Independent vs. Conditional Probabilities.

1) $P(M) = 90\%$
$P(M \cap S) = 60\%$
$P(S \mid M) = \frac{P(M \cap S)}{P(M)}$
$= \frac{0.6}{0.9}$
$= 0.667$
$= 66.7\%$

2) a) $P(+IBP \mid DE) = \frac{P(+IBP \cap DE)}{P(DE)}$
$= \frac{2500}{3000}$
$= 0.833$
$= 83.3\%$
b) $P(DE \mid +IBF) = \frac{P(+IBP \cap DE)}{P(+IBF)}$
$= \frac{2500}{5000}$
$= 0.500$
$= 50.0\%$

3) a) i) $P(LP) = \frac{n(LP)}{n(S)}$
$= \frac{133}{406}$
$= 0.328$
$= 32.8\%$
ii) $P(ON) = \frac{n(ON)}{n(S)}$
$= \frac{231}{406}$
$= 0.569$
$= 56.9\%$
iii) $P(LP \cap LN) = \frac{n(LP \cap LN)}{n(S)}$
$= \frac{123}{406}$
$= 0.303$
$= 30.3\%$
iv) $P(LP \mid LN) = \frac{P(LP \cap LN)}{P(LN)}$
$= \frac{0.303}{0.431}$
$= 0.703$
$= 70.3\%$

b) They are not independent because $P(LP) \neq P(LP \mid LN)$

4) i) $P(G \mid R) = 36\%$
ii) $P(G \mid N) = 59\%$
iii) $P(B \mid R) = 64\%$
iv) $P(B \mid N) = 41\%$
b) $P(N \cap B) = P(N) \cdot P(B \mid N)$
$= (0.7)(0.41)$
$= 0.287$
$= 28.7\%$

5) a) $P(D \mid F) = \frac{1}{2}$
$P(F) = \frac{1}{15}$
$\therefore P(F) \neq P(D \mid F)$,
$\therefore$ failing and dropping aren't independent events.
b) $P(D \cap F) = P(F) \cdot P(D \mid F)$
$= (\frac{1}{15}) \cdot (\frac{1}{2})$
$= \frac{1}{30}$

6) a) not independent
b) independent
c) not independent
d) not independent

7) a) independent because $P(A) = P(A \mid B)$.
b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
$= \frac{1}{3} / \frac{2}{3}$
$= \frac{1}{2}$

not independent because $P(A) \neq P(A \mid B)$

8) a) $P(F \cap 7) = P(F) \cdot P(7)$
$= \frac{12}{52} \cdot \frac{4}{52}$
$= \frac{3}{169}$
independent. so
b) $P(E \cap A) = P(E) \cdot P(A \mid E)$
$= \frac{10}{52} \cdot \frac{4}{51}$
$= \frac{20}{663}$
not independent.
c) $P(K \cap K) = P(K) \cdot P(K \mid K)$
$= (\frac{4}{52}) \cdot (\frac{3}{51})$
$= \frac{1}{221}$
not independent.

9) a) Yes
b) $n((C, D, a)) = n(C) \cdot n(D) \cdot n(a)$
$= 2 \cdot 6 \cdot 52$
$= 624$
c) $P(H \cap 6 \cap K) = P(H) \cdot P(6) \cdot P(K)$
$= (\frac{1}{2}) \cdot (\frac{1}{6}) \cdot (\frac{4}{52})$
$= \frac{1}{156}$
d) $P(T \cap rA) = P(T) \cdot P(rA)$
$= (\frac{1}{2}) \cdot (\frac{2}{52})$
$= \frac{1}{52}$
e) $P(O \cap O \cap O) = P(O) \cdot P(O) \cdot P(O)$
$= (\frac{1}{2}) \cdot (\frac{3}{6}) \cdot (\frac{28}{52})$
$= \frac{7}{52}$
f) $P(E \cap 24) = P(E) \cdot P(24)$
$= (\frac{1}{2}) \cdot (\frac{3}{6})$
$= \frac{1}{4}$
g) $P(3) = (\frac{1}{2}) \cdot (\frac{1}{6}) \cdot (\frac{4}{52})$
$= \frac{1}{156}$