HW # 5

Problem 1

$$\frac{dR_s}{dt} = -k_f L R_s + k_r R_s^* - k_e R_s + V_s + k_{rec} R_i \qquad 1)$$

$$\frac{dR_s^*}{dt} = k_f L R_s - k_r R_s^* - k_e^* R_s^* + k_{rec} R_i^* \qquad 2)$$

$$\frac{dR_i}{dt} = k_e R_s - k_{deg} R_i - k_{rec} R_i \qquad 3)$$

$$\frac{dR_i^*}{dt} = k_e^* R_s^* - k_{deg} R_i^* - k_{rec} R_i^* \qquad 4)$$

$$V_s + k_{rec}(R_i + R_i^*) = k_e R_s + k_e^* R_s^* \qquad 5)$$

Solve 5) for R_s

$$R_s = \frac{V_s + k_{rec}(R_i + R_i^*) - k_e^* R_s^*}{k_e} \qquad 6)$$

Solve 2, 3, 4 for species @ SS

$$R_s^* = \frac{k_f L R_s + k_{rec} R_i^*}{k_r + k_e^*} \qquad 7)$$

$$R_i = \frac{k_e R_s}{k_{deg} + k_{rec}} \qquad 8)$$

$$R_i^* = \frac{k_e^* R_s^*}{k_{deg} + k_{rec}} \qquad 9)$$

Plug 8+9 into 5 & solve for R_s

$$V_s + \frac{k_{rec}(k_e R_s + k_e^* R^*)}{k_{deg} + k_{rec}} = k_e R_s + k_e^* R_s^*$$

$$V_s - \frac{k_{rec}(k_e^* R_s^*)}{k_{deg} + k_{rec}} - k_e^* R_s^* = k_e R_s - \frac{k_{rec}(k_e R_s)}{k_{deg} + k_{rec}}$$

$$R_s = \frac{V_s(k_{deg} + k_{rec})}{k_e k_{rec}} - \frac{k_e^* R_s^*}{k_e}$$

$$V_s + k_{rec}(R_i + R_i^*) = k_e R_s + k_e^* R_s^*$$

$$V_s + \frac{k_{rec}}{k_{rec}+k_{deg}}(k_e^* R_s^* + k_e R_s)$$

$$V_s + \frac{k_e^* k_{rec}}{k_{rec}+k_{deg}}(R_s^*) - k_e^* R_s^* = k_e R_s - \frac{k_e k_{rec}}{k_{rec}+k_{deg}}(R_s)$$

$$V_s + \left(\frac{k_e^* k_{rec}}{k_{rec}+k_{deg}} - k_e^*\right) R_s^* = \left(k_e - \frac{k_e k_{rec}}{k_{rec}+k_{deg}}\right) R_s$$

$$\frac{V_s}{k_e} \Big/ \left(1 - \frac{k_e k_{rec}}{k_{rec}+k_{deg}}\right) - \frac{k_e^* R_s^*}{k_e} = R_s$$

$$R_s = -\frac{R_s^* k_e^*}{k_e} + V_s \frac{(k_{rec}+k_{deg})}{k_e k_{deg}} \quad 10)$$

Plug 9 + 10 into 7

$$R_s^* = \frac{k_f L}{k_r + k_e^*}\left(\frac{-R_s^* k_e^*}{k_e} + V_s \frac{(k_{rec}+k_{deg})}{k_e k_{deg}}\right) + \frac{k_{rec}}{k_r + k_e^*}\left(\frac{k_e^* R_s^*}{k_{deg} + k_{rec}}\right)$$

soln for R_s^*

$$R_s^* = \frac{k_f L V_s (k_{rec}+k_{deg})^2}{k_{deg}\left[(k_e^* k_f L + k_e(k_r + k_e^*))(k_{rec}+k_{deg}) - k_{rec} k_e^* k_e\right]}$$

$$R_{T_{max}}^* = R_s^* k_e^* \left(\frac{1}{k_e} + \frac{1}{k_{rec}+k_{deg}}\right)$$

$$R_{T_{max}}^* = k_e^* \left(\frac{1}{k_e} + \frac{1}{k_{rec}+k_{deg}}\right) \frac{k_f L V_s (k_{rec}+k_{deg})^2}{k_{deg}\left[k_e^* k_f L (k_{rec}+k_{deg})\right]} \quad L \gg 0$$

$$R_{T_{max}}^* = \left(\frac{k_{rec}+k_{deg}}{k_e^* k_{deg}} + \frac{1}{k_{deg}}\right) V_s$$

w/o rec $\quad R_{T_{max}} = \left(\frac{1}{k_e^*} + \frac{1}{k_{rec}+k_{deg}}\right) V_s$

w/ rec it is larger ~~since~~ since you are reusing product you will have a better maximum

Problem 2)

a) A is an activator of A & R
R is an inhibitor of A but n/A w/ R
d_a is degradation of A

b) A basal = ~~α~~ $-d_a$ A max = $r_{0a} + r_a$
R basal = ~~α~~ -1 R max = $r_{0r} + r_r$

b) it is unstable

c) attachment

d) it loops back around depending on the initial conditions
the system loops around the fixed point

e) see attachment

Problem 3

a) i. $u + v$

ii. α

iii. n

iv. -1

b) 1 SS sol'n for $n=1$

3 SS sol'n for $n=2$

for all higher n there appears to be 3 or more solutions

c) for $n=1$ middle is table

for $n=2$ middle is stable others are not

d)

$$\begin{vmatrix} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} \\ \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \end{vmatrix} = \begin{vmatrix} -1 & \frac{-\alpha v^n \ln v}{(v^n+1)^2} \\ \frac{-\alpha u^n \ln u}{(u^n+1)^2} & -1 \end{vmatrix}$$

$$1 - \frac{\alpha^2 u^n v^n \ln u \ln v}{(u^n+1)^2 (v^n+1)^2}$$

$$\begin{vmatrix} -1 & \frac{-n v^{n-1} \alpha}{(1+v^n)^2} \\ \frac{-n u^{n-1} \alpha}{(1+u^n)^2} & -1 \end{vmatrix}$$

e) $\begin{pmatrix} -1 & \dfrac{-n v^{n-1} \alpha}{(1+v^n)^2} \\ \dfrac{-n u^{n-1} \alpha}{(1+u^n)^2} & -1 \end{pmatrix}$

$\begin{vmatrix} -1-\lambda & \cancel{\alpha} \\ \cancel{u} & -1-\lambda \end{vmatrix}$

$$\lambda^2 + 2\lambda + 1 - \frac{n^2 u^{n-1} v^{n-1} \alpha^2}{(1+v^n)^2 (1+u^n)^2}$$

$n = 2$ $\quad \alpha = 10 \quad v + u = 2$

$$\lambda^2 + 2\lambda + 1 - \frac{4(2)(2)\,100}{25(25)}$$

$\lambda^2 + 2\lambda - 1.56$

$\lambda = 0.6, -2.6$ Saddle

$n = 1 \quad \alpha = 10 \quad v + u = \cancel{1}\ 2.7$

$\lambda^2 + 2\lambda - 2.89 \qquad \dfrac{1\,(\cancel{2.9})(\cancel{2.7})}{(3.7)^2}\;\dfrac{(100)}{(3.7)^2}$

$\lambda = \cancel{0.97}, \cancel{-2.97}$

$-0.26, -1.73$ stable