2. 图像去畸变

见代码

3. 双目视差的使用

见代码

4. 矩阵运算微分

①

$$\mathrm{d}(\mathbf{Ax})/\mathrm{d}\mathbf{x} = \frac{\mathbf{A}\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{x}} = \mathbf{A}$$

②

证法一:

$$\mathrm{d}(\mathbf{x^TAx})/\mathrm{d}\mathbf{x} = \frac{\mathrm{d}(\mathbf{x})^T\mathbf{Ax} + \mathbf{x}^T\mathbf{A}\mathrm{d}(\mathbf{x})}{\mathrm{d}\mathbf{x}} = (\mathbf{Ax})^T + \mathbf{x}^T\mathbf{A} = \mathbf{x}^T(\mathbf{A}^T + \mathbf{A})$$

证法二:

$$\begin{aligned} &\mathrm{d}(\mathbf{x^TAx})/\mathrm{d}\mathbf{x} = \frac{\partial(\mathbf{x^TAx})}{\partial \mathbf{x}^T} = P_{1\times N} \\ &P_k = \frac{\partial(\mathbf{x}_i\mathbf{A}_{ij}\mathbf{x}_j)}{\partial \mathbf{x}_k} = \mathbf{A}_{kj}\mathbf{x}_j + \mathbf{x}_i\mathbf{A}_{ik} \\ &\Rightarrow P = (\mathbf{Ax})^T + \mathbf{x}^T\mathbf{A} \end{aligned}$$

③

证法一:
对于左边

$$\mathbf{x^TAx} = \mathbf{x}_i\mathbf{A}_{ij}\mathbf{x}_j$$

对于右边

$$\begin{aligned} (\mathbf{xx^T})_{ij} &= \mathbf{x}_i\mathbf{x}_j \\ (\mathbf{Axx^T})_{ik} &= \mathbf{A}_{ij}\mathbf{x}_j\mathbf{x}_k \\ \mathrm{trace}(\mathbf{Axx^T}) &= \sum_i (\mathbf{Axx^T})_{ii} = \mathbf{A}_{ij}\mathbf{x}_j\mathbf{x}_i \end{aligned}$$

故而相等.
注: 这里有些地方使用了张量的表示法, 省略了求和符号.

5. 高斯牛顿法的曲线拟合

见代码

6. 批量最大似然估计

①

$$H = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

②

$$W = \begin{bmatrix} Q & 0 & 0 & 0 & 0 & 0 \\ 0 & Q & 0 & 0 & 0 & 0 \\ 0 & 0 & Q & 0 & 0 & 0 \\ 0 & 0 & 0 & R & 0 & 0 \\ 0 & 0 & 0 & 0 & R & 0 \\ 0 & 0 & 0 & 0 & 0 & R \end{bmatrix}$$

③ 有唯一解.

对$E = \frac{1}{2}(\mathbf{z} - \mathbf{Hx})^{\mathrm{T}} \mathbf{W}^{-1} (\mathbf{z} - \mathbf{Hx})$求偏导, 令其等于0:

$$\begin{aligned} -(Z - H\mathbf{x})^T W^{-1} H = 0 \\ \Rightarrow (H^T W^{-1} H)^T \mathbf{x} = (Z W^{-1} H)^T \\ \Rightarrow \mathbf{x} = (H^T W^{-1} H)^{-T} (Z W^{-1} H)^T \end{aligned}$$