Homework 1

Stephanie Schneider
AA 274 - Principles of Robotic Autonomy

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Problem 1.

Part (i)

a) $J = \int_0^{t_f} [\lambda + v(t)^2 + \omega(t)^2]\, dt$ $\qquad \lambda > 0$

$x(0) = 0 \qquad y(0) = 0 \qquad \theta(0) = -\pi/2 \qquad t_f$ free

$x(t_f) = 5 \qquad y(t_f) = 5 \qquad \theta(t_f) = -\pi/2$

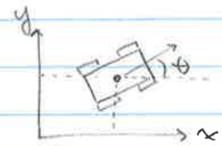
control inputs $|v(t)| \leq 0.5 m/s$

$u = (v, \omega) \qquad |\omega(t)| \leq 1$ rad/s

$\dot{x}(t) = v\cos\theta$

$\dot{y}(t) = v\sin\theta$

$\dot{\theta}(t) = \omega$



let $\tau = t/t_f$ $\qquad r = t_f$

$d\tau = \frac{1}{t_f} dt \qquad \frac{dx}{d\tau} = t_f \frac{df}{dt}$

$J = \int_0^1 (\lambda + v^2 + \omega^2) t_f \, d\tau$

$H = t_f(\lambda + v^2 + \omega^2) + [p_1 \; p_2 \; p_3] \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} t_f$

$= t_f(\lambda + v^2 + \omega^2 + p_1\dot{x} + p_2\dot{y} + p_3\dot{\theta})$

$\rightarrow H = t_f(\lambda + v^2 + \omega^2 + p_1 v\cos\theta + p_2 v\sin\theta + p_3\omega)$

$\dot{\vec{x}} = [\dot{x}, \dot{y}, \dot{\theta}]\, t_f$

$\dot{\vec{p}} = -\frac{\partial H}{\partial \vec{x}} = \begin{bmatrix} -\partial H/\partial x \\ -\partial H/\partial y \\ -\partial H/\partial \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ p_1 v\sin\theta - p_2 v\cos\theta \end{bmatrix}$

$\frac{\partial H}{\partial u} = 0 = \begin{bmatrix} \partial H/\partial v \\ \partial H/\partial \omega \end{bmatrix} = \begin{bmatrix} 2t_f v + p_1 t_f\cos\theta + p_2 t_f \sin\theta \\ 2t_f\omega + p_3 t_f \end{bmatrix}$

$2v + p_1\cos\theta + p_2\sin\theta = 0 \qquad 2\omega + p_3 = 0$

$\rightarrow \underline{v = \frac{1}{2}(-p_1\cos\theta - p_2\sin\theta)} \qquad \underline{\omega = -p_3/2}$

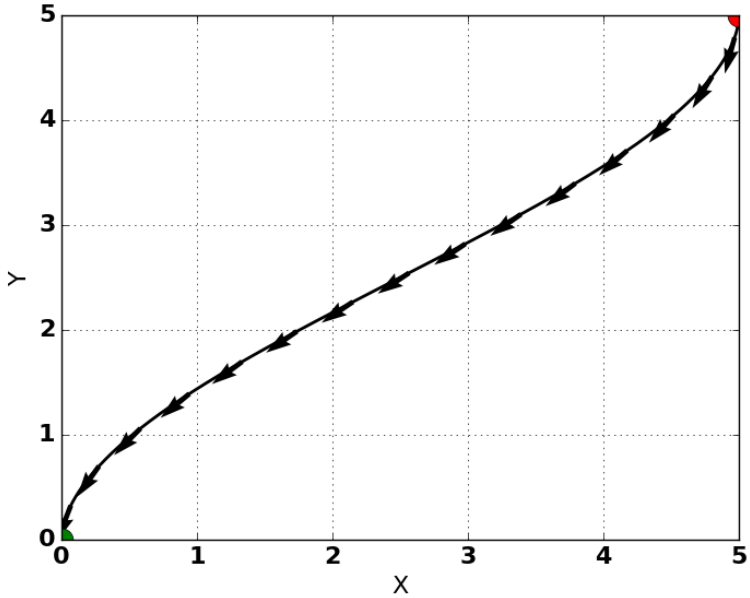
Part (ii)

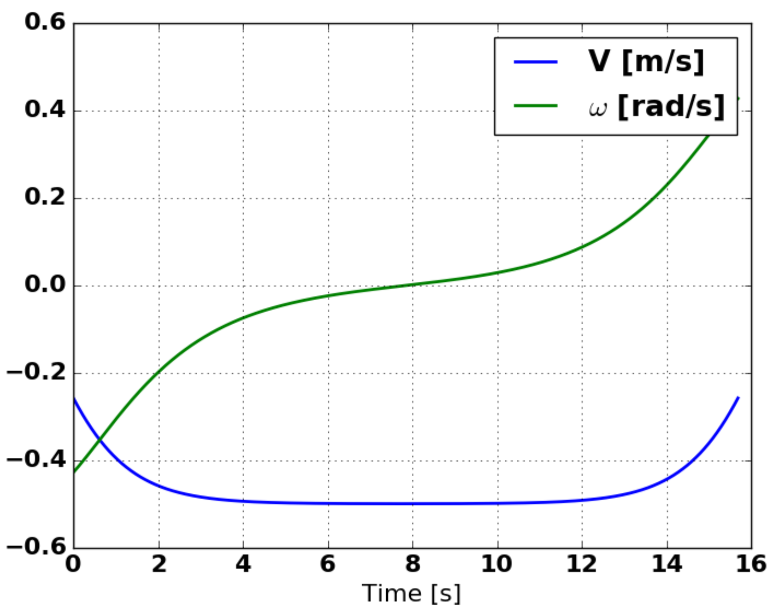
See `P1_optimal_control.py`. $\lambda = 0.249$

Part (iii)

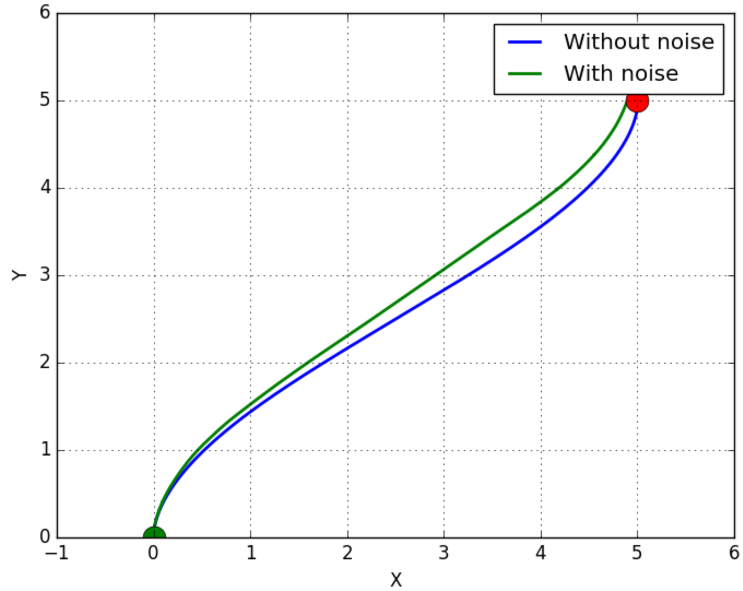
Since lambda is integrated over time, using the largest feasible lambda allocates more cost to a solution that takes more time. Therefore, by maximizing lambda, we force the optimal solution to minimize elapsed time.

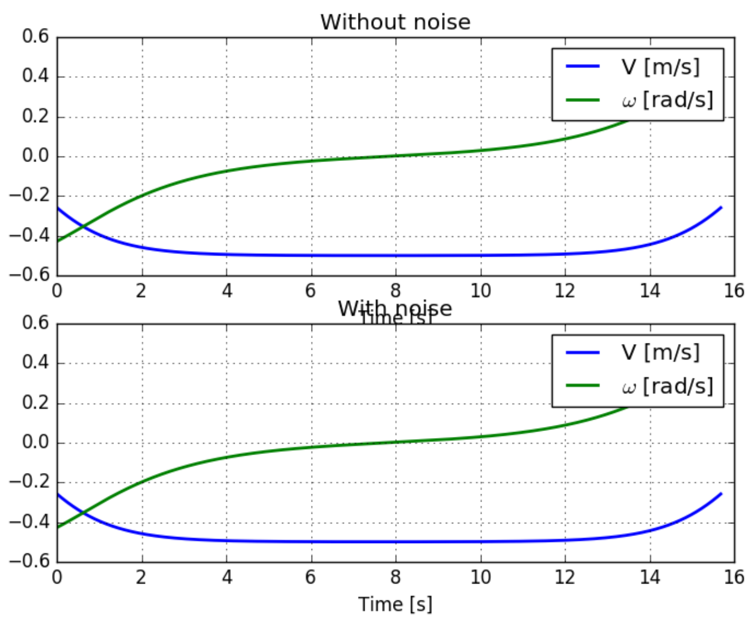
Part (iv)





Part (v)


Without noise
With noise
Y
X
6
5
4
3
2
1
0
−1
0
1
2
3
4
5
6


Without noise
V [m/s]
ω [rad/s]
Time [s]
With noise
V [m/s]
ω [rad/s]
Time [s]
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
2
4
6
8
10
12
14
16

Problem 2.

Part (i)

$$\begin{bmatrix} \psi_1(0) & \psi_2(0) & \psi_3(0) & \psi_4(0) \\ \dot\psi_1(0) & \dot\psi_2(0) & \dot\psi_3(0) & \dot\psi_4(0) \\ \psi_1(t_f) & \psi_2(t_f) & \psi_3(t_f) & \psi_4(t_f) \\ \dot\psi_1(t_f) & \dot\psi_2(t_f) & \dot\psi_3(t_f) & \dot\psi_4(t_f) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} \vec z(0) \\ \dot{\vec z}(0) \\ \vec z(t_f) \\ \dot{\vec z}(t_f) \end{bmatrix}$$

$\psi_1 = 1$ $\quad \dot\psi_1 = 0$ $\quad \vec z(0) = [x(0), y(0)] = [0, 0]$

$\psi_2 = t$ $\quad \dot\psi_2 = 1$ $\quad \dot{\vec z}(0) = [\dot x(0), \dot y(0)]$

$\psi_3 = t^2$ $\quad \dot\psi_3 = 2t$ $\quad = [V(0)\cos(\theta(0)), V(0)\sin(\theta(0))]$

$\psi_4 = t^3$ $\quad \dot\psi_4 = 3t^2$ $\quad = [0, -0.5]$

$\alpha_i = [x_i, y_i]$ $\quad \vec z(t_f) = [5, 5]$

$\dot{\vec z}(t_f) = [0, -0.5]$

1. $\alpha_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
2. $\alpha_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}$
3. $\alpha_1 + \alpha_2 t_f + \alpha_3 t_f^2 + \alpha_4 t_f^3 = [5, 5]$
4. $\alpha_2 + 2\alpha_3 t_f + 3\alpha_4 t_f^2 = [0, -0.5]$

$$\vec\alpha = \begin{array}{c} \begin{bmatrix} 0 & 0 \\ 0 & -0.5 \\ 0.067 & 0.167 \\ -0.00296 & -0.0074 \end{bmatrix} \\ \uparrow \quad\quad\quad \uparrow \\ x_i \quad\quad\quad y_i \end{array}$$

$x(t) = \sum x_i \psi_i(t)$

$y(t) = \sum y_i \psi_i(t)$

$\theta = \operatorname{atan}(\dot y / \dot x)$

$\dot\theta = \dfrac{\dot x \ddot y - \ddot x \dot y}{\dot x^2 + \dot y^2}$ (from wolfram)

In the problem definition, the matrix J is only invertible under the condition that $V > 0$. Therefore, we cannot set $V(tf) = 0$ because that violates the condition for differential flatness.

Part (ii)

See `P2_differential_flatness.py`.

Part (iii)

$\dot{s}(t) = V(t)$

$s(0) = 0$

$\tilde{V}(s) = \frac{ds}{d\tau}$

$\tau(s=0) = 0$

$$\tau(s) = \int_0^s \frac{ds'}{\tilde{V}(s')}$$

$$\tilde{\omega}(s) = \frac{\omega(s)}{V(s)} \tilde{V}(s) \leq 1$$

$|\tilde{V}(s)| \leq 0.5$

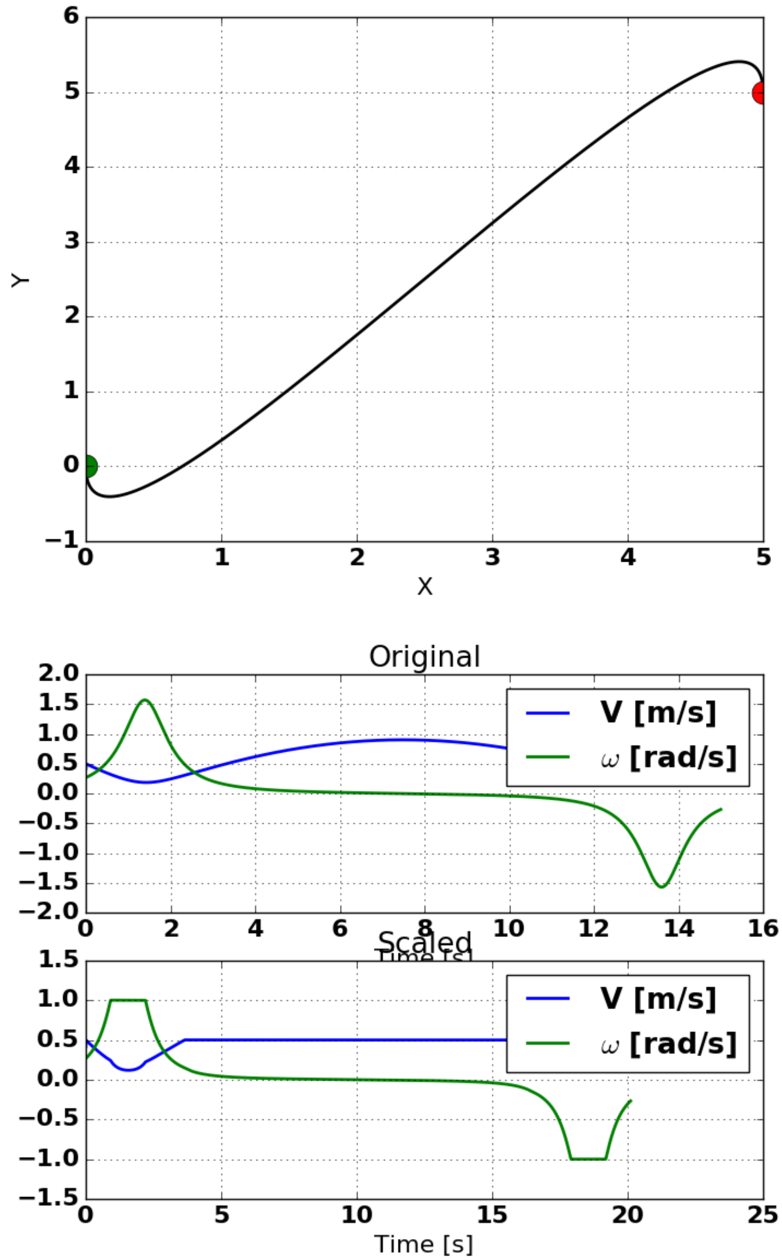
$|\tilde{\omega}(s)| \leq 1$

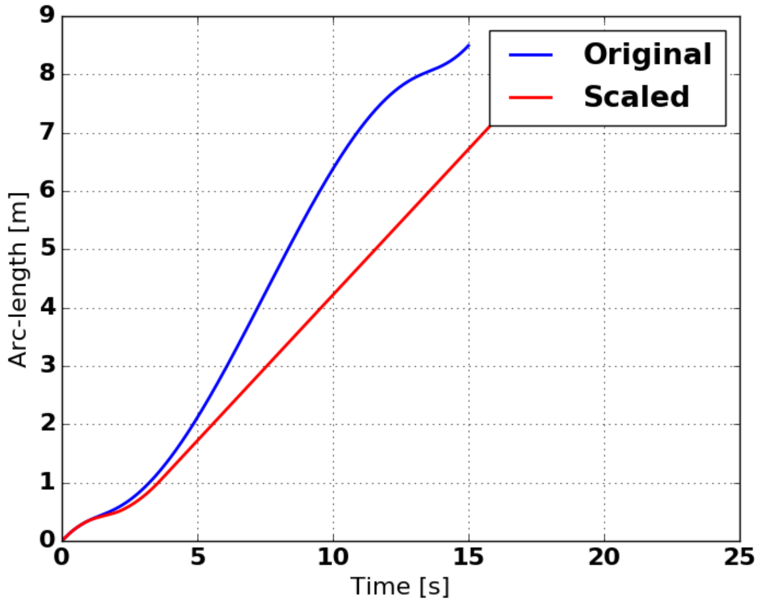
$$\tilde{V}(s) \leq \frac{\omega(s)}{V(s)}$$

$\dot{s}(t) = V(t)$

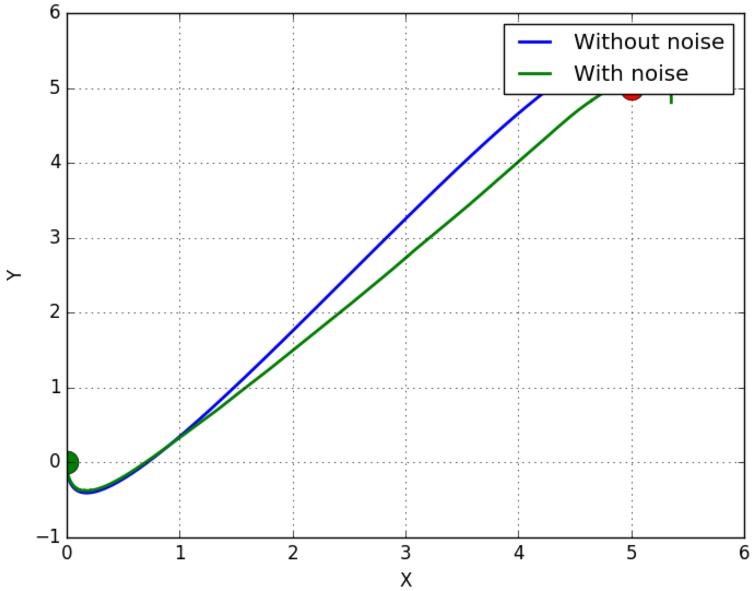
$$s(t) = \int_0^{tf} V(t)\, dt$$

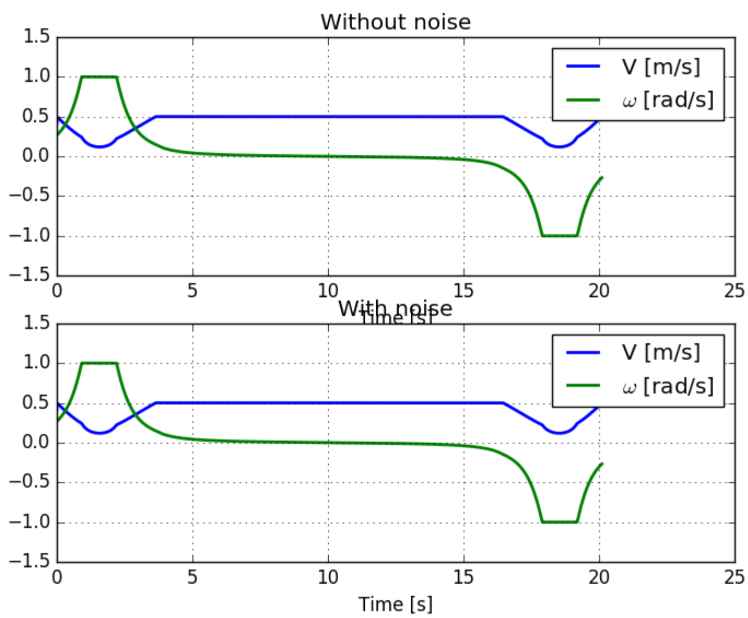
Part (iv)




Original
Scaled
Arc-length [m]
Time [s]
0
1
2
3
4
5
6
7
8
9
0
5
10
15
20
25

Part (v)


Without noise
With noise
X
Y


Without noise
V [m/s]
ω [rad/s]
Time [s]
With noise
V [m/s]
ω [rad/s]
Time [s]

Problem 3.

Part (i)

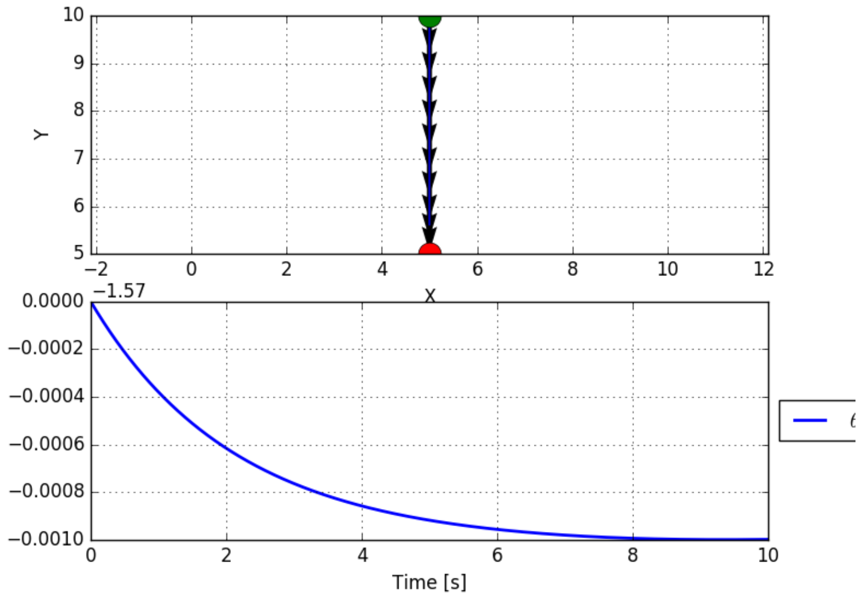
$$\rho = \sqrt{x^2 + y^2}$$
$$\alpha = atan2(y, x) - \theta + \pi$$
$$\delta = \alpha + \theta$$

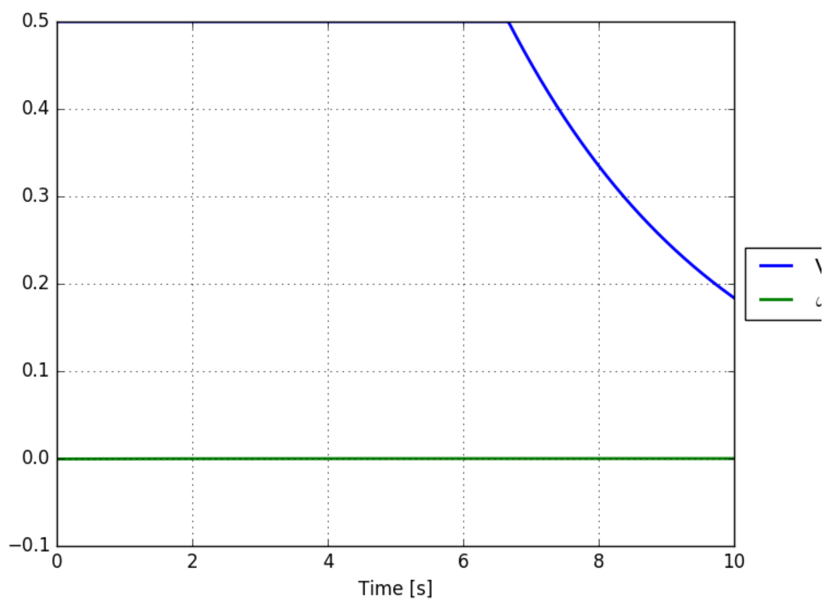
$$V = k_1 * \rho * cos(\alpha)$$
$$\omega = k_2 * \alpha + k_1 * cos(\alpha) * sinc(\frac{\alpha}{\pi}) * (\alpha + k_3 * \delta)$$

See `P3_pose_stabilization.py`.

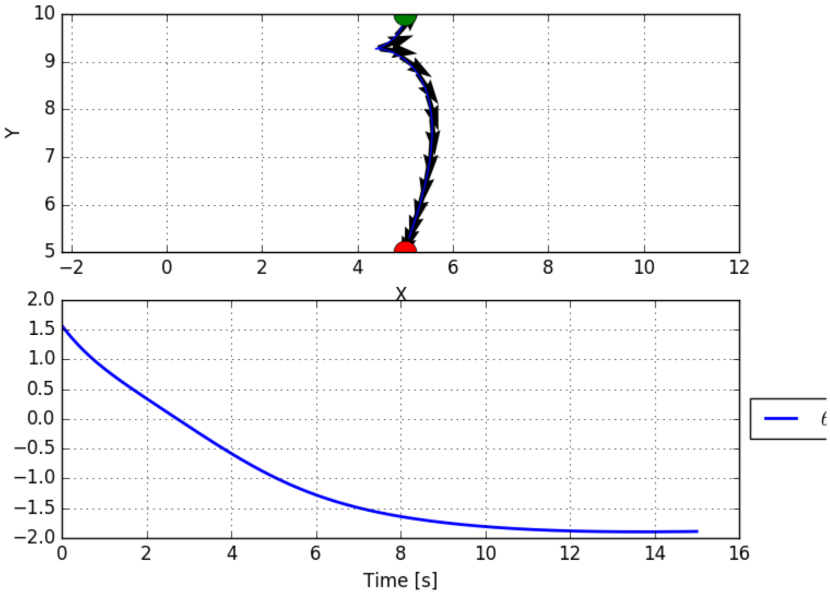
Part (ii)

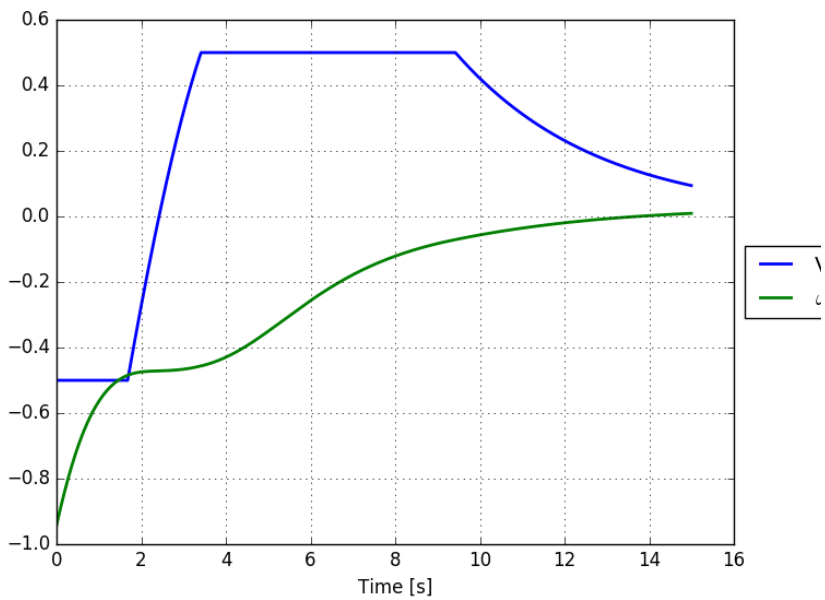
forward arguments: `5 10 -1.57 10`



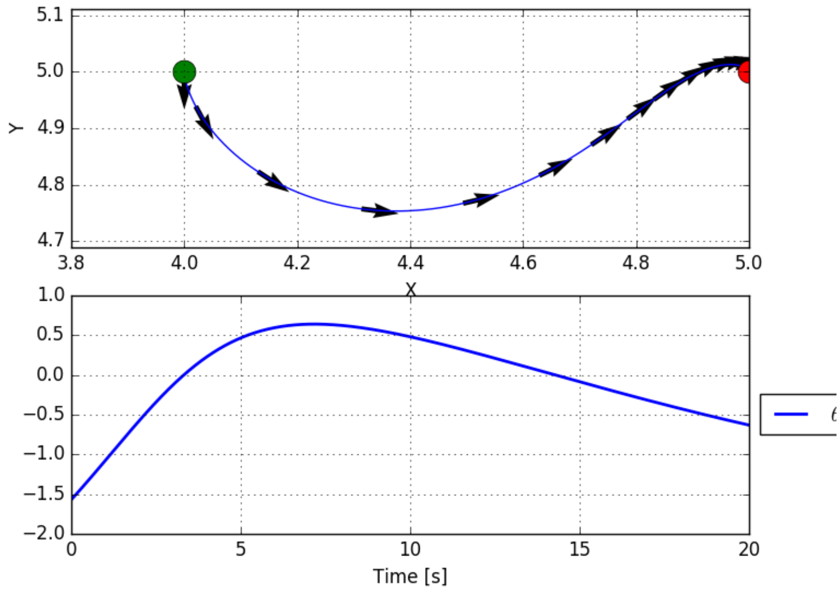


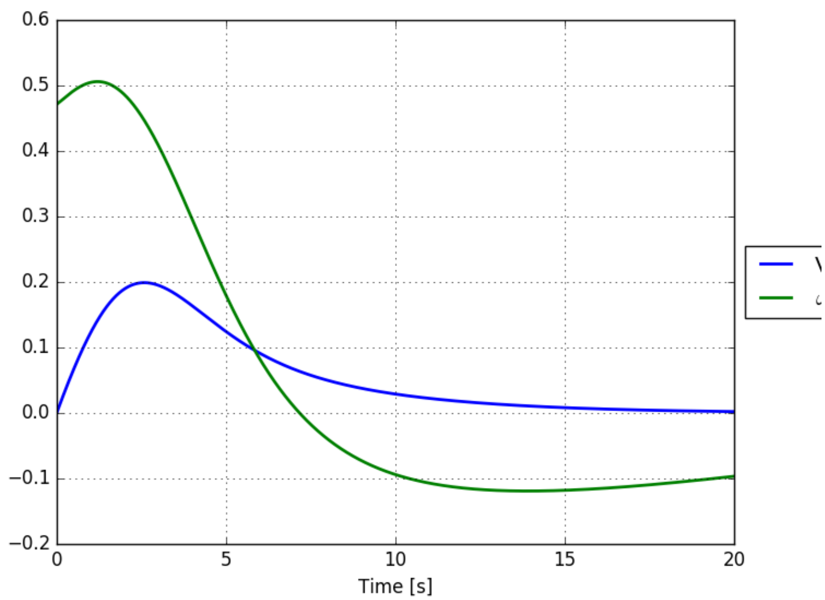
reverse arguments: `5 10 1.57 15`





parallel arguments: `4 5 -1.57 20`





Problem 4.

Part (i)

④ $u_1 = \ddot{x}_d + k_{px}(x_d - x) + k_{dx}(\dot{x}_d - \dot{x})$

$u_2 = \ddot{y}_d + k_{py}(y_d - y) + k_{dy}(\dot{y}_d - \dot{y})$

$(u_1, u_2) = (\ddot{x}, \ddot{y}) \quad \leftarrow$ virtual controls

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -V\sin\theta \\ \sin\theta & V\cos\theta \end{bmatrix} \begin{bmatrix} \dot{V} \\ \omega \end{bmatrix}$$

$$u_1 = \dot{V}\cos\theta - V\omega\sin\theta$$
$$u_2 = \dot{V}\sin\theta + V\omega\cos\theta$$

$$\dot{V} = \frac{u_1}{\cos\theta} + V\omega\tan\theta = \frac{u_2}{\sin\theta} - V\omega\cot\theta$$

$$V\omega(\tan\theta + \cot\theta) = \frac{u_2}{\sin\theta} - \frac{u_1}{\cos\theta}$$

$$V\omega\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) = \frac{u_2}{\sin\theta} - \frac{u_1}{\cos\theta} \quad \cdot \cos\theta\sin\theta$$

$$V\omega(\cancel{\sin^2\theta + \cos^2\theta}) = u_2\cos\theta - u_1\sin\theta$$

$$\boxed{\omega = \frac{u_2\cos\theta - u_1\sin\theta}{V}}$$

$$\dot{V} = \frac{u_1}{\cos\theta} + \cancel{V}\tan\theta\left[\frac{u_2\cos\theta - u_1\sin\theta}{\cancel{V}}\right]$$

$$= \frac{u_1}{\cos\theta} + u_2\sin\theta - \frac{u_1\sin^2\theta}{\cos\theta}$$

$$= \frac{u_1(1-\sin^2\theta)}{\cos\theta} + u_2\sin\theta$$

$$\boxed{\dot{V} = u_1\cos\theta + u_2\sin\theta}$$

$$\left.\begin{array}{ll} V_{new} = V_{prev} + \dot{V}dt & \text{(Euler step)} \\ x, y, \theta \sim \text{constant} & \end{array}\right\} \text{ctrl\_pose}$$

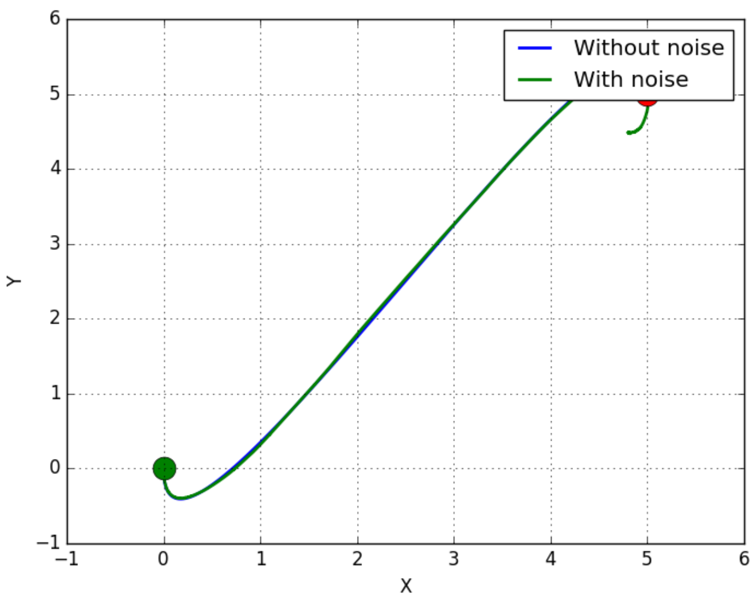
Part (ii)

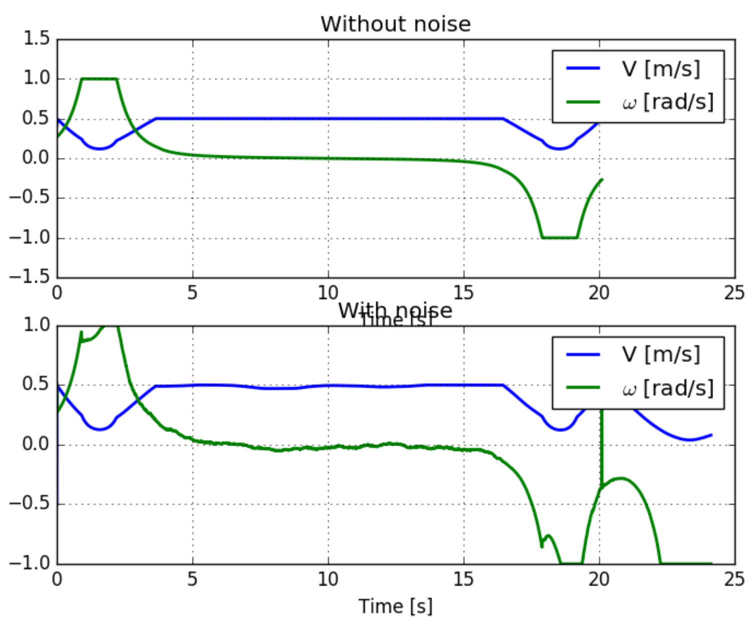
See `P4_trajectory_tracking.py`.

Part (iii)

See `P4_trajectory_tracking.py`.

Part (iv)





Problem 5.

Part (i)

See `random_strings.bag`

Part (ii)

`rosbag play filename.bag`.
This will publish all the messages recorded in `filename.bag`. In order to see the contents of the messages, you must subscribe to them, e.g. with `rostopic echo <topic>`.

Part (iii)

see `turtlebot.bag`