Bryce Stovall, 912179113 William Herr, 998103103

Part 1 Exponential Heuristic

We mainly use two different measurements in order to determine our estimate for the heuristic.

1. For determining the distance to travel we used the chebyshev distance:
Δd = MAX(|goal.x - current.x|, |goal.y - current.y|)
The reason we use this is because chebyshev distance allows us to move in 8 directions.

2. For determining the height difference we used the built in height function getTile(Point p1) therefore:
Δh = getTile(endPoint) - getTile(currentPoint)

For the exponential heuristic we used:

```
if (Δh < Δd):
    return MAX(Δd + Δh, 0)
else if (Δh == Δd)
    if (Δh > 0)
        return 2*Δd;
    else
        return Δd / 2;
else
    return Δh*2
```

Consider the two cases for ($\Delta h < \Delta d$):

1. $\Delta h \geq 0$ - you are going up and then flat or simply flat the entire way:
In this case you are going up and $\Delta h < \Delta d$. The best case in this situation is to use $2^1 * \Delta h$ which results in moving up until the current node and end node are at the same height. Then from here it is optimal to go straight until you reach the end node which results in $2^0 * (\Delta d - \Delta h)$. Therefore the best case is $2^1 * \Delta h + 2^0 * (\Delta d - \Delta h)$.

We claim that:
$\Delta d + \Delta h \leq 2^1 * \Delta h + 2^0 * (\Delta d - \Delta h)$

$\leq 2^1 * \Delta h + 2^0 \Delta d - 2^0 \Delta h$

$\leq 2^1 * \Delta h + \Delta d - \Delta h$

$\Delta d + \Delta h = \Delta d + \Delta h$

2. $\Delta h < 0$ - you are going down:
The best case is moving down using $2^{-1} * |\Delta h|$ and then moving over laterally a distance of MAX ($\Delta d + \Delta h$, 0). In the case where $|\Delta h| > \Delta d$ the resulting location of $2^{-1} * |\Delta h|$ is at the goal. Note that 2^{-1} must change to 2^{-x} where $x \in \mathbb{Z}^{+}$ *and* $x < \Delta d$ after each move. This allows for moves with height changes of greater than 1. In the case where we haven't reached the location of the goal after reaching the height of the goal, we must travel $2^0 * (\Delta d + \Delta h)$

We claim that:

$\Delta d + \Delta h \leq 2^{-1} * |\Delta h| + \text{MAX}(\Delta d + \Delta h, 0)$

$\leq ½ * |\Delta h| + \Delta d + \Delta h$ where $\Delta d + \Delta h > 0$

$\leq ½ * |\Delta h| + \Delta d - \Delta h$ since $\Delta h < 0$

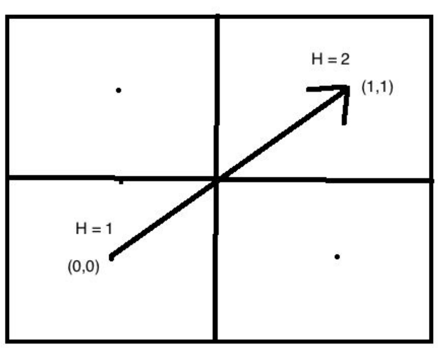
$\leq \Delta d - ½ * \Delta h$

$\Delta d - \Delta h \leq \Delta d - ½ * \Delta h$ since $\Delta h < 0$

$\Delta d \leq \Delta d + ½ * \Delta h$

Consider the two cases of ($\Delta h < \Delta d$):

1. If you are going up then the best case is traveling Δd with a height increase of 1 for each move. This costs 2 for each increment in Δd, so the total cost is $2 * \Delta d$.



In this picture we are moving from a height of 1 to a height of 2 and the states are 1 distance apart. Therefore the best option is to move directly there at a cost of 2^1. This same idea can be expanded to any distance meaning each move uses 2^1.

2. If you are going down then Δ h is not 0 and if $\Delta h = \Delta d$ then you are at the goal. For going down you must travel the same change in distance as height so if each move is 2^{-1} then the total cost is Δ h / 2. This equation also returns 0 for the flat case because 0 / 2 = 0.

Consider the final else case which is (Δ h > Δ d):

In the best case we are moving 2^1 for every increment of Δ h for a total cost of 2* Δ h. However, this is an overestimate of the change in distance because Δ h > Δ d. In other words we don't have enough distance between the current state and end state in order to move only upward a distance of 1 each time. Therefore we know that every move will cost 2^x where $x \in N^+$ *and* $x < \Delta$ d in order to accommodate the extra vertical distance. Therefore we can see that it will cost at least 2 * Δ h.

<u>Part 2 Bizarro World Heuristic</u>

For the bizarro heuristic we used:

```
if current height == 0:
        return 0
return MAX(|goal.x - current.x|, |goal.y - current.y| ) / 2
```

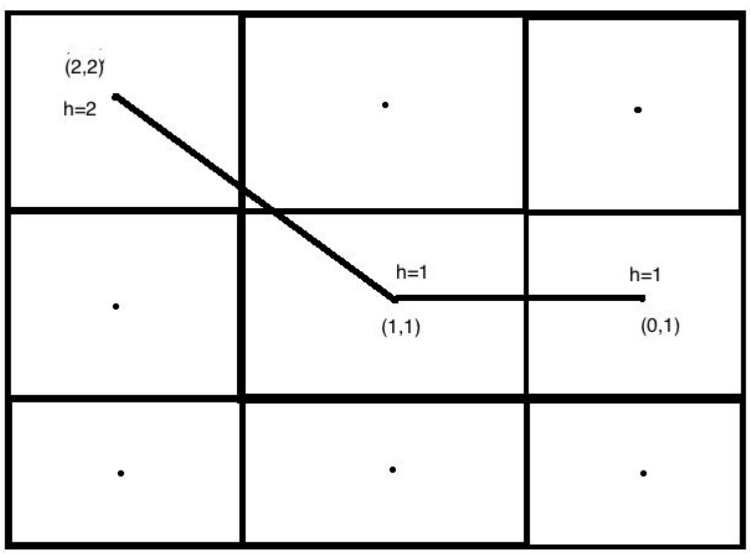
For the heuristic the first thing we do is to check if you are at a height of zero because if you are it costs nothing for your initial move. This means that we can not use MAX(|goal.x - current.x|, |goal.y - current.y|)*.5 because the first jump was not taken into account. Consider starting at point (0,0) with height 0 and going to point (5,5) at height 5. We can get to the goal state with the following costs: (0/1+1) + (1/3+1) + (3/5+1) + (5/5+1) + (5/5+1) which is a total of 2.416 which would have been less than our estimate of 2.5.

We can break the remaining part down into two cases:

1. Δ h $\geq$ 0 - going up:

Let a be the current height and b be the next height then the cost function is $\frac{a}{b+1}$. Since $\Delta h \geq 0$ we know that a $\leq$ b for all cases. It must also be true that when traveling upwards $1/2 < \frac{a}{b+1} \ \ \forall a, b$ and when traveling flat $1/2 \leq \frac{a}{b+1} \ \ \forall a, b$. So for all a and b the cost function is greater than or equal to 1/2. Since we run the cost function exactly as many times as the Chebyshev distance, then $\Delta d \ / \ 2$ must be admissible.

2. Δ h < 0 - going down:
Let a be the current height and b be the next height then the cost function is $\frac{a}{b+1}$. Since $\Delta h < 0$ we know that a > b for all cases. It must also be true that when traveling downwards $1 < \frac{a}{b+1} \ \ \forall a, b$ and when traveling flat $1/2 \leq \frac{a}{b+1} \ \ \forall a, b$. So for all a and b the cost function is greater than or equal to 1/2. Since we run the cost function exactly as many times as the Chebyshev distance, then $\Delta d \ / \ 2$ must be admissible.



In the case above our guess would be 1. It would cost 1 to go from point (2,2) to point (1,1) then would cost ½ to go from (1,1) to (0,1). This example could be extended to if the end goal was further down.