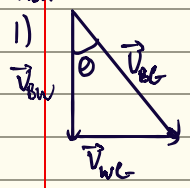
2-D Relative motion

1)



$|\vec{V}_{BG}|^2 = |\vec{V}_{BW}|^2 + |\vec{V}_{WG}|^2$

$= (5\text{m/s})^2 + (2\text{m/s})^2$

$= 29$

$|\vec{V}_{BG}| = 5.39 \text{ m/s}$

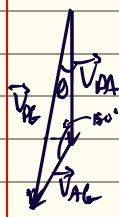
$\theta = \tan^{-1}\left(\frac{|\vec{V}_{WG}|}{|\vec{V}_{BW}|}\right)$

$= \tan^{-1}\left(\frac{2\text{m/s}}{5\text{m/s}}\right)$

$= 22^\circ$

$\therefore \vec{V}_{BG} = 5.39\text{m/s}\ [S22^\circ E]$

2)



$|\vec{V}_{PG}|^2 = |\vec{V}_{PA}|^2 + |\vec{V}_{AG}|^2 - 2|\vec{V}_{PA}||\vec{V}_{AG}|\cos\theta$

$= 300^2 + 80^2 - 2(200)(80)\cos 150^\circ$

$= 137909.2194$

$|\vec{V}_{PG}| = 371.44 \text{ km/h}$

$\frac{\sin\theta}{|\vec{V}_{AG}|} = \frac{\sin 150^\circ}{|\vec{V}_{PG}|}$

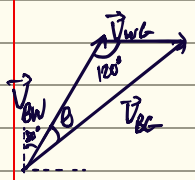
$\sin\theta = |\vec{V}_{AG}|\left(\frac{\sin 150^\circ}{|\vec{V}_{PG}|}\right)$

$\sin\theta = 0.107888839$

$\theta = 6^\circ$

$\therefore \vec{V}_{PG} = 371.44 \text{ km/h}\ [S6^\circ W]$

3)



$|\vec{V}_{BG}|^2 = |\vec{V}_{BW}|^2 + |\vec{V}_{WG}|^2 - 2|\vec{V}_{BW}||\vec{V}_{WG}|\cos\theta$

$= (10)^2 + (5)^2 - 2(10)(5)\cos 120$

$= 175$

$|\vec{V}_{BG}| = 13.23 \text{ km/h}$

$\alpha = 30^\circ + 19^\circ$

$= 49^\circ$

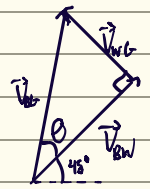
$\frac{\sin\theta}{|\vec{V}_{WG}|} = \frac{\sin 120^\circ}{|\vec{V}_{BG}|}$

$\sin\theta = 0.327328835$

$\theta = 19^\circ$

$\therefore \vec{V}_{BG} = 13.23 \text{ km/h}\ [N49^\circ E]$

4)



$|\vec{V}_{BG}|^2 = |\vec{V}_{BW}|^2 + |\vec{V}_{WG}|^2$

$= 4^2 + 3^2$

$= 25$

$|\vec{V}_{BG}| = 5 \text{ m/s}$

$\alpha = 37^\circ + 45^\circ$

$= 82^\circ$

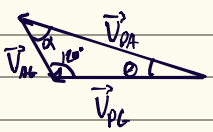
$\theta = \tan^{-1}\left(\frac{|\vec{V}_{WG}|}{|\vec{V}_{BW}|}\right)$

$= \tan^{-1}\left(\frac{3}{4}\right)$

$= 37^\circ$

$\therefore \vec{V}_{BG} = 5\text{m/s}\ [N8^\circ E]$

5)



$$\frac{\sin\theta}{|\vec{V}_{AC}|} = \frac{\sin 120°}{|\vec{V}_{OA}|}$$

$\sin\theta = 0.20784096$

$\theta = 12°$

$\therefore$ the plane must fly at [W12°N]

$\alpha = 180° - 120° - 12°$

$= 48°$

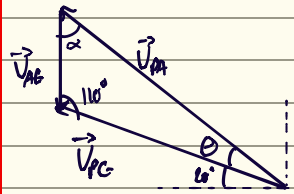
$$\frac{|\vec{V}_{PC}|}{\sin 48°} = \frac{|\vec{V}_{OA}|}{\sin 120°}$$

$|\vec{V}_{PC}| = 214.53 \text{ km/h}$

$\Delta t = \frac{d}{v_{PC}}$

$= 2.80 \text{ hr.}$

6)



$$\frac{\sin\theta}{|\vec{V}_{AC}|} = \frac{\sin 110°}{|\vec{V}_{OA}|}$$

$\sin\theta = 0.234923155$

$\theta = 14°$

$\beta = \theta + 20°$

$= 14° + 20°$

$= 34°$

$\therefore$ the plane must fly at [W34°N]

$\alpha = 180° - 110° - 14°$

$= 56°$

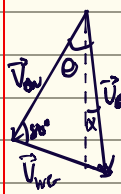
$$\frac{|\vec{V}_{PC}|}{\sin 56°} = \frac{|\vec{V}_{OA}|}{\sin 110°}$$

$|\vec{V}_{PC}| = 264.67 \text{ km/h}$

$t = \frac{d}{v_{PC}}$

$= 15.11 \text{ hr.}$

7)



$|\vec{V}_{SC}|^2 = |\vec{V}_{SW}|^2 + |\vec{V}_{WC}|^2 - 2|\vec{V}_{SW}||\vec{V}_{WC}|\cos 80°$

$= 5^2 + 2^2 - 2(5)(2)\cos 80°$

$= 25.527$

$|\vec{V}_{SC}| = 5.05 \text{ m/s}$

$$\frac{\sin\theta}{|\vec{V}_{WC}|} = \frac{\sin 80}{|\vec{V}_{SC}|}$$

$\sin\theta = 0.3898583$

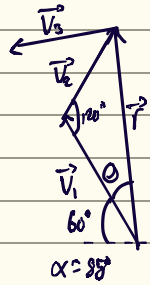
$\theta = 23°$

$\alpha = 30° - 23°$

$= 7°$

$\therefore \vec{V}_{SC} = 5.05 \text{ m/s} [S7°W]$

8)



$|\vec{r}|^2 = |\vec{V}_1|^2 + |\vec{V}_2|^2 - 2|\vec{V}_1||\vec{V}_2|\cos 120°$

$= 200^2 + 150^2 - 2(200)(150)\cos 120°$

$= 92500$

$|\vec{r}| = 304.14 \text{m}$

$$\frac{\sin\theta}{|\vec{V}_2|} = \frac{\sin 120°}{|\vec{r}|}$$

$\theta = 25°$

$|\vec{u}|^2 = |\vec{r}|^2 + |\vec{V}_3|^2 - 2(|\vec{r}|)(|\vec{V}_3|)\cos 75°$

$= 304.14^2 + 100^2 - 2(304.14)(100)\cos 75°$

$= 97198.62$

$\alpha = 0 + 5°$ $|\vec{u}| = 311.77 \text{ m}$

$= 19° + 5°$

$= 24°$

$$\frac{\sin\theta}{|\vec{V}_3|} = \frac{\sin 86°}{|\vec{u}|}$$

$\theta = 19°$

$\therefore \vec{u} = 311.77 \text{m} [N24°W]$