🤖

Robotics assignment 1

- Robotics 2023-24
- R11631045 Wei-Hsuan Cheng 程偉軒

In this assignment, the MATLAB package `NxRLab` / `ModernRobotics` is used.

Part A-1

$$R = \mathrm{Rot}(\hat{u}, \phi) = e^{[\hat{u}]\phi} \in SO(3), \text{where } \hat{u} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} \| \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} \|^{-1}, \ \phi = \frac{\pi}{2}, \ [\hat{u}]\phi \in so(3). \tag{1}$$

$$\Rightarrow R \approx \begin{bmatrix} 0.0333 & -0.1985 & 0.9795 \\ 0.5318 & 0.8333 & 0.1508 \\ -0.8462 & 0.5159 & 0.1333 \end{bmatrix} \blacksquare \tag{2}$$

```
%% A-1
clear; clc; close all;
%%%%%%%%%% Units
d2r = pi / 180;
r2d = 1 / d2r;

%%%%%%%%%% u, phi to R
theta = 90 * d2r;
omg = [1 5 2].'; omg = omg / norm(omg);
omg_theta = omg * theta;
so3mat = VecToso3(omg_theta);
R = MatrixExp3(so3mat);
```

Part A-2

$$R = \begin{bmatrix} 0.911 & -0.244 & 0.333 \\ 0.333 & 0.911 & -0.244 \\ -0.244 & 0.333 & 0.911 \end{bmatrix} = R_{ij} \equiv \mathrm{Rot}(\hat{u}, \phi) = e^{[\hat{u}]\phi} \in SO(3) \tag{3}$$

$$\Rightarrow \begin{cases} \phi = \cos^{-1}(\frac{\mathrm{Tr}(R)-1}{2}) \approx 0.5226 = 29.9456^\circ \\ \hat{u} = \frac{1}{2\sin\phi} \begin{bmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{bmatrix} \approx \begin{bmatrix} 0.5780 \\ 0.5780 \\ 0.5780 \end{bmatrix} \end{cases} \blacksquare \tag{4}$$

```
%% A-2
clear; clc; close all;
%%%%%%%%%% Units
d2r = pi / 180;
r2d = 1 / d2r;

%%%%%%%%%% R to u, phi
R = [.911 -.244 .333;
     .333 .911 -.244;
     -.244 .333 .911];
phi = acos((trace(R) - 1) / 2); % rad
phi_d = phi * r2d;
u_hat = (1 / (2 * sin(phi))) * [R(3,2) - R(2,3);
                                R(1,3) - R(3,1);
                                R(2,1) - R(1,2)];

%%%%%%%%%% Check if R stays in the manifold
detR = det(R);
R_transpose = R.';
R_inverse = inv(R);
```

> ⚠️ ***Note***
>
> There's some numeric error in the rotation matrix R since that,
>
> $$\det(R) = 1.005 \neq 1 \text{ and } R^T \neq R^{-1}.$$
>
> To make R stays in the manifold, the correct answer should be,
>
> $$R = \text{Rot}(\hat{u}, \phi), \text{where } \hat{u} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \phi = \frac{\pi}{6} = 30^\circ.$$

Part B-1

$$ {}^A_BR = \text{Rot}(\hat{z}, 45^\circ)\text{Rot}(\hat{y}, 30^\circ)\text{Rot}(\hat{x}, 60^\circ) \approx \begin{bmatrix} 0.6124 & -0.0474 & 0.7891 \\ 0.6124 & 0.6597 & -0.4356 \\ -0.5 & 0.75 & 0.4330 \end{bmatrix} \in SO(3),\ {}^Ap_B = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \in \mathbb{R}^3 \tag{5}$$

$$\Rightarrow {}^A_BT = \begin{bmatrix} {}^A_BR & {}^Ap_B, \\ 0 & 1 \end{bmatrix} \in SE(3) \blacksquare \tag{6}$$

```
%% B-1
clear; clc; close all;
%%%%%%%%%% Units
d2r = pi / 180;
r2d = 1 / d2r;

%%%%%%%%%% Homogeneous coordinates
A_BR = rotz(45) * roty(30) * rotx(60);
A_Bp = [3 6 9].';
A_BT = [A_BR A_Bp;
        zeros(1,3) 1];
```

Part B-2

- If frame $\{C\}$ rotates about the axis described by *body-frame*, then,

$$ {}^{\text{origin}}_CR = \text{Rot}(\hat{u}_1, \phi_1)\text{Rot}(\hat{u}_2, \phi_2)\text{Rot}(\hat{u}_3, \phi_3) \approx \begin{bmatrix} 0.6982 & -0.7152 & -0.0328 \\ 0.2743 & 0.3095 & -0.9105 \\ 0.6613 & 0.6267 & 0.4123 \end{bmatrix} \in SO(3), \tag{7}$$

$$\text{where } (\hat{u}_1, \phi_1) = (\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \| \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \|^{-1}, 60^\circ),\ (\hat{u}_2, \phi_2) = (\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \| \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \|^{-1}, 30^\circ),\ (\hat{u}_3, \phi_3) = (\begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} \| \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} \|^{-1}, 45^\circ), \tag{8}$$

$$\text{and } {}^{\text{origin}}p_C = 0 \in \mathbb{R}^3 \tag{9}$$

$$\Rightarrow {}^{\text{origin}}_CT = \begin{bmatrix} {}^{\text{origin}}_CR & {}^{\text{origin}}p_C, \\ 0 & 1 \end{bmatrix} \in SE(3) \blacksquare \tag{10}$$

- If frame $\{C\}$ rotates about the axis described by *world-frame*, then,

$$ {}^{\text{origin}}_CR = \text{Rot}(\hat{u}_3, \phi_3)\text{Rot}(\hat{u}_2, \phi_2)\text{Rot}(\hat{u}_1, \phi_1) \approx \begin{bmatrix} 0.1694 & -0.9855 & -0.0103 \\ 0.9816 & 0.1678 & 0.0906 \\ -0.0876 & -0.0254 & 0.9958 \end{bmatrix} \in SO(3), \tag{11}$$

$$\text{where } (\hat{u}_1, \phi_1) = (\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \| \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \|^{-1}, 60^\circ),\ (\hat{u}_2, \phi_2) = (\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \| \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \|^{-1}, 30^\circ),\ (\hat{u}_3, \phi_3) = (\begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} \| \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} \|^{-1}, 45^\circ), \tag{12}$$

$$\text{and } {}^{\text{origin}}p_C = 0 \in \mathbb{R}^3 \tag{13}$$

$$\Rightarrow {}^{\text{origin}}_CT = \begin{bmatrix} {}^{\text{origin}}_CR & {}^{\text{origin}}p_C, \\ 0 & 1 \end{bmatrix} \in SE(3) \blacksquare \tag{14}$$

```
%% B-2
clear; clc; close all;
%%%%%%%%%% Units
d2r = pi / 180;
r2d = 1 / d2r;

%%%%%%%%%% Rotation matrices
theta1 = 60 * d2r;
omg1 = [1 1 1].'; omg1 = omg1 / norm(omg1);
omg_theta1 = omg1 * theta1;
so3mat1 = VecToso3(omg_theta1);
R1 = MatrixExp3(so3mat1);

theta2 = 30 * d2r;
omg2 = [1 -1 2].'; omg2 = omg2 / norm(omg2);
omg_theta2 = omg2 * theta2;
so3mat2 = VecToso3(omg_theta2);
R2 = MatrixExp3(so3mat2);

theta3 = 45 * d2r;
omg3 = [-1 -3 1].'; omg3 = omg3 / norm(omg3);
omg_theta3 = omg3 * theta3;
so3mat3 = VecToso3(omg_theta3);
R3 = MatrixExp3(so3mat3);

%%%%%%%%%% If rotate about its body-frame at each time
R_body = R1 * R2 * R3;
origin_CT_body = [R_body zeros(3,1);
                  zeros(1,3) 1];

%%%%%%%%%% If rotate about the world-frame at each time
R_world = R3 * R2 * R1;
origin_CT_world = [R_world zeros(3,1);
                  zeros(1,3) 1];
```