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Humanoid Sensors and Actuators - Tutorial 4
Part 1

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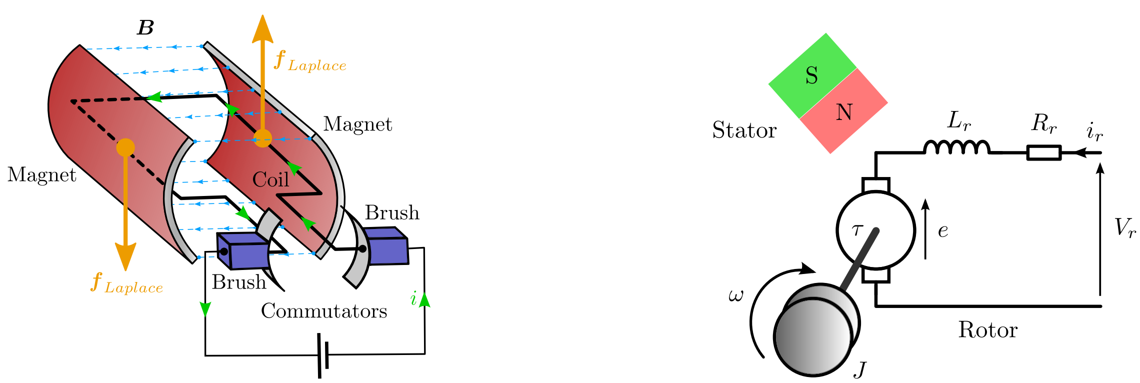
DC Motors: Simulation and Frequency response (47 points)

In this tutorial we will learn:

- How to simulate a DC motor on Matlab/Octave (without Simulink)

1 Modeling and simulation of a DC motor on Matlab/Octave

Direct-Current motor are electro-mechanical converters with linear torque-current and voltage-angular velocity mappings. On such machines, the rotor is made out of a set of electromagnets, connected to an external power source through sliding contacts, or "brushes" (c.f. Fig.1a). The stator usually consists of a permanent magnet although it can also be made of an electromagnet on bigger machines. DC motors are widely used in industry and – to some extent – in robotics[1] as they are cheap and relatively easy to control. This tutorial will cover some aspects of the simulation and frequency response of such machines. The electro-mechanical model of a DC motor usually consists of an RL circuit (c.f. Fig.1b), accounting for the rotor winding resistance and inductance, coupled to a generator, producing a velocity-dependent counter-electromotive force e induced – according to the Lenz's law of induction – by the relative changes of magnetic flux in the moving rotor winding.



(a) Working principle of a DC motor

(b) Electro-mechanical model of a DC motor.

Figure 1: Electro-mechanical model of a DC motor

[1] As a reminder from L3, the NAO humanoid robot from Softbank robotics uses DC motors (c.f. Fig. 2).

Accordingly the electro-machanical equations of a DC motor can be written as follows:

- **Electrical Equation:**

$$\begin{aligned} V_r &= R_r i_r + L_r \frac{di_r}{dt} + e && \text{(1a)} \\ e &= \frac{\omega}{K_V} = K_e \omega && \text{(1b)} \end{aligned}$$

- **Mechanical Equation:**

$$\begin{aligned} \tau_{mot} &= J\frac{d\omega}{dt} + \underbrace{\beta\omega}_{friction} + \tau_{load} && \text{(2a)} \\ &= K_\tau i_r \quad (K_\tau = K_e) && \text{(2b)} \end{aligned}$$

- **Power Equation:**

$$\begin{aligned} P_{elec} &= P_{heat} + P_{mech} && \text{(3a)} \\ &= R_r i_r^2 + \tau_{mot}\omega && \text{(3b)} \end{aligned}$$

- $V_r | V_s \in \mathbb{R}$: External rotor|stator bias $[V]$
- $R_r | R_s \in \mathbb{R}_+$: Rotor|stator winding resistance $[\Omega]$
- $L_r | L_s \in \mathbb{R}_+$: Rotor|stator winding inductance $[H]$
- $e \in \mathbb{R}$: Counter electromotive force $[V]$
- $J \in \mathbb{R}_+$: Rotor moment of inertia $[kg.m^2]$
- $\tau_{mot} | \tau_{load} \in \mathbb{R}$: Motor|load torques $[N.m]$
- $\omega \in \mathbb{R}$: Rotor angular velocity $[rad.s^{-1}]$
- $K_V \in \mathbb{R}_+$: Velocity constant $[rad.s^{-1}.V^{-1}]$
- $K_\tau \in \mathbb{R}_+$: Torque constant $[N.m.A^{-1}]$

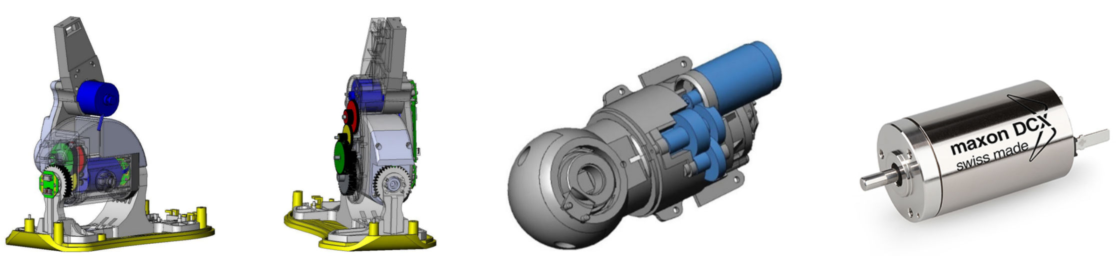
Rewriting these equations in the state-space form yields:

$$\underbrace{\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{di_r}{dt} \end{bmatrix}}_{\dot{\boldsymbol{x}}} = \underbrace{\begin{bmatrix} -\frac{\beta}{J} & \frac{K_\tau}{J} \\ -\frac{1}{L_r K_V} & -\frac{R_r}{L_r} \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \omega \\ i_r \end{bmatrix}}_{\boldsymbol{x}} + \underbrace{\begin{bmatrix} -\frac{1}{J} & 0 \\ 0 & \frac{1}{L_r} \end{bmatrix}}_{\boldsymbol{B}} \underbrace{\begin{bmatrix} \tau_{load} \\ V_r \end{bmatrix}}_{\boldsymbol{u}} \quad \text{(4a)}$$

$$\underbrace{\omega}_{\boldsymbol{y}} = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\boldsymbol{C}} \underbrace{\begin{bmatrix} \omega \\ i_r \end{bmatrix}}_{\boldsymbol{x}} \quad \text{(4b)}$$

After first-order temporal discretization, with a time step Δt, one can write:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{x}_k + \boldsymbol{A}\boldsymbol{x}_k \Delta t + \boldsymbol{B}\boldsymbol{u}_k \Delta t && \text{(5a)} \\ \boldsymbol{y}_k &= \boldsymbol{C}\boldsymbol{x}_k && \text{(5b)} \end{aligned}$$



(a) Detail of the NAO ankle module. (b) Arm drive chain on NAO. (c) NAO uses Maxon DC-motors.

Figure 2: Detail of NAO drive chain.

1.1 Setup

- We here wish to simulate a Maxon DC-motor from scratch on Matlab/Octave. In this tutorial we will consider the **Maxon DCX19S** motor (in its 24V flavor), which is reasonably close to the motors used on the NAO robot[2].
- Start by downloading the motor documentation from the manufacturer's website:

```
wget https://www.maxongroup.com/medias/sys\_master/root
    /8841086730270/EN-87.pdf
```

- Please use the template project `SimulatorMatlab` as basis for the tasks introduced in this tutorial. As in the previous tutorial, you are free to submit as many code files as you wish provided that you also include a `README.md` file indicating the structure of your homework. You can clone the template project as follows:

```
git -c http.sslVerify=false clone "https://gitlab.ics.ei.tum.de
    /quenlebo/SimulatorMatlab.git"
```

1.2 Code (21 points)

T.1.1 **(11 points)** Using the discrete state space equation (4) of the DC-motor and the data you find in the motor data-sheet, build a model of the Maxon DCX19S in Matlab/Octave:

- Implement the DC-motor state-space model in the function `dcMotorDynamics`. (4 points)
- Simulate the system's behavior in the *Main simulation loop* section. To that end, you should use the provided function handle and fixed-step Runge-Kutta `rk4_IntegrationStep` integration routine. (5 points)
- Compute the electrical and mechanical powers $P_{elec} = V_r i_r$ and $P_{mech} = \omega\tau$ of the system and store them in dedicated variables for each simulation epoch. (2 points)

T.1.2 **(10 points)** Simulate the behavior of your DCX19S motor over a $1s$ time horizon with a fixed time-step $\Delta t = 1\mu s$. Using the *stairs* plot function, visualize the voltage V_r, velocity ω, current i_r, power and efficiency signals in the following cases:

- Constant input voltage $V_r = 24V$, zero load torque $\tau_{load} = 0$. (1 points)
- Constant input voltage $V_r = 24V$, stall torque $\tau_{load} = \tau_s$. (1 points)
- Constant input voltage $V_r = 24V$, sinusoidal load torque with a frequency $f_l = 10Hz$ and an amplitude equal to half the stall torque: $\tau_{load} = \tau_s \left(\frac{1}{2} + \sin(2\pi f_l t)\right)$ (2 points)
- PWM input signal with a frequency of $f_s = 100Hz$, an amplitude of $24V$ and a duty cycle of 60%. Apart from Simulink functions, you are free to use any PWM generation method[3]. (3 points)
- PWM input signal with a frequency of $f_s = 40kHz$ an amplitude of $24V$ and a duty cycle of 60%. Apart from Simulink functions, you are free to use any PWM generation method. (3 points)

[2] You can find some of the specifications of the DC-motors mounted on NAO at the following address: `http://doc.aldebaran.com/1-14/family/nao_h25/motors_h25.html`

[3] **Hint:** You may for example consider using the `square` Matlab/Octave function

1.3 Report (26 points)

R.1.1 **(2 points)** What is the limiting factor that causes the motor speed to saturate for a given input voltage V_r?

R.1.2 **(2 points)** Measure the no load current i_0 in steady state, when the motor is rotating at maximum velocity with a constant input voltage $V_r = 24V$ and no external load torque $\tau_{load} = 0$. Propose a physical interpretation to the torque τ_0 resulting from this current.

R.1.3 **(2 points)** Propose an explanation to the current spike observed when setting the motor input voltage from $V_r = 0V$ to $V_r = 24V$ starting with $\omega = 0rad.s^-1$.

R.1.4 **(5 points)** The motor efficiency η is defined as the ratio (in %) between the true mechanical power $P^{\star}_{mech} = \omega\,(\tau - \tau_0)$ generated by the motor and the total electrical power $P_{elec} = V_r i_r$ consumed by the motor:

$$\eta = 100\frac{P^{\star}_{mech}}{P_{elec}} \tag{6}$$

Determine experimentally the maximum efficiency of your motor for $V_r = 24V$, as well as its maximum power (give the detail of your experiment protocol). Provide the corresponding motor states in terms of angular velocity and torque.

R.1.5 **(2 points)** Qualify qualitatively the behavior of the DC motor with respect to current and velocity (high pass, low pass ...) by observing the time response of the motor when a PWM control signal is applied to it.

R.1.6 **(3 points)** Give the transfer function of your DC motor from Voltage-to-Velocity (assuming zero load torque). Provide the different steps of your computation (a photo of a handwritten justification is OK: you are not expected to write this on LaTeX).

R.1.7 **(3 points)** Give the transfer function of your DC motor from Voltage-to-Torque (assuming zero load torque). Provide the different steps of your computation (a photo of a handwritten justification is OK: you are not expected to write this on LaTeX).

R.1.8 **(5 points)** On Matlab, plot the Bode diagrams (gain and phase) associated with these two transfer functions.

R.1.9 **(2 points)** In practice, what is the physical factor limiting the implementation of too high switch frequencies?