

# Contents

MACHINE Collect	2
MACHINE Collect1	3
MACHINE Collect2	4

**MACHINE** Collect**VARIABLES**

coins

max\_odd

**INVARIANTS***inv\_1:*  $coins \subseteq \mathbb{N}$ *inv\_2:*  $finite(coins)$ *inv\_3:*  $max\_odd \in \mathbb{N}$ **EVENTS****Initialisation****begin***init\_1:*  $coins := \emptyset$ *init\_2:*  $max\_odd := 0$ **end****Event** add  $\langle ordinary \rangle \hat{=}$ **any**

c

**where***grd\_1:*  $c \in \mathbb{N}$ **then***act\_1:*  $coins := coins \cup \{c\}$ **end****Event** findMax  $\langle ordinary \rangle \hat{=}$ **any**

odds

**where***grd\_1:*  $coins \neq \emptyset$ *grd\_2:*  $\exists x \cdot x \in coins \wedge x \bmod 2 = 1$ *grd\_3:*  $odds = \{x \cdot x \in coins \wedge x \bmod 2 = 1 \mid x\}$ **then***act\_1:*  $max\_odd := max(odds)$ **end****END**

**MACHINE** Collect1**REFINES** Collect**VARIABLES**

collected\_odds

max\_odd

**INVARIANTS****inv1.1:**  $collected\_odds \subseteq \mathbb{N}$ **inv1.2:**  $finite(collected\_odds)$ **inv1.3:**  $collected\_odds = \{x \cdot x \in coins \wedge x mod 2 = 1 | x\}$ **EVENTS****Initialisation****begin****init1.1:**  $collected\_odds := \emptyset$ **init1.2:**  $max\_odd := 0$ **end****Event** addOdd  $\langle ordinary \rangle \hat{=}$ **refines** add**any**

c

**where****grd1.1:**  $c \in \mathbb{N}$ **grd1.2:**  $c mod 2 = 1$ **then****act1.1:**  $collected\_odds := collected\_odds \cup \{c\}$ **end****Event** ignoreEven  $\langle ordinary \rangle \hat{=}$ **refines** add**any**

c

**where****grd1.1:**  $c \in \mathbb{N}$ **grd1.2:**  $c mod 2 = 0$ **then***skip***end****Event** findMax  $\langle ordinary \rangle \hat{=}$ **refines** findMax**when****grd1.2:**  $collected\_odds \neq \emptyset$ **with****odds:**  $odds = collected\_odds$ **then****act1.1:**  $max\_odd := max(collected\_odds)$ **end****END**

**MACHINE** Collect2**REFINES** Collect1**VARIABLES**

current\_max  
max\_odd  
found\_odd

**INVARIANTS**

inv2.1:  $current\_max \in \mathbb{N}$   
inv2.2:  $found\_odd \in \text{BOOL}$   
inv2.3:  $found\_odd = \text{FALSE} \Rightarrow collected\_odds = \emptyset$   
inv2.4:  $found\_odd = \text{TRUE} \Rightarrow collected\_odds \neq \emptyset \wedge \max(collected\_odds) = current\_max$   
thm2.1:  $\langle \text{theorem} \rangle \forall p, s \cdot p \in \mathbb{N} \wedge s \subseteq \mathbb{N} \wedge s \neq \emptyset \wedge finite(s) \wedge p \leq \max(s) \Rightarrow \max(s) = \max(s \cup \{p\})$   
thm2.2:  $\forall p, s \cdot p \in \mathbb{N} \wedge s \subseteq \mathbb{N} \wedge s \neq \emptyset \wedge finite(s) \wedge p > \max(s) \Rightarrow p = \max(s \cup \{p\})$

**EVENTS****Initialisation****begin**

init2.0:  $current\_max := 0$   
init2.1:  $max\_odd := 0$   
init2.2:  $found\_odd := \text{FALSE}$

**end****Event** firstOdd  $\langle \text{ordinary} \rangle \hat{=}$ **refines** addOdd**any**

c

**where**

grd2.0:  $c \in \mathbb{N}$   
grd2.1:  $found\_odd = \text{FALSE}$   
grd2.2:  $c \bmod 2 = 1$

**then**

act2.0:  $current\_max := c$   
act2.1:  $found\_odd := \text{TRUE}$

**end****Event** addOdd  $\langle \text{ordinary} \rangle \hat{=}$ **refines** addOdd**any**

c

**where**

grd2.0:  $c \in \mathbb{N}$   
grd2.1:  $found\_odd = \text{TRUE}$   
grd2.2:  $c > current\_max$   
grd2.3:  $c \bmod 2 = 1$

**then**act2.0:  $current\_max := c$ **end****Event** ignoreEven  $\langle \text{ordinary} \rangle \hat{=}$ **refines** ignoreEven**any**

c

**where**

grd1.1:  $c \in \mathbb{N}$   
grd1.2:  $c \bmod 2 = 0$

**then**

skip

**end****Event** findMax  $\langle \text{ordinary} \rangle \hat{=}$ **refines** findMax

```
    when
      grd1_1: found_odd = TRUE
    then
      act1_1: max_odd := current_max
    end
  END
```