervous system. It is made up of two main components, namely, gray matter and white matter. Researchers have discerned a great deal about gray and white matter and distinct brain regions through autopsies and imaging techniques. The study of the human brain in diseased states or under conditions associated with brain damage have resulted in major insights into this complex organ.

1 BRAIN ANATOMY :FIBER PATHWAYS

White matter refers to areas of the central nervous system that are mainly made up of myelinated axons, also called tracts or fiber pathways [**?**]. It is composed of bundles which connect various gray matter areas of the brain to each other and carry nerve impulses between neurons. Myelin acts as an insulator allowing electrical signals to jump rather than course through axons, increasing the speed of transmission of all nerve signals through a phenomenon known as saltatory conduction [**?**].
Long thought to be passive tissue, white matter affects learning and brain functions, modulates the distribution of action potentials, and acts as a relay and coordinator of communication between different brain regions [**?**].

The human brain consists of the following tracts on the left and right sides:

Thalamic Radiation (ATR) refers to fiber pathways that connect the anterior nuclear group of the thalamus and the midline nuclear group of the thalamus with the frontal lobe through the anterior thalamic peduncle, the anterior limb of the internal capsule and other parts of the cerebral white matter [**?**][**?**]. ATR abnormalities have a possible link with cognitive abnormalities and negative symptoms in schizophrenia[**?**].

Corpus Callosum (CC) is a wide, flat bundle of nerve fibers, located at the longitudinal fissure beneath the cortex, which acts a link between the two hemispheres of the brain and facilitates communication between them. The term corpus callosum means 'tough body' in Latin. With approximately 200 - 250 million contralateral axonal projections to its credit, it is the largest among the various white matter structures in the central nervous system. The anterior

Figure 1: The Human Brain Fiber Pathway

portion of this structure is called *'genu'*, while the posterior structure is called *'splenium'*. In between its anterior and posterior portions, lies the 'truncus' or its *'body'*. Studies have revealed that the anterior of corpus callosum in left-handed people is eleven percent (100%) larger than that of right-handed people [**?**].

Genu of the Corpus Callosum (genu) refers to the rostral most portion of the corpus callosum. It is bounded caudally by the body of the corpus callosum and ventrocaudally by the rostrum of the corpus callosum [**?**].

Splenium of the Corpus Callosum (splenium) refers to the caudal most portion of the corpus callosum. It is bounded rostrally by the body of the corpus callosum [**?**]. It overlaps the tela chorioidea of the third ventricle and the mid-brain, and ends in a thick, convex, free border. A sagittal section of the splenium shows that the posterior end of the corpus callosum is acutely bent forward, the upper and lower parts being applied to each other [**?**].

Body of Corpus Callosum (truncus) refers to the portion of the corpus callosum located between the genu of the corpus callosum and the splenium of the corpus callosum. In a common parcellation, corpus callosum, it is divided into four parts: the rostral body of the corpus callosum, the anterior midbody of the corpus callosum, the posterior midbody of the corpus callosum and the isthmus of the corpus callosum [**?**]. **Cingulum (Cing)** refers to a fiber pathway that runs longitudinally in the cingulate white matter; it connects portions of the cingulate gyrus, the parietal lobe and the prefrontal cortex with the parahippocampal gyrus and adjacent structures of the temporal lobe. ”All connectios entering and exiting the cingulate gyrus pass through the cingulum bundle”. It is composed of the Cingulum ammonale and the Cingulum limitans [**?**].

Corticospinal Tract (CST) refers to a fiber pathway from the cerebral cortex to the spinal cord. Its fibers originate from pyramidal neurons of the precentral gyrus, and on their way to the spinal cord, they pass through parts of the cerebral white matter (including the posterior limb of the internal capsule), the crus cerebri, the longitudinal pontine fibers, the pyramid of the medulla (where they are known as the pyramidal tract) and the pyramidal decussation. In the ducussation, some fibers cross to the other side of the brainstem to form the lateral corticospinal tract. Those fibers that do not cross split to form the anterolateral corticospinal tract and the anterior corticospinal tract [**?**].

Fornix (Fornix) The fornix (Latin, ”vault” or ”arch”) is a C-shaped bundle of fibers (axons) in the brain, and carries signals from the hippocampus to the hypothalamus. The fibres begin in the hippocampus on each side of the brain (where they are also known as the fimbria); the separate left and right sides are each called the crus of the fornix. The bundles of fibres come together in the midline of the brain, forming the body of the fornix. The inferior edge of the septum pellucidum (a membrane that separates the two lateral ventricles) is attached to the upper face of the fornix body. The body of the fornix travels anteriorly and divides again near the anterior commissure. The left and right parts separate, but there is also an anterior/posterior divergence. The posterior fibres (called the postcommissural fornix) of each side continue through the hypothalamus to the mammillary bodies; then to the anterior nuclei of thalamus, which project to the cingulate cortex. The anterior fibers (precommissural fornix) end at the septal nuclei and nucleus accumbens of each half of the brain [**?**].

While its exact function and importance in the physiology of the brain are still not entirely clear, it has been demonstrated that surgical transection – the cutting of the fornix along its body – can cause memory loss [**?**]. There is some debate over what type of memory is affected by this damage, but it has been found to most closely correlate with recall memory rather than recognition memory. This means that damage to the fornix can cause difficulty in recalling long-term information such as details of past events, but it has little effect on the ability to recognize objects or familiar situations [**?**].

Inferior Fronto-occipital Fasciculus (IFO) The occipitofrontal fasciculus passes backward from the frontal lobe, along the lateral border of the caudate nucleus, and on the medial aspect of the corona radiata; its fibers radiate in a fan-like manner and pass into the occipital and temporal lobes lateral to the posterior and inferior cornua [**?**].

Inferior Longitudinal Fasciculus (ILF) The inferior longitudinal fasciculus connects the temporal lobe and occipital lobe, running along the lateral walls of the inferior and posterior cornua of the lateral ventricle. The existence of this fasciculus independent from the occipitotemporal fasciculus has been questioned for the human being, such that it has been proposed that the term inferior longitudinal fasciculus be replaced by the term "occipitotemporal projection" [**?**].

Superior Longitudinal Fasciculus (SLF) refers to a fiber pathway in the cerebral white matter. It is composed of fibers that connect the cortex of the frontal lobe with cortex of the occipital lobe and the temporal lobe. Some authors refer to the connection with the temporal lobe as the arcuate fasciculus [**?**].

is a pair of long bi-directional bundles of neurons connecting the front and the back of the cerebrum. Each association fiber bundle is lateral to the centrum ovale of a cerebral hemisphere and connects the frontal, occipital, parietal, and temporal lobes. The neurons pass from the frontal lobe through the operculum to the posterior end of the lateral sulcus where numerous neurons radiate into the occipital lobe and other neurons turn downward and forward around the putamen and radiate to anterior portions of the temporal lobe [**?**].

Ventral Tegmental Area (VTA) After substantia nigra, VTA which is situated adjacent to the substantia nigra in the midbrain is one of the major dopaminergic areas in the brain. Even though there is no clear anatomical separation between the two, there are areas that seem to differ slightly.

Together with an integral part of a network of structures, it is known as the reward system involved in reinforcing behavior. VTA is also thought to play major role in motivation, reward, emotional and cognitive processes. VTA is activated with experiencing something rewarding which may be necessary to the development of addiction.

Other than addiction VTA is involved in pathophysiology of disorders as in case of schizophrenia where dopaminergic neurons in the VTA have been proposed to play a role and attention-deficit hyperactivity disorder (ADHD) has been linked to low dopamine activity in the VTA [**?**].

2 REGISTRATION

The registration problem or sometimes called alignment or absolute orientation is one of fundamental problem in computer vision. If we have two objects and we need to align them that means we should reduce the distance between them by making one object fix and move the other one to the closest distance, this simple form of alignment called rigid transformation, if we add scaling then it call non-rigid transformation and it will extend the size of the object as well. Due to its fundamental importance in computer vision, it is necessary step in many different applications, for instance: object recognition, tracking, range data fusion, graphics, medical image alignment, robotics and structural bioinformatics etc [**?**].

2.1 *Iterative closest point (ICP)*

ICP, which is an algorithm employed to minimize the distance between two or more points clouds, is one of the most widely used algorithms in aligning three dimensional models given an initial guess of the rigid body transformation required [**?**]. In ICP (in our case) one points cloud (i.e., vertex cloud), the reference, or target, is kept fixed, while the other one, the source, is transformed to best match the reference. The algorithm iteratively revises the transformation (combination of translation, rotation and scaling) needed to minimize a distance from the source to the reference points cloud.

3 PCA TRANSFORMATION

PCA is mathematically defined as an orthogonal linear transformation that transforms the data to a new coordinate system such that the greatest variance by some projection of the data comes to lie on the first coordinate (called the first principal component), the second greatest variance on the second coordinate, and so on[**?**]. PCA is used in the code as a preliminary step, so that the template and target are aligned as much as possible before the registration can begin.

4 LEAST SQUARES (LSQR)

LSQR uses an iterative method to approximate the solution. The number of iterations required to reach a certain accuracy depends strongly on the scaling of the problem ($||Ax - b||^2$). Poor scaling of the rows or columns of A should therefore be avoided where possible [**?**].

2

METHOD

1 THE ICP FRAMEWORK

The main subject of this thesis is demonstrating how the ICP framework can be extended to nonrigid registration, whilst keeping properties of the original algorithm. The principle idea is the application of the work presented in [**?**]. The extended ICP framework uses different regularisations to control the incrementally deformation of template towards the target whilst decreasing the distance and stiffness weight iteratively, recovering the whole range of global and local deformations.

As has been defined in the introduction, vertex registration is a problem in which two or more datasets of points are given and the task is to optimally align them by estimating a best transformation. In our case, we use a dense registration method to find a mapping from each point in the template onto the target while sparse methods find correspondence only for selected feature points. This is done by deforming the template, locally moving it closer in each iteration to the target in order to wrap them together with respect to stiffness.

ICP moves the template S towards the target T step by step. In each iteration, it minimizes the difference between the template S and the target, T as illustrated in figure {**??**}, to reach the minimal value by solving the main equation (**??**):

$$||Ax - b||^2 \tag{1}$$

In equation (**??**) x is a list of X_i, each X_i is a 3×4 affine matrix which uses homogeneous coordinates $[x, y, z, 1]$ in 3D Euclidean space. By stacking X_i together we get matrix x with size $4n \times 3$ as shown in equation (**??**):

$$X = [X_1, X_2, ..., X_n]^T \tag{2}$$

If we were to simply solve the equation (**??**) by assigning A as the template and b as the target, we will get exactly $Ax = b$, which means the difference between them is zero. That leads to the deforming of A and a complete loss of its shape, therefore, we need to add the stiffness part to the equation that will prevent this deformation by keeping the vertices originally as close to each other as possible which we will later explain in this chapter.

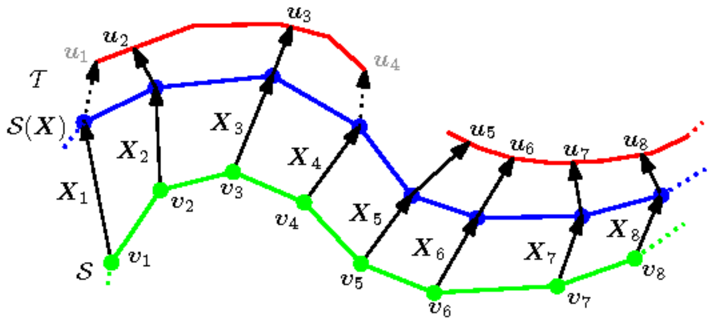


Figure 2: The template graph S (green) is deformed by locally affine transformations (X_i) onto the target graph T (red). The algorithm determines closest points (u_i) for each displaced source vertex $(X_i v_i)$ and finds the optimal deformation for the stiffness used in this iteration. This is repeated until a stable state is found. The process then continues with a lower stiffness. Due to the stiffness constraint the vertices do not move directly towards the target graph, but may move parallel along it. The correspondences u_1 and u_4 are dropped as they lie on the border of the target [**?**].

2 SETUP DATA

The method presented in [**?**] deals with surface graph representations of the data $S = (V, E)$ where S, V and E are assigned to graph, vertices and edges, respectively. As described in the introduction, our data contains human brain fiber bundles (pathways) saved in a ***ply*** data format, as shown in the simplified figure {**??**}. The ***ply*** format consists of three parts; the first, uppermost part is the header of the file which contains the file description (i.e. format, comment, etc) and the major components of the header element parts which describe how many elements each part has. In the sample in figure {**??**}, there are 69283 vertices in x,y,z and 603 fibers. The second part of the ***ply*** files contains vertices in 3D Euclidean coordinate space while the last part of the ***ply*** file represents the end index of each fiber. As the method in [**?**] require the template to be in graph format, we develop a tool to read ***ply*** files into a numpy array of arrays which can subsequently be used as a graph. To be able to use ***numpy ndarray*** as a graph, we put each tract in a separate array with respect to the link between points (Edges E). To do so, we put each point linked together next to each other, and then wrap all the tracts belonging to the same bundle (i.e ATR, CC, genu, splenium, etc) in a new ***numpy ndarray***.

3 PREPARING GRAPHS FOR ICP

After we read the data in graph format, we must align the template graph to the target graph as closely as possible. In order to do so, we use *Principal*

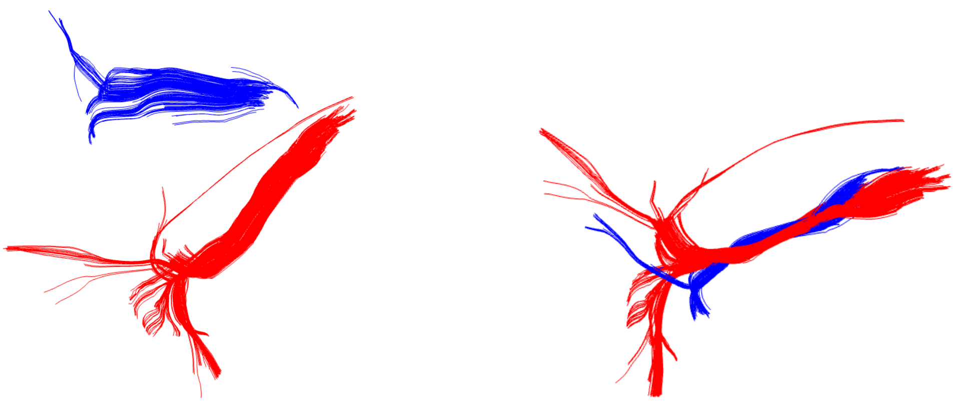
```
ply
format ascii 1.0
comment DTI Tractography, produced by fiber-track
element vertices 69283
property float x
property float y
property float z
element fiber 603
property int endindex
end_header
-4.71338558197 -19.9100589752 4.76097154617
-4.73113059998 -19.3581771851 4.68979740143
-4.72901630402 -18.8120174408 4.5764541626
-4.79067516327 -18.2503032684 4.52395439148
......
152
299
364
494
637
767
```

Figure 3: Simplified ***ply*** file sample

Components Analysis (PCA). Before applying PCA, we scale the template graph and the target graph to a $[0,1]$ scale to match each other. This is done by subtracting all points from the minimum value in the graph and then dividing them by the maximum value in the graph. Next, we apply PCA and use $[-1,1]^3$ cube combination and measure the distance between each point in the template graph to the closest point in the target graph $\sum ||v_i - v_j||_F^2$ where $v_i \in S, v_j \in T$ eight times to select the combination with the minimum distance as is illustrated in table {**??**}.

1	2	3	4	5	6	7	8
(x,y,z)	(x,y,-z)	(x,-y,z)	(x,-y,-z)	(-x,-y,z)	(-x,y,-z)	(-x,y,z)	(-x,-y,-z)

Table 1: $[-1,1]^3$ cube combination



(a) Original position (b) After PCA

Figure 4: PCA alignment

4 DISTANCE MEASUREMENT

In all processes in our code, we use K-D tree (or k-dimensional) to measure the euclidean distance between the points, including in PCA transformation, ICP or any other step in which we measure the distance.

4.1 *The K-D tree*

"I will right something about K-D tree"

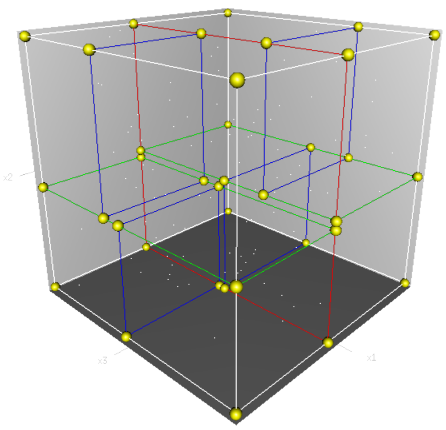


Figure 5: A 3-dimensional k-d tree. The first split (the red vertical plane) cuts the root cell (white) into two subcells, each of which is then split (by the green horizontal planes) into two subcells. Finally, four cells are split (by the four blue vertical planes) into two subcells. Since there is no more splitting, the final eight are called leaf cells [**?**].

5 FINDING THE AFFINE TRANSFORMATION MATRICES

After we prepare our data, we refer back to [**?**] and determine how we need to build the cost function. As we are going to explain, it consists of three parts: the distance term, stiffness term and landmark term.

As we mention above, the cost function, as shown in equation (**??**), consists of three terms, each of which represent different concepts: the distance term, stiffness term and landmark term. The distance term represents the distance between the template graph S and target graph T which has to be as minimal as possible (i.e., which must be minimized). The stiffness term regularizes the template graph to prevent it from being exactly the target graph. The final term is the landmark term which gives the initial case for the template graph.

$$E(X) = E_d(X) + \alpha E_s(X) + \beta E_t(X) \tag{3}$$

Let us illustrate the first term, **the distance term**. As we mentioned before, it is the euclidean distance between points in the template graph S and points in the target graph T, which can be written as represented in equation (**??**). This distance has to be minimized as much as possible to align the template graph S to the target graph T with respect to the original shape of the template graph S. That means we need to ensure (by using the stiffness term, which we will discuss later in this section) that all points which are close to each other stay as near to each other as possible.

$$E_d(X) = \sum_{v_i \in V} w_i dist^2(T, X_i v_i) \tag{4}$$

In equation (**??**), v_i represent the vertices V in the template graph S in homogeneous coordinate $v_i = [x, y, z, 1]^T$, such that they can be multiplied by affine matrices X_i. These affine matrices are 3×4 matrices including the transformation part, as illustrated in equation (**??**). The distance between each vertex $v_i \in V$ in the template graph S and its closest vertex $u_i \in U$ in target graph T noted with $dist^2(T, X_i v_i)$, and $w \in [0, 1]$ is weight which is *one* if the correspondence for this vertex v_i is found in the target graph S, and *zero* otherwise. This concept will let each vertex v_i in the template graph T moves toward its correspondence vertex u_i in the target graph T if it is found, otherwise, it will move with its neighbors because of the stiffness. Essentially, we use a distance threshold as a parameter to distinguish correspondances.

Now we come to the second term, **the stiffness term**, which regularizes the shape deformation, represented in equation (**??**):

$$E_s(X) = \sum_{i,j \in E} ||(X_i - X_j)G||_F^2 \tag{5}$$

Regularizing the deformation occurs by penalising the weighted difference of the transformations of neighbouring vertices under the Frobenius norm $||.||_F$ using a weighting matrix $G := diag(1, 1, 1, \gamma)$.

The parameter γ is used to balance the rotational and skew against translation while transforming the template graph S [**?**]. The value of parameter γ depends on the units of the data and on the type of deformation that shall be expressed. In our case we select it to be one as we have already scaled our data into the $[-1, 1]^3$ cube.

We use twelve parameters per vertex in 3×4 shape. This will allow us to write the cost function as a quadratic function which can be solved directly [**?**].

The Frobenius norm in equation (**??**) defined for matrix A is demonstrated in equation (**??**):

$$||A||_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2} \tag{6}$$

By applying this equation we keep the vertices which are linked together (i.e., there is an edge between them or they are in the same tract) or neighbors close to each other.

The α parameter in equation (**??**) is a constant that manages the effect of the stiffness term; when it is high the neighbor vertices doesn't move far from eachother and when it is low, a greater deformation can occur.

The final part of the equation (**??**) is the **Stiffness term** which is demonstrated in the equation below (**??**):

$$E_l(X) = \sum_{(vi,l)\in L} ||(X_i v_i - l)||^2 \tag{7}$$

Amberg et al. mention the landmark initializes and guides the registration. Given a set of landmarks $L = \{(v_{i1}, l_1), (v_{i2}, l_2), ..., (v_{il}, l_l)\}$ mapping template graph S vertices V_i into the target graph T vertices U_i.

The registration can be found even without landmarks. Without landmarks, the cost function has global minima where the template is collapsed onto a point on the target surface, but the local minimum corresponding to the correct registration can be found for a wide range of initial conditions.

In our case, we use PCA transformation as the initial step to align two graphs, therefore, we omit the landmark term later on from our cost function This is considered to be a first optimization to the cost function due to a reduction of computational effort and time to calculate the landmark term in our code.

We illustrate now the cost function in more depth and transform the graphs in matrices that suit our cost function to ease implementation of the method.

The **distance term** in equation (**??**) and (**??**) become:

$$\bar{E}_d(X) = \sum_{v_i \in V} w_i ||X_i v_i - u_i||^2 \tag{8}$$

$$= \left\| (W \otimes I_3) \left(\begin{bmatrix} X_1 & & \\ & \ddots & \\ & & X_n \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} - \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \right) \right\|^2$$

where $W = diag(w_1, \dots, w_n)$ corresponds to the weight in the diagonal matrix multiplied by Kronecker product $\otimes$ to I_3 identity matrix. X is diagonal matrix of X_i that we need to solve and that is multiplied by V which are vertices of template graph S, subtracted by the corresponding vertices U of the target graph T.

the **Kronecker product**, denoted by $\otimes$, is an operation on two matrices of arbitrary size resulting in a block matrix. It is a generalization of the outer product, (denoted by the same symbol) from vectors to matrices, and gives the matrix of the tensor product with respect to a standard choice of basis [**?**].

For instance, If A is an $m \times n$ matrix and B is a $p \times q$ matrix, then the Kronecker product $A \otimes B$ is the $mp \times nq$ block matrix:

$$A \otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix}$$

Thus, the result of $W \otimes I_3$ is a $3n \times 3n$ matrix, where matrix X has size $3n \times 4n$, V has a shape $4n \times 1$ and U has shape $3n \times 1$.

We continue to reform the equation to be easily differentiated by converting it into canonical form. We swap the position of the X and V matrices. For matrix V we define a sparse matrix D which contains the the vertices $v_i \in V$ in diagonal position as demonstrated below:

$$D = \begin{bmatrix} v_1^T & & & \\ & v_2^T & & \\ & & \ddots & \\ & & & v_n^T \end{bmatrix} \tag{9}$$

The new format of the equation becomes:

$$\bar{E}_d(X) = ||W(DX - U)||_F^2 \tag{10}$$

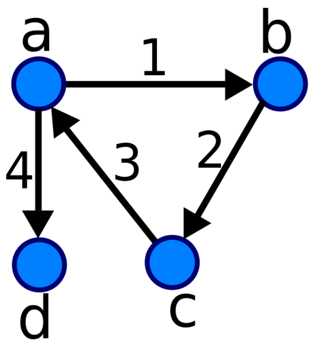
Where the matrix W has size $n \times n$, the matrix D has size $n \times 4n$ (vertices v_i are in homogeneous coordinates $[x, y, z, 1]^T$), X has size $4n \times 3$, and U doesn't change with shape $n \times 3$ (vertices u_i are in 3D coordinate).

We continue to reform **the stiffness term** which penalises differences between the transformation matrices (affine matrices) X assigned to neighboring vertices. The simplified form of the stiffness term will be:

$$E_s(X) = ||(M \otimes G)X||_F^2 \tag{11}$$

where M is a node-arc incidence matrix defined for directed graphs. The number of rows in this matrix equal the number of nodes (vertices) and the number of columns equal the number of arcs (edges) on the graph and the values has to be *zeros, ones and/or minus ones*. The value will be *zero* if the edge on this column is not connected to the vertex on this row where the value lies. Otherwise, the value must be either 1 or -1; it will be 1 if the edge direction is coming towards the vertex and -1 otherwise. As illustrated in the figure {**??**}, the matrix M has size $e \times n$ where $eandn$ are the number of edges and vertices respectively. $G = diag(1, 1, 1, y)$ is the diagonal matrix and the result of $M \otimes G$ is a matrix with size $4e \times 4n$.

The final term is **the landmark term** which needs to be reformed as well; the process of doing so is similar to that of the distance term. We only consider



	1	2	3	4
a	1	0	-1	1
b	-1	1	0	0
c	0	-1	1	0
d	0	0	0	-1

Figure 6: Oriented graph with corresponding incidence matrix [**?**]

those vertices from D (*D from the distance term (**??**)*) which are landmarks in a new matrix called D_L, and vertices from U (which are vertices from target graph T) which are corresponded to those landmarks. These in turn are put in a new matrix denoted $U_L = [l_1, l_1, \ldots, l_l]^T$.

The final shape of the cost function is a quadratic function, as shown below:

$$\bar{E}(X) = \left\| \begin{bmatrix} \alpha M \otimes G \\ WD \\ \beta D_L \end{bmatrix} X - \begin{bmatrix} 0 \\ WU \\ U_L \end{bmatrix} \right\|^2 = ||AX - B||_F^2 \tag{12}$$

The current form of the function can be minimized by setting its derivative to zero and solving it as linear equation. The minimum value of $\bar{E}(X)$ is when $X = (A^T A)^{-1} A^T B$. A is also a sparse matrix and most of its values are zeros. However, it is still a large matrix and requires the most time in solving the equation. In order to solve this problem, we will remove the landmark term as mentioned before because we already have the initial step by applying PCA. Once we omit the landmark term, the function will look like this:

$$\bar{E}(X) = \left\| \begin{bmatrix} \alpha M \otimes G \\ WD \end{bmatrix} X - \begin{bmatrix} 0 \\ WU \end{bmatrix} \right\|^2 = ||AX - B||_F^2 \tag{13}$$

6 APPLYING THE TRANSFORMATIONS

Let us review the principle of affine transformation briefly. If we have a point $[x, y, 1]$ in homogeneous coordinate in 2D euclidean space and we want to move this point three units through the X *axis*, $[x, y, 1]^T a = [x+3, y, 1]^T$, then in this case, a is called the affine matrix.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x+3 \\ y \\ 1 \end{bmatrix}$$

The example mentioned above is only for translation; we still have some other moves (i.e. sheer, rotation and scale) which can be performed, as demonstrated in figure {??} for 2D shape.

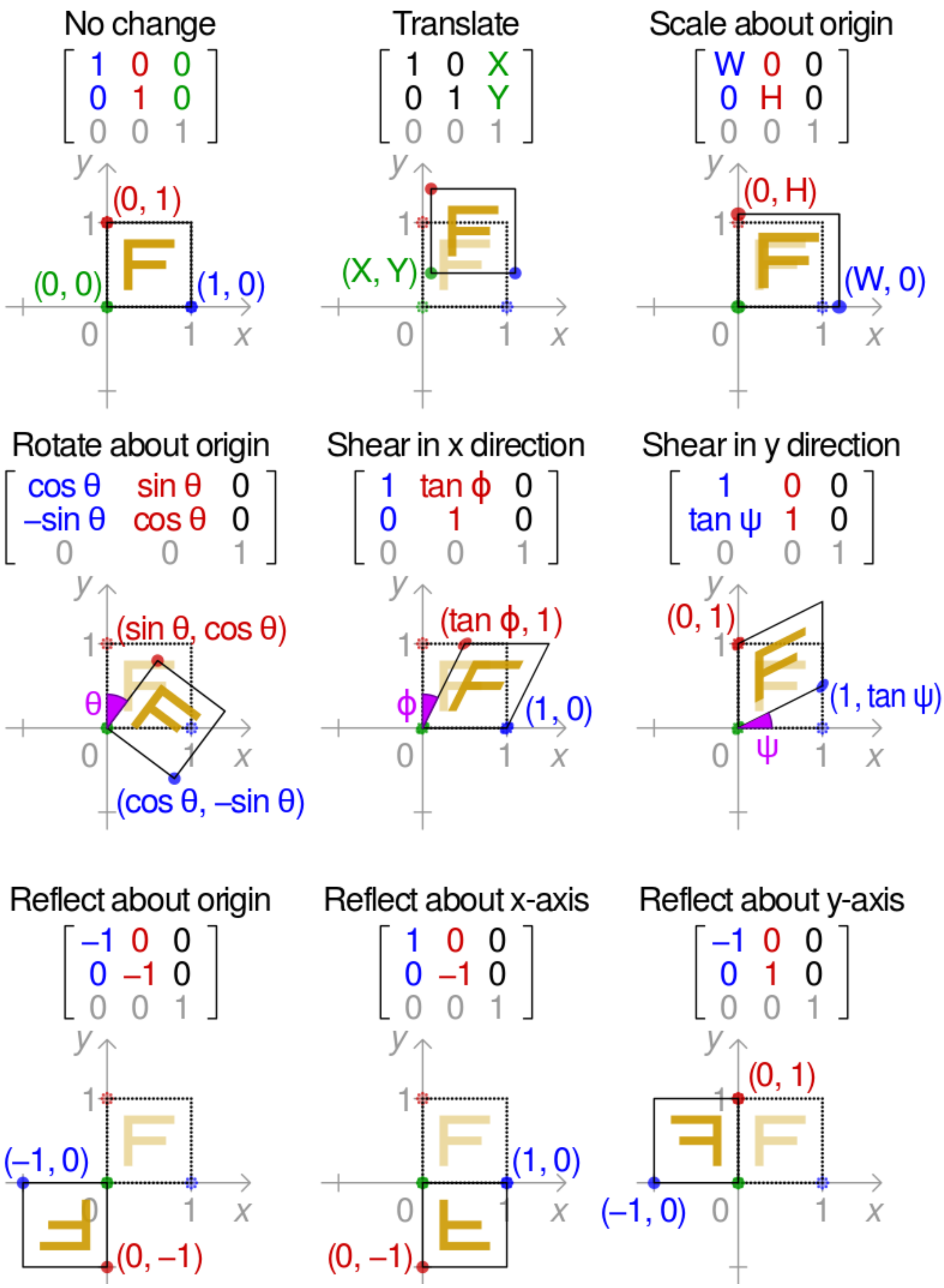


Figure 7: 2D affine transformation matrices [?]

We have shown how 2D transformation occurs; the same principles apply to 3D as well, as demonstrated below:

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} \begin{bmatrix} a & b & c & e \\ f & g & h & i \\ j & k & l & m \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

However, this is still for one point; if we have more points as in our case, the equation above becomes:

$$\begin{bmatrix} \hat{x_1} & \hat{x_2} & \dots & \hat{x_n} \\ \hat{y_1} & \hat{y_2} & \dots & \hat{y_n} \\ \hat{z_1} & \hat{z_2} & \dots & \hat{z_n} \\ 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} a & b & c & e \\ f & g & h & i \\ j & k & l & m \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \\ z_1 & z_2 & \dots & z_n \\ 1 & 1 & \dots & 1 \end{bmatrix}$$

7 NONRIGID OPTIMAL ICP ALGORITHMS

As we mentioned in the introduction, ***Iterative Closest Point (ICP)*** is an algorithm employed to minimize the distance between two or more points clouds. To achieve this goal, we use ***Sparse Least Squares algorithm (LSQR)*** which is an iterative method for solving $Ax = b$ and $min||Ax - b||^2$, where the matrix A is large and sparse (as in our case). The method is based on the bidiagonalization procedure of Golub and Kahan [**?**]. It is analytically equivalent to the standard method of conjugate gradients, but possesses more favorable numerical properties [**?**].

We will demonstrait the idea of using *LSQR* to solve our cost function in the next chapter implementation.

3

IMPLEMENTATION

To implement the method we have discussed before, we wrote the code in *Python Programming Language* and *Spyder* version 3 and *Integrated Development Environment (IDE)* are used.

As we mentioned before, our data is in ***ply*** format as shown in figure {**??**}, to read this format we wrote our own method using *plyfile* library due to special case of our data which has different arrangement files. The method we wrote return data in *numpy ndarray* structure where each array tract is putted in an array and all tracts belong to the same bundle were putted together with respect to the sequences of the neighboring vertices.

Then we apply PCA transformation (*sklearn.decomposition* library) to the data and generate histogram (*matplotlib.pyplot* library) for distances between each vertex in the template graph S and its closet vertex in target graph T, before and after PCA transformation to compare the distance difference, because in some cases where the two graphs (template and target) are already in the best alignment before ICP, PCA transformation increase the distance between them as we will show in the result chapter. If the distance between two graphs is increased we drop the PCA transformation step. For distance measurement we use *K-D tree* from *scikit learn*.

We continue to build variables as shown in equation (**??**), to do so, we use histograms generated in the previous step to determine the threshold for the maximum distance we need to consider for W in the **distance term** for cost function, if the value is equal or below the threshold $w_i = 1$, otherwise $w_i = 0$. Then we build W as diaginal sparse matrix using *scipy.sparse* library. Then we continue using the same library (*scipy.sparse*) to build D sparse matrix and calculate WD and WU.

Now we have **the distance term** of the cost function, we need to build the **stiffness term**, we use the same library (*scipy.sparse*) to build M sparse matrix and G diagonal sparse matrix and calculate Kronecker product ($M \otimes G$).

The final step of building the cost function is to vertically stack MG and WD to have A, and vertically stack *zero* sparse matrix and WU to have B as require $||Ax - b||_F^2$.

The last step is solving the cost fuction $||Ax - b||_F^2$ by using ***LSQR*** from *scipy.sparse.linalg* library. As mentioned in the *scipy.sparse.linalg.lsqr* documentation [?], b in equation $Ax - b$ must be a vector, and as b in our case is a matrix size $n \times 3$, we use matrices fundamental concept to solve this problem as we use **lsqrt** three times for each column and horizontally stack the result to get the affine matrices combination.

Finally for visualization we customize functions from *open3d* library to view brain bundles due to special case of our data.

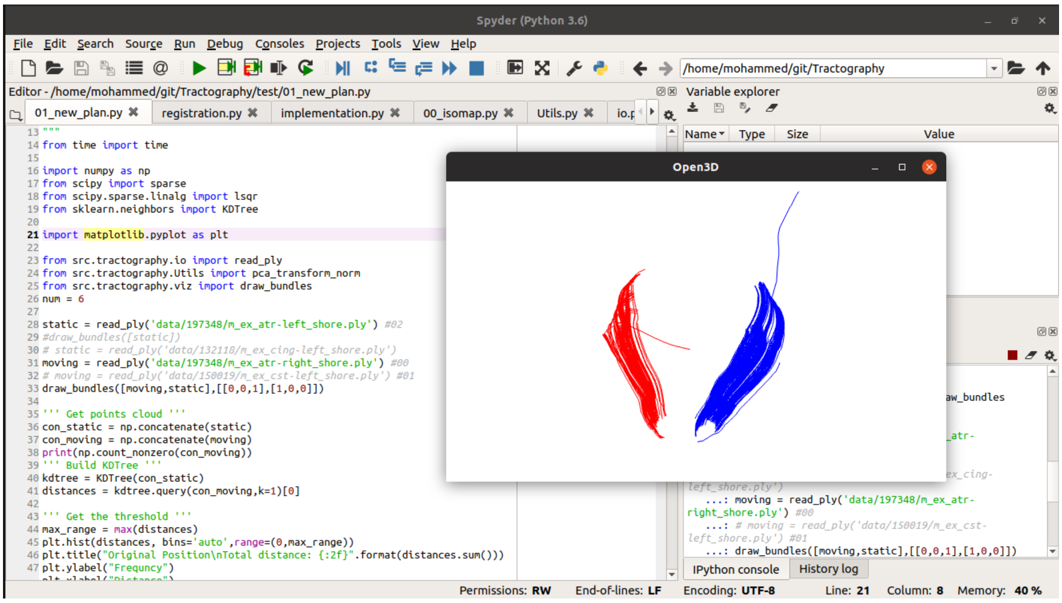


Figure 8: IDE and visualization tool

4

RESULTS

Outline

- The device specification
- Number of sample and description
- Data source (from where we get it)
- Number of points and tracts
- The algorithm and method to compare with
- Comparison :
  - ISO MAP [to be discuss]
  - Time
  - Distance
  - Visual

Figure 9: To be edited

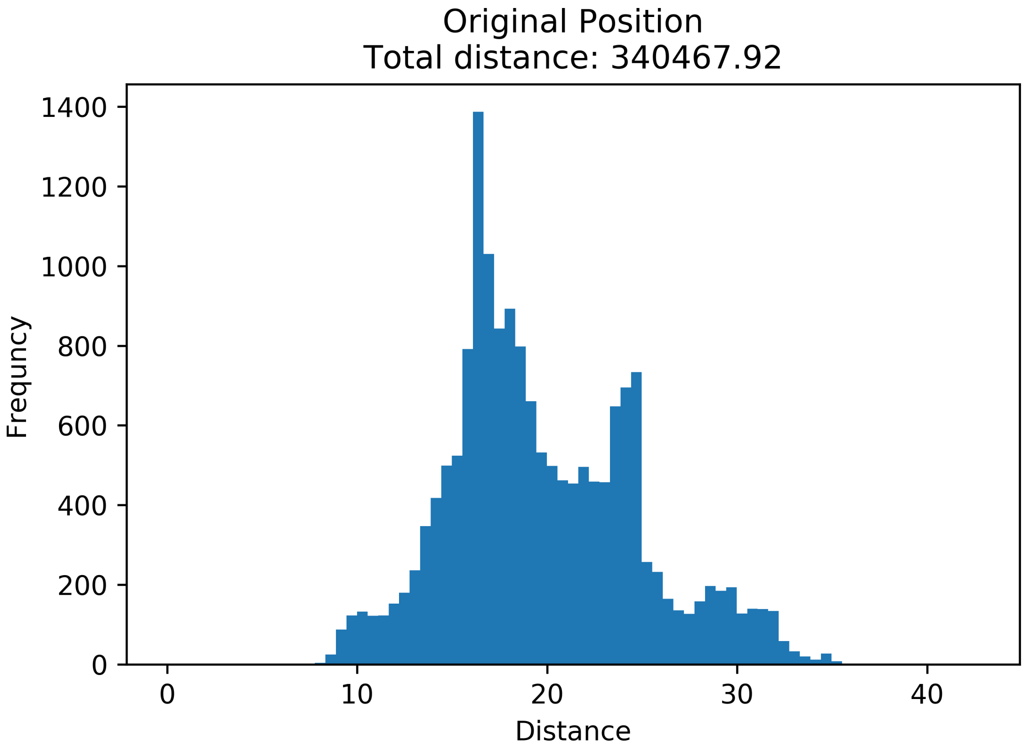


Figure 10: To be edited

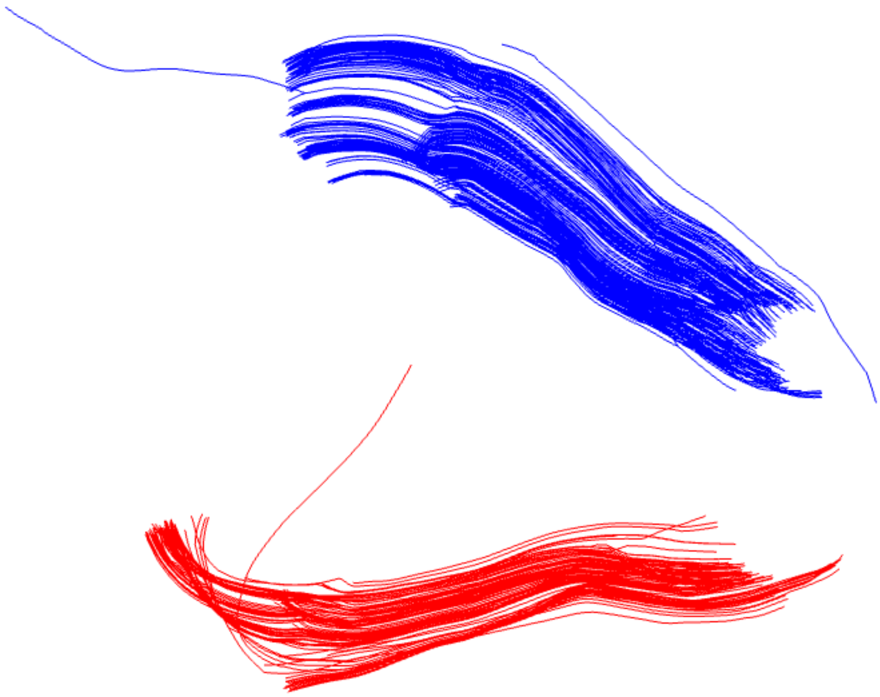


Figure 11: To be edited

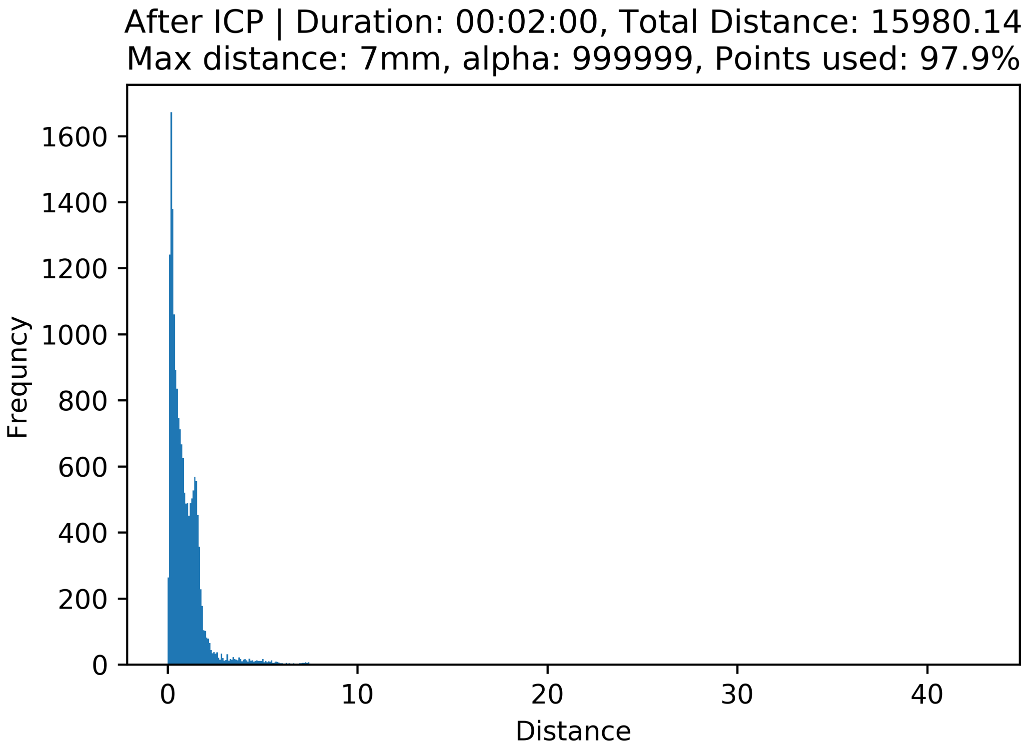


Figure 12: To be edited

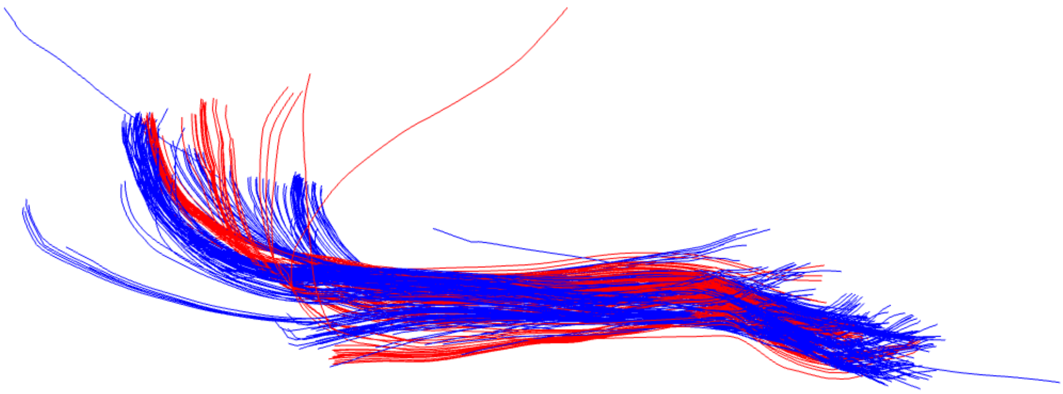


Figure 13: To be edited

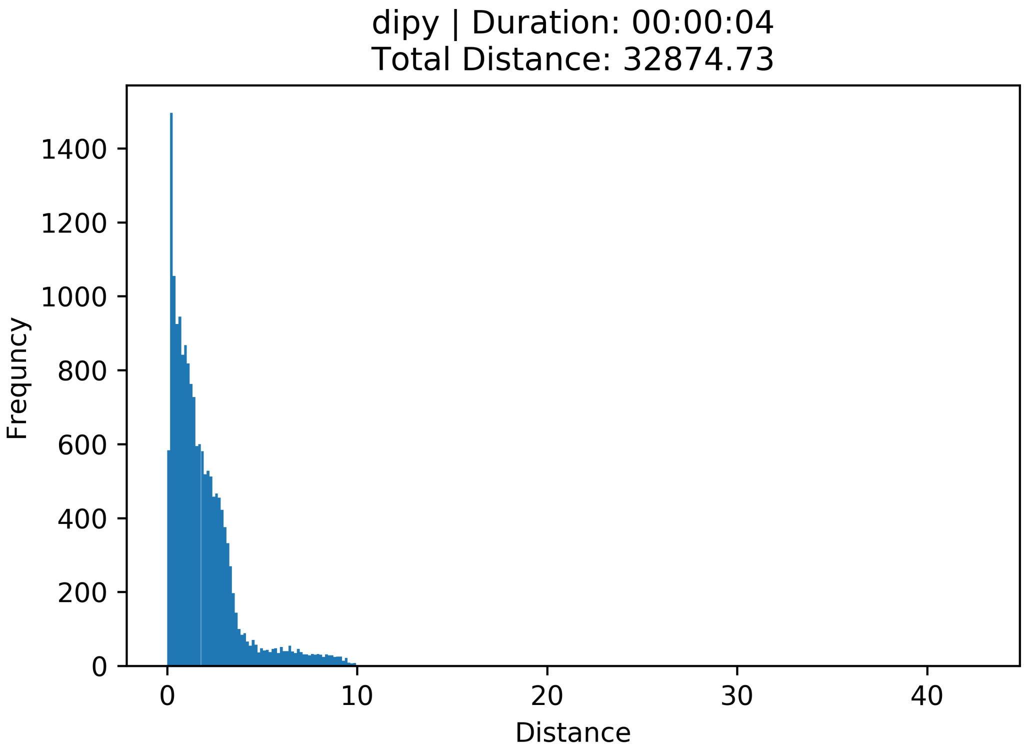


Figure 14: To be edited

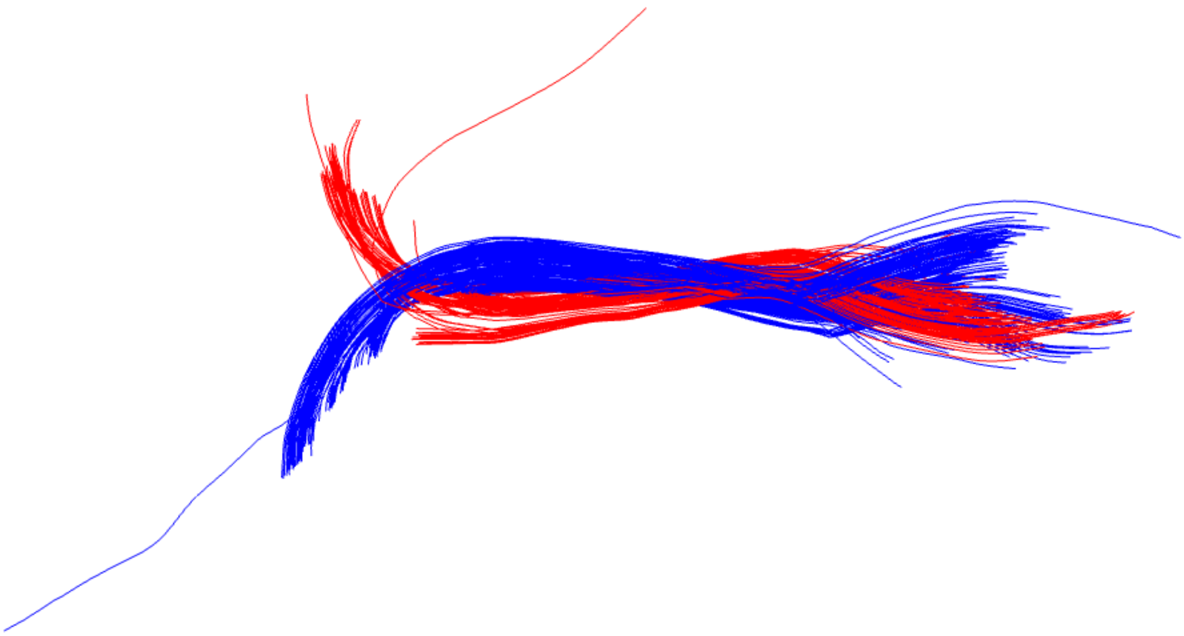


Figure 15: To be edited

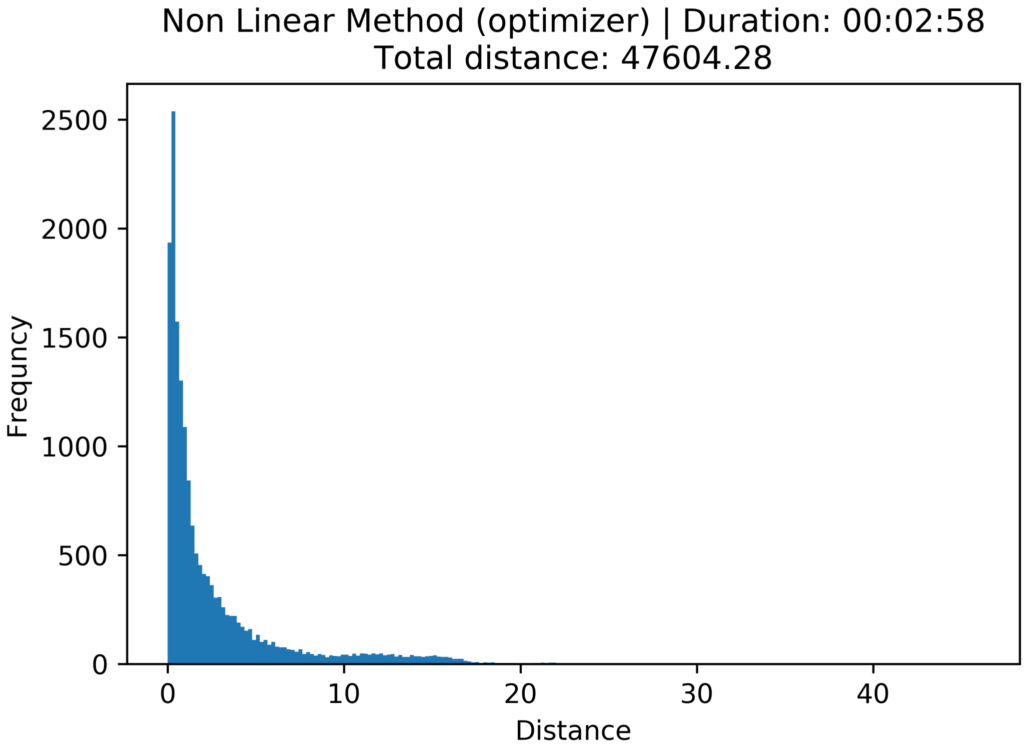


Figure 16: To be edited

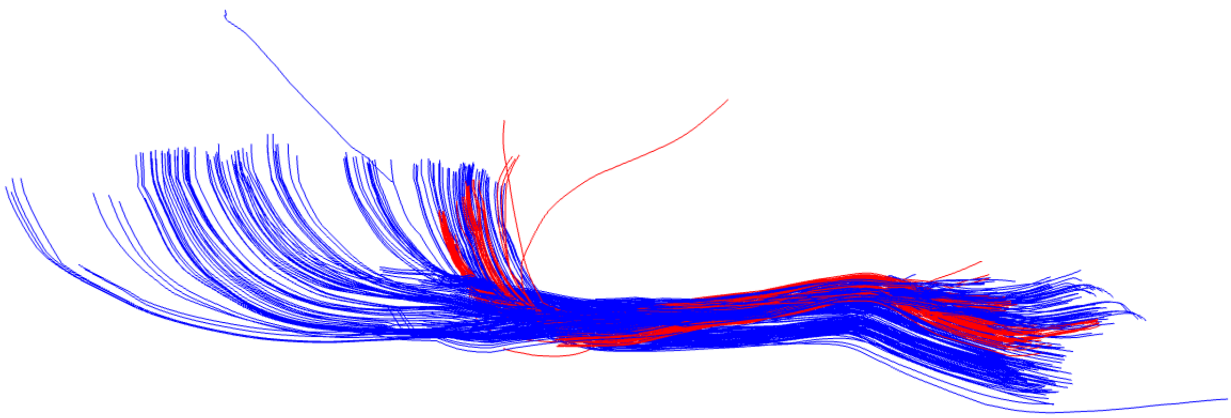


Figure 17: To be edited

5

CONCLUSION