Empirical Performance Investigation of a Büchi Complementation Construction

Daniel Weibel

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Abstract

This will be the abstract.

Acknowledgements

Contents

Chapter 1

Introduction

At the beginning of the 1960s, a Swiss logician named Julius Richard Büchi at Michigan University was looking for a way to prove the decidability of the satisfiability of monadic second order logic with one successor (S1S). Büchi applied a trick that truly founded a new paradigm in the application of logic to theoretical computer science. He thought of interpretations of a S1S formula as infinitly long words of a formal language and designed a type of finite state automaton that accepts such a word if and only if the interpretation it represents satisfies the formula. After proving that every S1S formula can be translated to such an automaton and vice versa (Büchi's Theorem), the satisfiabilty problem of an S1S formula could be reduced to testing the non-emptiness of the corresponding automaton.

This special type of finite state automaton was later called Büchi automaton.

1.1 Context

1.1.1 Büchi Automata and Büchi Complementation

Büchi automata are finite state automata that process words of infinite length, so called ω-words. If Σ is the alphabet of a Büchi automaton, then the set of all the possible ω-words that can be generated from this alphabet is denoted by Σ^ω. A word $\alpha \in \Sigma^\omega$ is accepted by a Büchi automaton if it results in at least one run that contain infinitely many occurrences of at least one accepting state. A run of a Büchi automaton on a word is an infinite sequence of states. Deterministic Büchi automata have at most one run for each word in Σ^ω, whereas non-deterministic Büchi automata may have multiple runs for a word.

The complement of a Büchi automaton A is another Büchi automaton[1] denoted by $\overline{A}$. Both, A and $\overline{A}$, share the same alphabet Σ. Regarding a word $\alpha \in \Sigma^\omega$, the relation between an automaton and its complement is as follows:

$$\alpha \text{ accepted by } A \iff \alpha \text{ not accepted by } \overline{A}$$

That is, all the words of Σ^ω that are *accepted* by an automaton are *rejected* by its complement, and all the words of Σ^ω that are *rejected* by an automaton are *accepted* by its complement. In other words, there is no single word of Σ^ω that is accepted or rejected by *both* of an automaton and its complement.

A Büchi complementation construction is an algorithm that, given a Büchi automaton, creates the complement of this Büchi automaton. The difficulty of this operation depends on whether the input automaton is determinstic or non-deterministic. The complementation of deterministic Büchi automata is "easy" and can be done in polynomial time and linear space [17]. The complementation of non-deterministic Büchi automata, however, is very complex. The understanding and reduction of its complexity is a domain of active research and lies at the centre of this thesis.

Consequently, when in the following we talk about "Büchi complementation", we specifically mean the complementation of *non-determinstic* Büchi automata. The main problem with the complexity of Büchi complementation is the so-called state growth or state complexity (sometimes also called state blow-up or state explosion). This is the number of states of the output automaton in relation to the number of states of the input automaton. In simple words, Büchi complementation constructions produce complements that may be very, very large.

This inhibits the practial applicaion of Büchi complementation, because in this case the limited computing and time resources may not be high enough to accommodate for this high complexity. In the following subsections we highlight an important application that Büchi complementation has in practice, and thereby motivate the research on Büchi complementation and of this thesis.

[1] Büchi automata are closed under complementation. This has been proved by Büchi [4], who, to this end, described the first Büchi complementation construction in history.

1.1.2 Applications of Büchi Complementation

Language Containment of ω-Regular Languages

Büchi complementation is used for testing language containment of ω-regular languages. The ω-regular languages are the class of formal languages that is equivalent to non-deterministic Büchi automata[2]. At this point, we briefly describe the language containment in general, before in turn describing an application of the language containment problem in the next subsection.

Given two ω-regular languages L_1 and L_2 over alphabet Σ^ω the language containment problem consists in testing whether $L_1 \subseteq L_2$. This is true if all the words of L_1 are also in L_2, and false if L_1 contains at least one word that is not in L_2. The way this problem is algorithmically solved is by testing $L_1 \cap \overline{L_2} = \varnothing$. Here, $\overline{L_2}$ denotes the complement language of L_2, which means $\overline{L_2}$ contains all the words of Σ^ω that are *not* in L_2. The steps for testing $L_1 \cap \overline{L_2} = \varnothing$ are the following:

- Create the complement language $\overline{L_2}$ of L_2
- Create the intersection language $L_{1,\overline{2}}$ of L_1 and $\overline{L_2}$
- Test whether $L_{1,\overline{2}}$ is empty (that is, contains no words at all)

Thus, the language containment problem is reduced to three operations on languages, *complementation*, *intersection*, and *emptiness testing*. The common way to work with formal languages is not to handle the languages themselves, but more compact structures that represent them, such as automata. In the case of ω-regular languages, these are non-deterministic Büchi automata.

For solving $L_1 \subseteq L_2$, one thus works with two Büchi automata A_1 and A_2 that represent L_1 and L_2, respectively. The problem then becomes $L(A_1) \subseteq L(A_2)$, and equivalently, $L(A_1) \cap \overline{L(A_2)} = \varnothing$. This is automata-theoretically solved as $\mathsf{empty}(A_1 \cap \overline{A_2})$, which includes the three following steps:

- Construct the complement automaton $\overline{A_2}$ of A_2
- Construct the intersection automaton $A_{1,\overline{2}}$ of A_1 and A_2
- Test whether $A_{1,\overline{2}}$ is empty (that is, accepts no words at all)

If the final emptiness test on automaton $A_{1,\overline{2}}$ is true, then $L_1 \subseteq L_2$ is true, and if the emptiness test is false, then $L_1 \subseteq L_2$ is false. In this way, the language containment problem of ω-regular languages is reduced to three operations on non-deterministic Büchi automata, *complementation*, *intersection*, and *emptiness testing*, the first of them being the subject of this thesis.

Automata-Theoretic Model Checking via Language Containment

In the last subsection, we have seen that Büchi complementation is used for testing language containment of ω-regular languages. In this subsection, we will see what in turn language containment of ω-regular languages is used for. To this end, we describe one important application of it, namely the language containment approach to automata-theoretic model checking. In the following, we first describe basic working of this technique in general, and then point out the significance that Büchi complementation bears for it.

Basics

The language containment approach to automata-theoretic model checking is an approach to automata-theoretic model checking, which is an approach to model checking, which in turn is an approch to formal verification [**?**]. Figure 1.1 shows the branch of the family of formal verification techniques that contains the language containment approach to automata-theoretic model checking.

Formal verification is the use of mathematical techniques for proving the correctness of a system (software of hardware) with respect to a specified property [**?**]. A typical example is to verify that a program is

[2]Note that deterministic Büchi automata have a lower expressivity than non-deterministic Büchi automata, and are equivalent to only a subset of the ω-regular languages.

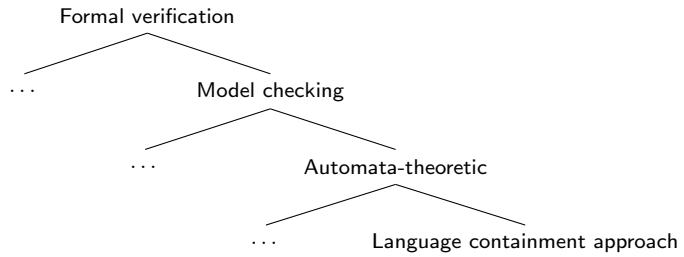

Figure 1.1: Branch of the family of formal verification techniques that contains the language containment approach to automata-theoretic model checking.

deadlock-free (in which case the property would be "deadlock-freeness"). In general, formal verification techniques consist of the following three parts [**?**]:

1. A framework for modelling the system to verify
2. A framework for specifying the property to be verified
3. A verification method for testing whether the system satisfies the property

For the language containment approach to automata-theoretic model checking, the frameworks for points 1 and 2 are Büchi automata representing ω-regular languages. The verification method (point 3) is testing language containment of the system automaton's language in the property automaton's language. In some more detail, this works as follows. [49][**?**]

The system s to verify is modelled as a Büchi automaton, say S. This Büchi automaton represents an ω-regular language $L(S)$, and each word of $L(S)$ represents in turn a possible *computation trace* of the system. A computation trace is an infinite[3] sequence of "situations" that the system is in at any point in time. Such a situation consists of a finite amount of information of, for example, the values of variables, registers, or buffers. The observation that such a trace can be represented as a word of an ω-regular langauges comes from the fact that it can be represented intuitively as a linear Kripke structure (which in turn is an interpretation for a temporal logic formula that can also be used to represent computations), which in turn can be represented by a word of a language whose alphabet is ranges over the powerset of the atomic propositions of the Kripke strucutre. A work that explains these intimate relations between computation, temporal logic, formal languages, and automata in more detail is [49]. In simple words, the language $L(S)$, represented by the system automaton S, represents everything that the system *can* do.

Similarly, a property p to be verified is represented as a Büchi automaton, say P, which represents the ω-regular language $L(P)$, whose words represent computation traces. These computation traces are all the computations of a system like s that satisfy the property p. If for example p is "deadlock-freeness", then the words of $L(P)$ represent all the possible computation traces that do *not* contain a deadlock. In this way, the language $L(P)$ represents everything that the system is *allowed* to do.

The verification step is finally done by testing $L(S) \subseteq L(P)$. If this is true, then everything that the system *can* do is *allowed* to do, and the system satisfies the property p. If the language containment test is false, then the system *can* do a computation that is *not allowed* to do, and the system does not satisify the property p.

Summarising, the language containment approach to automata-theoretic model checking requires language containment of ω-regular languages, which, as we have seen, requires Büchi complementation. In the following, we will highlight the particular importance of Büchi complementation for this type of formal verification.

[3]The infinity of computation traces suggests that this type of formal verification (and model checking in general) is used for systems that are not expected to terminate and may run indefinitely. This type of systems is called *reactive* systems. They contrast with systems that are expected to terminate and produce a result. For this latter type of systems other formal verification techniques than model checking are used. See for example [**?**] and [**?**] for works that cover the formal verification of both types of systems.

1.1.3 Significance of Büchi Complementation

The complexity of Büchi complementation makes the just described model checking approach nearly inapplicable in practice [**?**]. According to [**?**], there are so far no tools that include this approach. This is unfortunate, because the other Büchi operations for language containment, intersection and emptiness testing, have highly efficient solutions [**?**] (cf. [49]), and thus Büchi complementation is the only bottleneck. Existing practical applications are thus forced to circumvent the need for Büchi complementation. This is possible, has however certain disadvantages as we will see in the following.

One way to circumvent the complementation of non-deterministic Büchi automata is to specify the property as a deterministic Büchi automaton [**?**][**?**]. As we have mentioned, the complementation of deterministic Büchi automata has an efficient solution. The disadvantage of this approach is, however, that the property automaton may become exponentially larger, and that it is generally more complicated and less intuitive to represent a language as a deterministic automaton [**?**].

Another way is to use a more classical approach model checking, that leads us back to the fundamental model checking question. In this approach, the system is represented by a Kripke structure K, and the property is specified as a formula φ of a temporal logic, typically linear temporal logic (LTL). The verification step then consists in checking whether K is a *model* of φ. This is also expressed as "K satisfies φ", written as $K \models \varphi$, which means very informally that the interpretation K makes the formula φ true. It is from this fundamental question that *model checking* has its name.

The model checking test can be done automata-theoretically [**?**] as follows (in Figure 1.1, this would be a sibling to the language containment approach). First, the Kripke structure K is translated to a Büchi automaton A_K, then the property formula φ is negated and translated to the Büchi automaton $A_{\neg\varphi}$, and finally $\mathsf{empty}(A_K \cap A_{\neg\varphi})$ is tested. This effectively tests $L(K) \subseteq L(\varphi)$, which is equivalent to $K \models \varphi$. By negating the formula φ, the complementation of the property, that is required for the automata-theoretic verification step, is pushed off from Büchi automata to temporal logic formulas, in which case it is trivial. This approach of model checking is used by the SPIN model checker [**?**]. The disadvantage of this approach is that the typically used temporal logic, LTL, is less expressive than Büchi automata, Hence this approach does not allow to specify many properties that could be expressed by Büchi automata. It has even been stated that the expressivity of LTL is unsufficient for industrial verification applications [**?**].

For more information on model checking, as well as other formal verification techniques, we refer to the following works: [**?**][**?**][**?**].

As can be seen from these elaborations, having efficient procedures for Büchi complementation would be of great practical value. Even though handling the "worst-cases" will forever be unefficient,

1.2 Motivation

In the previous section we have seen that Büchi complementation has a high complexity, and that it would be of great practical value to better understand this complexity. In this section, we advocate a way to study this complexity that has not received much attention in the past, namely its empirical rather than theoretical investigation. In the following, we first present the traditional theoretical way of analysing the complexity of Büchi complementation constructions. Then, we will motivate the need for empirical investigations, including a review of what has been done so far in this direction.

1.2.1 Theoretical Investigation of Worst-Case Performance

The traditional performance measure for Büchi complementation constructions is their *worst-case state growth*[4]. This is the maximum number of states the construction *can* generate, in relation to the number of states of the input automaton.

[4] As mentioned previously, also known as state complexity, state blow-up, or state explosion.

For example, the initial complementation construction by Büchi (1962) [4] has a worst-case state growth of $2^{2^{O(n)}}$, where n is the number of states of the input automaton. At this point, two short comments. First, the state growth is often not given as an exact function, but uses the big-O notation. Second, for notating state growths, we will consistenly use the variable n, whose meaning is the number of states of the input automaton. This means, for example, that for an input automaton with 8 states, the maximum number of states that the output automaton of Büchi's construction can have is $1.16 \times 10^{7}7$ (if assuming the concrete function 2^{2^n}).

Different constructions exhibit different worst-case state growths, and one of the main objectives of construction creators is to reduce this number. For example, the much more recent construction by Schewe (2009) [35] has a worst-case state growth as low as $(0.76n)^n + n^2$. Given an input automaton with 8 states, the maximum number of states of the output automaton is approximately 119.5 million.

A related objective of research is the quest for the theoretical worst-case state growth of Büchi complementation *per se*. A first result of $n!$ has been proposed in 1988 by Michel [20]. He proved that there exists a family of automata whose complement *cannot* have less than $n!$ states (these automata are known as Michel automata, and we will use them as part of the test data for our experiments). This proves a *lower bound* for the fundamental worst-case complexity of Büchi complementation, as it is not known whether the Michel automata are the real worst cases, or if there are even worse cases. Indeed, in 2007, Yan [55] proved a new higher lower bound of $(0.76n)^n$ (Michel's $n!$ corresponds to approximately $(0.36n)^n$). The worst-case state growth of a concrete construction naturally serves as an *upper bound* to a known lower bound. Given Schewe's number $(0.76n)^n + n^2$, the lower bound of $(0.76n)^n$ by Yan is regarded as "sharp", as the gap between the lower and upper bound is very narrow, and consequently, the lower bound canot rise much anymore.

Many construction developers aim at bringing the worst-case state growth of their construction close to the currently known lower bound. It goes so far that a construction matching this lower bound is regarded as "optimal".

1.2.2 Need for Empirical Investigation of Actual Performance

Worst-Case State Growth as Main Performance Measure

Since the introduction of Büchi automata in 1962, many different Büchi complementation constructions have been proposed (see our review in Section **??**). The main performance measure for these constructions has usually been their so-called *worst-case state growth* or *worst-case state complexity* (in the following, we will use these two terms interchangeably[5]).

State growth basically denotes the number of states of the output automaton in relation to the number of states of the input automaton to a complementation construction. Each automaton has thus its specific own state growth with each construction. The worst-case state growth of a construction results from a theoretical worst-case automaton, which has a higher state growth than any other automaton. In different words, the worst-case state growth is the maximum number of states that a concstruction *can* generate.

The state growth and the resulting time and space complexity is the biggest issue of Büchi complementation. The worst-case state growth seems to "distill" this issue to a single number which is practical to use as a concise performance measure and to compare different constructions with each other. Thus, much of the research effort in Büchi complementation construction has gone into reducing the worst-case state growth.

For example, the complementation construction that has been described in 1962 by Büchi himself [4] has a doubly exponential worst-case state growth of $2^{2^{O(n)}}$, where n is the number of states of the input automaton[6]. Note that worst-case state growths are often not given as exact functions, but include the big O notation. A later construction from 1987 by Sistla, Vardi, and Wolper [37] reduces this worst-case

[5] Further terms used in the literature are state blow-up or state explosion.

[6] In the following, we will always notate state growths as a function of n, and n will always be the number of states of the input automaton.

state complexity to a singly exponential funtion of $O\left(2^{4n^2}\right)$. More recently, a construction from 2006 by Friedgut, Kupferman and Vardi from 2006 [7] has a worst-caste state complexity of only $(0.96n)^n$.

In parallel to the quest for complementation constructions with a low worst-case state complexity, there is a quest for finding the worst-caste state complexity of Büchi complementation itself. This is done by showing a theoretical minimum state growth of certain automata which even an ideal complementation construction could not undermatch. In this way, one proves a *lower bound* for the worst-case complexity of Büchi complementation (it is still possible that there exist automata with an even higher theoretical minimum state growth). In 1988 Michel proved such a lower bound of $n!$ [20] (in a different notation approximately $(0.36n)^n$). In 2008, Yan proved a new lower bound of $(0.76n)^n$ [55]. This result is still valid at the time of this writing.

A lower bound of $(0.76n)^n$ means that no complementation construction can ever have a worst-case state growth lower than $(0.76n)^n$. Consequently, a construction that achieves this worst-case state growth is commonly regarded as "optimal".

Importance of Empirical Performance Investigations

Worst-case state growths are certainly interesting from a theoretical point of view. However, they can be seen as only one aspect of the performance of Büchi complementation constructions. The reality is certainly much more complex. If, for example, one is interested in practical questions (like how does a construction peform on a concrete automaton?), then worst-case state growths are a poor guide to the concrete performance of the construction [42].

For example, if we have a concrete automaton, then the worst-case state growth does not reveal anything about how big the complement of this concrete automaton will be. It is not even clear whether in a concrete case a construction with a lower worst-case state growth produces a smaller complement than a construction with a higher worst-case state growth.

Worst-case state growths are mainly meaningful for the worst cases. However, if these worst cases would occur, then all constructions would be far from practical anyway [**?**]. If we take for example Friedgut, Kupferman, and Vardi's construction [7] with a worst-case complexity of $(0.96n)^n$, Sistla, Vardi, and Wolper's construction with a worst-case complexity of $O\left(2^{4n^2}\right)$, and a "worst-case" automata of, say, 15 states, then the complements of the two constructions would have $2.73 \times 10^1 7$ (23.7 million billion) and $8.45 \times 10^2 70$ states, respectively. While this difference is still huge, in practice this does not matter much, because both cases are most likely anyway not feasible in practice.

Hence, with regard to the practical application of Büch complementation (see Section 1.1), we believe that it is important to investigate the *actual* performance of Büchi complementation constructions on *concrete* input automata. This is done by *empirical* performance investigations that includes an implementation of the investigated construction, and a set of test data.

Review of Empirical Performance Investigations

There have bee not many empirical investigations of Büchi complementation constructions, and most of the time they have been used as a proof of concept that a given construction *can* be applied in practice. Tasiran et al. (1995) [**?**] created an efficient implementation of Safra's complementation construction and used it with the HSIS formal verification tool [**?**] for automata-theoretic model checking tasks. They could easily complement property automata with some hundreds of states. However, they do not statistically evaluate the results.

In 2003, Gurumurthy et al. [**?**] implemented Friedgut, Kupferman, and Vardi's complementation construction [16] with different optimisations as part of the tool Wring [**?**]. They complemented 1000 small automata, that were generated by translation from LTL formulas, and made basic statistical evaluation comparing the effect of the different optimisations on the performance.

In 2006, Althoff et al. [2] implemented Safra's [33] and Muller and Schupp's [24] determinisation constructions (that can be used as the base for complementation, see Section **??**) in a tool called OmegaDet,

determinsed the Michel automata [20] with 2 to 6 states, and compared the number of states of the determinised automata.

In 2008, Tsay et al. [45] did a comparative experiment with the GOALtool [44][45][46][43] including the constructions by Safra [33], Piterman [28], Thomas [41] (called WAPA[7] in GOAL), and Kupferman and Vardi[16] (called WAA[8] in GOAL). These constructions are pre-implemented in the publicly available GOALtool[9]. As the test data, they used 300 Büchi automata with an average size of 5.4 that have been translated from LTL formulas. They evaluated the number of states and transitions of the complements, and the execution times.

In 2009, Kamarkar and Chakraborty [**?**] proposed an improvement of Schewe's complementation construction [35] and implemented it, as well as Schewe's original construction, on top of the model checker NuSMV [**?**][**?**]. They ran the constructions on 12 test automata and compared the sizes of the output automata with each other. Furthermore, by the means of the GOALtool, they evaluated the constructions by Friedgut, Kupferman, and Vardi [16] (called WAA in GOAL) and Piterman [28] with the same test data.

In 2010, Tsai et al. [42] carried out the first "larger-scale" comparative experiment by using the GOALtool (paper entitled "State of Büchi Compelentation"). They compared the constructions by Piterman [28], Schewe [35] (called *Rank* in GOAL), and Vardi and Wilke [52] (called *Slice* in GOAL), with different sets of optimsiations. As the test data, they use 11,000 randomly generated automata with 15 states and an alphabet size of 2, that are split into 110 classes of 11 transition densities and 10 acceptancd densities.

In the same year, 2010, Breuers [**?**] proposed an improvment for the construction by Sistla, Vardi, and Wolper [37], implemented it, and tested it on randomly generated test sets of "easy", "medium", and "hard" automata of sizes 5, 10, and 15, and an alphabet size of 2. The difficulty classes of the test automat are based on different combinations of transition and acceptance densities. The generation of the test automata is inspired by the experiments by Tsai et al. [42].

In 2012, Breuers et al. [3] implemented their improvement of Sistla, Vardi, and Wolper's construction [37] as the publicly available tool Alekto[10], and tested it on the test set 11,000 automata used by Tsai et al. [42]. They compare the performance of their construction on this test set with the results of the three constructions evaluated and reported in [42]

Regarding the state complexity of Büchi complementation constructions, only the worst-case state growths have been investigated. However, they are a poor guide to actual peformance of constructions [42]. Need for empirical complexity investigations to see the *actual* performance of complementation constructions.

1.3 Aim and Scope

Aim: empirical performance investigation of a specific Büchi complementaiton construction, comparison with other constructions

Scope: two test sets, relatively small automata, no real world or "typical" examples,

1.4 Overview

[7] Via **W**eak **A**lternating **P**arity **A**utomaton.
[8] Via Weak Alternating Automaton.
[9] http://goal.im.ntu.edu.tw/wiki/doku.php
[10] http://www.automata.rwth-aachen.de/research/Alekto/

Appendix A

Plugin Installation and Usage

Since between the 2014–08–08 and 2014–11–17 releases of GOAL certain parts of the plugin interfaces have changed, and we adapted our plugin accordingly, the currently maintained version of the plugin works only with GOAL versions 2014-11-17 or newer. It is thus essential for any GOAL user to update to this version in order to use our plugin.

Appendix B

Median Complement Sizes of the GOAL Test Set

Bla bla bla

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	269	308	254	236	238	297	266	156	207	68
1.2	960	1,407	1,479	2,150	1,152	1,090	942	1,206	718	104
1.4	3,426	2,915	2,752	3,393	2,693	3,265	2,263	2,425	1,844	154
1.6	3,799	3,698	4,901	3,926	3,960	3,655	2,580	1,905	2,124	155
1.8	3,375	3,169	3,420	3,967	3,943	3,132	2,246	1,144	971	114
2.0	1,906	2,261	2,383	2,884	2,354	2,096	1,169	932	568	98
2.2	1,467	1,633	1,795	1,942	1,611	1,640	569	499	330	78
2.4	924	1,232	1,319	1,317	1,056	886	514	314	182	59
2.6	625	763	880	945	828	684	316	175	132	44
2.8	483	584	836	690	575	395	240	151	103	41
3.0	319	450	557	523	367	313	155	116	84	32

(a) Fribourg

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	269	308	254	236	238	297	266	156	207	68
1.2	960	1,407	1,479	2,150	1,152	1,090	942	1,206	718	104
1.4	3,426	2,915	2,752	3,393	2,693	3,265	2,263	2,425	1,844	154
1.6	3,799	3,698	4,901	3,926	3,960	3,655	2,580	1,905	2,124	155
1.8	3,375	3,169	3,420	3,967	3,943	3,093	2,246	1,144	971	114
2.0	1,906	2,184	2,383	2,818	2,354	1,989	1,127	885	568	97
2.2	1,410	1,561	1,639	1,884	1,609	1,588	496	464	284	78
2.4	884	1,200	1,234	1,184	939	806	373	256	165	55
2.6	575	731	815	860	751	575	246	162	114	43
2.8	431	530	672	466	371	274	174	120	85	36
3.0	232	325	344	360	269	169	91	85	53	27

(b) Fribourg+R2C

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	390	438	434	324	328	459	337	204	227	40
1.2	1,576	2,394	2,505	2,996	1,613	1,551	1,166	1,542	1,002	58
1.4	5,007	4,336	4,652	4,877	3,458	3,956	3,169	3,380	1,868	86
1.6	5,067	5,032	6,444	4,868	4,575	3,864	3,211	1,731	1,892	85
1.8	4,016	3,701	3,647	4,523	3,548	3,009	1,808	451	336	62
2.0	1,663	2,276	2,676	3,035	1,925	1,932	464	307	150	54
2.2	989	1,514	1,621	1,826	1,121	846	155	127	93	45
2.4	560	821	919	771	529	267	133	87	55	32
2.6	388	519	524	441	259	219	84	50	41	26
2.8	311	317	396	242	165	95	64	44	33	22
3.0	173	224	211	169	102	72	41	34	27	18

(c) Fribourg+R2C+C

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	225	223	195	181	187	199	189	124	161	68
1.2	731	971	946	1,071	629	562	488	568	388	104
1.4	2,228	1,701	1,543	1,732	1,241	1,287	945	944	727	154
1.6	2,489	2,263	2,331	2,133	1,777	1,443	964	757	889	155
1.8	2,381	2,027	2,009	2,075	1,618	1,243	1,005	592	515	114
2.0	1,390	1,569	1,416	1,573	1,093	1,008	594	464	330	98
2.2	1,118	1,197	1,150	1,151	879	809	317	330	241	78
2.4	712	885	836	809	580	535	316	231	145	59
2.6	498	569	601	627	497	412	217	137	113	44
2.8	391	455	578	456	374	263	173	119	90	41
3.0	258	350	392	354	253	208	119	97	74	32

(d) Fribourg+M1

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	215	213	189	174	175	192	186	121	156	68
1.2	712	914	913	1,075	619	563	526	620	416	104
1.4	2,075	1,620	1,503	1,650	1,254	1,339	1,003	1,006	848	154
1.6	2,344	2,062	2,340	2,016	1,755	1,520	1,053	858	986	155
1.8	2,205	1,873	1,920	2,040	1,689	1,315	1,080	664	598	114
2.0	1,290	1,485	1,405	1,522	1,134	1,044	652	531	392	98
2.2	1,023	1,119	1,092	1,127	868	875	376	359	262	78
2.4	674	849	790	807	617	544	355	251	156	59
2.6	478	549	594	597	510	431	231	147	116	44
2.8	370	439	559	455	382	283	182	124	93	41
3.0	249	341	388	348	260	225	123	101	77	32

(e) Fribourg+M1+M2

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	225	223	195	181	187	199	189	124	161	68
1.2	731	971	946	1,071	629	562	488	568	388	104
1.4	2,228	1,701	1,543	1,732	1,241	1,287	945	944	727	154
1.6	2,489	2,263	2,331	2,133	1,777	1,443	964	757	889	155
1.8	2,381	2,027	2,009	2,075	1,618	1,215	1,005	592	515	114
2.0	1,390	1,513	1,416	1,542	1,093	1,003	594	441	330	97
2.2	1,019	1,156	1,064	1,104	859	785	304	303	221	78
2.4	672	867	789	772	544	478	269	191	139	55
2.6	466	542	572	568	452	348	183	129	99	43
2.8	368	407	480	337	260	197	129	96	75	36
3.0	201	261	266	272	199	136	83	74	50	27

(f) Fribourg+M1+R2C

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	329	303	279	240	229	288	230	157	160	40
1.2	988	1,392	1,356	1,352	751	741	608	704	516	58
1.4	2,939	2,581	2,066	2,190	1,351	1,622	1,132	1,261	932	86
1.6	3,150	2,900	2,842	2,218	1,885	1,563	1,177	821	896	85
1.8	2,782	2,485	2,047	2,180	1,625	1,269	855	395	309	62
2.0	1,338	1,638	1,544	1,566	979	957	349	261	147	54
2.2	838	1,125	993	1,027	667	521	153	125	93	45
2.4	494	700	624	524	296	214	126	87	55	32
2.6	327	434	383	334	212	163	82	50	41	26
2.8	283	273	305	202	144	95	60	44	33	22
3.0	164	200	173	142	92	72	41	34	27	18

(g) Fribourg+M1+R2C+C

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	126	118	97	60	51	52	62	36	48	30
1.2	432	517	345	262	160	126	92	120	109	40
1.4	1,044	331	133	89	45	22	19	31	27	20
1.6	358	24	11	5	4	6	5	3	3	4
1.8	19	5	1	1	1	1	1	1	1	1
2.0	1	1	1	1	1	1	1	1	1	1
2.2	1	1	1	1	1	1	1	1	1	1
2.4	1	1	1	1	1	1	1	1	1	1
2.6	1	1	1	1	1	1	1	1	1	1
2.8	1	1	1	1	1	1	1	1	1	1
3.0	1	1	1	1	1	1	1	1	1	1

(h) Fribourg+R

Figure B.1: Median complement sizes of the 10,939 effective samples of the internal tests on the GOAL test set. The rows (1.0 to 3.0) are the transition densities, and the columns (0.1 to 1.0) are the acceptance densities.

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	130	117	109	77	69	61	56	40	40	29
1.2	387	456	352	281	155	136	101	105	75	45
1.4	822	683	394	376	230	204	151	120	105	63
1.6	890	594	458	321	237	178	134	114	113	61
1.8	624	507	324	275	196	136	110	92	89	41
2.0	362	286	211	176	117	103	79	64	59	34
2.2	248	222	124	116	82	73	56	52	50	28
2.4	147	145	114	87	56	48	43	39	35	19
2.6	115	117	67	61	47	42	32	29	29	15
2.8	95	71	52	45	38	29	27	25	23	13
3.0	59	60	47	35	32	27	22	21	20	10

(a) Piterman+EQ+RO

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	171	174	166	124	118	117	100	67	84	35
1.2	622	833	803	877	529	398	320	372	215	53
1.4	2,086	1,618	1,367	1,676	1,065	967	664	682	494	78
1.6	2,465	2,073	2,182	1,959	1,518	1,259	767	545	623	78
1.8	2,310	1,963	1,950	1,988	1,485	1,095	746	418	346	57
2.0	1,318	1,482	1,393	1,461	981	871	434	338	228	50
2.2	1,068	1,145	1,085	1,067	772	747	263	235	158	40
2.4	689	838	809	751	524	466	240	159	93	30
2.6	469	531	555	565	437	360	169	94	71	23
2.8	369	421	536	405	329	224	130	81	58	21
3.0	244	327	360	322	219	176	85	64	49	16

(b) Slice+P+RO+MADJ+EG

Figure B.2: Median complement sizes of the 10,998 effective samples of the external tests without the Rank construction. The rows (1.0 to 3.0) are the transition densities, and the columns (0.1 to 1.0) are the acceptance densities.

Appendix C

Execution Times

Construction	Mean	Min.	P25	Median	P75	Max.	Total	≈ hours
Fribourg	8.5	2.5	3.3	4.9	7.3	586.0	93,351.2	259
Fribourg+R2C	6.6	2.2	2.9	4.2	6.4	219.7	72,545.7	202
Fribourg+R2C+C	8.5	2.2	2.6	3.5	6.4	582.9	93,396.2	259
Fribourg+M1	4.9	2.5	3.2	4.1	5.9	55.1	54,061.3	150
Fribourg+M1+M2	4.6	2.2	2.9	3.8	5.1	38.4	49,848.0	138
Fribourg+M1+R2C	4.4	2.2	2.8	3.6	5.3	42.5	48,572.0	135
Fribourg+M1+R2C+C	5.6	2.5	3.2	4.0	6.5	147.4	60,918.9	169
Fribourg+R	7.5	2.2	3.0	3.9	6.3	470.5	82,387.3	229

Table C.1: Execution times in CPU time seconds for the 10,939 effectvie samples of the GOAL test set.

Construction	Mean	Min.	P25	Median	P75	Max.	Total	≈ hours
Piterman+EQ+RO	3.0	2.2	2.6	2.8	3.0	42.9	21,410.6	59
Slice+P+RO+MADJ+EG	3.7	2.2	2.7	3.2	4.1	36.7	26,398.9	73
Rank+TR+RO	16.0	2.3	2.8	3.7	9.3	443.3	115,563.9	321
Fribourg+M1+R2C	4.0	2.2	2.7	3.1	4.4	410.4	28,970.8	80

Table C.2: Execution times in CPU time seconds for the 7,204 effectvie samples of the GOAL test set.

Construction	Mean	Min.	P25	Median	P75	Max.	Total	≈ hours
Piterman+EQ+RO	3.6	2.2	2.7	2.9	3.4	365.7	39,663.4	110
Slice+P+RO+MADJ+EG	4.3	2.2	2.9	3.7	5.0	42.4	47,418.2	132
Fribourg+M1+R2C	4.7	2.2	2.8	3.6	5.3	410.4	52,149.0	145

Table C.3: Execution times in CPU time seconds for the 10,998 effectvie samples of the GOAL test set without the Rank construction.

Construction	Michel 1	Michel 2	Michel 3	Michel 4	Fitted curve	Std. error
Fribourg	2.3	4.0	88.8	100,976.0	$(1.14n)^n$	0.64%
Fribourg+R2C	2.3	3.4	27.4	27,938.3	$(0.92n)^n$	0.64%
Fribourg+M1	2.2	3.6	17.9	6,508.4	$(0.72n)^n$	0.63%
Fribourg+M1+M2	2.3	3.5	13.8	2,707.4	$(0.62n)^n$	0.62%
Fribourg+M1+M2+R2C	2.5	3.5	10.8	2,332.6	$(0.61n)^n$	0.62%
Fribourg+R	2.4	3.7	86.0	101,809.6	$(1.14n)^n$	0.64%

Table C.4: Execution times in CPU time seconds for the four Michel automata.

Construction	Michel 1	Michel 2	Michel 3	Michel 4	Fitted curve	Std. error
Piterman+EQ+RO	2.5	3.8	42.6	75,917.4	$(1.08n)^n$	0.64%
Slice+P+RO+MADJ+EG	2.3	3.6	11.4	159.5	$(0.39n)^n$	0.38%
Rank+TR+RO	2.2	3.0	6.4	30.0	$(0.29n)^n$	0.18%
Fribourg+M1+M2+R2C	2.5	3.5	10.8	2,332.6	$(0.61n)^n$	0.62%

Table C.5: Execution times in CPU time seconds for the four Michel automata.

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