KDD2020- Tutorials

Robust, Deep and Inductive Anomaly Detection

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Definition

Anomaly Detection

- Anomalies are objects : **different from most other objects.**

Application

Anomaly Detection: Video Surveillance.

- Detecting: **Background activities.**

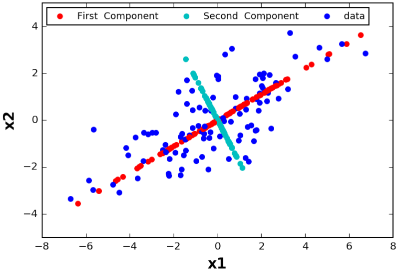
Anomaly Detection: By Spectral Techniques

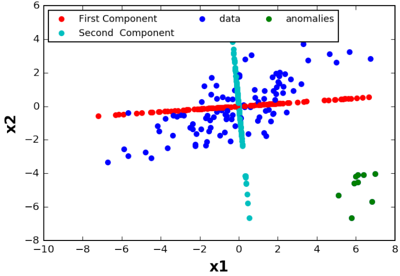
- Analysis based on **Eigen-Decomposition** of data.
- **Key Idea:**
  - Find combination of attributes **capturing bulk of variability**.
  - Reduced set of attributes can **explain only normal data well**.
- Several methods use Principal Component Analysis.
  - **Top few principal components** capture variability: normal data.
  - **Outliers have variability** in the smallest component.

Motivation and Challenges

Anomaly Detection: PCA

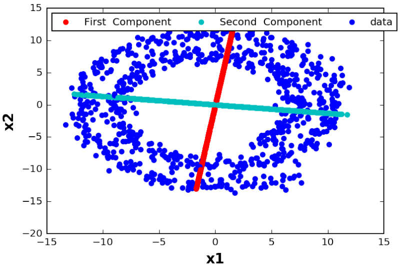
- PCA is highly sensitive to **data perturbation.**

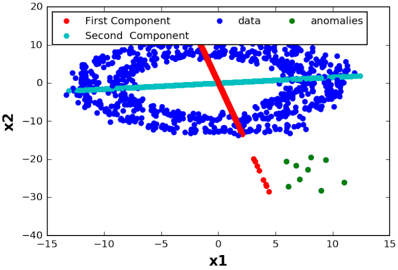




Anomaly Detection: PCA

- PCA, Robust PCA **fails to capture non-linear projections.**

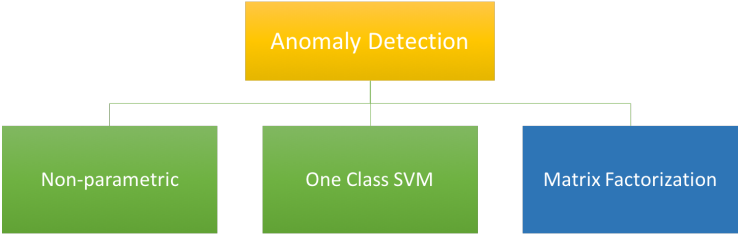




- We propose **Robust (Convolutional) Auto-encoder** to overcome these limitations.

Related Work

Conventional Anomaly Detection Techniques



Matrix Factorization Approach: PCA

- PCA interpreted as **matrix factorisation.**
- Factorise $\mathbf{X} \in \mathbb{R}^{N \times D}$ into $\mathbf{Z} \in \mathbb{R}^{N \times K}$ and $\mathbf{W} \in \mathbb{R}^{K \times D}$

$$\min_{\mathbf{W}\mathbf{W}^T=\mathbf{I},\mathbf{Z}} \|\mathbf{X} - \mathbf{Z}\mathbf{W}\|_F^2 = \min_{\mathbf{W}\mathbf{W}^T=\mathbf{I}} \|\mathbf{X} - \mathbf{X}\mathbf{W}^T\mathbf{W}\|_F^2$$

Limitations:

- **Does not handle data perturbations.**
- **Does not capture nonlinear projections.**

Robust PCA

- RPCA generalizes PCA with **tuning parameter** $\lambda > 0$.

$$\min_{\mathbf{S},\mathbf{N}} \|\mathbf{S}\|_* + \lambda\|\mathbf{N}\|_\mathbf{1} : \mathbf{X} = \mathbf{S} + \mathbf{N} \quad (1)$$

- **S** is signal, **N** is noise matrix.
- Points with **high value of** N are considered anomalous.

Limitations:

- Does not capture **nonlinear projections.**

Robust (Convolutional) Autoencoders

Auto-encoders for anomaly detection.

- Auto encoder with single **hidden layer**

$$\min_{\mathbf{U},\mathbf{V}} \|\mathbf{X} - \mathbf{f}(\mathbf{XU})\mathbf{V}\|_{\mathbf{F}}^{\mathbf{2}} \tag{2}$$

- $\hat{\mathbf{X}} = f(\mathbf{XU})\mathbf{V}$ is **reconstruction error measure.**

- **XU** projects $\mathbf{X}$ into K dimensional space
$\mathbf{U} \in \mathbf{R}^{\mathbf{D}\times\mathbf{K}}$, $\mathbf{V} \in \mathbf{R}^{\mathbf{K}\times\mathbf{D}}$.
- Non linear projection: **sigmoid** $f(\cdot) := (1 + \exp(-a))^{-1}$

Auto-encoders for anomaly detection.

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  - $\mathbf{U}$: Weights (input $\rightarrow$ hidden),

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  - $\mathbf{U}$: Weights (input $\rightarrow$ hidden),
  - $\mathbf{V}$: Weights (hidden $\rightarrow$ output),
  - activation function: $f : \mathbb{R} \rightarrow \mathbb{R}$

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- Non linear projection: **sigmoid** $f(\cdot) := (1 + \exp(-a))^{-1}$

Comparison: Conventional Anomaly Detection Methods

- Deep (convolution) Robust Auto encoder **are versatile.**

	Handles Data Perturbation	Captures Non-linear Structure	Learns Non-linear Structure from data
PCA	No	No	No
RPCA	Yes	No	No
RKPCA	Yes	Yes	No
RCAE	Yes	Yes	Yes

Robust (convolution) Auto-Encoders [RCAE]

- For activation function $f : \mathbb{R} \to \mathbb{R}$

$$\min_{\mathbf{U},\mathbf{V},\mathbf{N}} \|\mathbf{X} - \mathbf{f}(\mathbf{XU})\mathbf{V} + \mathbf{N}\|_{\mathbf{F}}^{\mathbf{2}} + \frac{\mu}{\mathbf{2}} \cdot (\|\mathbf{U}\|_{\mathbf{F}}^{\mathbf{2}} + \|\mathbf{V}\|_{\mathbf{F}}^{\mathbf{2}}) + \lambda\|\mathbf{N}\|_{\mathbf{1}} \tag{3}$$

- $\hat{X} = f(\mathbf{XU})\mathbf{V}$ is **reconstruction error measure**
- $\lambda, \mu > 0$ are tuning parameters.
- $\mathbf{Z} = \mathbf{f}(\mathbf{XU})$ **latent representation decoded by V weights.**
- $\mathbf{N}$ captures **gross outliers.**
- $0 < \lambda < +\infty$, models a standard auto encoder robust to noise.

RCAE Vs Robust PCA (1)

- **RPCA objective** function with basic equality constraints:

$$\min_{S,N} \|\mathbf{S}\|_* + \lambda \|\mathbf{N}\|_\mathbf{1} \tag{4}$$

- **RCAE** objective function with basic equality constraints:

$$\min_{U,V,N} \|\mathbf{X} - \mathbf{f}(\mathbf{XU})\mathbf{V} + \mathbf{N}\|_\mathbf{F}^\mathbf{2} + \frac{\mu}{\mathbf{2}} \cdot (\|\mathbf{U}\|_\mathbf{F}^\mathbf{2} + \|\mathbf{V}\|_\mathbf{F}^\mathbf{2}) + \lambda \|\mathbf{N}\|_\mathbf{1} \tag{5}$$

- Conceptual Similarity between **Equation** 4 and **Equation** 5 established [5].

Training RCAE (1)

- Consider the Objective function of **Robust Autoencoder**.

$$\min_{\mathbf{U},\mathbf{V},\mathbf{N}} \|\mathbf{X} - \mathbf{f}(\mathbf{X}\mathbf{U})\mathbf{V} + \mathbf{N}\|_{\mathbf{F}}^{\mathbf{2}} + \frac{\mu}{\mathbf{2}} \cdot (\|\mathbf{U}\|_{\mathbf{F}}^{\mathbf{2}} + \|\mathbf{V}\|_{\mathbf{F}}^{\mathbf{2}}) + \lambda\|\mathbf{N}\|_{\mathbf{1}} \quad (6)$$

- More generally objective could be rewritten as below.

$$\min_{\theta,\mathbf{N}} \|\mathbf{X} - \hat{\mathbf{X}}(\theta) + \mathbf{N}\|_{\mathbf{F}}^{\mathbf{2}} + \frac{\mu}{\mathbf{2}} \cdot \mathbf{\Omega}(\theta) + \lambda\|\mathbf{N}\|_{\mathbf{1}} \quad (7)$$

- where $\hat{\mathbf{X}}(\theta)$ is some generic predictor with parameters θ.
- $\mathbf{\Omega}(\cdot)$: regularization function.
- Equation 7 is **non-convex but unconstrained and sub-differentiable**

Training RCAE (2)

- For differentiable function $\hat{\mathbf{X}}(\theta)$ back-propagation is employed.
- We follow **soft thresholding approach** to optimize $\mathbf{N}$.
- For fixed $\mathbf{N}$,θ,$\mathbf{U}, \mathbf{V}$ the objective is:

$$\min_{\theta,\mathbf{N}} \|\mathbf{N} - (\mathbf{X} - \hat{\mathbf{X}}(\theta)) + \lambda\|\mathbf{N}\|_{\mathbf{1}} \quad (8)$$

- Applying **Soft-thresholding**[1] to compute **N**

$$N_{ij} = \begin{cases} (\mathbf{X} - \hat{\mathbf{X}}(\theta))_{ij} - \frac{\lambda}{2} & \text{if } (\mathbf{X} - \hat{\mathbf{X}}(\theta))_{ij} > \frac{\lambda}{2} \\ (\mathbf{X} - \hat{\mathbf{X}}(\theta))_{ij} + \frac{\lambda}{2} & \text{if } (\mathbf{X} - \hat{\mathbf{X}}(\theta))_{ij} < -\frac{\lambda}{2} \\ 0 & \text{else.} \end{cases} \quad (9)$$

[1] Bach, F., Jenatton, R., Mairal, J., Obozinski, G. Convex Optimization with Sparsity-Inducing Norms.

Experimental Setup

Summary of Datasets

- We compare all methods on three real-world datasets for anomaly detection:
  - **Restaurant**, comprising video background modeling and activity detection consisting of snapshots of restaurant activities.
  - **USPS**, comprising the USPS handwritten digits.
  - **CIFAR-10** consisting of 60000 32×32 colour images of 10 classes,with 6000 images per class.

Dataset	# instances	# anomalies	# features
`restaurant`	200	Unknown (foreground)	19200
`usps`	231	11 (‘7’)	256
`cifar-10`	5000	50 (cats)	1024

Anomaly Detection: Methods Compared

- Compare empirical effectiveness of:
  - **Truncated SVD**: zero-mean features is equivalent to PCA.[2]
  - **Robust PCA .**[2]
  - **Robust kernel PCA (RKPCA)**[2]**.**
  - **Autoencoder (AE)**[3]**.**
  - **Convolutional Autoencoder (CAE)**[3] **where** $\lambda = +\infty$**.**
  - **Robust Convolutional Autoencoder (RCAE).**[3]

[2] Publicly available implementations[3][5][1]

[3] Tensorflow Implementation: https://github.com/raghavchalapathy/rcae

Experiment Settings

- Experiments were conducted **for three scenarios:**
  - **Non-inductive anomaly detection.**
  - **Inductive anomaly detection.**
  - **Image denoising.**
- **Four network parameters** were tuned for best performance:
  - number of convolutional filters.
  - filter size.
  - strides of convolution operation.
  - activation applied.
- Number of **hidden nodes** $H \in [3, 64, 128]$, regularisation parameters $\lambda \in [0, 100]$ and $\mu \in [0.05, 0.1]$.
- Initial **weight matrices** were from uniform distribution in $[-1, 1]$.

Evaluation Methodology

- Predictive performance is measured against the ground truth anomaly labels **using three standard metrics:**
  - **Area under the precision-recall curve (AUPRC)**
  - **Area under the ROC curve (AUROC))**
  - **Precision at 10 (P@10)**
- **AUPRC and AUROC** measure ranking performance.
- **P@10** measures classification performance.(actual anomalies among top-10 scored instances).

Results

Non Inductive: Top anomalous Images Detected

- USPS : 220 images of ‘1’s, and 11 images of ‘7’(anomalous)

(a)RCAE

(b) RPCA

- CIFAR: 5000 images of ‘dog’s, and 50 images of ‘cat’s(anomalous)

(a) RCAE

(b) RPCA

Non Inductive Anomaly Detection: Performance

- The **Robust convolution autoencoder** outperforms the state of the art methods.

(a) usps

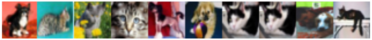
Methods	AUPRC	AUROC	P@10
RCAE	0.9614 ± 0.0025	0.9988± 0.0243	0.9108 ± 0.0113
CAE	0.7003 ± 0.0105	0.9712 ± 0.0002	0.8730 ± 0.0023
AE	0.8533 ± 0.0023	0.9927 ± 0.0022	0.8108 ± 0.0003
RKPCA	0.5340 ± 0.0262	0.9717 ± 0.0024	0.5250 ± 0.0307
DRMF	0.7737 ± 0.0351	0.9928 ± 0.0027	0.7150 ± 0.0342
RPCA	0.7893 ± 0.0195	0.9942 ± 0.0012	0.7250 ± 0.0323
SVD	0.6091 ± 0.1263	0.9800 ± 0.0105	0.5600 ± 0.0249

(b) cifar-10

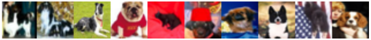
Methods	AUPRC	AUROC	P@10
RCAE	0.9934 ± 0.0003	0.6255 ± 0.0055	0.8716 ± 0.0005
CAE	0.9011 ± 0.0000	0.6191 ± 0.0000	0.0000 ± 0.0000
AE	0.9341 ± 0.0029	0.5260 ± 0.0003	0.2000 ± 0.0003
RKPCA	0.0557 ± 0.0037	0.5026 ± 0.0123	0.0550 ± 0.0185
DRMF	0.0034 ± 0.0000	0.4847 ± 0.0000	0.0000 ± 0.0000
RPCA	0.0036 ± 0.0000	0.5211 ± 0.0000	0.0000 ± 0.0000
SVD	0.0024 ± 0.0000	0.5299 ± 0.0000	0.0000 ± 0.0000

Inductive: Top anomalous Images Detected

- First train model on **only normal** 5000 dog images.
- Evaluate it on a test set 500 dogs and 50 'cat's(anomalous).



(a) RCAE



(b) CAE

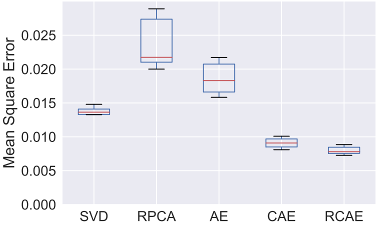
	SVD	RKPCA	AE	CAE	RCAE
AUPRC	0.1752 ± 0.0051	0.1006 ± 0.0045	0.6200 ± 0.0005	0.6423 ± 0.0005	0.6908 ± 0.0001
AUROC	0.4997 ± 0.0066	0.4988 ± 0.0125	0.5007 ± 0.0010	0.4708 ± 0.0003	0.5576 ± 0.0005
P@10	0.2150 ± 0.0310	0.0900 ± 0.0228	0.1086 ± 0.0001	0.2908 ± 0.0001	0.5986 ± 0.0001

Image De-noising Capability: RCAE vs RPCA

- Top anomalous images in original form (first row), noisy form (second row), image denoising task on `cifar-10`.

(a) RCAE

(b) RPCA



Conclusion

Conclusion

- Extended robust PCA model **to the nonlinear autoencoder setting**.
- Our approach is **robust, deep and inductive.**
- Not oversensitive besides captures **subtle anomalies**.
- Extend deep autoencoders for **outlier description**.

References

- [1] Clevert, D.A., Unterthiner, T., Hochreiter, S.: Fast and accurate deep network learning by exponential linear units (elus). arXiv preprint arXiv:1511.07289 (2015)
- [2] Goodfellow, I., Bengio, Y., Courville, A. Deep Learning. MIT Press (2016), http://www.deeplearningbook.org
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- [4] Zhao, M., Saligrama, V. Anomaly detection with score functions based on nearest neighbor graphs. In Advances in Neural Information Processing Systems (NIPS). pp. 2250 2258 (2009)
- [5]Chalapathy, Raghavendra, Aditya Krishna Menon, and Sanjay Chawla. Robust, Deep and Inductive Anomaly Detection.arXiv:1704.06743 (2017).
- [6] Candes, E., Li, X., Ma, Y., Wright, J.: Robust principal component analysis: Recovering low-rank matrices from sparse errors. In: Sensor Array and Multichannel Signal Processing Workshop (SAM), 2010 IEEE. pp. 201204. IEEE (2010)


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