Thesis:

$$\sum_{n=1}^{\infty}(-1)^n \ln\left(1+\frac{1}{n}\right) = \ln\left(\frac{2}{\pi}\right)$$

Transforming:

$$\begin{aligned}
\sum_{n=1}^{\infty}(-1)^n \ln\left(1+\frac{1}{n}\right) &= \sum_{n=1}^{\infty}(-1)^n \ln\left(\frac{n+1}{n}\right) \\
&= -\ln\left(\frac{2}{1}\right)+\ln\left(\frac{3}{2}\right)-\ln\left(\frac{4}{3}\right)+\ln\left(\frac{5}{4}\right)-\cdots \\
&= \ln\left(\frac{1}{2}\right)+\ln\left(\frac{3}{2}\right)+\ln\left(\frac{3}{4}\right)+\ln\left(\frac{5}{4}\right)+\cdots \\
&= \ln\left(\frac{1\cdot 3}{2\cdot 2}\right)+\ln\left(\frac{3\cdot 5}{4\cdot 4}\right)+\ln\left(\frac{5\cdot 7}{6\cdot 6}\right)+\cdots \\
&= -\ln\left(\frac{2\cdot 2}{1\cdot 3}\cdot\frac{4\cdot 4}{3\cdot 5}\cdot\frac{5\cdot 7}{6\cdot 6}\cdot\cdots\right)
\end{aligned}$$

Wallis product for π:

$$\frac{\pi}{2} = \frac{2\cdot 2}{1\cdot 3}\cdot\frac{4\cdot 4}{3\cdot 5}\cdot\frac{5\cdot 7}{6\cdot 6}\cdot\cdots$$

Hence:

$$\sum_{n=1}^{\infty}(-1)^n \ln\left(1+\frac{1}{n}\right) = -\ln\left(\frac{\pi}{2}\right) = \ln\left(\frac{2}{\pi}\right)$$