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Humanoid Sensors and Actuators - Tutorial 4
Part 2

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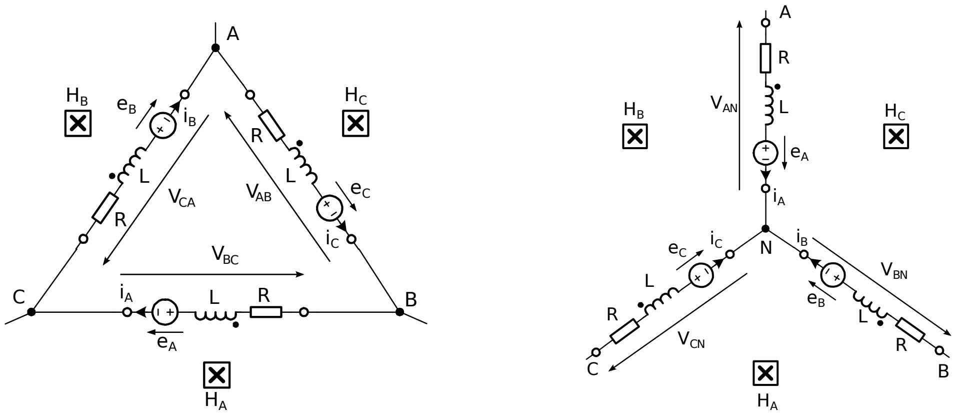
DC Motors: Velocity and Torque Control (34 points)

In this tutorial we will learn:

- How to simulate a BLDC motor on Matlab/Octave (without Simulink)

2 Modeling and simulation of a BLDC motor on Matlab/Octave

A Brushless Direct Current (BLDC) motor consists of a permanent magnet rotor and of a set of wound stator poles. The electrical energy is converted into mechanical energy by the magnetic attraction forces between the permanent magnet rotor and a rotating magnetic field induced in the poles of the wound stator[1]. BLDC motors are widely used in robotics because of their high efficiency, mainly resulting from the absence of sliding contacts between the rotor and stator. The counterpart of this high yield lies in the complexity of their control strategy, as a state-dependent rotating magnetic field has to be generated using fixed stator coils.



(a) BLDC Delta (Δ) configuration: No Neutral Point

(b) BLDC Star (Y) configuration: Neutral Point with a voltage V_N

Figure 1: Star topology favors high torques at the expense of velocity: it is often preferred in robotics.

[1] One could understand it in the same way as a compass placed in a rotating magnetic field: the magnetic needle (rotor) must rotate at the same speed as the outer field.

Most BLDC motors have a three-phase winding topology with star (Y) connection (c.f. Fig.1b). A motor with this topology is driven by energizing two phases at the same time. The electro-machanical equations of a BLDC motor can be written as follows[2]:

- **Electrical Equation:**
  Observing the circuit diagram of Fig.1b, one can write:

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} V_A - V_N \\ V_B - V_N \\ V_C - V_N \end{bmatrix} = \begin{bmatrix} R_A & 0 & 0 \\ 0 & R_B & 0 \\ 0 & 0 & R_C \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} L_{AA} & L_{AB} & L_{AC} \\ L_{BA} & L_{BB} & L_{BC} \\ L_{CA} & L_{CB} & L_{CC} \end{bmatrix} \begin{bmatrix} \frac{di_A}{dt} \\ \frac{di_B}{dt} \\ \frac{di_C}{dt} \end{bmatrix} + \begin{bmatrix} e_A \\ e_B \\ e_C \end{bmatrix} \tag{1}$$

where $L_{AB}, L_{AC}, \cdots$ denote self-inductance terms between the stator coils. The voltage V_N at neutral point is defined as $V_N = \frac{1}{3}(V_A + V_B + V_C) - \sum BEMF$. We here consider as a first approximation that the stator winding have the same physical properties. Therefore it is possible to write $R_A = R_B = R_C = R$, $L_A = L_B = L_C = L$ and $L_{AB} = L_{AC} = \cdots = M$, where M is referred to as the *mutual inductance*, resulting in:

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix} \begin{bmatrix} \frac{di_A}{dt} \\ \frac{di_B}{dt} \\ \frac{di_C}{dt} \end{bmatrix} + \begin{bmatrix} e_A \\ e_B \\ e_C \end{bmatrix} \tag{2}$$

Since $i_A + i_B + i_C = 0$, we have $Mi_A = -Mi_B - Mi_C$ allowing to further simplify equation (2):

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{bmatrix} \begin{bmatrix} \frac{di_A}{dt} \\ \frac{di_B}{dt} \\ \frac{di_C}{dt} \end{bmatrix} + \begin{bmatrix} e_A \\ e_B \\ e_C \end{bmatrix} \tag{3}$$

In BLDC or PMSM motors, the counter electromotive forces e_A, e_B and e_C induced in the stator windings by the rotor motions are periodic functions of the rotor angle θ, with a linear dependence in the angular velocity ω of the rotor:

$$e_A = \omega \frac{f_A(\theta)}{K_V} = K_e \omega f_A(\theta) \tag{4a}$$

$$e_B = \omega \frac{f_B(\theta)}{K_V} = K_e \omega f_B(\theta) \tag{4b}$$

$$e_C = \omega \frac{f_C(\theta)}{K_V} = K_e \omega f_C(\theta) \tag{4c}$$

Electrical rotor speed ω and position θ are related by:

$$\frac{d\theta}{dt} = \frac{P\omega}{2} \tag{5}$$

where P is the number of poles in the motor. In a **PMSM**[3], the terms $f_A(\theta)$, $f_B(\theta)$ and $f_C(\theta)$ denote sinus functions with a phase shift $\varphi = \frac{2\pi}{3}$:

$$f_A(\theta) = \sin(\theta), \quad f_B(\theta) = \sin(\theta + \frac{2\pi}{3}), \quad f_C(\theta) = \sin(\theta + \frac{4\pi}{3}) \tag{6}$$

[2] For more details, see Pragasen et al. (1989) *Modeling, Simulation, and Analysis of Permanent-Magnet Motor Drives, Part II: The Brushless DC Motor Drive*

[3] PMSM = Permanent Magnet Synchronous Machine

In a **BLDC** motor, $f_A(\theta)$, $f_B(\theta)$ and $f_C(\theta)$ denote trapezoidal functions with a phase shift $\varphi = \frac{2\pi}{3}$:

$$f_A(\theta) = \begin{bmatrix} 1 & (0 \le \theta < \pi/6) \\ 2 - 6\theta/\pi & (\pi/6 \le \theta < 3\pi/6) \\ -1 & (3\pi/6 \le \theta < 7\pi/6) \\ 6\theta/\pi - 8 & (7\pi/6 \le \theta < 9\pi/6) \\ 1 & (9\pi/6 \le \theta < 2\pi) \end{bmatrix}, f_B(\theta) = \begin{bmatrix} -1 & (0 \le \theta < 3\pi/6) \\ 6\theta/\pi - 4 & (3\pi/6 \le \theta < 5\pi/6) \\ 1 & (5\pi/6 \le \theta < 9\pi/6) \\ -6\theta/\pi + 10; & (9\pi/6 \le \theta < 11\pi/6) \\ -1 & (11\pi/6 \le \theta < 2\pi) \end{bmatrix},$$
$$f_C(\theta) = \begin{bmatrix} 6\theta/\pi & (0 \le \theta < \pi/6) \\ 1 & (\pi/6 \le \theta < 5\pi/6) \\ 6 - 6\theta/\pi & (5\pi/6 \le \theta < 7\pi/6) \\ -1 & (7\pi/6 \le \theta < 11\pi/6) \\ 6\theta/\pi - 12 & (11\pi/6 \le \theta < 2\pi) \end{bmatrix} \tag{7}$$

- **Mechanical Equation:**

$$\tau_{mot} = J\frac{d\omega}{dt} + \underbrace{\beta\omega}_{friction} + \tau_{load} \tag{8a}$$
$$= K_\tau \left(f_A(\theta) i_A + f_B(\theta) i_B + f_C(\theta) i_C \right) \quad K_\tau \simeq K_e \tag{8b}$$

- **Power Equation:**

$$P_{elec} = P_{heat} + P_{mech} \tag{9a}$$
$$= R\left(i_A^2 + i_B^2 + i_C^2\right) + \tau_{mot}\omega \tag{9b}$$

Rewriting these equations in the state-space form yields:

$$\underbrace{\begin{bmatrix} \frac{d\theta}{dt} \\ \frac{d\omega}{dt} \\ \frac{di_A}{dt} \\ \frac{di_B}{dt} \\ \frac{di_C}{dt} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & \frac{P}{2} & 0 & 0 & 0 \\ 0 & -\frac{\beta}{J} & \frac{K_\tau f_A(\theta)}{J} & \frac{K_\tau f_B(\theta)}{J} & \frac{K_\tau f_C(\theta)}{J} \\ 0 & -\frac{f_A(\theta)}{(L-M)K_V} & -\frac{R}{(L-M)} & 0 & 0 \\ 0 & -\frac{f_B(\theta)}{(L-M)K_V} & 0 & -\frac{R}{(L-M)} & 0 \\ 0 & -\frac{f_C(\theta)}{(L-M)K_V} & 0 & 0 & -\frac{R}{(L-M)} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \theta \\ \omega \\ i_A \\ i_B \\ i_C \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ -\frac{1}{J} & 0 & 0 & 0 \\ 0 & \frac{1}{(L-M)} & 0 & 0 \\ 0 & 0 & \frac{1}{(L-M)} & 0 \\ 0 & 0 & 0 & \frac{1}{(L-M)} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \tau_{load} \\ V_A \\ V_B \\ V_C \end{bmatrix}}_{u}$$

where:

- $V_A, V_B, V_C \in \mathbb{R}$: Input voltages $[V]$
- $V_{AN}, V_{BN}, V_{CN} \in \mathbb{R}$: Phase voltages $[V]$
- $R \in \mathbb{R}_+$: Stator winding resistance $[\Omega]$
- $L|M \in \mathbb{R}_+$: Stator winding inductance and mutual inductance $[H]$
- $e_A, e_B, e_C \in \mathbb{R}$: Phase counter-electromotive forces $[V]$
- $J \in \mathbb{R}_+$: Rotor moment of inertia $[kg.m^2]$
- $\tau_{mot}|\tau_{load} \in \mathbb{R}$: Motor|load torques $[N.m]$
- $\omega \in \mathbb{R}$: Rotor angular velocity $[rad.s^{-1}]$
- $K_V \in \mathbb{R}_+$: Velocity constant $[rad.s^{-1}.V^{-1}]$
- $K_\tau \in \mathbb{R}_+$: Torque constant $[N.m.A^{-1}]$

2.1 Setup

- We here wish to simulate a Maxon BLDC-motor from scratch on Matlab/Octave. In this tutorial we will consider the **Maxon EC-i 52** motor (in its 24V flavor).
- Start by downloading the motor documentation from the manufacturer's website:

```
wget https://www.maxongroup.com/medias/sys_master/root
    /8841184935966/EN-269.pdf
```

- Please use the template project `SimulatorMatlab` as basis for the tasks introduced in this tutorial. As in the previous tutorial, you are free to submit as many code files as you wish provided that you also include a `README.md` file indicating the structure of your homework. You can clone the template project as follows:

```
git -c http.sslVerify=false clone "https://gitlab.ics.ei.tum.de/
    quenlebo/SimulatorMatlab.git"
```

2.2 Code (34 points)

T.2.1 **(20 points)** Using the discrete state space equation (10a) of the BLDC-motor and the data you find in the motor data-sheet, build a model of the Maxon EC-i 52 in Matlab/Octave:

T.2.1.1 **(9 points)** Start by implementing the Back Electro-Motive Force (BEMF) waveforms generation routines $f_A(\theta)$, $f_B(\theta)$ and $f_C(\theta)$ in the form of three Matlab/Octave functions, denoted as `BEMF_A`, `BEMF_B` and `BEMF_C`:

* `BEMF_A`, `BEMF_B` and `BEMF_C` should take the electrical angle θ as input and provide the BEMF waveform as output.
* For each function, you should implement the sinusoidal and trapezoidal BEMF waves.
* Code a selection routine allowing to easily switch between sinusoidal and trapezoidal BEMF.

T.2.1.2 **(6 points)** Implement the BLDC-motor state-space model in the Matlab/Octave function `bldcMotorDynamics`. This function is actually very similar to that of the DC motor, in the last tutorial session.

T.2.1.3 **(5 points)** Simulate the system's behavior in the *Main simulation loop* section. To that end, you should use the provided function handle and fixed-step Runge-Kutta `rk4_IntegrationStep` integration routine.

T.2.2 **(14 points)** Simulate the behavior of your Maxon EC-i 52 motor over a $1s$ time horizon with a fixed time-step $\Delta t = 1\mu s$. Using the *stairs* plot function, visualize the BEMF e_A, e_B, e_C, velocity ω, phase currents i_A, i_B, i_C, an torque signals in the following cases:

- Zero input voltage $V_A = V_B = V_C = 0V$, stall torque $\tau_{load} = \tau_s$, sinusoidal BEMF. (3 points)
- Zero input voltage $V_A = V_B = V_C = 0V$, stall torque $\tau_{load} = \tau_s$, trapezoidal BEMF. (3 points)
- Sinusoidal input voltages, zero load torque $\tau_{load} = \tau_s$, sinusoidal BEMF. (4 points)
- Sinusoidal input voltages, zero load torque $\tau_{load} = \tau_s$, trapezoidal BEMF. (4 points) In both cases, the input voltages should have a frequency $f_v = 10Hz$ and an amplitude equal to 12V: $V_A = 12\sin(2\pi f_v t), V_B = 12\sin(2\pi f_v t + 2\frac{\pi}{3}), V_C = 12\sin(2\pi f_v t + 4\frac{\pi}{3})$.