ECE C147/247 HW4 Q1: Optimization for Fully Connected Networks

In this notebook, we will implement different optimization rules for gradient descent. We have provided starter code; however, you will need to copy and paste your code from your implementation of the modular fully connected nets in HW #3 to build upon this.

`utils` has built a solid API for building these modular frameworks and training them, and we will use their very well implemented framework as opposed to "reinventing the wheel." This includes using their Solver, various utility functions, and their layer structure. This also includes `nndl.fc_net`, `nndl.layers`, and `nndl.layer_utils`.

In [1]:
```
## Import and setups

import time
import numpy as np
import matplotlib.pyplot as plt
from nndl.fc_net import *
from utils.data_utils import get_CIFAR10_data
from utils.gradient_check import eval_numerical_gradient, eval_numerical_gradient_array
from utils.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

In [2]:
```
# Load the (preprocessed) CIFAR10 data.

data = get_CIFAR10_data()
for k in data.keys():
  print('{}: {} '.format(k, data[k].shape))
```

```
X_train: (49000, 3, 32, 32)
y_train: (49000,)
X_val: (1000, 3, 32, 32)
y_val: (1000,)
X_test: (1000, 3, 32, 32)
y_test: (1000,)
```

Building upon your HW #3 implementation

Copy and paste the following functions from your HW #3 implementation of a modular FC net:

- `affine_forward` in `nndl/layers.py`
- `affine_backward` in `nndl/layers.py`

- `relu_forward` in `nndl/layers.py`
- `relu_backward` in `nndl/layers.py`
- `affine_relu_forward` in `nndl/layer_utils.py`
- `affine_relu_backward` in `nndl/layer_utils.py`
- The `FullyConnectedNet` class in `nndl/fc_net.py`

Test all functions you copy and pasted

In [3]:

```
from nndl.layer_tests import *

affine_forward_test(); print('\n')
affine_backward_test(); print('\n')
relu_forward_test(); print('\n')
relu_backward_test(); print('\n')
affine_relu_test(); print('\n')
fc_net_test()
```

```
If affine_forward function is working, difference should be less than 1e-9:
difference: 9.769849468192957e-10


If affine_backward is working, error should be less than 1e-9::
dx error: 4.517717286767966e-10
dw error: 2.953555218603546e-10
db error: 1.3004066660747692e-11


If relu_forward function is working, difference should be around 1e-8:
difference: 4.999999798022158e-08


If relu_forward function is working, error should be less than 1e-9:
dx error: 3.2756114574567796e-12


If affine_relu_forward and affine_relu_backward are working, error should be less than 1
e-9::
dx error: 1.797929234036535e-10
dw error: 1.510465039305725e-10
db error: 3.2756019865460813e-12


Running check with reg = 0
Initial loss: 2.300223304820917
W1 relative error: 4.631040062839554e-08
W2 relative error: 1.2098517344039791e-06
W3 relative error: 3.738792456207884e-07
b1 relative error: 2.0538755808609935e-08
b2 relative error: 4.805233546659617e-09
b3 relative error: 1.4364540241108383e-10
Running check with reg = 3.14
Initial loss: 6.742053383557732
W1 relative error: 2.0360662169265662e-08
W2 relative error: 1.170895493191328e-06
W3 relative error: 5.248572999147182e-09
b1 relative error: 2.778938800619773e-07
b2 relative error: 5.317300046122514e-09
b3 relative error: 1.296243952406403e-10
```

Training a larger model

In general, proceeding with vanilla stochastic gradient descent to optimize models may be fraught with problems and limitations, as discussed in class. Thus, we implement optimizers that improve on SGD.

SGD + momentum

In the following section, implement SGD with momentum. Read the `nndl/optim.py` API, and be sure you understand it. After, implement `sgd_momentum` in `nndl/optim.py` . Test your implementation of `sgd_momentum` by running the cell below.

In [4]:
```
from nndl.optim import sgd_momentum

N, D = 4, 5
w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
v = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)

config = {'learning_rate': 1e-3, 'velocity': v}
next_w, _ = sgd_momentum(w, dw, config=config)

expected_next_w = np.asarray([
  [ 0.1406,      0.20738947,  0.27417895,  0.34096842,  0.40775789],
  [ 0.47454737,  0.54133684,  0.60812632,  0.67491579,  0.74170526],
  [ 0.80849474,  0.87528421,  0.94207368,  1.00886316,  1.07565263],
  [ 1.14244211,  1.20923158,  1.27602105,  1.34281053,  1.4096    ]])
expected_velocity = np.asarray([
  [ 0.5406,      0.55475789,  0.56891579, 0.58307368,  0.59723158],
  [ 0.61138947,  0.62554737,  0.63970526,  0.65386316,  0.66802105],
  [ 0.68217895,  0.69633684,  0.71049474,  0.72465263,  0.73881053],
  [ 0.75296842,  0.76712632,  0.78128421,  0.79544211,  0.8096    ]])

print('next_w error: {}'.format(rel_error(next_w, expected_next_w)))
print('velocity error: {}'.format(rel_error(expected_velocity, config['velocity'])))
```

```
next_w error: 8.882347033505819e-09
velocity error: 4.269287743278663e-09
```

SGD + Nesterov momentum

Implement `sgd_nesterov_momentum` in `ndl/optim.py` .

In [5]:
```
from nndl.optim import sgd_nesterov_momentum

N, D = 4, 5
w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
v = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)

config = {'learning_rate': 1e-3, 'velocity': v}
next_w, _ = sgd_nesterov_momentum(w, dw, config=config)

expected_next_w = np.asarray([
  [0.08714,      0.15246105,  0.21778211,  0.28310316,  0.34842421],
  [0.41374526,   0.47906632,  0.54438737,  0.60970842,  0.67502947],
  [0.74035053,   0.80567158,  0.87099263,  0.93631368,  1.00163474],
  [1.06695579,   1.13227684,  1.19759789,  1.26291895,  1.32824    ]])
expected_velocity = np.asarray([
  [ 0.5406,      0.55475789,  0.56891579,  0.58307368,  0.59723158],
  [ 0.61138947,  0.62554737,  0.63970526,  0.65386316,  0.66802105],
  [ 0.68217895,  0.69633684,  0.71049474,  0.72465263,  0.73881053],
  [ 0.75296842,  0.76712632,  0.78128421,  0.79544211,  0.8096    ]])
```

```
print('next_w error: {}'.format(rel_error(next_w, expected_next_w)))
print('velocity error: {}'.format(rel_error(expected_velocity, config['velocity'])))
```

```
next_w error: 1.0875186845081027e-08
velocity error: 4.269287743278663e-09
```

Evaluating SGD, SGD+Momentum, and SGD+NesterovMomentum

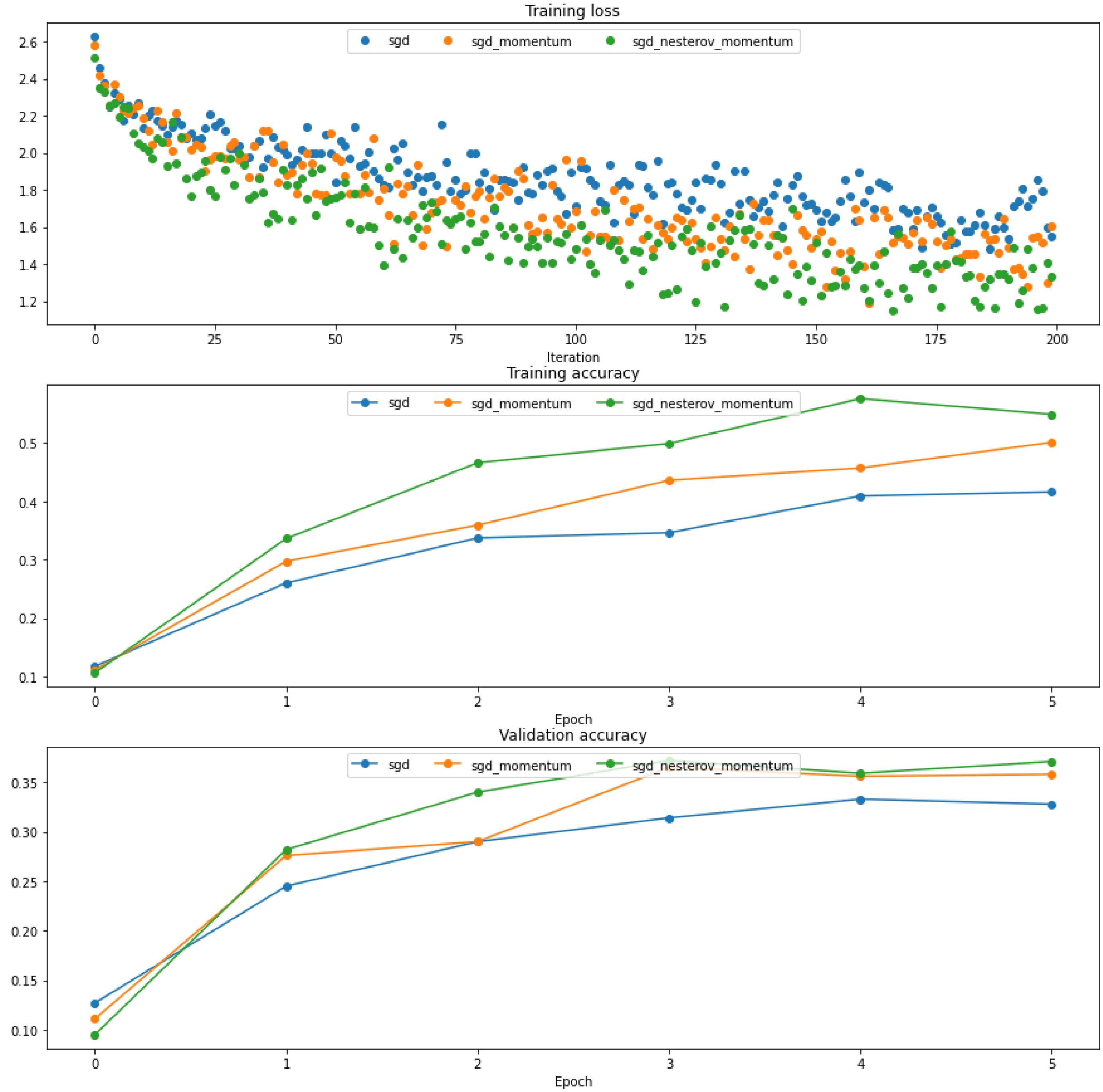
Run the following cell to train a 6 layer FC net with SGD, SGD+momentum, and SGD+Nesterov momentum. You should see that SGD+momentum achieves a better loss than SGD, and that SGD+Nesterov momentum achieves a slightly better loss (and training accuracy) than SGD+momentum.

In [6]:

```
num_train = 4000
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

solvers = {}

for update_rule in ['sgd', 'sgd_momentum', 'sgd_nesterov_momentum']:
  print('Optimizing with {}'.format(update_rule))
  model = FullyConnectedNet([100, 100, 100, 100, 100], weight_scale=5e-2)

  solver = Solver(model, small_data,
                  num_epochs=5, batch_size=100,
                  update_rule=update_rule,
                  optim_config={
                    'learning_rate': 1e-2,
                  },
                  verbose=False)
  solvers[update_rule] = solver
  solver.train()
  print

plt.subplot(3, 1, 1)
plt.title('Training loss')
plt.xlabel('Iteration')

plt.subplot(3, 1, 2)
plt.title('Training accuracy')
plt.xlabel('Epoch')

plt.subplot(3, 1, 3)
plt.title('Validation accuracy')
plt.xlabel('Epoch')

for update_rule, solver in solvers.items():
  plt.subplot(3, 1, 1)
  plt.plot(solver.loss_history, 'o', label=update_rule)

  plt.subplot(3, 1, 2)
  plt.plot(solver.train_acc_history, '-o', label=update_rule)

  plt.subplot(3, 1, 3)
  plt.plot(solver.val_acc_history, '-o', label=update_rule)

for i in [1, 2, 3]:
```

```
    plt.subplot(3, 1, i)
    plt.legend(loc='upper center', ncol=4)
  plt.gcf().set_size_inches(15, 15)
  plt.show()
```

```
Optimizing with sgd
Optimizing with sgd_momentum
Optimizing with sgd_nesterov_momentum
```



RMSProp

Now we go to techniques that adapt the gradient. Implement `rmsprop` in `nndl/optim.py` . Test your implementation by running the cell below.

In [7]:

```
from nndl.optim import rmsprop

N, D = 4, 5
w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
a = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)
```

```
config = {'learning_rate': 1e-2, 'a': a}
next_w, _ = rmsprop(w, dw, config=config)

expected_next_w = np.asarray([
  [-0.39223849, -0.34037513, -0.28849239, -0.23659121, -0.18467247],
  [-0.132737,   -0.08078555, -0.02881884,  0.02316247,  0.07515774],
  [ 0.12716641,  0.17918792,  0.23122175,  0.28326742,  0.33532447],
  [ 0.38739248,  0.43947102,  0.49155973,  0.54365823,  0.59576619]])
expected_cache = np.asarray([
  [ 0.5976,      0.6126277,   0.6277108,   0.64284931,  0.65804321],
  [ 0.67329252,  0.68859723,  0.70395734,  0.71937285,  0.73484377],
  [ 0.75037008,  0.7659518,   0.78158892,  0.79728144,  0.81302936],
  [ 0.82883269,  0.84469141,  0.86060554,  0.87657507,  0.8926    ]])

print('next_w error: {}'.format(rel_error(expected_next_w, next_w)))
print('cache error: {}'.format(rel_error(expected_cache, config['a'])))
```

```
next_w error: 9.524687511038133e-08
cache error: 2.6477955807156126e-09
```

Adaptive moments

Now, implement `adam` in `nndl/optim.py`. Test your implementation by running the cell below.

In [8]:

```
# Test Adam implementation; you should see errors around 1e-7 or less
from nndl.optim import adam

N, D = 4, 5
w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
v = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)
a = np.linspace(0.7, 0.5, num=N*D).reshape(N, D)

config = {'learning_rate': 1e-2, 'v': v, 'a': a, 't': 5}
next_w, _ = adam(w, dw, config=config)

expected_next_w = np.asarray([
  [-0.40094747, -0.34836187, -0.29577703, -0.24319299, -0.19060977],
  [-0.1380274,  -0.08544591, -0.03286534,  0.01971428,  0.0722929],
  [ 0.1248705,   0.17744702,  0.23002243,  0.28259667,  0.33516969],
  [ 0.38774145,  0.44031188,  0.49288093,  0.54544852,  0.59801459]])
expected_a = np.asarray([
  [ 0.69966,     0.68908382,  0.67851319,  0.66794809,  0.65738853,],
  [ 0.64683452,  0.63628604,  0.6257431,   0.61520571,  0.60467385,],
  [ 0.59414753,  0.58362676,  0.57311152,  0.56260183,  0.55209767,],
  [ 0.54159906,  0.53110598,  0.52061845,  0.51013645,  0.49966,   ]])
expected_v = np.asarray([
  [ 0.48,        0.49947368,  0.51894737,  0.53842105,  0.55789474],
  [ 0.57736842,  0.59684211,  0.61631579,  0.63578947,  0.65526316],
  [ 0.67473684,  0.69421053,  0.71368421,  0.73315789,  0.75263158],
  [ 0.77210526,  0.79157895,  0.81105263,  0.83052632,  0.85      ]])

print('next_w error: {}'.format(rel_error(expected_next_w, next_w)))
print('a error: {}'.format(rel_error(expected_a, config['a'])))
print('v error: {}'.format(rel_error(expected_v, config['v'])))
```

```
next_w error: 1.1395691798535431e-07
a error: 4.208314038113071e-09
v error: 4.214963193114416e-09
```

Comparing SGD, SGD+NesterovMomentum, RMSProp, and

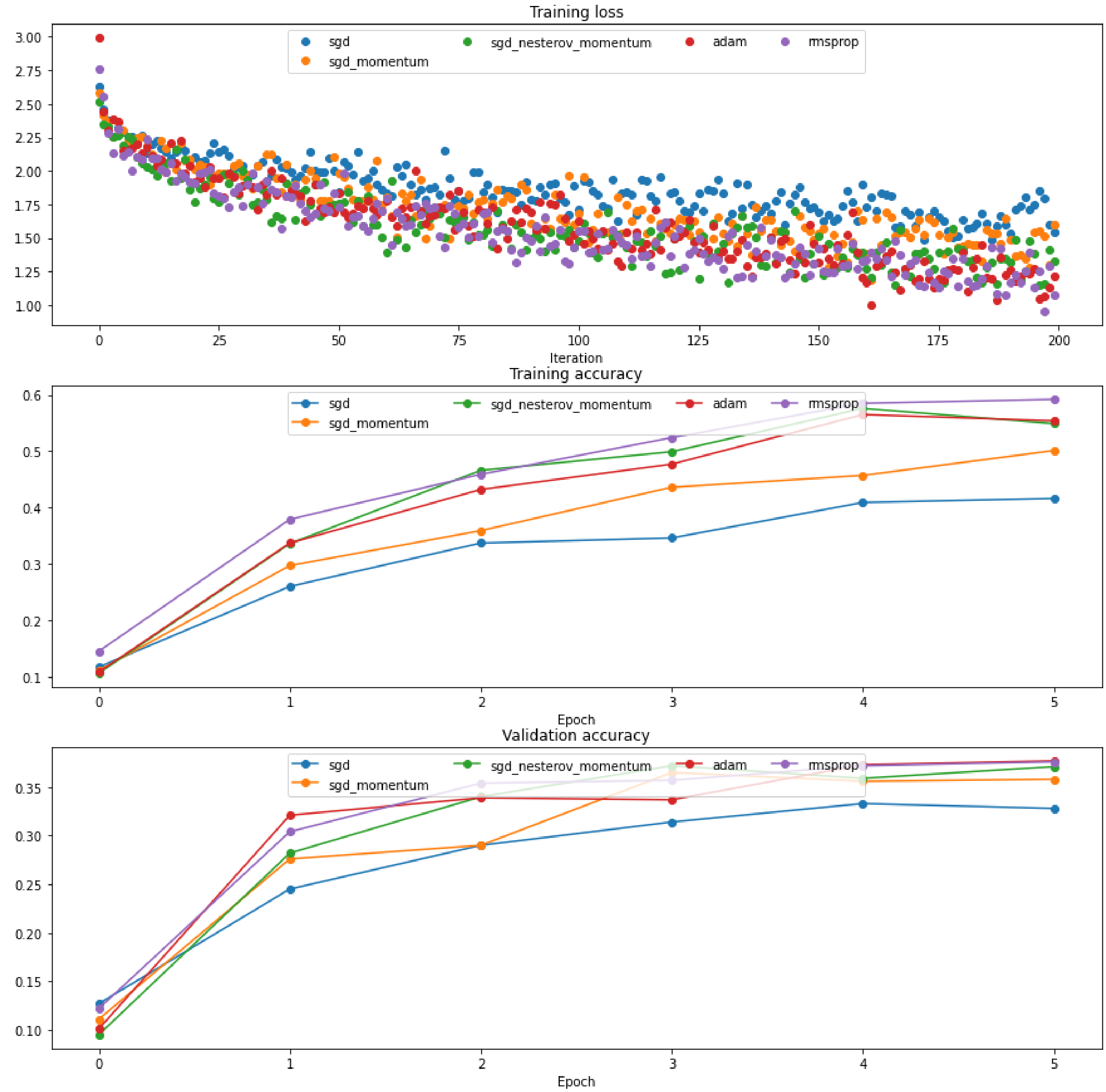
Adam

The following code will compare optimization with SGD, Momentum, Nesterov Momentum, RMSProp and Adam. In our code, we find that RMSProp, Adam, and SGD + Nesterov Momentum achieve approximately the same training error after a few training epochs.

In [9]:

```
learning_rates = {'rmsprop': 2e-4, 'adam': 1e-3}

for update_rule in ['adam', 'rmsprop']:
  print('Optimizing with {}'.format(update_rule))
  model = FullyConnectedNet([100, 100, 100, 100, 100], weight_scale=5e-2)

  solver = Solver(model, small_data,
                  num_epochs=5, batch_size=100,
                  update_rule=update_rule,
                  optim_config={
                    'learning_rate': learning_rates[update_rule]
                  },
                  verbose=False)
  solvers[update_rule] = solver
  solver.train()
  print

plt.subplot(3, 1, 1)
plt.title('Training loss')
plt.xlabel('Iteration')

plt.subplot(3, 1, 2)
plt.title('Training accuracy')
plt.xlabel('Epoch')

plt.subplot(3, 1, 3)
plt.title('Validation accuracy')
plt.xlabel('Epoch')

for update_rule, solver in solvers.items():
  plt.subplot(3, 1, 1)
  plt.plot(solver.loss_history, 'o', label=update_rule)

  plt.subplot(3, 1, 2)
  plt.plot(solver.train_acc_history, '-o', label=update_rule)

  plt.subplot(3, 1, 3)
  plt.plot(solver.val_acc_history, '-o', label=update_rule)

for i in [1, 2, 3]:
  plt.subplot(3, 1, i)
  plt.legend(loc='upper center', ncol=4)
plt.gcf().set_size_inches(15, 15)
plt.show()
```

```
Optimizing with adam
Optimizing with rmsprop
```



Easier optimization

In the following cell, we'll train a 4 layer neural network having 500 units in each hidden layer with the different optimizers, and find that it is far easier to get up to 50+% performance on CIFAR-10. After we implement batchnorm and dropout, we'll ask you to get 55+% on CIFAR-10.

In [10]:

```
optimizer = 'adam'
best_model = None

layer_dims = [500, 500, 500]
weight_scale = 0.01
learning_rate = 1e-3
lr_decay = 0.9

model = FullyConnectedNet(layer_dims, weight_scale=weight_scale,
                          use_batchnorm=True)

solver = Solver(model, data,
```

```python
                num_epochs=10, batch_size=100,
                update_rule=optimizer,
                optim_config={
                  'learning_rate': learning_rate,
                },
                lr_decay=lr_decay,
                verbose=True, print_every=50)
solver.train()
```

```
(Iteration 1 / 4900) loss: 2.300475
(Epoch 0 / 10) train acc: 0.165000; val_acc: 0.181000
(Iteration 51 / 4900) loss: 1.932997
(Iteration 101 / 4900) loss: 1.698234
(Iteration 151 / 4900) loss: 1.781077
(Iteration 201 / 4900) loss: 1.744266
(Iteration 251 / 4900) loss: 1.705927
(Iteration 301 / 4900) loss: 1.526387
(Iteration 351 / 4900) loss: 1.702914
(Iteration 401 / 4900) loss: 1.566511
(Iteration 451 / 4900) loss: 1.707089
(Epoch 1 / 10) train acc: 0.399000; val_acc: 0.421000
(Iteration 501 / 4900) loss: 1.556329
(Iteration 551 / 4900) loss: 1.699830
(Iteration 601 / 4900) loss: 1.511032
(Iteration 651 / 4900) loss: 1.479937
(Iteration 701 / 4900) loss: 1.394786
(Iteration 751 / 4900) loss: 1.428443
(Iteration 801 / 4900) loss: 1.461454
(Iteration 851 / 4900) loss: 1.469750
(Iteration 901 / 4900) loss: 1.500166
(Iteration 951 / 4900) loss: 1.564140
(Epoch 2 / 10) train acc: 0.475000; val_acc: 0.470000
(Iteration 1001 / 4900) loss: 1.559267
(Iteration 1051 / 4900) loss: 1.301459
(Iteration 1101 / 4900) loss: 1.494965
(Iteration 1151 / 4900) loss: 1.533820
(Iteration 1201 / 4900) loss: 1.460437
(Iteration 1251 / 4900) loss: 1.357185
(Iteration 1301 / 4900) loss: 1.399734
(Iteration 1351 / 4900) loss: 1.423651
(Iteration 1401 / 4900) loss: 1.289892
(Iteration 1451 / 4900) loss: 1.413948
(Epoch 3 / 10) train acc: 0.492000; val_acc: 0.457000
(Iteration 1501 / 4900) loss: 1.088179
(Iteration 1551 / 4900) loss: 1.374607
(Iteration 1601 / 4900) loss: 1.378255
(Iteration 1651 / 4900) loss: 1.387306
(Iteration 1701 / 4900) loss: 1.296451
(Iteration 1751 / 4900) loss: 1.402440
(Iteration 1801 / 4900) loss: 1.545424
(Iteration 1851 / 4900) loss: 1.205041
(Iteration 1901 / 4900) loss: 1.341622
(Iteration 1951 / 4900) loss: 1.407368
(Epoch 4 / 10) train acc: 0.533000; val_acc: 0.494000
(Iteration 2001 / 4900) loss: 1.243661
(Iteration 2051 / 4900) loss: 1.226753
(Iteration 2101 / 4900) loss: 1.248218
(Iteration 2151 / 4900) loss: 1.168664
(Iteration 2201 / 4900) loss: 1.329747
(Iteration 2251 / 4900) loss: 1.091494
(Iteration 2301 / 4900) loss: 1.298749
(Iteration 2351 / 4900) loss: 1.169799
(Iteration 2401 / 4900) loss: 1.291062
(Epoch 5 / 10) train acc: 0.568000; val_acc: 0.517000
(Iteration 2451 / 4900) loss: 1.090569
(Iteration 2501 / 4900) loss: 1.040105
(Iteration 2551 / 4900) loss: 1.196661
(Iteration 2601 / 4900) loss: 1.184482
(Iteration 2651 / 4900) loss: 1.248305
(Iteration 2701 / 4900) loss: 0.966165
(Iteration 2751 / 4900) loss: 1.073918
(Iteration 2801 / 4900) loss: 1.219766
(Iteration 2851 / 4900) loss: 1.084969
(Iteration 2901 / 4900) loss: 1.214645
(Epoch 6 / 10) train acc: 0.576000; val_acc: 0.527000
```

```
(Iteration 2951 / 4900) loss: 1.353390
(Iteration 3001 / 4900) loss: 1.289739
(Iteration 3051 / 4900) loss: 0.962614
(Iteration 3101 / 4900) loss: 1.228209
(Iteration 3151 / 4900) loss: 1.111609
(Iteration 3201 / 4900) loss: 1.097480
(Iteration 3251 / 4900) loss: 1.163739
(Iteration 3301 / 4900) loss: 0.973220
(Iteration 3351 / 4900) loss: 1.050828
(Iteration 3401 / 4900) loss: 1.176953
(Epoch 7 / 10) train acc: 0.629000; val_acc: 0.522000
(Iteration 3451 / 4900) loss: 1.084450
(Iteration 3501 / 4900) loss: 1.093541
(Iteration 3551 / 4900) loss: 1.387569
(Iteration 3601 / 4900) loss: 1.125746
(Iteration 3651 / 4900) loss: 1.075082
(Iteration 3701 / 4900) loss: 1.066931
(Iteration 3751 / 4900) loss: 0.909745
(Iteration 3801 / 4900) loss: 0.947617
(Iteration 3851 / 4900) loss: 1.034305
(Iteration 3901 / 4900) loss: 1.090769
(Epoch 8 / 10) train acc: 0.667000; val_acc: 0.558000
(Iteration 3951 / 4900) loss: 0.995495
(Iteration 4001 / 4900) loss: 1.008144
(Iteration 4051 / 4900) loss: 0.943819
(Iteration 4101 / 4900) loss: 0.969136
(Iteration 4151 / 4900) loss: 1.047509
(Iteration 4201 / 4900) loss: 0.908712
(Iteration 4251 / 4900) loss: 0.874539
(Iteration 4301 / 4900) loss: 0.944562
(Iteration 4351 / 4900) loss: 0.856182
(Iteration 4401 / 4900) loss: 0.962699
(Epoch 9 / 10) train acc: 0.660000; val_acc: 0.531000
(Iteration 4451 / 4900) loss: 0.945672
(Iteration 4501 / 4900) loss: 0.901853
(Iteration 4551 / 4900) loss: 0.789863
(Iteration 4601 / 4900) loss: 0.997731
(Iteration 4651 / 4900) loss: 0.946927
(Iteration 4701 / 4900) loss: 0.762023
(Iteration 4751 / 4900) loss: 0.799917
(Iteration 4801 / 4900) loss: 0.892969
(Iteration 4851 / 4900) loss: 0.892236
(Epoch 10 / 10) train acc: 0.668000; val_acc: 0.534000
```

In [11]:

```
y_test_pred = np.argmax(model.loss(data['X_test']), axis=1)
y_val_pred = np.argmax(model.loss(data['X_val']), axis=1)
print('Validation set accuracy: {}'.format(np.mean(y_val_pred == data['y_val'])))
print('Test set accuracy: {}'.format(np.mean(y_test_pred == data['y_test'])))
```

```
Validation set accuracy: 0.558
Test set accuracy: 0.533
```

```python
import numpy as np


"""
This file implements various first-order update rules that are commonly used for
training neural networks. Each update rule accepts current weights and the
gradient of the loss with respect to those weights and produces the next set of
weights. Each update rule has the same interface:

def update(w, dw, config=None):

Inputs:
  - w: A numpy array giving the current weights.
  - dw: A numpy array of the same shape as w giving the gradient of the
    loss with respect to w.
  - config: A dictionary containing hyperparameter values such as learning rate,
    momentum, etc. If the update rule requires caching values over many
    iterations, then config will also hold these cached values.

Returns:
  - next_w: The next point after the update.
  - config: The config dictionary to be passed to the next iteration of the
    update rule.

NOTE: For most update rules, the default learning rate will probably not perform
well; however the default values of the other hyperparameters should work well
for a variety of different problems.

For efficiency, update rules may perform in-place updates, mutating w and
setting next_w equal to w.
"""


def sgd(w, dw, config=None):
  """
  Performs vanilla stochastic gradient descent.

  config format:
  - learning_rate: Scalar learning rate.
  """
  if config is None: config = {}
  config.setdefault('learning_rate', 1e-2)

  w -= config['learning_rate'] * dw
  return w, config


def sgd_momentum(w, dw, config=None):
  """
  Performs stochastic gradient descent with momentum.

  config format:
  - learning_rate: Scalar learning rate.
  - momentum: Scalar between 0 and 1 giving the momentum value.
```

```
      Setting momentum = 0 reduces to sgd.
    - velocity: A numpy array of the same shape as w and dw used to store a moving
      average of the gradients.
    """
    if config is None: config = {}
    config.setdefault('learning_rate', 1e-2)
    config.setdefault('momentum', 0.9) # set momentum to 0.9 if it wasn't there
    v = config.get('velocity', np.zeros_like(w))   # gets velocity, else sets it to zero.

    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the momentum update formula.  Return the updated weights
    #   as next_w, and the updated velocity as v.
    # ================================================================ #

    alpha = config['momentum']
    eps = config['learning_rate']

    v = alpha * v - eps * dw
    w += v

    next_w = w

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    config['velocity'] = v

    return next_w, config

def sgd_nesterov_momentum(w, dw, config=None):
    """
    Performs stochastic gradient descent with Nesterov momentum.

    config format:
    - learning_rate: Scalar learning rate.
    - momentum: Scalar between 0 and 1 giving the momentum value.
      Setting momentum = 0 reduces to sgd.
    - velocity: A numpy array of the same shape as w and dw used to store a moving
      average of the gradients.
    """
    if config is None: config = {}
    config.setdefault('learning_rate', 1e-2)
    config.setdefault('momentum', 0.9) # set momentum to 0.9 if it wasn't there
    v = config.get('velocity', np.zeros_like(w))   # gets velocity, else sets it to zero.

    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the momentum update formula.  Return the updated weights
    #   as next_w, and the updated velocity as v.
    # ================================================================ #

    alpha = config['momentum']
    eps = config['learning_rate']

```

```python
  v_old = v
  v = alpha * v - eps * dw
  w += (v + alpha * (v - v_old))

  next_w = w

  # =========================================================================== #
  # END YOUR CODE HERE
  # =========================================================================== #

  config['velocity'] = v

  return next_w, config

def rmsprop(w, dw, config=None):
  """
  Uses the RMSProp update rule, which uses a moving average of squared gradient
  values to set adaptive per-parameter learning rates.

  config format:
  - learning_rate: Scalar learning rate.
  - decay_rate: Scalar between 0 and 1 giving the decay rate for the squared
    gradient cache.
  - epsilon: Small scalar used for smoothing to avoid dividing by zero.
  - beta: Moving average of second moments of gradients.
  """
  if config is None: config = {}
  config.setdefault('learning_rate', 1e-2)
  config.setdefault('decay_rate', 0.99)
  config.setdefault('epsilon', 1e-8)
  config.setdefault('a', np.zeros_like(w))

  next_w = None

  # =========================================================================== #
  # YOUR CODE HERE:
  #   Implement RMSProp.  Store the next value of w as next_w.  You need
  #   to also store in config['a'] the moving average of the second
  #   moment gradients, so they can be used for future gradients. Concretely,
  #   config['a'] corresponds to "a" in the lecture notes.
  # =========================================================================== #

  a = config['a']
  beta = config['decay_rate']
  eps = config['learning_rate']
  nu = config['epsilon']

  a = beta * a + (1 - beta) * dw * dw
  w -= (eps * dw) / (np.sqrt(a) + nu)

  config['a'] = a
  next_w = w

  # =========================================================================== #
  # END YOUR CODE HERE
  # =========================================================================== #

  return next_w, config

```

```python

def adam(w, dw, config=None):
  """
  Uses the Adam update rule, which incorporates moving averages of both the
  gradient and its square and a bias correction term.

  config format:
  - learning_rate: Scalar learning rate.
  - beta1: Decay rate for moving average of first moment of gradient.
  - beta2: Decay rate for moving average of second moment of gradient.
  - epsilon: Small scalar used for smoothing to avoid dividing by zero.
  - m: Moving average of gradient.
  - v: Moving average of squared gradient.
  - t: Iteration number.
  """
  if config is None: config = {}
  config.setdefault('learning_rate', 1e-3)
  config.setdefault('beta1', 0.9)
  config.setdefault('beta2', 0.999)
  config.setdefault('epsilon', 1e-8)
  config.setdefault('v', np.zeros_like(w))
  config.setdefault('a', np.zeros_like(w))
  config.setdefault('t', 0)

  next_w = None

  # ============================================================== #
  # YOUR CODE HERE:
  #   Implement Adam.  Store the next value of w as next_w.  You need
  #   to also store in config['a'] the moving average of the second
  #   moment gradients, and in config['v'] the moving average of the
  #   first moments.  Finally, store in config['t'] the increasing time.
  # ============================================================== #

  t = config['t']
  v = config['v']
  a = config['a']
  eps = config['learning_rate']
  nu = config['epsilon']
  beta1 = config['beta1']
  beta2 = config['beta2']

  t += 1
  v = beta1 * v + (1 - beta1) * dw
  a = beta2 * a + (1 - beta2) * dw * dw
  v_u = v / (1 - beta1**t)
  a_u = a / (1 - beta2**t)
  w -= (eps * v_u) / (np.sqrt(a_u) + nu)

  config['t'] = t
  config['v'] = v
  config['a'] = a
  next_w = w

  # ============================================================== #
  # END YOUR CODE HERE
  # ============================================================== #

  return next_w, config

```

ECE C147/247 HW4 Q2: Batch Normalization

In this notebook, you will implement the batch normalization layers of a neural network to increase its performance. Please review the details of batch normalization from the lecture notes.

`utils` has built a solid API for building these modular frameworks and training them, and we will use their very well implemented framework as opposed to "reinventing the wheel." This includes using their Solver, various utility functions, and their layer structure. This also includes `nndl.fc_net`, `nndl.layers`, and `nndl.layer_utils`.

In [1]:

```
## Import and setups

import time
import numpy as np
import matplotlib.pyplot as plt
from nndl.fc_net import *
from nndl.layers import *
from utils.data_utils import get_CIFAR10_data
from utils.gradient_check import eval_numerical_gradient, eval_numerical_gradient_array
from utils.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

In [2]:

```
# Load the (preprocessed) CIFAR10 data.

data = get_CIFAR10_data()
for k in data.keys():
  print('{}: {} '.format(k, data[k].shape))
```

```
X_train: (49000, 3, 32, 32)
y_train: (49000,)
X_val: (1000, 3, 32, 32)
y_val: (1000,)
X_test: (1000, 3, 32, 32)
y_test: (1000,)
```

Batchnorm forward pass

Implement the training time batchnorm forward pass, `batchnorm_forward`, in `nndl/layers.py`. After that, test your implementation by running the following cell.

In [3]:

```
# Check the training-time forward pass by checking means and variances
# of features both before and after batch normalization
```

```python
# Simulate the forward pass for a two-layer network
N, D1, D2, D3 = 200, 50, 60, 3
X = np.random.randn(N, D1)
W1 = np.random.randn(D1, D2)
W2 = np.random.randn(D2, D3)
a = np.maximum(0, X.dot(W1)).dot(W2)

print('Before batch normalization:')
print('  means: ', a.mean(axis=0))
print('  stds: ', a.std(axis=0))

# Means should be close to zero and stds close to one
print('After batch normalization (gamma=1, beta=0)')
a_norm, _ = batchnorm_forward(a, np.ones(D3), np.zeros(D3), {'mode': 'train'})
print('  mean: ', a_norm.mean(axis=0))
print('  std: ', a_norm.std(axis=0))

# Now means should be close to beta and stds close to gamma
gamma = np.asarray([1.0, 2.0, 3.0])
beta = np.asarray([11.0, 12.0, 13.0])
a_norm, _ = batchnorm_forward(a, gamma, beta, {'mode': 'train'})
print('After batch normalization (nontrivial gamma, beta)')
print('  means: ', a_norm.mean(axis=0))
print('  stds: ', a_norm.std(axis=0))
```

```
Before batch normalization:
  means:  [-27.52804285   3.23108626 -39.84281267]
  stds:  [32.45802149 34.16563075 41.61402063]
After batch normalization (gamma=1, beta=0)
  mean:  [6.63913369e-16 2.27595720e-17 4.26603197e-16]
  std:  [1. 1. 1.]
After batch normalization (nontrivial gamma, beta)
  means:  [11. 12. 13.]
  stds:  [1.         1.99999999 2.99999999]
```

Implement the testing time batchnorm forward pass, `batchnorm_forward`, in `nndl/layers.py`. After that, test your implementation by running the following cell.

In [4]:

```python
# Check the test-time forward pass by running the training-time
# forward pass many times to warm up the running averages, and then
# checking the means and variances of activations after a test-time
# forward pass.

N, D1, D2, D3 = 200, 50, 60, 3
W1 = np.random.randn(D1, D2)
W2 = np.random.randn(D2, D3)

bn_param = {'mode': 'train'}
gamma = np.ones(D3)
beta = np.zeros(D3)
for t in np.arange(50):
  X = np.random.randn(N, D1)
  a = np.maximum(0, X.dot(W1)).dot(W2)
  batchnorm_forward(a, gamma, beta, bn_param)
bn_param['mode'] = 'test'
X = np.random.randn(N, D1)
a = np.maximum(0, X.dot(W1)).dot(W2)
a_norm, _ = batchnorm_forward(a, gamma, beta, bn_param)

# Means should be close to zero and stds close to one, but will be
# noisier than training-time forward passes.
print('After batch normalization (test-time):')
print('  means: ', a_norm.mean(axis=0))
print('  stds: ', a_norm.std(axis=0))
```

```
After batch normalization (test-time):
  means:  [-0.07597434  0.00739562  0.07691946]
  stds:  [0.88788487 1.04432885 0.99089243]
```

Batchnorm backward pass

Implement the backward pass for the batchnorm layer, `batchnorm_backward` in `nndl/layers.py` . Check your implementation by running the following cell.

In [5]:

```
# Gradient check batchnorm backward pass

N, D = 4, 5
x = 5 * np.random.randn(N, D) + 12
gamma = np.random.randn(D)
beta = np.random.randn(D)
dout = np.random.randn(N, D)

bn_param = {'mode': 'train'}
fx = lambda x: batchnorm_forward(x, gamma, beta, bn_param)[0]
fg = lambda a: batchnorm_forward(x, gamma, beta, bn_param)[0]
fb = lambda b: batchnorm_forward(x, gamma, beta, bn_param)[0]

dx_num = eval_numerical_gradient_array(fx, x, dout)
da_num = eval_numerical_gradient_array(fg, gamma, dout)
db_num = eval_numerical_gradient_array(fb, beta, dout)

_, cache = batchnorm_forward(x, gamma, beta, bn_param)
dx, dgamma, dbeta = batchnorm_backward(dout, cache)
print('dx error: ', rel_error(dx_num, dx))
print('dgamma error: ', rel_error(da_num, dgamma))
print('dbeta error: ', rel_error(db_num, dbeta))
```

```
dx error:  1.3987316119808057e-09
dgamma error:  8.256068052448124e-12
dbeta error:  3.2755634106925108e-12
```

Implement a fully connected neural network with batchnorm layers

Modify the `FullyConnectedNet()` class in `nndl/fc_net.py` to incorporate batchnorm layers. You will need to modify the class in the following areas:

(1) The gammas and betas need to be initialized to 1's and 0's respectively in `__init__` .

(2) The `batchnorm_forward` layer needs to be inserted between each affine and relu layer (except in the output layer) in a forward pass computation in `loss` . You may find it helpful to write an `affine_batchnorm_relu()` layer in `nndl/layer_utils.py` although this is not necessary.

(3) The `batchnorm_backward` layer has to be appropriately inserted when calculating gradients.

After you have done the appropriate modifications, check your implementation by running the following cell.

Note, while the relative error for W3 should be small, as we backprop gradients more, you may find the relative error increases. Our relative error for W1 is on the order of 1e-4.

In [6]:

```python
N, D, H1, H2, C = 2, 15, 20, 30, 10
X = np.random.randn(N, D)
y = np.random.randint(C, size=(N,))

for reg in [0, 3.14]:
  print('Running check with reg = ', reg)
  model = FullyConnectedNet([H1, H2], input_dim=D, num_classes=C,
                            reg=reg, weight_scale=5e-2, dtype=np.float64,
                            use_batchnorm=True)

  loss, grads = model.loss(X, y)
  print('Initial loss: ', loss)

  for name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    grad_num = eval_numerical_gradient(f, model.params[name], verbose=False, h=1e-5)
    print('{} relative error: {}'.format(name, rel_error(grad_num, grads[name])))
  if reg == 0: print('\n')
```

```
Running check with reg =  0
Initial loss:  2.191057485443417
W1 relative error: 8.308090754249485e-05
W2 relative error: 1.0217928713160541e-05
W3 relative error: 5.235349587857187e-10
b1 relative error: 1.7763568394002505e-07
b2 relative error: 2.220446049250313e-08
b3 relative error: 1.0585275403998414e-10
beta1 relative error: 7.306782539667682e-08
beta2 relative error: 8.096766141771687e-09
gamma1 relative error: 7.579108849046012e-08
gamma2 relative error: 3.089761818884722e-09


Running check with reg =  3.14
Initial loss:  6.988257449407779
W1 relative error: 1.8170001423524735e-06
W2 relative error: 1.5313005259248896e-06
W3 relative error: 1.3481894007180487e-08
b1 relative error: 1.7763568394002505e-07
b2 relative error: 8.881784197001252e-08
b3 relative error: 2.573882924097516e-10
beta1 relative error: 2.520345435204669e-09
beta2 relative error: 7.583300904467152e-09
gamma1 relative error: 2.485653322350572e-09
gamma2 relative error: 1.4850195551907493e-08
```

Training a deep fully connected network with batch normalization.

To see if batchnorm helps, let's train a deep neural network with and without batch normalization.

In [7]:

```python
# Try training a very deep net with batchnorm
hidden_dims = [100, 100, 100, 100, 100]

num_train = 1000
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

weight_scale = 2e-2
```

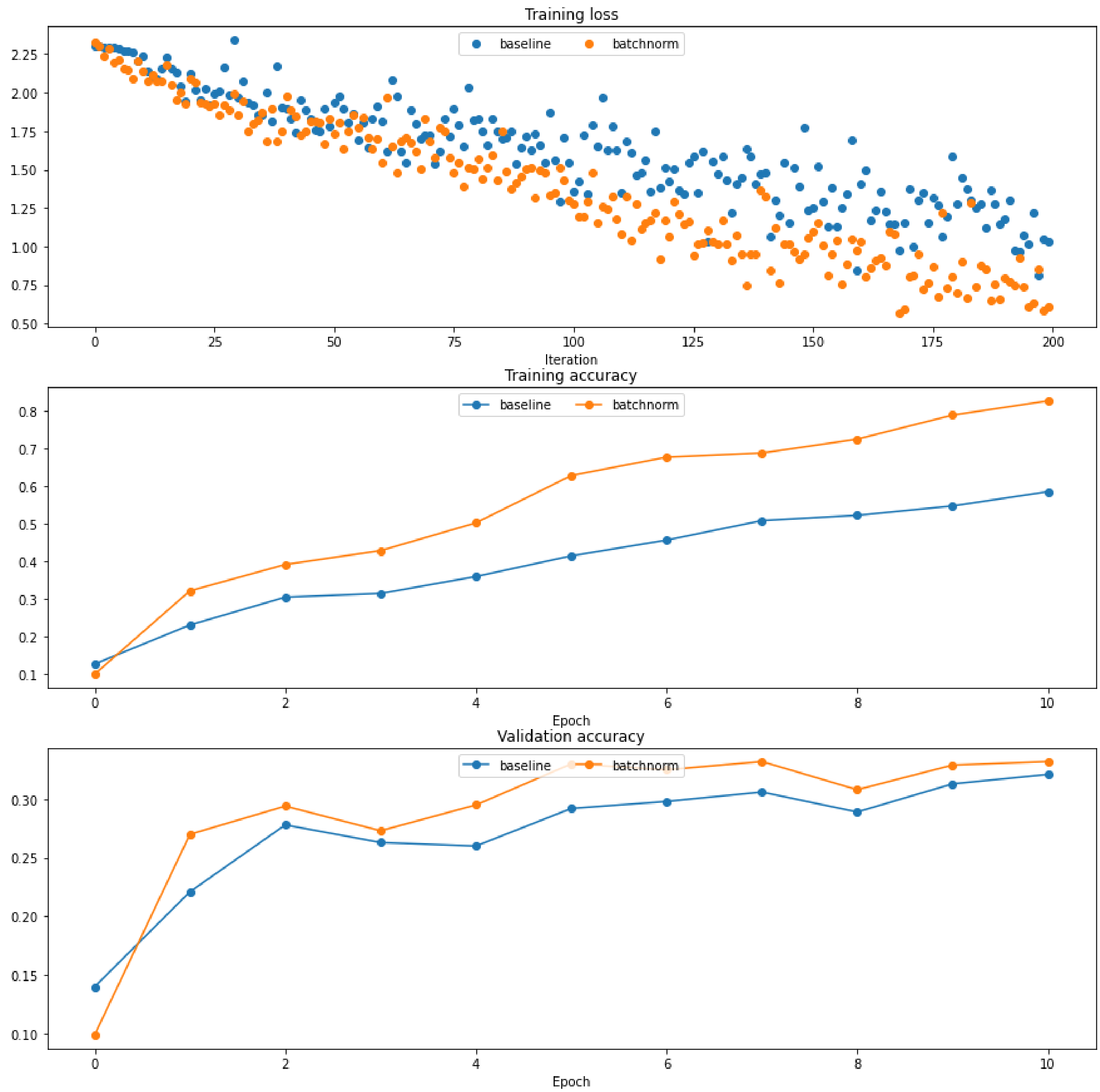
```
bn_model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, use_batchnorm=True)
model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, use_batchnorm=False)

bn_solver = Solver(bn_model, small_data,
                num_epochs=10, batch_size=50,
                update_rule='adam',
                optim_config={
                  'learning_rate': 1e-3,
                },
                verbose=True, print_every=200)
bn_solver.train()

solver = Solver(model, small_data,
                num_epochs=10, batch_size=50,
                update_rule='adam',
                optim_config={
                  'learning_rate': 1e-3,
                },
                verbose=True, print_every=200)
solver.train()
```

```
(Iteration 1 / 200) loss: 2.325430
(Epoch 0 / 10) train acc: 0.100000; val_acc: 0.099000
(Epoch 1 / 10) train acc: 0.322000; val_acc: 0.270000
(Epoch 2 / 10) train acc: 0.392000; val_acc: 0.294000
(Epoch 3 / 10) train acc: 0.429000; val_acc: 0.273000
(Epoch 4 / 10) train acc: 0.503000; val_acc: 0.295000
(Epoch 5 / 10) train acc: 0.629000; val_acc: 0.330000
(Epoch 6 / 10) train acc: 0.678000; val_acc: 0.325000
(Epoch 7 / 10) train acc: 0.689000; val_acc: 0.332000
(Epoch 8 / 10) train acc: 0.726000; val_acc: 0.308000
(Epoch 9 / 10) train acc: 0.790000; val_acc: 0.329000
(Epoch 10 / 10) train acc: 0.828000; val_acc: 0.332000
(Iteration 1 / 200) loss: 2.301877
(Epoch 0 / 10) train acc: 0.127000; val_acc: 0.140000
(Epoch 1 / 10) train acc: 0.231000; val_acc: 0.221000
(Epoch 2 / 10) train acc: 0.305000; val_acc: 0.278000
(Epoch 3 / 10) train acc: 0.315000; val_acc: 0.263000
(Epoch 4 / 10) train acc: 0.360000; val_acc: 0.260000
(Epoch 5 / 10) train acc: 0.415000; val_acc: 0.292000
(Epoch 6 / 10) train acc: 0.457000; val_acc: 0.298000
(Epoch 7 / 10) train acc: 0.509000; val_acc: 0.306000
(Epoch 8 / 10) train acc: 0.523000; val_acc: 0.289000
(Epoch 9 / 10) train acc: 0.548000; val_acc: 0.313000
(Epoch 10 / 10) train acc: 0.586000; val_acc: 0.321000
```

In [8]:

```
plt.subplot(3, 1, 1)
plt.title('Training loss')
plt.xlabel('Iteration')

plt.subplot(3, 1, 2)
plt.title('Training accuracy')
plt.xlabel('Epoch')

plt.subplot(3, 1, 3)
plt.title('Validation accuracy')
plt.xlabel('Epoch')

plt.subplot(3, 1, 1)
plt.plot(solver.loss_history, 'o', label='baseline')
plt.plot(bn_solver.loss_history, 'o', label='batchnorm')

plt.subplot(3, 1, 2)
plt.plot(solver.train_acc_history, '-o', label='baseline')
plt.plot(bn_solver.train_acc_history, '-o', label='batchnorm')
```

```
plt.subplot(3, 1, 3)
plt.plot(solver.val_acc_history, '-o', label='baseline')
plt.plot(bn_solver.val_acc_history, '-o', label='batchnorm')

for i in [1, 2, 3]:
  plt.subplot(3, 1, i)
  plt.legend(loc='upper center', ncol=4)
plt.gcf().set_size_inches(15, 15)
plt.show()
```



Batchnorm and initialization

The following cells run an experiment where for a deep network, the initialization is varied. We do training for when batchnorm layers are and are not included.

In [9]:

```
# Try training a very deep net with batchnorm
hidden_dims = [50, 50, 50, 50, 50, 50, 50]
```

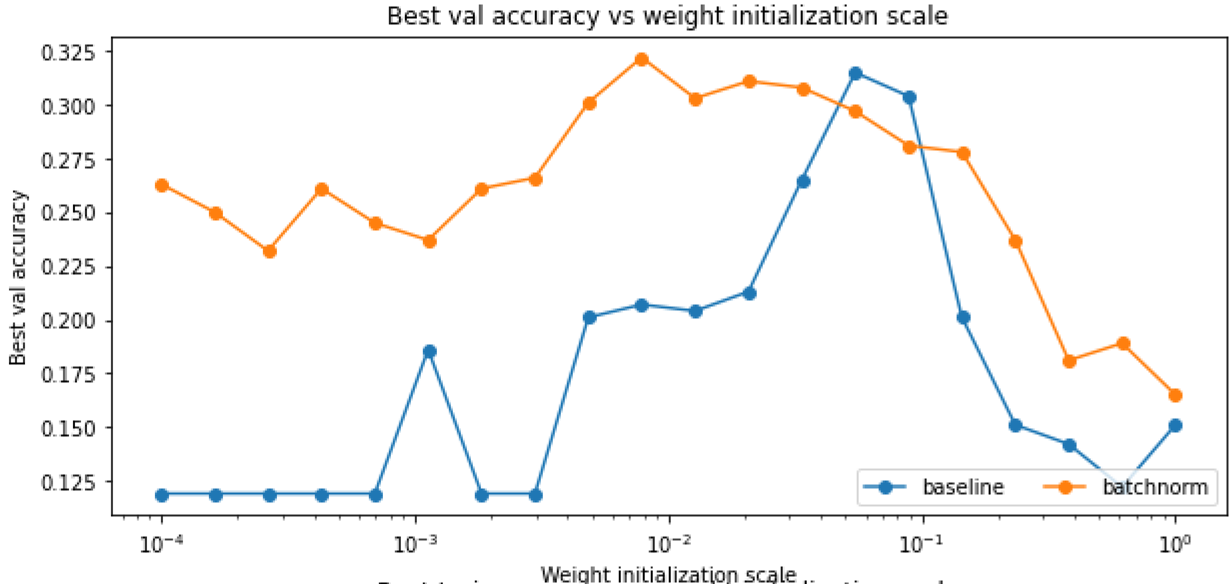
```
num_train = 1000
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

bn_solvers = {}
solvers = {}
weight_scales = np.logspace(-4, 0, num=20)
for i, weight_scale in enumerate(weight_scales):
  print('Running weight scale {} / {}'.format(i + 1, len(weight_scales)))
  bn_model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, use_batchnorm=Tru
  model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale, use_batchnorm=False)

  bn_solver = Solver(bn_model, small_data,
                  num_epochs=10, batch_size=50,
                  update_rule='adam',
                  optim_config={
                    'learning_rate': 1e-3,
                  },
                  verbose=False, print_every=200)
  bn_solver.train()
  bn_solvers[weight_scale] = bn_solver

  solver = Solver(model, small_data,
                  num_epochs=10, batch_size=50,
                  update_rule='adam',
                  optim_config={
                    'learning_rate': 1e-3,
                  },
                  verbose=False, print_every=200)
  solver.train()
  solvers[weight_scale] = solver
```

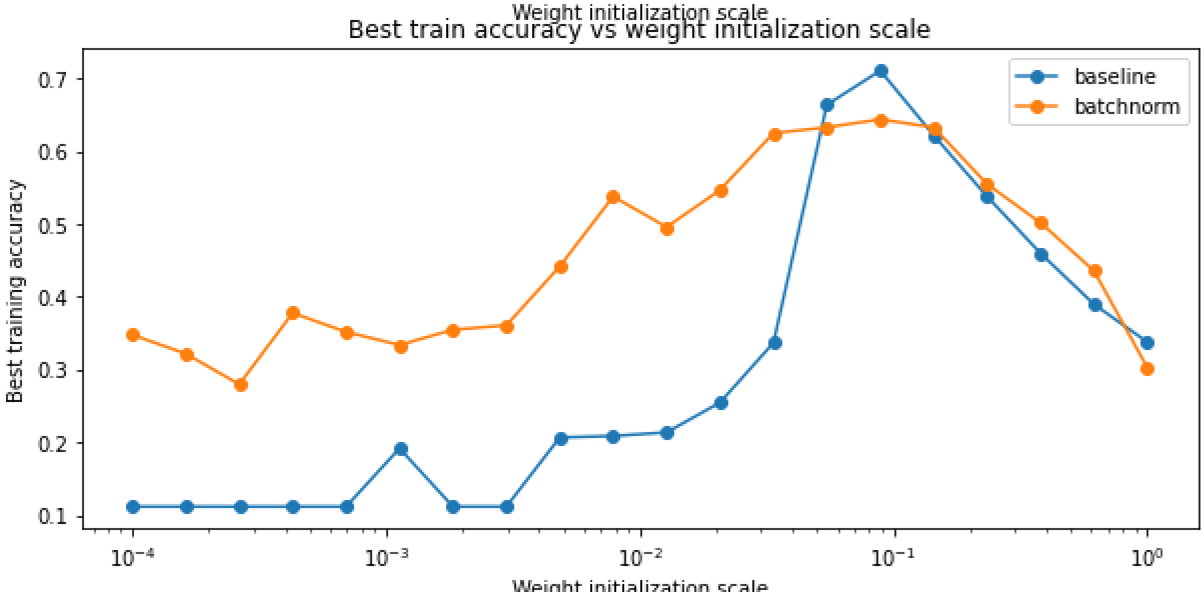
```
Running weight scale 1 / 20
Running weight scale 2 / 20
Running weight scale 3 / 20
Running weight scale 4 / 20
Running weight scale 5 / 20
Running weight scale 6 / 20
Running weight scale 7 / 20
Running weight scale 8 / 20
Running weight scale 9 / 20
Running weight scale 10 / 20
Running weight scale 11 / 20
Running weight scale 12 / 20
Running weight scale 13 / 20
Running weight scale 14 / 20
Running weight scale 15 / 20
Running weight scale 16 / 20
Running weight scale 17 / 20
Running weight scale 18 / 20
Running weight scale 19 / 20
Running weight scale 20 / 20
```

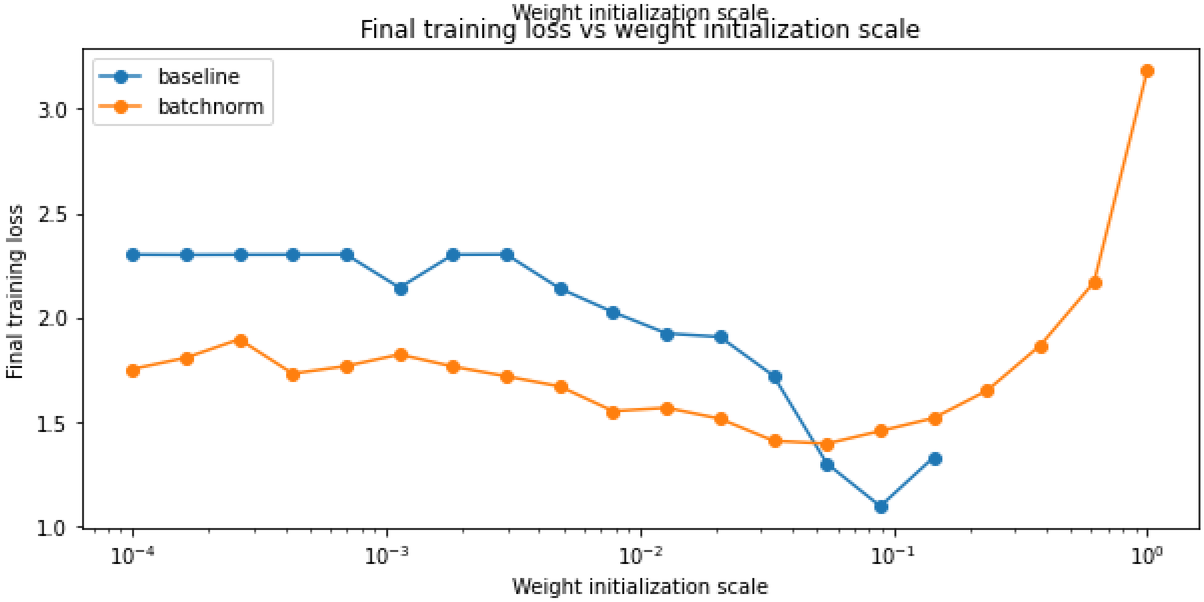
In [10]:

```
# Plot results of weight scale experiment
best_train_accs, bn_best_train_accs = [], []
best_val_accs, bn_best_val_accs = [], []
final_train_loss, bn_final_train_loss = [], []

for ws in weight_scales:
  best_train_accs.append(max(solvers[ws].train_acc_history))
  bn_best_train_accs.append(max(bn_solvers[ws].train_acc_history))
```

```
  best_val_accs.append(max(solvers[ws].val_acc_history))
  bn_best_val_accs.append(max(bn_solvers[ws].val_acc_history))

  final_train_loss.append(np.mean(solvers[ws].loss_history[-100:]))
  bn_final_train_loss.append(np.mean(bn_solvers[ws].loss_history[-100:]))

plt.subplot(3, 1, 1)
plt.title('Best val accuracy vs weight initialization scale')
plt.xlabel('Weight initialization scale')
plt.ylabel('Best val accuracy')
plt.semilogx(weight_scales, best_val_accs, '-o', label='baseline')
plt.semilogx(weight_scales, bn_best_val_accs, '-o', label='batchnorm')
plt.legend(ncol=2, loc='lower right')

plt.subplot(3, 1, 2)
plt.title('Best train accuracy vs weight initialization scale')
plt.xlabel('Weight initialization scale')
plt.ylabel('Best training accuracy')
plt.semilogx(weight_scales, best_train_accs, '-o', label='baseline')
plt.semilogx(weight_scales, bn_best_train_accs, '-o', label='batchnorm')
plt.legend()

plt.subplot(3, 1, 3)
plt.title('Final training loss vs weight initialization scale')
plt.xlabel('Weight initialization scale')
plt.ylabel('Final training loss')
plt.semilogx(weight_scales, final_train_loss, '-o', label='baseline')
plt.semilogx(weight_scales, bn_final_train_loss, '-o', label='batchnorm')
plt.legend()

plt.gcf().set_size_inches(10, 15)
plt.show()
```







Question:

In the cell below, summarize the findings of this experiment, and WHY these results make sense.

Answer:

- From the plots, we can know that batchnorm can achieve higher accuracy. When the weight initialization scale is smaller than 10^(-1), the model with batchnorm performs better accuracies and has lower training loss than the model without batchnorm.

- The model with batchnorm (the orange curve) has wider ranges on the weight initialization scale in the training loss plot. And, it is stable and consistant with the change of the weight initialization scale. It means that batchnorm makes the model less sensitive to the weight initialization scale.

- This is because batchnorm is used in regularization, and makes results more stable. Therefore, from the results shown above, the model with batchnorm (the orange curve) is less sensitive and less affected by the weight initialization scale.

ECE C147/247 HW4 Q3: Dropout

In this notebook, you will implement dropout. Then we will ask you to train a network with batchnorm and dropout, and acheive over 55% accuracy on CIFAR-10.

`utils` has built a solid API for building these modular frameworks and training them, and we will use their very well implemented framework as opposed to "reinventing the wheel." This includes using their Solver, various utility functions, and their layer structure. This also includes `nndl.fc_net` , `nndl.layers` , and `nndl.layer_utils` .

In [1]:

```
## Import and setups

import time
import numpy as np
import matplotlib.pyplot as plt
from nndl.fc_net import *
from nndl.layers import *
from utils.data_utils import get_CIFAR10_data
from utils.gradient_check import eval_numerical_gradient, eval_numerical_gradient_array
from utils.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

In [2]:

```
# Load the (preprocessed) CIFAR10 data.

data = get_CIFAR10_data()
for k in data.keys():
  print('{}: {} '.format(k, data[k].shape))
```

```
X_train: (49000, 3, 32, 32)
y_train: (49000,)
X_val: (1000, 3, 32, 32)
y_val: (1000,)
X_test: (1000, 3, 32, 32)
y_test: (1000,)
```

Dropout forward pass

Implement the training and test time dropout forward pass, `dropout_forward` , in `nndl/layers.py` . After that, test your implementation by running the following cell.

In [3]:

```
x = np.random.randn(500, 500) + 10
```

```
for p in [0.3, 0.6, 0.75]:
  out, _ = dropout_forward(x, {'mode': 'train', 'p': p})
  out_test, _ = dropout_forward(x, {'mode': 'test', 'p': p})

  print('Running tests with p = ', p)
  print('Mean of input: ', x.mean())
  print('Mean of train-time output: ', out.mean())
  print('Mean of test-time output: ', out_test.mean())
  print('Fraction of train-time output set to zero: ', (out == 0).mean())
  print('Fraction of test-time output set to zero: ', (out_test == 0).mean())
```

```
Running tests with p =  0.3
Mean of input:  10.001329958949242
Mean of train-time output:  10.000677630103738
Mean of test-time output:  10.001329958949242
Fraction of train-time output set to zero:  0.300204
Fraction of test-time output set to zero:  0.0
Running tests with p =  0.6
Mean of input:  10.001329958949242
Mean of train-time output:  9.995013082981687
Mean of test-time output:  10.001329958949242
Fraction of train-time output set to zero:  0.600344
Fraction of test-time output set to zero:  0.0
Running tests with p =  0.75
Mean of input:  10.001329958949242
Mean of train-time output:  9.99724972751136
Mean of test-time output:  10.001329958949242
Fraction of train-time output set to zero:  0.750256
Fraction of test-time output set to zero:  0.0
```

Dropout backward pass

Implement the backward pass, `dropout_backward`, in `nndl/layers.py`. After that, test your gradients by running the following cell:

In [4]:

```
x = np.random.randn(10, 10) + 10
dout = np.random.randn(*x.shape)

dropout_param = {'mode': 'train', 'p': 0.8, 'seed': 123}
out, cache = dropout_forward(x, dropout_param)
dx = dropout_backward(dout, cache)
dx_num = eval_numerical_gradient_array(lambda xx: dropout_forward(xx, dropout_param)[0],

print('dx relative error: ', rel_error(dx, dx_num))
```

```
dx relative error:  1.8929063206183813e-11
```

Implement a fully connected neural network with dropout layers

Modify the `FullyConnectedNet()` class in `nndl/fc_net.py` to incorporate dropout. A dropout layer should be incorporated after every ReLU layer. Concretely, there shouldn't be a dropout at the output layer since there is no ReLU at the output layer. You will need to modify the class in the following areas:

(1) In the forward pass, you will need to incorporate a dropout layer after every relu layer.

(2) In the backward pass, you will need to incorporate a dropout backward pass layer.

Check your implementation by running the following code. Our W1 gradient relative error is on the order of 1e-6 (the largest of all the relative errors).

In [5]:

```
N, D, H1, H2, C = 2, 15, 20, 30, 10
X = np.random.randn(N, D)
y = np.random.randint(C, size=(N,))

for dropout in [0, 0.25, 0.5]:
  print('Running check with dropout = ', dropout)
  model = FullyConnectedNet([H1, H2], input_dim=D, num_classes=C,
                            weight_scale=5e-2, dtype=np.float64,
                            dropout=dropout, seed=123)

  loss, grads = model.loss(X, y)
  print('Initial loss: ', loss)

  for name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    grad_num = eval_numerical_gradient(f, model.params[name], verbose=False, h=1e-5)
    print('{} relative error: {}'.format(name, rel_error(grad_num, grads[name])))
  print('\n')
```

```
Running check with dropout =  0
Initial loss:  2.3051948273987857
W1 relative error: 2.5272575344376073e-07
W2 relative error: 1.8875602996866792e-05
W3 relative error: 2.3446001188064074e-07
b1 relative error: 1.3413524910372306e-07
b2 relative error: 5.051339805546953e-08
b3 relative error: 1.4926760614954125e-10


Running check with dropout =  0.25
Initial loss:  2.29898614757146
W1 relative error: 9.737733598900162e-07
W2 relative error: 5.073657932383196e-08
W3 relative error: 2.8870225716091813e-08
b1 relative error: 9.618747087456625e-09
b2 relative error: 1.897778283870511e-09
b3 relative error: 8.933554101600298e-11


Running check with dropout =  0.5
Initial loss:  2.302437587710995
W1 relative error: 4.553387957138422e-08
W2 relative error: 1.8475263967389784e-08
W3 relative error: 4.3413247403122424e-07
b1 relative error: 1.872462967441693e-08
b2 relative error: 5.045591219274328e-09
b3 relative error: 1.2715825087959627e-10
```

Dropout as a regularizer

In class, we claimed that dropout acts as a regularizer by effectively bagging. To check this, we will train two small networks, one with dropout and one without dropout.

In [6]:

```
# Train two identical nets, one with dropout and one without

num_train = 500
small_data = {
```

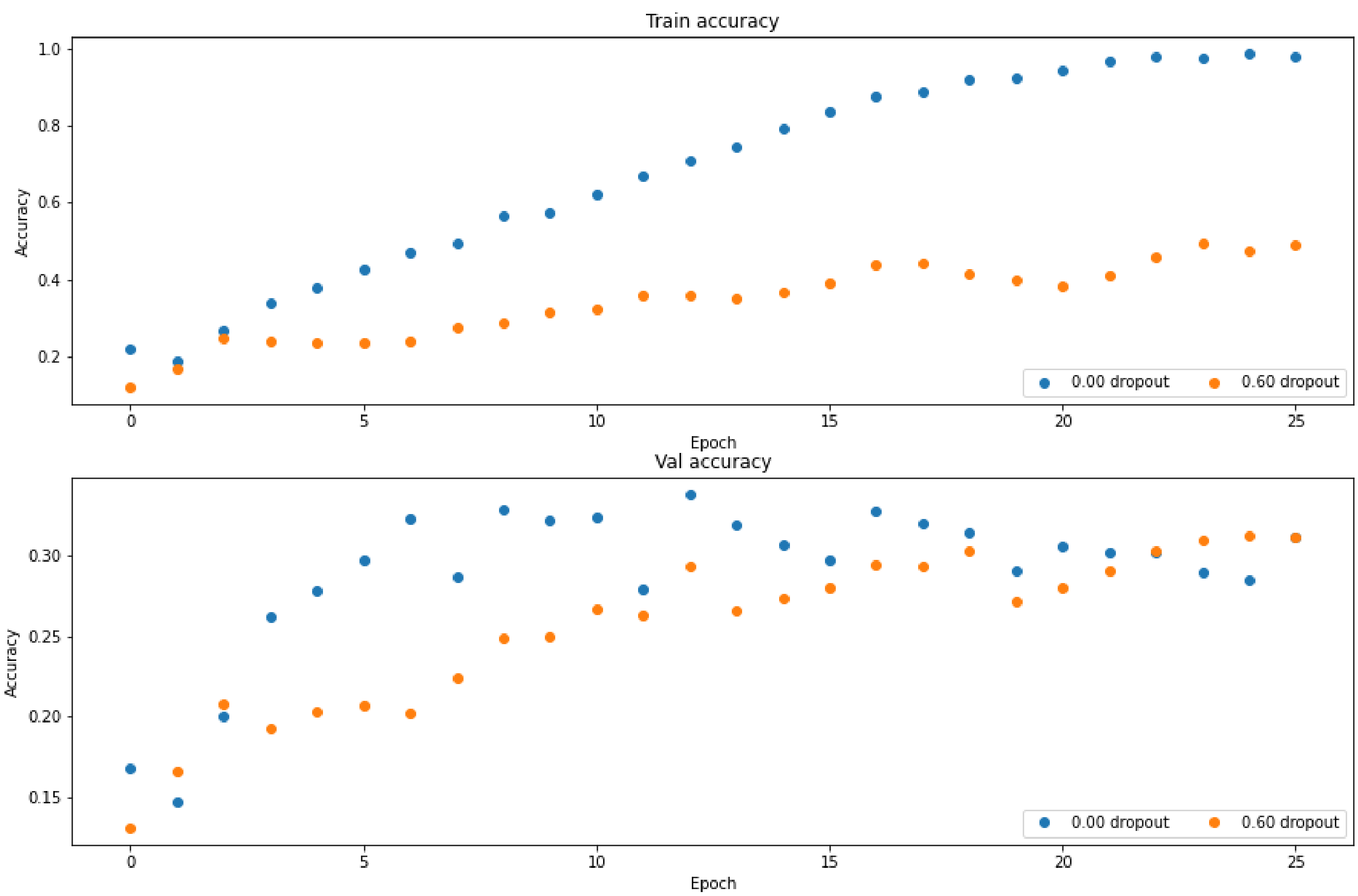
```python
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

solvers = {}
dropout_choices = [0, 0.6]
for dropout in dropout_choices:
  model = FullyConnectedNet([100, 100, 100], dropout=dropout)

  solver = Solver(model, small_data,
                  num_epochs=25, batch_size=100,
                  update_rule='adam',
                  optim_config={
                    'learning_rate': 5e-4,
                  },
                  verbose=True, print_every=100)
  solver.train()
  solvers[dropout] = solver
```

```
(Iteration 1 / 125) loss: 2.300804
(Epoch 0 / 25) train acc: 0.220000; val_acc: 0.168000
(Epoch 1 / 25) train acc: 0.188000; val_acc: 0.147000
(Epoch 2 / 25) train acc: 0.266000; val_acc: 0.200000
(Epoch 3 / 25) train acc: 0.338000; val_acc: 0.262000
(Epoch 4 / 25) train acc: 0.378000; val_acc: 0.278000
(Epoch 5 / 25) train acc: 0.428000; val_acc: 0.297000
(Epoch 6 / 25) train acc: 0.468000; val_acc: 0.323000
(Epoch 7 / 25) train acc: 0.494000; val_acc: 0.287000
(Epoch 8 / 25) train acc: 0.566000; val_acc: 0.328000
(Epoch 9 / 25) train acc: 0.572000; val_acc: 0.322000
(Epoch 10 / 25) train acc: 0.622000; val_acc: 0.324000
(Epoch 11 / 25) train acc: 0.670000; val_acc: 0.279000
(Epoch 12 / 25) train acc: 0.710000; val_acc: 0.338000
(Epoch 13 / 25) train acc: 0.746000; val_acc: 0.319000
(Epoch 14 / 25) train acc: 0.792000; val_acc: 0.307000
(Epoch 15 / 25) train acc: 0.834000; val_acc: 0.297000
(Epoch 16 / 25) train acc: 0.876000; val_acc: 0.327000
(Epoch 17 / 25) train acc: 0.886000; val_acc: 0.320000
(Epoch 18 / 25) train acc: 0.918000; val_acc: 0.314000
(Epoch 19 / 25) train acc: 0.922000; val_acc: 0.290000
(Epoch 20 / 25) train acc: 0.944000; val_acc: 0.306000
(Iteration 101 / 125) loss: 0.156105
(Epoch 21 / 25) train acc: 0.968000; val_acc: 0.302000
(Epoch 22 / 25) train acc: 0.978000; val_acc: 0.302000
(Epoch 23 / 25) train acc: 0.976000; val_acc: 0.289000
(Epoch 24 / 25) train acc: 0.986000; val_acc: 0.285000
(Epoch 25 / 25) train acc: 0.978000; val_acc: 0.311000
(Iteration 1 / 125) loss: 2.306395
(Epoch 0 / 25) train acc: 0.120000; val_acc: 0.131000
(Epoch 1 / 25) train acc: 0.170000; val_acc: 0.166000
(Epoch 2 / 25) train acc: 0.246000; val_acc: 0.208000
(Epoch 3 / 25) train acc: 0.240000; val_acc: 0.193000
(Epoch 4 / 25) train acc: 0.234000; val_acc: 0.203000
(Epoch 5 / 25) train acc: 0.234000; val_acc: 0.207000
(Epoch 6 / 25) train acc: 0.238000; val_acc: 0.202000
(Epoch 7 / 25) train acc: 0.276000; val_acc: 0.224000
(Epoch 8 / 25) train acc: 0.288000; val_acc: 0.249000
(Epoch 9 / 25) train acc: 0.314000; val_acc: 0.250000
(Epoch 10 / 25) train acc: 0.324000; val_acc: 0.267000
(Epoch 11 / 25) train acc: 0.360000; val_acc: 0.263000
(Epoch 12 / 25) train acc: 0.360000; val_acc: 0.293000
(Epoch 13 / 25) train acc: 0.352000; val_acc: 0.266000
(Epoch 14 / 25) train acc: 0.366000; val_acc: 0.273000
(Epoch 15 / 25) train acc: 0.390000; val_acc: 0.280000
(Epoch 16 / 25) train acc: 0.438000; val_acc: 0.294000
(Epoch 17 / 25) train acc: 0.442000; val_acc: 0.293000
(Epoch 18 / 25) train acc: 0.416000; val_acc: 0.303000
(Epoch 19 / 25) train acc: 0.400000; val_acc: 0.271000
(Epoch 20 / 25) train acc: 0.384000; val_acc: 0.280000
(Iteration 101 / 125) loss: 1.881304
(Epoch 21 / 25) train acc: 0.412000; val_acc: 0.290000
(Epoch 22 / 25) train acc: 0.460000; val_acc: 0.303000
(Epoch 23 / 25) train acc: 0.494000; val_acc: 0.309000
(Epoch 24 / 25) train acc: 0.474000; val_acc: 0.312000
(Epoch 25 / 25) train acc: 0.488000; val_acc: 0.311000
```

In [7]:

```python
# Plot train and validation accuracies of the two models

train_accs = []
val_accs = []
for dropout in dropout_choices:
  solver = solvers[dropout]
  train_accs.append(solver.train_acc_history[-1])
  val_accs.append(solver.val_acc_history[-1])
```

```
plt.subplot(3, 1, 1)
for dropout in dropout_choices:
  plt.plot(solvers[dropout].train_acc_history, 'o', label='%.2f dropout' % dropout)
plt.title('Train accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.legend(ncol=2, loc='lower right')

plt.subplot(3, 1, 2)
for dropout in dropout_choices:
  plt.plot(solvers[dropout].val_acc_history, 'o', label='%.2f dropout' % dropout)
plt.title('Val accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.legend(ncol=2, loc='lower right')

plt.gcf().set_size_inches(15, 15)
plt.show()
```



Question

Based off the results of this experiment, is dropout performing regularization? Explain your answer.

Answer:

- Yes, it is performing regularization. From the second plot, we can find that the model with or without dropout have similar validation accuracies. While in the first plot, the model without dropout has significantly higher training accuracies than the model with dropout. It means that the additional training accuracies of the model without dropout is overfitting, and the dropout regularized it.

Final part of the assignment

Get over 55% validation accuracy on CIFAR-10 by using the layers you have implemented. You will be graded according to the following equation:

```
min(floor((X - 32%)) / 28%, 1)
```

where if you get 60% or higher validation accuracy, you get full points.

In [8]:

```
# ================================================================ #
# YOUR CODE HERE:
#   Implement a FC-net that achieves at least 55% validation accuracy
#   on CIFAR-10.
# ================================================================ #

hidden_dims = [600, 600, 600, 600]
weight_scale = 0.04
dropout = 0.2
learning_rate = 3e-3
lr_decay = 0.95
update_rule = 'adam'

model = FullyConnectedNet(hidden_dims = hidden_dims, weight_scale = weight_scale, dropou
                          use_batchnorm = True, reg = 0.0)

solver = Solver(model, data,
                num_epochs = 100, batch_size = 500,
                update_rule = update_rule,
                optim_config = {
                  'learning_rate': learning_rate,
                },
                lr_decay = lr_decay,
                verbose=True, print_every = 100)
solver.train()

y_test_pred = np.argmax(model.loss(data['X_test']), axis = 1)
y_val_pred = np.argmax(model.loss(data['X_val']), axis = 1)
print("\nTest set accuracy: {}".format(np.mean(np.equal(y_test_pred, data['y_test']))))
print("Validation set accuracy: {}".format(np.mean(np.equal(y_val_pred, data['y_val'])))

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
```

```
(Iteration 1 / 9800) loss: 2.489383
(Epoch 0 / 100) train acc: 0.186000; val_acc: 0.200000
(Epoch 1 / 100) train acc: 0.436000; val_acc: 0.443000
(Iteration 101 / 9800) loss: 1.597815
(Epoch 2 / 100) train acc: 0.484000; val_acc: 0.498000
(Iteration 201 / 9800) loss: 1.366692
(Epoch 3 / 100) train acc: 0.537000; val_acc: 0.515000
(Iteration 301 / 9800) loss: 1.398289
(Epoch 4 / 100) train acc: 0.577000; val_acc: 0.566000
(Iteration 401 / 9800) loss: 1.248255
(Epoch 5 / 100) train acc: 0.599000; val_acc: 0.538000
(Iteration 501 / 9800) loss: 1.140919
(Epoch 6 / 100) train acc: 0.664000; val_acc: 0.548000
(Iteration 601 / 9800) loss: 1.117880
(Epoch 7 / 100) train acc: 0.613000; val_acc: 0.548000
(Iteration 701 / 9800) loss: 1.022194
(Epoch 8 / 100) train acc: 0.691000; val_acc: 0.580000
(Iteration 801 / 9800) loss: 1.053228
(Epoch 9 / 100) train acc: 0.702000; val_acc: 0.565000
(Iteration 901 / 9800) loss: 0.900552
(Epoch 10 / 100) train acc: 0.715000; val_acc: 0.581000
(Iteration 1001 / 9800) loss: 0.955007
(Epoch 11 / 100) train acc: 0.746000; val_acc: 0.593000
(Iteration 1101 / 9800) loss: 0.873861
(Epoch 12 / 100) train acc: 0.752000; val_acc: 0.579000
(Iteration 1201 / 9800) loss: 0.802197
(Epoch 13 / 100) train acc: 0.757000; val_acc: 0.585000
(Iteration 1301 / 9800) loss: 0.784768
(Epoch 14 / 100) train acc: 0.802000; val_acc: 0.597000
(Iteration 1401 / 9800) loss: 0.775939
(Epoch 15 / 100) train acc: 0.789000; val_acc: 0.583000
(Iteration 1501 / 9800) loss: 0.742718
(Epoch 16 / 100) train acc: 0.838000; val_acc: 0.596000
(Iteration 1601 / 9800) loss: 0.663400
(Epoch 17 / 100) train acc: 0.833000; val_acc: 0.585000
(Iteration 1701 / 9800) loss: 0.644984
(Epoch 18 / 100) train acc: 0.845000; val_acc: 0.597000
(Iteration 1801 / 9800) loss: 0.589513
(Epoch 19 / 100) train acc: 0.838000; val_acc: 0.597000
(Iteration 1901 / 9800) loss: 0.619806
(Epoch 20 / 100) train acc: 0.876000; val_acc: 0.595000
(Iteration 2001 / 9800) loss: 0.576475
(Epoch 21 / 100) train acc: 0.903000; val_acc: 0.586000
(Iteration 2101 / 9800) loss: 0.573623
(Epoch 22 / 100) train acc: 0.899000; val_acc: 0.600000
(Iteration 2201 / 9800) loss: 0.478417
(Epoch 23 / 100) train acc: 0.903000; val_acc: 0.585000
(Iteration 2301 / 9800) loss: 0.470228
(Epoch 24 / 100) train acc: 0.916000; val_acc: 0.599000
(Iteration 2401 / 9800) loss: 0.435533
(Epoch 25 / 100) train acc: 0.932000; val_acc: 0.588000
(Iteration 2501 / 9800) loss: 0.453185
(Epoch 26 / 100) train acc: 0.933000; val_acc: 0.596000
(Iteration 2601 / 9800) loss: 0.473915
(Epoch 27 / 100) train acc: 0.950000; val_acc: 0.603000
(Iteration 2701 / 9800) loss: 0.383685
(Epoch 28 / 100) train acc: 0.944000; val_acc: 0.606000
(Iteration 2801 / 9800) loss: 0.369992
(Epoch 29 / 100) train acc: 0.946000; val_acc: 0.595000
(Iteration 2901 / 9800) loss: 0.412054
(Epoch 30 / 100) train acc: 0.944000; val_acc: 0.594000
(Iteration 3001 / 9800) loss: 0.320266
(Epoch 31 / 100) train acc: 0.969000; val_acc: 0.600000
(Iteration 3101 / 9800) loss: 0.291068
(Epoch 32 / 100) train acc: 0.962000; val_acc: 0.598000
(Iteration 3201 / 9800) loss: 0.294330
```

```
(Epoch 33 / 100) train acc: 0.963000; val_acc: 0.608000
(Iteration 3301 / 9800) loss: 0.261495
(Epoch 34 / 100) train acc: 0.978000; val_acc: 0.597000
(Iteration 3401 / 9800) loss: 0.317429
(Epoch 35 / 100) train acc: 0.981000; val_acc: 0.609000
(Iteration 3501 / 9800) loss: 0.266649
(Epoch 36 / 100) train acc: 0.974000; val_acc: 0.598000
(Iteration 3601 / 9800) loss: 0.309518
(Epoch 37 / 100) train acc: 0.981000; val_acc: 0.596000
(Iteration 3701 / 9800) loss: 0.271904
(Epoch 38 / 100) train acc: 0.984000; val_acc: 0.597000
(Iteration 3801 / 9800) loss: 0.221919
(Epoch 39 / 100) train acc: 0.980000; val_acc: 0.610000
(Iteration 3901 / 9800) loss: 0.247744
(Epoch 40 / 100) train acc: 0.990000; val_acc: 0.601000
(Iteration 4001 / 9800) loss: 0.211057
(Epoch 41 / 100) train acc: 0.986000; val_acc: 0.602000
(Iteration 4101 / 9800) loss: 0.215980
(Epoch 42 / 100) train acc: 0.989000; val_acc: 0.602000
(Iteration 4201 / 9800) loss: 0.189695
(Epoch 43 / 100) train acc: 0.990000; val_acc: 0.600000
(Iteration 4301 / 9800) loss: 0.237263
(Epoch 44 / 100) train acc: 0.989000; val_acc: 0.593000
(Iteration 4401 / 9800) loss: 0.149792
(Epoch 45 / 100) train acc: 0.981000; val_acc: 0.603000
(Iteration 4501 / 9800) loss: 0.195684
(Epoch 46 / 100) train acc: 0.994000; val_acc: 0.605000
(Iteration 4601 / 9800) loss: 0.225804
(Epoch 47 / 100) train acc: 0.990000; val_acc: 0.608000
(Iteration 4701 / 9800) loss: 0.227298
(Epoch 48 / 100) train acc: 0.993000; val_acc: 0.604000
(Iteration 4801 / 9800) loss: 0.155038
(Epoch 49 / 100) train acc: 0.997000; val_acc: 0.601000
(Epoch 50 / 100) train acc: 0.995000; val_acc: 0.611000
(Iteration 4901 / 9800) loss: 0.179200
(Epoch 51 / 100) train acc: 0.994000; val_acc: 0.603000
(Iteration 5001 / 9800) loss: 0.148308
(Epoch 52 / 100) train acc: 0.996000; val_acc: 0.601000
(Iteration 5101 / 9800) loss: 0.203314
(Epoch 53 / 100) train acc: 0.998000; val_acc: 0.602000
(Iteration 5201 / 9800) loss: 0.133153
(Epoch 54 / 100) train acc: 0.998000; val_acc: 0.613000
(Iteration 5301 / 9800) loss: 0.179897
(Epoch 55 / 100) train acc: 0.997000; val_acc: 0.603000
(Iteration 5401 / 9800) loss: 0.138111
(Epoch 56 / 100) train acc: 0.998000; val_acc: 0.606000
(Iteration 5501 / 9800) loss: 0.120119
(Epoch 57 / 100) train acc: 0.999000; val_acc: 0.607000
(Iteration 5601 / 9800) loss: 0.167840
(Epoch 58 / 100) train acc: 0.996000; val_acc: 0.610000
(Iteration 5701 / 9800) loss: 0.188360
(Epoch 59 / 100) train acc: 0.996000; val_acc: 0.603000
(Iteration 5801 / 9800) loss: 0.190754
(Epoch 60 / 100) train acc: 0.999000; val_acc: 0.612000
(Iteration 5901 / 9800) loss: 0.189417
(Epoch 61 / 100) train acc: 0.998000; val_acc: 0.603000
(Iteration 6001 / 9800) loss: 0.206282
(Epoch 62 / 100) train acc: 0.997000; val_acc: 0.597000
(Iteration 6101 / 9800) loss: 0.140797
(Epoch 63 / 100) train acc: 0.997000; val_acc: 0.604000
(Iteration 6201 / 9800) loss: 0.153129
(Epoch 64 / 100) train acc: 0.999000; val_acc: 0.613000
(Iteration 6301 / 9800) loss: 0.166696
(Epoch 65 / 100) train acc: 0.997000; val_acc: 0.605000
(Iteration 6401 / 9800) loss: 0.113103
(Epoch 66 / 100) train acc: 0.999000; val_acc: 0.604000
```

```
(Iteration 6501 / 9800) loss: 0.103705
(Epoch 67 / 100) train acc: 0.998000; val_acc: 0.613000
(Iteration 6601 / 9800) loss: 0.100708
(Epoch 68 / 100) train acc: 0.999000; val_acc: 0.607000
(Iteration 6701 / 9800) loss: 0.108514
(Epoch 69 / 100) train acc: 0.999000; val_acc: 0.608000
(Iteration 6801 / 9800) loss: 0.097242
(Epoch 70 / 100) train acc: 0.999000; val_acc: 0.608000
(Iteration 6901 / 9800) loss: 0.119275
(Epoch 71 / 100) train acc: 0.999000; val_acc: 0.606000
(Iteration 7001 / 9800) loss: 0.092603
(Epoch 72 / 100) train acc: 0.999000; val_acc: 0.612000
(Iteration 7101 / 9800) loss: 0.140257
(Epoch 73 / 100) train acc: 0.999000; val_acc: 0.611000
(Iteration 7201 / 9800) loss: 0.101904
(Epoch 74 / 100) train acc: 0.999000; val_acc: 0.614000
(Iteration 7301 / 9800) loss: 0.109674
(Epoch 75 / 100) train acc: 0.999000; val_acc: 0.611000
(Iteration 7401 / 9800) loss: 0.111727
(Epoch 76 / 100) train acc: 0.998000; val_acc: 0.615000
(Iteration 7501 / 9800) loss: 0.128696
(Epoch 77 / 100) train acc: 1.000000; val_acc: 0.613000
(Iteration 7601 / 9800) loss: 0.145574
(Epoch 78 / 100) train acc: 0.999000; val_acc: 0.607000
(Iteration 7701 / 9800) loss: 0.134654
(Epoch 79 / 100) train acc: 0.997000; val_acc: 0.604000
(Iteration 7801 / 9800) loss: 0.114184
(Epoch 80 / 100) train acc: 0.999000; val_acc: 0.612000
(Iteration 7901 / 9800) loss: 0.173852
(Epoch 81 / 100) train acc: 0.999000; val_acc: 0.613000
(Iteration 8001 / 9800) loss: 0.126730
(Epoch 82 / 100) train acc: 1.000000; val_acc: 0.612000
(Iteration 8101 / 9800) loss: 0.085527
(Epoch 83 / 100) train acc: 0.997000; val_acc: 0.613000
(Iteration 8201 / 9800) loss: 0.090590
(Epoch 84 / 100) train acc: 0.999000; val_acc: 0.613000
(Iteration 8301 / 9800) loss: 0.109501
(Epoch 85 / 100) train acc: 0.999000; val_acc: 0.616000
(Iteration 8401 / 9800) loss: 0.103534
(Epoch 86 / 100) train acc: 0.999000; val_acc: 0.613000
(Iteration 8501 / 9800) loss: 0.096822
(Epoch 87 / 100) train acc: 1.000000; val_acc: 0.615000
(Iteration 8601 / 9800) loss: 0.095079
(Epoch 88 / 100) train acc: 0.998000; val_acc: 0.614000
(Iteration 8701 / 9800) loss: 0.125121
(Epoch 89 / 100) train acc: 0.999000; val_acc: 0.611000
(Iteration 8801 / 9800) loss: 0.127577
(Epoch 90 / 100) train acc: 0.999000; val_acc: 0.614000
(Iteration 8901 / 9800) loss: 0.101643
(Epoch 91 / 100) train acc: 0.999000; val_acc: 0.619000
(Iteration 9001 / 9800) loss: 0.125605
(Epoch 92 / 100) train acc: 0.999000; val_acc: 0.611000
(Iteration 9101 / 9800) loss: 0.133782
(Epoch 93 / 100) train acc: 0.999000; val_acc: 0.609000
(Iteration 9201 / 9800) loss: 0.095340
(Epoch 94 / 100) train acc: 0.999000; val_acc: 0.615000
(Iteration 9301 / 9800) loss: 0.129781
(Epoch 95 / 100) train acc: 0.997000; val_acc: 0.614000
(Iteration 9401 / 9800) loss: 0.121475
(Epoch 96 / 100) train acc: 0.999000; val_acc: 0.612000
(Iteration 9501 / 9800) loss: 0.133388
(Epoch 97 / 100) train acc: 0.999000; val_acc: 0.615000
(Iteration 9601 / 9800) loss: 0.118243
(Epoch 98 / 100) train acc: 0.998000; val_acc: 0.615000
(Iteration 9701 / 9800) loss: 0.100336
(Epoch 99 / 100) train acc: 1.000000; val_acc: 0.611000
```

```
(Epoch 100 / 100) train acc: 1.000000; val_acc: 0.613000

Test set accuracy: 0.593
Validation set accuracy: 0.615
```

```python
import numpy as np


def affine_forward(x, w, b):
  """
  Computes the forward pass for an affine (fully-connected) layer.

  The input x has shape (N, d_1, ..., d_k) and contains a minibatch of N
  examples, where each example x[i] has shape (d_1, ..., d_k). We will
  reshape each input into a vector of dimension D = d_1 * ... * d_k, and
  then transform it to an output vector of dimension M.

  Inputs:
  - x: A numpy array containing input data, of shape (N, d_1, ..., d_k)
  - w: A numpy array of weights, of shape (D, M)
  - b: A numpy array of biases, of shape (M,)

  Returns a tuple of:
  - out: output, of shape (N, M)
  - cache: (x, w, b)
  """

  # ================================================================ #
  # YOUR CODE HERE:
  #   Calculate the output of the forward pass.  Notice the dimensions
  #   of w are D x M, which is the transpose of what we did in earlier
  #   assignments.
  # ================================================================ #

  x_reshape = x.reshape((x.shape[0], w.shape[0])) #N x D
  out = np.dot(x_reshape, w) + b.reshape((1, b.shape[0])) #N x M

  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  cache = (x, w, b)
  return out, cache


def affine_backward(dout, cache):
  """
  Computes the backward pass for an affine layer.

  Inputs:
  - dout: Upstream derivative, of shape (N, M)
  - cache: Tuple of:
    - x: A numpy array containing input data, of shape (N, d_1, ..., d_k)
    - w: A numpy array of weights, of shape (D, M)
    - b: A numpy array of biases, of shape (M,)

  Returns a tuple of:
  - dx: Gradient with respect to x, of shape (N, d1, ..., d_k)
  - dw: Gradient with respect to w, of shape (D, M)
  - db: Gradient with respect to b, of shape (M,)
  """
  x, w, b = cache
  dx, dw, db = None, None, None

```

```python
  # ================================================================ #
  # YOUR CODE HERE:
  #   Calculate the gradients for the backward pass.
  # Notice:
  #   dout is N x M
  #   dx should be N x d1 x ... x dk; it relates to dout through multiplication with w, which is D x M
  #   dw should be D x M; it relates to dout through multiplication with x, which is N x D after reshaping
  #   db should be M; it is just the sum over dout examples
  # ================================================================ #

  x_reshape = np.reshape(x, (x.shape[0], w.shape[0])) #N x D
  dx_reshape = np.dot(dout, w.T)

  dx = np.reshape(dx_reshape, x.shape) #N x D
  dw = np.dot(x_reshape.T, dout) #D x M
  db = np.dot(dout.T, np.ones(x.shape[0]))  #M x 1

  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  return dx, dw, db

def relu_forward(x):
  """
  Computes the forward pass for a layer of rectified linear units (ReLUs).

  Input:
  - x: Inputs, of any shape

  Returns a tuple of:
  - out: Output, of the same shape as x
  - cache: x
  """
  # ================================================================ #
  # YOUR CODE HERE:
  #   Implement the ReLU forward pass.
  # ================================================================ #

  out = np.maximum(x, 0)

  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  cache = x
  return out, cache


def relu_backward(dout, cache):
  """
  Computes the backward pass for a layer of rectified linear units (ReLUs).

  Input:
  - dout: Upstream derivatives, of any shape
  - cache: Input x, of same shape as dout

  Returns:
```

```python
  - dx: Gradient with respect to x
  """
  x = cache

  # ================================================================ #
  # YOUR CODE HERE:
  #   Implement the ReLU backward pass
  # ================================================================ #

  dx = dout * (x > 0)

  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  return dx

def batchnorm_forward(x, gamma, beta, bn_param):
  """
  Forward pass for batch normalization.

  During training the sample mean and (uncorrected) sample variance are
  computed from minibatch statistics and used to normalize the incoming data.
  During training we also keep an exponentially decaying running mean of the mean
  and variance of each feature, and these averages are used to normalize data
  at test-time.

  At each timestep we update the running averages for mean and variance using
  an exponential decay based on the momentum parameter:

  running_mean = momentum * running_mean + (1 - momentum) * sample_mean
  running_var = momentum * running_var + (1 - momentum) * sample_var

  Note that the batch normalization paper suggests a different test-time
  behavior: they compute sample mean and variance for each feature using a
  large number of training images rather than using a running average. For
  this implementation we have chosen to use running averages instead since
  they do not require an additional estimation step; the torch7 implementation
  of batch normalization also uses running averages.

  Input:
  - x: Data of shape (N, D)
  - gamma: Scale parameter of shape (D,)
  - beta: Shift paremeter of shape (D,)
  - bn_param: Dictionary with the following keys:
    - mode: 'train' or 'test'; required
    - eps: Constant for numeric stability
    - momentum: Constant for running mean / variance.
    - running_mean: Array of shape (D,) giving running mean of features
    - running_var Array of shape (D,) giving running variance of features

  Returns a tuple of:
  - out: of shape (N, D)
  - cache: A tuple of values needed in the backward pass
  """
  mode = bn_param['mode']
  eps = bn_param.get('eps', 1e-5)
  momentum = bn_param.get('momentum', 0.9)
```

```python

  N, D = x.shape
  running_mean = bn_param.get('running_mean', np.zeros(D, dtype=x.dtype))
  running_var = bn_param.get('running_var', np.zeros(D, dtype=x.dtype))

  out, cache = None, None
  if mode == 'train':

    # ======================================================================= #
    # YOUR CODE HERE:
    #   A few steps here:
    #     (1) Calculate the running mean and variance of the minibatch.
    #     (2) Normalize the activations with the running mean and variance.
    #     (3) Scale and shift the normalized activations.  Store this
    #         as the variable 'out'
    #     (4) Store any variables you may need for the backward pass in
    #         the 'cache' variable.
    # ======================================================================= #

    mean = np.mean(x, axis = 0)
    var = np.var(x, axis = 0)
    normalize_x = (x - mean) / np.sqrt(var + eps)

    running_mean = momentum * running_mean + (1 - momentum) * mean
    running_var = momentum * running_var + (1 - momentum) * var

    out = gamma * normalize_x + beta

    cache = {'normalize_x': normalize_x,
             'x_minus_mean': (x - mean),
             'sqrt_var_eps': np.sqrt(var + eps),
             'gamma': gamma
            }

    # ======================================================================= #
    # END YOUR CODE HERE
    # ======================================================================= #

  elif mode == 'test':

    # ======================================================================= #
    # YOUR CODE HERE:
    #   Calculate the testing time normalized activation.  Normalize using
    #   the running mean and variance, and then scale and shift appropriately.
    #   Store the output as 'out'.
    # ======================================================================= #

    normalize_x = (x - running_mean) / np.sqrt(running_var + eps)
    out = gamma * normalize_x + beta

    # ======================================================================= #
    # END YOUR CODE HERE
    # ======================================================================= #

  else:
    raise ValueError('Invalid forward batchnorm mode "%s"' % mode)

  # Store the updated running means back into bn_param
  bn_param['running_mean'] = running_mean
```

```python
    bn_param['running_var'] = running_var

    return out, cache

def batchnorm_backward(dout, cache):
    """
    Backward pass for batch normalization.

    For this implementation, you should write out a computation graph for
    batch normalization on paper and propagate gradients backward through
    intermediate nodes.

    Inputs:
    - dout: Upstream derivatives, of shape (N, D)
    - cache: Variable of intermediates from batchnorm_forward.

    Returns a tuple of:
    - dx: Gradient with respect to inputs x, of shape (N, D)
    - dgamma: Gradient with respect to scale parameter gamma, of shape (D,)
    - dbeta: Gradient with respect to shift parameter beta, of shape (D,)
    """
    dx, dgamma, dbeta = None, None, None

    # =============================================================== #
    # YOUR CODE HERE:
    #   Implement the batchnorm backward pass, calculating dx, dgamma, and
dbeta.
    # =============================================================== #

    normalize_x = cache.get('normalize_x')
    x_minus_mean = cache.get('x_minus_mean')
    sqrt_var_eps = cache.get('sqrt_var_eps')
    gamma = cache.get('gamma')
    N = dout.shape[0]

    dbeta = np.sum(dout, axis = 0)
    dgamma = np.sum(dout * normalize_x, axis = 0)

    dnormalize_x = dout * gamma

    db = x_minus_mean * dnormalize_x # b = 1 / sqrt_var_eps
    dc = (-1 / (sqrt_var_eps * sqrt_var_eps)) * db # c = sqrt_var_eps
    de = (1 / (2 * sqrt_var_eps)) * dc # e = sqrt_var_eps * sqrt_var_eps
    dvar = np.sum(de, axis = 0)

    da = dnormalize_x / sqrt_var_eps # a = x - mu
    dmu = -np.sum(da, axis = 0) - 2 * np.sum(x_minus_mean, axis = 0) * dvar / N

    dx = da + 2 * x_minus_mean * dvar / N + dmu / N

    # =============================================================== #
    # END YOUR CODE HERE
    # =============================================================== #

    return dx, dgamma, dbeta

def dropout_forward(x, dropout_param):
    """
    Performs the forward pass for (inverted) dropout.

```

```python
  Inputs:
  - x: Input data, of any shape
  - dropout_param: A dictionary with the following keys:
    - p: Dropout parameter. We drop each neuron output with probability p.
    - mode: 'test' or 'train'. If the mode is train, then perform dropout;
      if the mode is test, then just return the input.
    - seed: Seed for the random number generator. Passing seed makes this
      function deterministic, which is needed for gradient checking but not in
      real networks.

  Outputs:
  - out: Array of the same shape as x.
  - cache: A tuple (dropout_param, mask). In training mode, mask is the dropout
    mask that was used to multiply the input; in test mode, mask is None.
  """
  p, mode = dropout_param['p'], dropout_param['mode']
  if 'seed' in dropout_param:
    np.random.seed(dropout_param['seed'])

  mask = None
  out = None

  if mode == 'train':
    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the inverted dropout forward pass during training time.
    #   Store the masked and scaled activations in out, and store the
    #   dropout mask as the variable mask.
    # ================================================================ #

    mask = (np.random.rand(*x.shape) < (1 - p)) / (1 - p)
    out = x * mask

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

  elif mode == 'test':

    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the inverted dropout forward pass during test time.
    # ================================================================ #

    out = x

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

  cache = (dropout_param, mask)
  out = out.astype(x.dtype, copy=False)

  return out, cache

def dropout_backward(dout, cache):
  """
  Perform the backward pass for (inverted) dropout.
```

```python

  Inputs:
  - dout: Upstream derivatives, of any shape
  - cache: (dropout_param, mask) from dropout_forward.
  """
  dropout_param, mask = cache
  mode = dropout_param['mode']

  dx = None
  if mode == 'train':
    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the inverted dropout backward pass during training time.
    # ================================================================ #

    dx = dout * mask

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #
  elif mode == 'test':
    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the inverted dropout backward pass during test time.
    # ================================================================ #

    dx = dout

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #
  return dx

def svm_loss(x, y):
  """
  Computes the loss and gradient using for multiclass SVM classification.

  Inputs:
  - x: Input data, of shape (N, C) where x[i, j] is the score for the jth class
    for the ith input.
  - y: Vector of labels, of shape (N,) where y[i] is the label for x[i] and
    0 <= y[i] < C

  Returns a tuple of:
  - loss: Scalar giving the loss
  - dx: Gradient of the loss with respect to x
  """
  N = x.shape[0]
  correct_class_scores = x[np.arange(N), y]
  margins = np.maximum(0, x - correct_class_scores[:, np.newaxis] + 1.0)
  margins[np.arange(N), y] = 0
  loss = np.sum(margins) / N
  num_pos = np.sum(margins > 0, axis=1)
  dx = np.zeros_like(x)
  dx[margins > 0] = 1
  dx[np.arange(N), y] -= num_pos
  dx /= N
  return loss, dx

```

```python

def softmax_loss(x, y):
  """
  Computes the loss and gradient for softmax classification.

  Inputs:
  - x: Input data, of shape (N, C) where x[i, j] is the score for the jth
class
    for the ith input.
  - y: Vector of labels, of shape (N,) where y[i] is the label for x[i] and
    0 <= y[i] < C

  Returns a tuple of:
  - loss: Scalar giving the loss
  - dx: Gradient of the loss with respect to x
  """

  probs = np.exp(x - np.max(x, axis=1, keepdims=True))
  probs /= np.sum(probs, axis=1, keepdims=True)
  N = x.shape[0]
  loss = -np.sum(np.log(probs[np.arange(N), y])) / N
  dx = probs.copy()
  dx[np.arange(N), y] -= 1
  dx /= N
  return loss, dx

```

```python
from .layers import *


def affine_relu_forward(x, w, b):
  """
  Convenience layer that performs an affine transform followed by a ReLU

  Inputs:
  - x: Input to the affine layer
  - w, b: Weights for the affine layer

  Returns a tuple of:
  - out: Output from the ReLU
  - cache: Object to give to the backward pass
  """
  a, fc_cache = affine_forward(x, w, b)
  out, relu_cache = relu_forward(a)
  cache = (fc_cache, relu_cache)
  return out, cache


def affine_relu_backward(dout, cache):
  """
  Backward pass for the affine-relu convenience layer
  """
  fc_cache, relu_cache = cache
  da = relu_backward(dout, relu_cache)
  dx, dw, db = affine_backward(da, fc_cache)
  return dx, dw, db

```

```python
import numpy as np
from .layers import *
from .layer_utils import *


class TwoLayerNet(object):
  """
  A two-layer fully-connected neural network with ReLU nonlinearity and
  softmax loss that uses a modular layer design. We assume an input dimension
  of D, a hidden dimension of H, and perform classification over C classes.

  The architecure should be affine - relu - affine - softmax.

  Note that this class does not implement gradient descent; instead, it
  will interact with a separate Solver object that is responsible for running
  optimization.

  The learnable parameters of the model are stored in the dictionary
  self.params that maps parameter names to numpy arrays.
  """

  def __init__(self, input_dim=3*32*32, hidden_dims=100, num_classes=10,
               dropout=0, weight_scale=1e-3, reg=0.0):
    """
    Initialize a new network.

    Inputs:
    - input_dim: An integer giving the size of the input
    - hidden_dims: An integer giving the size of the hidden layer
    - num_classes: An integer giving the number of classes to classify
    - dropout: Scalar between 0 and 1 giving dropout strength.
    - weight_scale: Scalar giving the standard deviation for random
      initialization of the weights.
    - reg: Scalar giving L2 regularization strength.
    """
    self.params = {}
    self.reg = reg

    # ================================================================ #
    # YOUR CODE HERE:
    #   Initialize W1, W2, b1, and b2.  Store these as self.params['W1'],
    #   self.params['W2'], self.params['b1'] and self.params['b2']. The
    #   biases are initialized to zero and the weights are initialized
    #   so that each parameter has mean 0 and standard deviation weight_scale.
    #   The dimensions of W1 should be (input_dim, hidden_dim) and the
    #   dimensions of W2 should be (hidden_dims, num_classes)
    # ================================================================ #

    self.params['W1'] = weight_scale * np.random.randn(input_dim, hidden_dims) + 0
    self.params['W2'] = weight_scale * np.random.randn(hidden_dims, num_classes) + 0

    self.params['b1'] = np.zeros((hidden_dims, 1))
    self.params['b2'] = np.zeros((num_classes, 1))

    # ================================================================ #
    # END YOUR CODE HERE
```

```
    # ================================================================ #

  def loss(self, X, y=None):
    """
    Compute loss and gradient for a minibatch of data.

    Inputs:
    - X: Array of input data of shape (N, d_1, ..., d_k)
    - y: Array of labels, of shape (N,). y[i] gives the label for X[i].

    Returns:
    If y is None, then run a test-time forward pass of the model and return:
    - scores: Array of shape (N, C) giving classification scores, where
      scores[i, c] is the classification score for X[i] and class c.

    If y is not None, then run a training-time forward and backward pass and
    return a tuple of:
    - loss: Scalar value giving the loss
    - grads: Dictionary with the same keys as self.params, mapping parameter
      names to gradients of the loss with respect to those parameters.
    """
    scores = None

    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the forward pass of the two-layer neural network. Store
    #   the class scores as the variable 'scores'.  Be sure to use the layers
    #   you prior implemented.
    # ================================================================ #

    out_l1, cache_l1 = affine_forward(X, self.params['W1'], self.params['b1'])
    out_relu, cache_relu = relu_forward(out_l1)
    scores, cache_l2 = affine_forward(out_relu, self.params['W2'], self.params['b2'])

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    # If y is None then we are in test mode so just return scores
    if y is None:
      return scores

    loss, grads = 0, {}
    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the backward pass of the two-layer neural net.  Store
    #   the loss as the variable 'loss' and store the gradients in the
    #   'grads' dictionary.  For the grads dictionary, grads['W1'] holds
    #   the gradient for W1, grads['b1'] holds the gradient for b1, etc.
    #   i.e., grads[k] holds the gradient for self.params[k].
    #
    #   Add L2 regularization, where there is an added cost 0.5*self.reg*W^2
    #   for each W.  Be sure to include the 0.5 multiplying factor to
    #   match our implementation.
    #
    #   And be sure to use the layers you prior implemented.
    # ================================================================ #

```

```python
    loss, dx2 = softmax_loss(scores, y)
    reg_loss = 0.5 * self.reg * (np.linalg.norm(self.params['W1'], 'fro')**2 + np.linalg.norm(self.params['W2'], 'fro')**2)
    loss += reg_loss

    dh1, dW2, db2 = affine_backward(dx2, cache_l2)
    da = relu_backward(dh1, cache_relu)
    dx1, dW1, db1 = affine_backward(da, cache_l1)

    grads['W1'] = dW1 + self.reg * self.params['W1']
    grads['b1'] = db1.T

    grads['W2'] = dW2 + self.reg * self.params['W2']
    grads['b2'] = db2.T

    # ============================================================== #
    # END YOUR CODE HERE
    # ============================================================== #

    return loss, grads


class FullyConnectedNet(object):
  """
  A fully-connected neural network with an arbitrary number of hidden layers,
  ReLU nonlinearities, and a softmax loss function. This will also implement
  dropout and batch normalization as options. For a network with L layers,
  the architecture will be

  {affine - [batch norm] - relu - [dropout]} x (L - 1) - affine - softmax

  where batch normalization and dropout are optional, and the {...} block is
  repeated L - 1 times.

  Similar to the TwoLayerNet above, learnable parameters are stored in the
  self.params dictionary and will be learned using the Solver class.
  """

  def __init__(self, hidden_dims, input_dim=3*32*32, num_classes=10,
               dropout=0, use_batchnorm=False, reg=0.0,
               weight_scale=1e-2, dtype=np.float32, seed=None):
    """
    Initialize a new FullyConnectedNet.

    Inputs:
    - hidden_dims: A list of integers giving the size of each hidden layer.
    - input_dim: An integer giving the size of the input.
    - num_classes: An integer giving the number of classes to classify.
    - dropout: Scalar between 0 and 1 giving dropout strength. If dropout=0 then
      the network should not use dropout at all.
    - use_batchnorm: Whether or not the network should use batch normalization.
    - reg: Scalar giving L2 regularization strength.
    - weight_scale: Scalar giving the standard deviation for random
      initialization of the weights.
    - dtype: A numpy datatype object; all computations will be performed using
      this datatype. float32 is faster but less accurate, so you should use
      float64 for numeric gradient checking.
```

```python
    - seed: If not None, then pass this random seed to the dropout layers. This
      will make the dropout layers deteriminstic so we can gradient check the
      model.
    """
    self.use_batchnorm = use_batchnorm
    self.use_dropout = dropout > 0
    self.reg = reg
    self.num_layers = 1 + len(hidden_dims)
    self.dtype = dtype
    self.params = {}

    # ================================================================ #
    # YOUR CODE HERE:
    #   Initialize all parameters of the network in the self.params dictionary.
    #   The weights and biases of layer 1 are W1 and b1; and in general the
    #   weights and biases of layer i are Wi and bi. The
    #   biases are initialized to zero and the weights are initialized
    #   so that each parameter has mean 0 and standard deviation weight_scale.
    #
    #   BATCHNORM: Initialize the gammas of each layer to 1 and the beta
    #   parameters to zero.  The gamma and beta parameters for layer 1 should
    #   be self.params['gamma1'] and self.params['beta1'].  For layer 2, they
    #   should be gamma2 and beta2, etc. Only use batchnorm if self.use_batchnorm
    #   is true and DO NOT do batch normalize the output scores.
    # ================================================================ #

    dims = []
    dims.append(input_dim)
    dims.extend(hidden_dims)
    dims.append(num_classes)

    for i in np.arange(self.num_layers):
      num = str(i+1)
      self.params['W'+num] = weight_scale * np.random.randn(dims[i], dims[i+1]) + 0
      self.params['b'+num] = np.zeros((dims[i+1]))

      if i == (self.num_layers - 1):
        break

      if self.use_batchnorm:
        self.params['gamma'+num] = np.ones((dims[i+1]))
        self.params['beta'+num] = np.zeros((dims[i+1]))

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    # When using dropout we need to pass a dropout_param dictionary to each
    # dropout layer so that the layer knows the dropout probability and the mode
    # (train / test). You can pass the same dropout_param to each dropout layer.
    self.dropout_param = {}
    if self.use_dropout:
      self.dropout_param = {'mode': 'train', 'p': dropout}
```

```python
      if seed is not None:
        self.dropout_param['seed'] = seed

    # With batch normalization we need to keep track of running means and
    # variances, so we need to pass a special bn_param object to each batch
    # normalization layer. You should pass self.bn_params[0] to the forward pass
    # of the first batch normalization layer, self.bn_params[1] to the forward
    # pass of the second batch normalization layer, etc.
    self.bn_params = []
    if self.use_batchnorm:
      self.bn_params = [{'mode': 'train'} for i in np.arange(self.num_layers - 1)]

    # Cast all parameters to the correct datatype
    for k, v in self.params.items():
      self.params[k] = v.astype(dtype)


  def loss(self, X, y=None):
    """
    Compute loss and gradient for the fully-connected net.

    Input / output: Same as TwoLayerNet above.
    """
    X = X.astype(self.dtype)
    mode = 'test' if y is None else 'train'

    # Set train/test mode for batchnorm params and dropout param since they
    # behave differently during training and testing.
    if self.dropout_param is not None:
      self.dropout_param['mode'] = mode
    if self.use_batchnorm:
      for bn_param in self.bn_params:
        bn_param['mode'] = mode

    scores = None

    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the forward pass of the FC net and store the output
    #   scores as the variable "scores".
    #
    #   BATCHNORM: If self.use_batchnorm is true, insert a bathnorm layer
    #   between the affine_forward and relu_forward layers.  You may
    #   also write an affine_batchnorm_relu() function in layer_utils.py.
    #
    #   DROPOUT: If dropout is non-zero, insert a dropout layer after
    #   every ReLU layer.
    # ================================================================ #

    fc_outs = {}
    fc_caches = {}

    batchnorm_outs = {}
    batchnorm_caches = {}

    h = {}
    h[0] = X
```

```python
    relu_caches = {}

    dropout_caches = {}

    for i in np.arange(self.num_layers):
      num = str(i+1)
      # fc
      fc_outs[i+1], fc_caches[i+1] = affine_forward(h[i], self.params['W'+num], self.params['b'+num])
      if i == (self.num_layers - 1):
        break
      relu_input = fc_outs[i+1]

      # batch-norm
      if self.use_batchnorm:
        batchnorm_outs[i+1], batchnorm_caches[i+1] = batchnorm_forward(fc_outs[i+1], self.params['gamma'+num], self.params['beta'+num], self.bn_params[i])
        relu_input = batchnorm_outs[i+1]

      # relu
      h[i+1], relu_caches[i+1] = relu_forward(relu_input)

      # dropout
      if self.use_dropout:
        h[i+1], dropout_caches[i+1] = dropout_forward(h[i+1], self.dropout_param)

    scores = fc_outs[self.num_layers]

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    # If test mode return early
    if mode == 'test':
      return scores

    loss, grads = 0.0, {}
    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the backwards pass of the FC net and store the gradients
    #   in the grads dict, so that grads[k] is the gradient of self.params[k]
    #   Be sure your L2 regularization includes a 0.5 factor.
    #
    #   BATCHNORM: Incorporate the backward pass of the batchnorm.
    #
    #   DROPOUT: Incorporate the backward pass of dropout.
    # ================================================================ #

    loss, dx = softmax_loss(scores, y)
    reg_loss_sum = 0
    for i in np.arange(self.num_layers):
      num = str(i+1)
      reg_loss_sum += np.linalg.norm(self.params['W'+num], 'fro')**2
    loss += 0.5 * self.reg * reg_loss_sum

    dict_dW = {}
    dict_db = {}

```

```python
    dict_dgamma = {}
    dict_dbeta = {}

    # fc - last layer
    dh, dW, db = affine_backward(dx, fc_caches[self.num_layers])
    dict_dW[self.num_layers] = dW
    dict_db[self.num_layers] = db

    for i in np.arange(self.num_layers - 1, 0, -1):
      #dropout
      if self.use_dropout:
        dh = dropout_backward(dh, dropout_caches[i])

      # relu
      dx_relu = relu_backward(dh, relu_caches[i])
      fc_input = dx_relu

      # batch-norm
      if self.use_batchnorm:
        dx_batchnorm, dgamma, dbeta = batchnorm_backward(dx_relu,
batchnorm_caches[i])
        dict_dgamma[i] = dgamma
        dict_dbeta[i] = dbeta
        fc_input = dx_batchnorm

      # fc
      dh, dW, db = affine_backward(fc_input, fc_caches[i])
      dict_dW[i] = dW
      dict_db[i] = db

    for i in np.arange(self.num_layers):
      num = str(i+1)
      grads['W'+num] = dict_dW[i+1] + self.reg * self.params['W'+num]
      grads['b'+num] = dict_db[i+1]
      if i == (self.num_layers - 1):
        break
      if self.use_batchnorm:
        grads['gamma'+num] = dict_dgamma[i+1]
        grads['beta'+num] = dict_dbeta[i+1]

    # ============================================================== #
    # END YOUR CODE HERE
    # ============================================================== #

    return loss, grads
```