Thesis (Leibniz formula for π):

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \frac{\pi}{4}$$

Take:

$$y = \arctan(x)$$

then:

$$y' = \frac{1}{1+x^2}$$

For $-1 < x < 1$ we have:

$$\frac{1}{1+x^2} \quad = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$$

Integrating both sides from 0 to u:

$$\int_0^u \frac{1}{1+x^2} = \arctan(u) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} u^{2n+1}$$

It is clear that the formula above holds for $u \in (-1, 1)$. How about $u = 1$? Note that the sequence (a_n) given by:

$$a_n := \frac{(-1)^n}{2n+1}$$

is decreasing and bounded. Thus, by monotone convergence theorem the formula holds for $u = 1$, too.
Consequently, since $\arctan(1) = \pi/2$:

$$\frac{\pi}{2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$