Technical Report: Paying Less for More? Combo Plans for Edge-Computing Services

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In this technical report, we provide proofs for Theorem 1 and Theorem 2 in our paper. Recall that Case 1 corresponds to the inexpensive combo plan price and Case 2 corresponds to the expensive combo plan price. Without ambiguity, we have $\sqrt{C_1(C_1+C_2)} < C_3 \le \sqrt{C_2(C_1+C_2)}$ in Case 3 and $\sqrt{C_2(C_1+C_2)} < C_3 \le \sqrt{C_1(C_1+C_2)}$ in Case 4.

As our work focuses on fine-grained pricing, the PAYG cost of each task can be arbitrarily small compared with the plan reservation fee throughout the report.

1 Proof of Theorem 1

To prove Theorem 1, we will prove the competitive ratio of SOR scheme under 4 cases (Cases 1-4) is $1+\frac{C_1+C_2}{C_3}$. Recall that x and y are the typical data and computing costs.

Lemma 1. *The competitive ratio of SOR scheme is* 2 *for all cases when $x < C_1$ and $y < C_2$.*

Proof. According to the SOR scheme, the user will reserve a combo plan when $x + y = C_3$. Let t_c denote the time when the user reserves the combo plan. Let a_c and b_c denote the communication volume and computing workload of the task arriving at t_c, $(a_c, b_c) = \{(a_i, b_i)|(a_i, b_i, t_i) \in \mathbf{S}, t_i = t_c\}$. Let E_1, E_2 and E_2 denote the expiration time of latest data, computing and combo plans reserved before t_c. Then x and y at t_c can be expressed as $x = \sum_{i:\max(E_1,E_3,t_c-T)\le t_i<t_c} a_i p_1$ and $y = \sum_{i:\max(E_2,E_3,t_c-T)\le t_i<t_c} b_i p_2$. Without further notification, we assume x and y are calculated at t_c in this proof. Let t_s denote the arrival time of the first task whose cost is counted in either x or y, $t_s = \min\{t_i | t_i \ge \min\big(\max(E_1, E_3, t_c - T), \max(E_2, E_3, t_c - T)\big)\}$. We further define the corresponding validation period as $[t_s, t_c+T)$. The validation period starts from the arrival time of the first task not covered by the previous plan and ends at the expiration time of the combo plan.

We will first show that the worst case is that the arrival of tasks stops after the user reserves a combo plan. We call this case as case w.

The cost under SOR scheme in the validation period in case w is

$$C_{SOR,w} = C_3 + x + y. \tag{1}$$

The corresponding optimal offline cost in the validation period in case w is

$$C_{OPT,w} = C_3, \tag{2}$$

as $C_3 \le C_1 + C_2$ and $x + y + a_c p_1 + b_c p_2 > C_3$. The combo plan is reserved at t_s by optimal offline strategy.

The competitive ratio in case w in the validation period is

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_3 + x + y}{C_3} \tag{3}$$

We will show that any counter example will not give a higher competitive ratio. We will use case u to denote the counter example where the tasks arrives after the user reserves the combo plan according to the SOR scheme. According to the SOR scheme, we have $t_c \le t_s + T$. If the task only arrives during the period $[t_c, t_s + T)$, then the competitive ratio is the same as $\frac{C_{SOR,w}}{C_{OPT,w}}$. If the task arrives after $t_s + T$, the optimal offline strategy will reserve a combo plan in $[t_s, t_c)$ as $x+y = C_3$, which means the reservation the combo plan by the optimal offline strategy might be delayed. Let t_{OPT} denote the time when the optimal offline strategy reserves a combo plan. We assume there is no task arrival in $[t_{OPT}+T, t_c+T)$, which is a optimistic scenario for the optimal offline strategy. In the same validation period, now the optimal offline cost in case u is

$$C_{OPT,u} = C_3 + x' + y', \tag{4}$$

where x' and y' are the accumulated PAYG cost for communication volume and computing workload in $[t_s, t_{OPT})$. The cost under SOR scheme in case u in the validation period is

$$C_{SOR,u} = C_3 + x + y. \tag{5}$$

The competitive ratio in case u is

$$\frac{C_{SOR,u}}{C_{OPT,u}} \leq \frac{C_3 + x + y}{C_3 + x' + y'} \leq \frac{C_{SOR,w}}{C_{OPT,w}}, \tag{6}$$

where the first inequality is due to the fact that we ignore the task arrivals in $[t_{OPT} + T, t_c + T)$.

Therefore, the competitive ratio in the validation period is

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_3 + x + y}{C_3} = 2. \tag{7}$$

We notice that this competitive ratio is for a single reservation of combo plan in a single validation period. However, as long as we have $x < C_1$ and $y < C_2$ for every validation period, the overall competitive ratio for SOR scheme is still the same. □

Lemma 2. *In Case 1 and Case 4, the competitive ratio of SOR scheme is less than or equal to $1 + \frac{C_1+C_2}{C_3}$ when $x \geq C_1$ and $y < C_2$.*

Proof. According to the SOR scheme, the user will reserve a combo plan when $x = C_1$. Let t_c denote the time when the user reserves a combo plan. We have the similar definition of a_c, b_c, x, y, t_s, and validation period as in Lemma 1. We will show that the worst case is the same as in Lemma 1. After the user buys a combo plan, the arrival of tasks stops. We call this case w. The cost under SOR scheme in case w in the validation period is

$$C_{SOR,w} = C_3 + x + y. \tag{8}$$

If $x + y \geq C_3$, then the SOR scheme should reserve a combo plan already. Therefore, we consider $x + y \leq C_3$. In this case, the optimal offline cost in case w in this validation period is

$$C_{OPT,w} = \min(C_1 + y + b_c p_2, C_3). \tag{9}$$

The optimal offline strategy reserves either data plan or combo plan at t_s. If $C_{OPT,w} = C_3$, then the worst case is already proved in Lemma 1. If $C_{OPT,w} = C_1 + y + b_c p_2$, we assume the task arrives after t_c. Then there will be extra computing workload cost in the optimal offline cost, which makes the competitive ratio smaller. Therefore, case w is the worst case.

Consider when $C_{OPT,w} = C_3$. Then the competitive ratio is

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_3 + x + y}{C_3} \leq 2, \tag{10}$$

as $x + y \leq C_3$. Consider when $C_{OPT} = (C_1 + y + b_c p_2)$. Then the competitive ratio is

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_3 + x + y}{C_1 + y + b_c p_2}. \tag{11}$$

This expression will get maximum value if $y = 0$ and $b_c = 0$. Then we have

$$\frac{C_{SOR,w}}{C_{OPT,w}} \leq \frac{C_3 + C_1}{C_1} \leq 1 + \frac{C_1 + C_2}{C_3}. \tag{12}$$

The last inequality is due to the fact that $C_3 \leq \sqrt{C_1(C_1 + C_2)}$. We notice that this competitive ratio is for a single reservation of combo plan in a single validation period. However, as long as we have $x \geq C_1$ and $y < C_2$ for every validation period in Case 1 or Case 4, the overall competitive ratio for SOR scheme is still the same. □

Lemma 3. *In Case 2 and Case 3, the competitive ratio of SOR scheme is $1 + \frac{C_1+C_2}{C_3}$ when $x \geq C_1$ and $y < C_2$.*

Proof. According to the SOR scheme, the user will first reserve an exclusive data plan when $x = C_1$ and then reserve a non-exclusive computing plan when $y = C_3 - C_1$ if the exclusive data plan is still on-going. Let t_d denote the time when the user reserves an exclusive data plan. Let t_m denote the time when the user reserves a non-exclusive computing plan. Let a_m and b_m denote the communication volume and computing workload of the task arriving at t_m, $(a_m, b_m) = \{(a_i, b_i)|(a_i, b_i, t_i) \in \mathbf{S}, t_i = t_m\}$. Let E_1, E_2 and E_3 denote the expiration time of the latest data, computing and combo plans reserved before t_d. Then x and y at t_d and t_m can be expressed as $x = \sum_{i:\max(E_1,E_3,t_d-T)\le t_i<t_d} a_i p_1$ and $y = \sum_{i:\max(E_2,E_3,t_m-T)\le t_i<t_m} b_i p_2$. Without further notification, we assume x and y are calculated at t_d and t_m in this proof. Let t_s denote the arrival time of the first task whose cost is counted in either x or y, $t_s = \min\{t_i|t_i \ge \min(\max(E_1, E_3, t_d - T), \max(E_2, E_3, t_m - T))\}$. We further define the corresponding validation period as $[t_s, t_m+T)$. The validation period starts from the arrival time of the first task not covered by the previous plan and ends at the expiration time of the non-exclusive computing plan.

We will show that the worst case is as follow. After the user reserves an exclusive data plan, the arrival of data volume stops. Within the reservation period of the exclusive data plan, the user reserves a non-exclusive computing plan, the arrival of task stops after that. We call this case as case w.

The cost under SOR scheme in case w in this validation period is

$$C_{SOR,w} = C_1 + x + C_2 + y. \tag{13}$$

The optimal offline cost in case w in this validation period is

$$C_{OPT,w} = \min(C_1 + y + b_m p_2, C_3). \tag{14}$$

The optimal offline strategy reserves either the data plan or the combo plan at t_s

Consider when $C_{OPT,w} = C_3$. Then we have

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_1 + x + C_2 + y}{C_3} = 1 + \frac{C_1 + C_2}{C_3}. \tag{15}$$

Consider when $C_{OPT,w} = (C_1+y+b_i p_2)$. Then we have

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_1 + x + C_2 + y}{C_1 + y + b_i p_2} \le 1 + \frac{C_1 + C_2}{C_3}, \tag{16}$$

as $y = C_3 - C_1$ and $x = C_1$.

The first type of the counter example for case w is that the non-exclusive computing plan is not reserved in the reservation period of exclusive data plan. If the non-exclusive computing plan is not reserved in the reservation period of exclusive data plan, then the worst case is a ski-rental problem and the competitive ratio is 2, which is less than $1 + \frac{C_1+C_2}{C_3}$.

The second type of the counter example for case w is that there are still tasks coming after the reservation of the non-exclusive computing plan. We will prove it will not result in a higher competitive ratio. If $C_{OPT,w} = C_3$, then the idea is similar to the proof in Lemma 1. The tasks arrived after t_s+T might delay the reservation of the combo plan. Therefore, there will be extra data and computing cost x' and y' before the combo plan reservation time, $C_{OPT,u} = C_3+x'+y'$, which makes the competitive ratio smaller. If $C_{OPT,w} = C_1 + y + b_m p_2$, the tasks arrived after t_s+T might delay the reservation of the data plan, there will be extra data cost x' before the combo plan reservation time, $C_{OPT,u} = C_1 + x' + y + b_m p_2$, which makes the competitive ratio smaller.

Therefore, case w is the worst case and the competitive ratio is $1 + \frac{C_1+C_2}{C_3}$. We notice that this competitive ratio is for a single reservation of combo plan in a single validation period. However, as long as we have $x \ge C_1$ and $y < C_2$ for every validation period in Case 2 or Case 3, the overall competitive ratio for SOR scheme is still the same. □

Lemma 4. *In Case 1 and Case 3, the competitive ratio of SOR scheme is less than or equal to* $1 + \frac{C_1+C_2}{C_3}$ *when* $x < C_1$ *and* $y \ge C_2$.

Proof. According to the SOR scheme, the user will reserve a combo plan when $y = C_2$. Let t_c denote the time when the user reserves a combo plan. We have the similar definitions of a_c, b_c, x, y, t_s, and validation period as in Lemma 1. The worst case is as follow. After the user reserves a combo plan, the arrival of tasks stops. Let w denote the the worst case. The proof of the worst case is similar to Lemma 2. The cost under SOR scheme in the worst case in this validation period is

$$C_{SOR,w} = C_3 + x + y. \tag{17}$$

Due to the similar reason mentioned in Lemma 2, we consider $x + y \le C_3$. In the worst case, the optimal offline cost in this validation period is

$$C_{OPT,w} = \min(C_2 + x + a_c p_1, C_3). \tag{18}$$

Consider when $C_{OPT,w} = C_3$. Then the competitive ratio is

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_3 + x + y}{C_3} \leq 2, \tag{19}$$

as $x+y \leq C_3$. Consider when $C_{OPT,w} = (C_2+x+a_c p_1)$. Then the competitive ratio is

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_3 + x + y}{C_2 + x + a_c p_1}. \tag{20}$$

This expression will get maximum value if $x = 0$ and $a_c = 0$. Then we have

$$\frac{C_{SOR,w}}{C_{OPT,w}} \leq \frac{C_3 + C_2}{C_2} \leq 1 + \frac{C_1 + C_2}{C_3}. \tag{21}$$

The last inequality is due to the fact that $C_3 \leq \sqrt{C_2(C_1 + C_2)}$. We notice that this competitive ratio is for a single reservation of combo plan in a single validation period. However, as long as we have $x < C_1$ and $y \geq C_2$ for every validation period in Case 1 or Case 3, the overall competitive ratio for SOR scheme is still the same. □

Lemma 5. *In Case 2 and Case 4, the competitive ratio of SOR scheme is* $1 + \frac{C_1+C_2}{C_3}$ *when* $x < C_1$ *and* $y \geq C_2$.

Proof. According to the SOR scheme, the user will first reserve an exclusive computing plan when $y = C_2$ and then reserve a non-exclusive data plan when $x = C_3 - C_2$ if the exclusive computing plan is still on-going. Let t_m denote the time when the user reserves an exclusive computing plan. Let t_d denote the time when the user reserves a non-exclusive data plan. Let a_d and b_d denote the communication volume and computing workload of the task arriving at t_d, $(a_d, b_d) = \{(a_i, b_i) | (a_i, b_i, t_i) \in \mathbf{S}, t_i = t_d\}$. Let E_1, E_2 and E_2 denote the expiration time of the latest data, computing and combo plans reserved before t_d. Then x and y at t_d and t_m can be expressed as $x = \sum_{i:\max(E_1,E_3,t_d-T) \leq t_i < t_d} a_i p_1$ and $y = \sum_{i:\max(E_2,E_3,t_m-T) \leq t_i < t_m} b_i p_2$. Without further notification, we assume x and y are calculated at t_d and t_m in this proof. Let t_s denote the arrival time of the first task whose cost is counted in either x or y, $t_s = \min\{t_i | t_i \geq \min\big(\max(E_1, E_3, t_d - T), \max(E_2, E_3, t_m - T)\big)\}$. We further define the corresponding validation period as $[t_s, t_d+T)$. The validation period starts from the arrival time of the first task and ends at the expiration time of the non-exclusive computing plan. The worst case w is as follow. After the user reserves an exclusive computing plan, the arrival of computing workloads stops. Within the same reservation period, $\max(E_2, E_3, t_m - T) = \max(E_1, E_3, t_d - T)$, the user reserves a non-exclusive data plan, the arrival of task stops after that. The proof of the worst case is similar to Lemma 3.

The cost under SOR scheme in the worst case in this validation period is

$$C_{SOR,w} = C_1 + x + C_2 + y. \tag{22}$$

The optimal offline cost in the worst case in this validation period is

$$C_{OPT,w} = \min(C_2 + x + a_d p_1, C_3). \tag{23}$$

Consider when $C_{OPT,w} = C_3$. Then the competitive ratio is

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_1 + x + C_2 + y}{C_3} = 1 + \frac{C_1 + C_2}{C_3}. \tag{24}$$

Consider when $C_{OPT.w} = (C_2 + x + a_d p_1)$. Then the competitive ratio is

$$\frac{C_{SOR,w}}{C_{OPT,w}} = \frac{C_1 + x + C_2 + y}{C_2 + x + a_d p_1} \leq 1 + \frac{C_1 + C_2}{C_3}, \tag{25}$$

as $y = C_2$ and $x = C_3 - C_2$. We notice that this competitive ratio is for a single reservation of combo plan in a single validation period. However, as long as we have $x < C_1$ and $y \geq C_2$ for every validation period in Case 2 or Case 4, the overall competitive ratio for SOR scheme is still the same. □

As the user definitely reserves a plan before $x \geq C_1$ and $y \geq C_2$, we have the following propositions.

Proposition 1. *In Case 1, the competitive ratio of SOR scheme is less than or equal to* $1 + \frac{C_1+C_2}{C_3}$.

Proof. The competitive ratio of SOR scheme is less than or equal to $1 + \frac{C_1+C_2}{C_3}$ in all situations according to Lemma 1, Lemma 2 and Lemma 4. □

Proposition 2. *In Case 2, the competitive ratio of SOR scheme is* $1 + \frac{C_1+C_2}{C_3}$.

Proof. The competitive ratio of SOR scheme is less than or equal to $1 + \frac{C_1+C_2}{C_3}$ in all situations according to Lemma 1, Lemma 3 and Lemma 5. $\square$

Proposition 3. *In Case 3, the competitive ratio of SOR scheme is* $1 + \frac{C_1+C_2}{C_3}$.

Proof. The competitive ratio of SOR scheme is less than or equal to $1 + \frac{C_1+C_2}{C_3}$ in all situations according to Lemma 1, Lemma 3 and Lemma 4. $\square$

Proposition 4. *In Case 4, the competitive ratio of SOR scheme is* $1 + \frac{C_1+C_2}{C_3}$.

Proof. The competitive ratio of SOR scheme is less than or equal to $1 + \frac{C_1+C_2}{C_3}$ in all situations according to Lemma 1, Lemma 2 and Lemma 5. $\square$

Theorem 1. *The competitive ratio of SOR scheme with fine-grained PAYG charging is* $1 + \frac{C_1+C_2}{C_3}$.

Proof. According to Propositions 1, 2, 3, and 4, the competitive ratio of SOR scheme is $1 + \frac{C_1+C_2}{C_3}$ for all Cases. $\square$

2 Proof of Theorem 2

Theorem 2. *The competitive ratio of SOR scheme is minimum among all deterministic online algorithms.*

Proof. We prove this theorem by showing that no online deterministic theorems can achieve a better competitive ratio than $1 + \frac{C_1+C_2}{C_3}$ in Case 2 when $x \geq C_1$ and $y < C_2$. In this specific situation, we will first show that reserving computing and data plans separately gives the best competitive ratio.

For the scheme which only satisfies the task by PAYG payments, its competitive ratio is unbounded as there can be infinite numbers of task arrivals during the same reservation period. This also applies for schemes only reserves the computing plan or only reserves the data plan. Once the user purchases a combo plan, reserving another plan during the reservation period of the combo plan gives a higher cost than reserving another plan after the reservation period of the combo plan. Therefore, only two options are left. The first one is to reserve a combo plan and the second one is to reserve computing and data plans separately.

According to Lemma 2, the cost to reserve a combo plan in the worst case is

$$C_{COM} = C_3 + x + y. \tag{26}$$

The optimal offline cost in the worst case according to Lemma 2 is

$$C_{OPT} = C_1 + y + b_c p_2, \tag{27}$$

when $C_1 + y \leq C_3$. The competitive ratio under this scheme is

$$\frac{C_{COM}}{C_{OPT}} = \frac{C_3 + x + y}{C_1 + y + b_c p_2}. \tag{28}$$

This expression gets to its maximum value when $y = 0$ and $b_c = 0$, which is $\frac{C_3+x}{C_1}$. To get the best competitive ratio under this situation, we needs set x to its minimum value which means to reserve the combo plan when $x = C_1$. Then the best competitive ratio under the worst situation is

$$\max\big(\min(\frac{C_{COM}}{C_{OPT}})\big) = \frac{C_3 + C_1}{C_1}. \tag{29}$$

As $C_3 \geq \sqrt{C_1(C_1+C_2)}$, $\frac{C_3+C_1}{C_1} \geq 1 + \frac{C_1+C_2}{C_3}$. Therefore, reserving a combo plan has a higher competitive ratio than reserving computing and data plans separately.

At this point, we already prove that reserving computing and data plans separately is the best option in this situation. Then we will prove that the SOR scheme gives the best competitive ratio in this option.

The cost to reserve computing and data plans separately in the worst case is

$$C_{SEP} = C_1 + x + C_2 + y. \tag{30}$$

The optimal offline cost in the worst case according to Lemma 3 is

$$C_{OPT} = C_1 + y + b_m p_2, \tag{31}$$

when $C_1 + y \leq C_3$. The competitive ratio under this scheme is

$$\frac{C_{SEP}}{C_{OPT}} = \frac{C_1 + x + C_2 + y}{C_1 + y + b_m p_2}. \tag{32}$$

In this option, the user can set values for both x and y as it reserves computing and data plans separately. To minimize this expression, we need to set x to its minimum (C_1) and y to its maximum ($C_3 - C_1$). Then the minimum competitive ratio is

$$\min\left(\frac{C_{SEP}}{C_{OPT}}\right) = \frac{C_1 + C_2 + C_3}{C_3} = \frac{C_{SOR}}{C_{OPT}}. \tag{33}$$

$b_m p_2$ is ignored in the above expression as it is arbitrarily small. $\square$