Empirical Performance Investigation of a Büchi Complementation Construction

Master's Thesis Presentation

Daniel Weibel

Foundations of Dependable Systems Group
Department of Informatics
University of Fribourg
daniel.weibel@unifr.ch

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UNIVERSITÉ DE FRIBOURG
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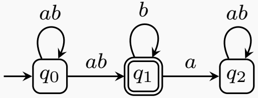
Outline

1. **Introduction**
2. **Implementation**
3. **Study Setup**
4. **Results**

Basics

Büchi automata



- Finite state automata running on infinite words (ω-words) $\in \Sigma^\omega$
- A word is accepted if it has an accepting run
- A run is accepting if it visits an accepting state infinitely often

Büchi complementation

The complement of a Büchi automaton A is another Büchi automaton B, such that:

B accepts a word if and only if it is not accepted by A

State Complexity of Büchi Complementation

A (n states) $\longrightarrow$ Büchi complementation construction $\longrightarrow$ B (m states)

- State complexity: m in relation to n
  - Size of complement in relation to size of input automaton
- Also known as *state growth*, *state blow-up*, or *state explosion*
- Can be **very high**
- Inhibits the application of Büchi complementation in practice
  - E.g. in automata-theoretic model checking
- The lower the state complexity, the higher the performance of a construction
- Importance to investigate the state complexity of Büchi complementation constructions

Worst-Case State Complexity

- Every construction has a specific worst-case state complexity
- Maximum number of states that a construction can produce
- Examples:

Complementation construction	Worst-case state complexity	Example value with $n = 15$
[Büchi, 1962]	$2^{2^{O(n)}}$	$1.4 \times 10^{9,864}$
[Piterman, 2007]	$O(n^{2n})$	1.9×10^{35}
[Vardi and Wilke, 2007]	$O((3n)^n)$	6.3×10^{24}
[Schewe, 2009]	$O((0.76n)^n)$	7.1×10^{15}

- Often used to assess the performance or efficiency of a construction, but. . .

Empirical Way to Investigate Performance

- Worst-case state complexity reflects only a **small** aspect of the state complexity of a construction
- From a practical point of view, we are interested in the performance of a construction in a **real-world** scenario
  - E.g. how does a construction perform on automata with 15 states?
- Such insights can be gained by **empirical** investigations
- Empirical performance investigation:
  1. **Implement construction**
  2. **Run the implementation on test automata**
  3. **Analyse generated complements**
- Aim of this thesis:
  - Empirically investigate the performance of the **Fribourg construction** (see next slide)

The Fribourg Construction

- Described by [Allred and Ultes-Nitsche, 2014]
- Slice-based complementation construction
  - See main complementation approaches: *Ramsey-based*, *determinisation-based*, *rank-based*, and *slice-based*
- Worst-case state complexity: $O((1.59n)^n)$
- Optimisations:

R2C If input automaton is complete, remove states whose rightmost component is 2-coloured

M1 Merge certain pairs of adjacent components
  - Worst-case state complexity: $O((1.195n)^n)$

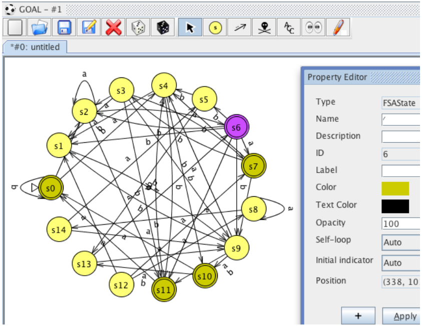
M2 Keep only one 2-coloured component in a state
  - Worst-case state complexity: $O((0.86n)^n)$

Roadmap

GOAL

- **G**raphical Tool for **O**mega-**A**utomata and **L**ogics
- http://goal.im.ntu.edu.tw/wiki/doku.php
- Allows to create and manipulate ω-automata

Graphical user interface



Command line interface

```
$ ./gc generate -t fsa -a nbw -s 15 -A classical -m density -dt 1.6 -da 0.3
<?xml version="1.0" encoding="UTF-8" standalone="no"?>
<Structure label-on="Transition" type="FiniteStateAutomaton">
    <Name/>
    <Description/>
    <Formula/>
    <Alphabet type="Classical">
        <Symbol>a</Symbol>
        <Symbol>b</Symbol>
    </Alphabet>
    <StateSet>
        <State sid="0">
            <Y>160</Y>
            <X>346</X>
            <Properties/>
        </State>
        <State sid="1">
            <Y>54</Y>
            <X>291</X>
            <Properties/>
        </State>
        <State sid="2">
            <Y>104</Y>
            <X>486</X>
            <Properties/>
        </State>
        <State sid="3">
            <Y>236</Y>
```

GOAL: Büchi Complementation Constructions

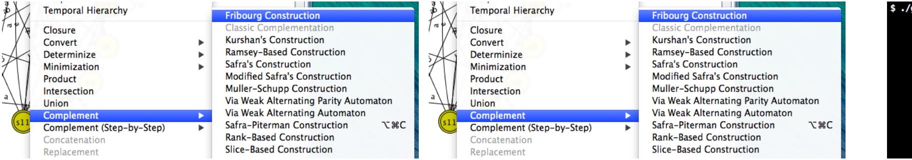
- GOAL contains implementations of several complementation constructions (GOAL version 2014–11–17):

GOAL Name	Reference
Ramsey	[Sistla et al., 1985, Sistla et al., 1987]
Safra	[Safra, 1988a, Safra, 1988b]
MS	[Muller and Schupp, 1995]
ModifiedSafra	[Althoff et al., 2006]
Piterman	[Piterman, 2006, Piterman, 2007]
WAA	[Kupferman and Vardi, 1997, Kupferman and Vardi, 2001]
WAPA	[Thomas, 1999]
Rank	[Schewe, 2009]
Slice+P	[Vardi and Wilke, 2007]
Slice	[Kähler and Wilke, 2008]

Fribourg Construction Plugin for GOAL

- GOAL is built with the Java Plugin Framework (JPF)[1]
  - Allows to create **plugins** containing **extensions** for pre-defined **extension points**
- We created a plugin that contains an extension with the implementation of the Fribourg construction

Graphical user interface



- Download: https://frico.s3.amazonaws.com/goal_plugins/ch.unifr.goal.complement.zip

[1] http://jpf.sourceforge.net/

GOAL and Plugin Demo

Roadmap

1. Introduction
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Roadmap

Study setup

- **Test data**
- **Test scenarios**
- **Execution**

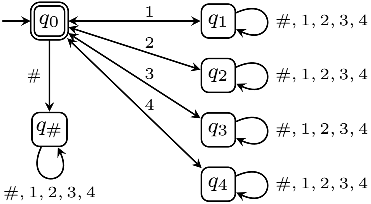
Test Data: GOAL Test Set

- Created and used by [Tsai et al., 2011]
- 11,000 automata
  - 15 states
  - Alphabet $\Sigma = \{0, 1\}$
  - 11 transition densities
    - $\mathcal{T} = (1.0,\ 1.2,\ 1.4,\ 1.6,\ 1.8,\ 2.0,\ 2.2,\ 2.4,\ 2.6,\ 2.8,\ 3.0)$
  - 10 acceptance densities
    - $\mathcal{A} = (0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 1.0)$
  - 110 classes at 100 automata for each combination $\mathcal{T} \times \mathcal{A}$
- Analysis
  - 61.8% universal automata
  - 0.6% empty automata
  - 9.0% complete automata
- Download: https://frico.s3.amazonaws.com/test_sets/goal.zip

Test Data: Michel Test Set

- Automata used by [Michel, 1988] to prove $n!$ lower bound
- Generally provoke very high state complexity
- Four Michel automata
  - **Michel 1:** 3 states, 2 symbols, 5 transitions
  - **Michel 2:** 4 states, 3 symbols, 8 transitions
  - **Michel 3:** 5 states, 4 symbols, 11 transitions
  - **Michel 4:** 6 states, 5 symbols, 14 transitions

Michel 4:



- Download: https://frico.s3.amazonaws.com/test_sets/michel.zip

Test Scenarios

- Internal tests
  - Compare different versions of the Fribourg construction
  - Combinations of optimisations R2C, M1, and M2
  - Further options:
    - C: make input automaton complete
    - R: remove unreachable and dead states from output automaton
- External tests
  - Compare Fribourg construction with other constructions
  - Choose best version of Fribourg construction for each test set
  - Other constructions (see Slide 10):
    - Piterman [Piterman, 2006, Piterman, 2007]
    - Rank [Schewe, 2009]
    - Slice [Vardi and Wilke, 2007]

Test Scenarios

	GOAL test set	Michel test set
Internal tests	– Fribourg – Fribourg+R2C – Fribourg+R2C+C – Fribourg+M1 – Fribourg+M1+R2C – Fribourg+M1+R2C+C – Fribourg+M1+M2 – Fribourg+R	– Fribourg – Fribourg+R2C – Fribourg+M1 – Fribourg+M1+M2 – Fribourg+M1+M2+R2C – Fribourg+R
External tests	– Piterman+EQ+RO – Rank+TR+RO – Slice+P+RO+MADJ+EG – Fribourg+M1+R2C	– Piterman+EQ+RO – Rank+TR+RO – Slice+P+RO+MADJ+EG – Fribourg+M1+M2+R2C

Test Scenarios: Shorthand Names

	GOAL test set	Michel test set
Internal	**IG**	**IM**
External	**EG**	**EM**

Execution: Resource Limits

- For IG and EG, we limit the resources for each complementation task (= complementation of 1 automaton)
  - Time: 600 seconds (CPU time)
  - Memory: 1 GB (Java heap)
- If a complementation task exceeds these limits, it is aborted
- Reason: prevent excessive resource requirements
- **Effective samples**
  - Automata which have been successfully complemented by **all** constructions of a test scenario
- Analysis of results of IG and EG is based on effective samples
- For IM and EM, there are no limits

Execution: Environment

- Execution on UBELIX: high-performance computing (HPC) cluster at the University of Bern[2]
- Through command line interface of GOAL
  - E.g. `gc complement -m fribourg 00001.gff`
  - 1 automaton = 1 process
- Usage of UBELIX hnodes 1–42 and jnodes:
  - Processor: Intel Xeon E5-2665 2.40GHz
  - Architecture: 64 bit
  - CPUs: 16
  - Memory (RAM): 64 GB (hnodes) or 256 GB (jnodes)
  - Operating System: Red Hat Enterprise Linux 6.6

[2] http://ubelix.unibe.ch

Roadmap

1. Introduction
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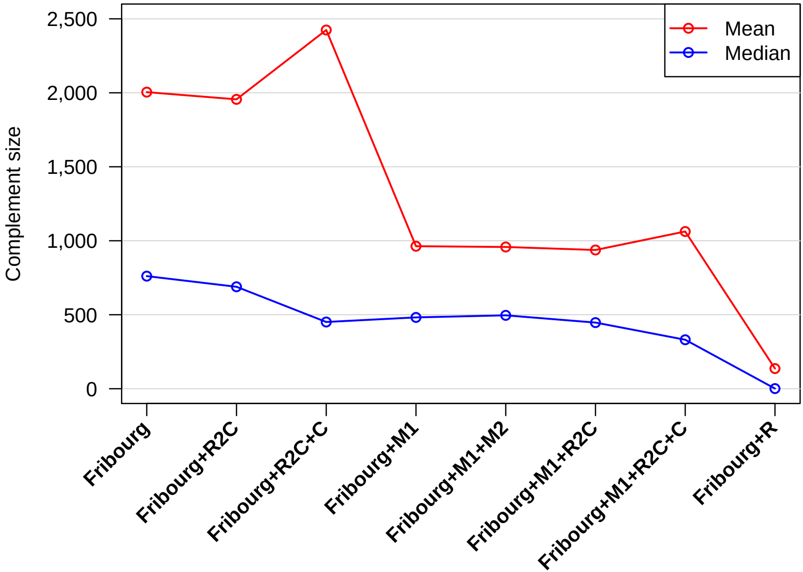
4. **Results**

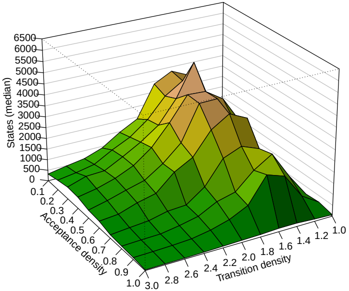
Results: Internal Tests on GOAL Test Set

	GOAL test set	Michel test set
Internal	**IG**	**IM**
External	**EG**	**EM**

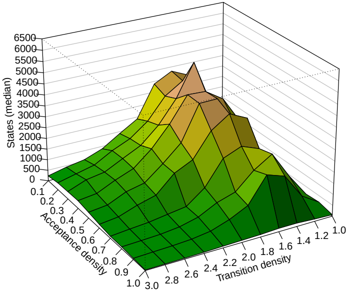
- Fribourg
- Fribourg+R2C
- Fribourg+R2C+C
- Fribourg+M1
- Fribourg+M1+R2C
- Fribourg+M1+R2C+C
- Fribourg+M1+M2
- Fribourg+R

IG: Complement Sizes (10,939 Eff. Samples)

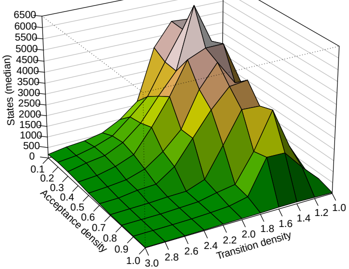




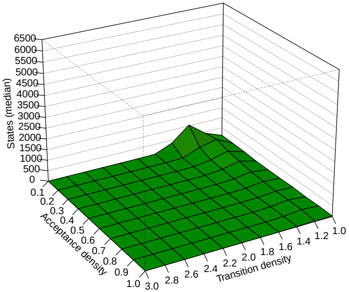
Fribourg



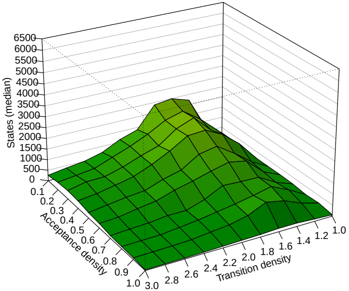
Fribourg+R2C



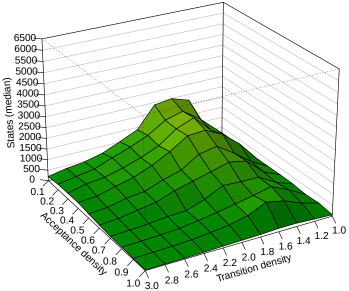
Fribourg+R2C+C



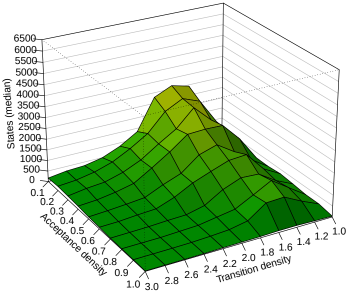
Fribourg+R



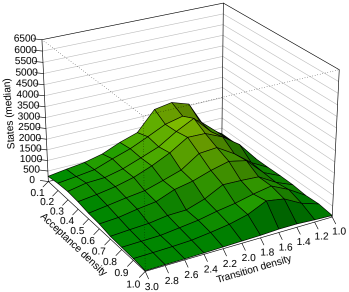
Fribourg+M1



Fribourg+M1+R2C

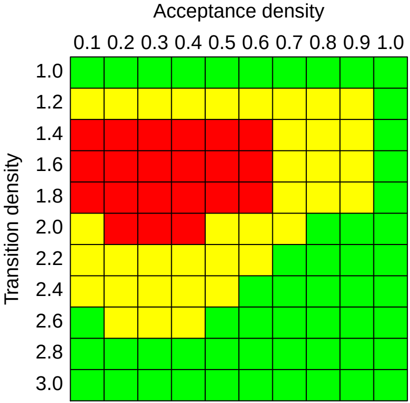


Fribourg+M1+R2C+C



Fribourg+M1+M2

IG: Difficulty Classes (10,939 Eff. Samples)



Green = easy, yellow = medium, red = hard

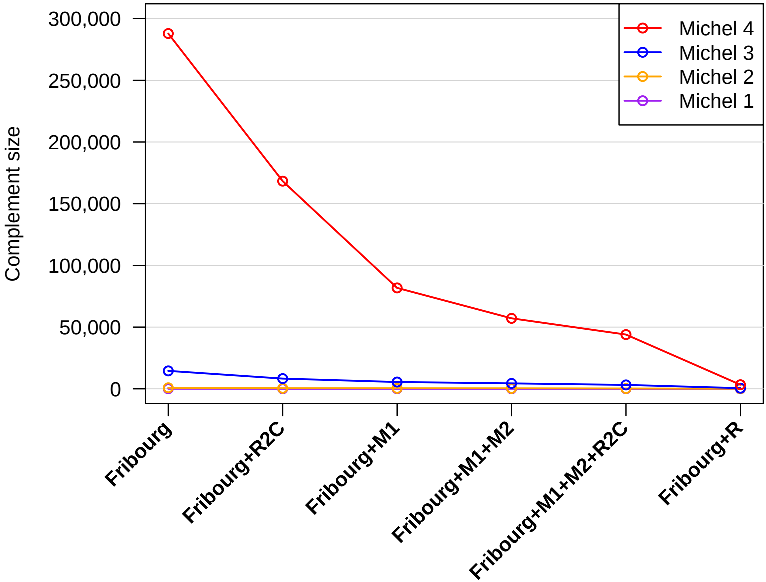
For each class: mean of median complement sizes of each construction, breakpoints at 500 and 1,600 states

Results: Internal Tests on Michel Test Set

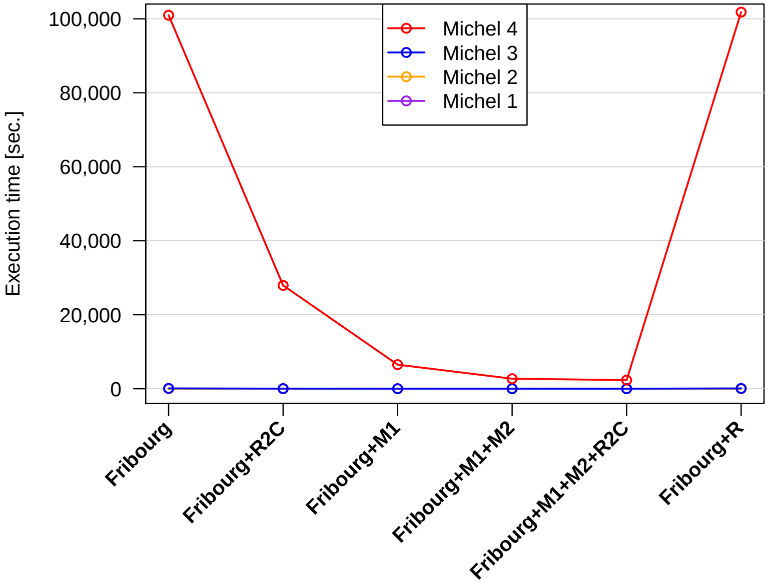
	GOAL test set	Michel test set
Internal	**IG**	**IM**
External	**EG**	**EM**

- Fribourg
- Fribourg+R2C
- Fribourg+M1
- Fribourg+M1+M2
- Fribourg+M1+M2+R2C
- Fribourg+R

IM: Complement Sizes



IM: Execution times

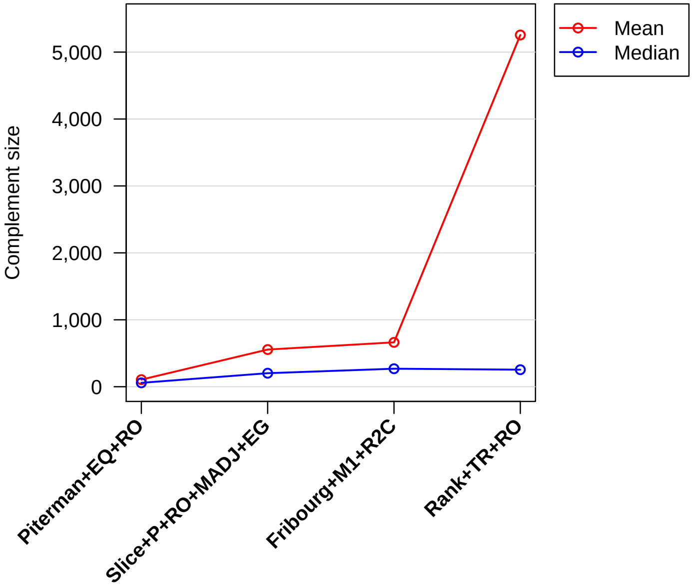


Results: External Tests on GOAL Test Set

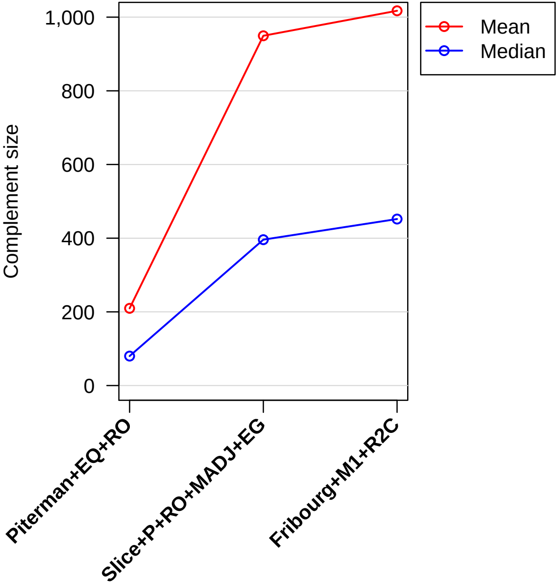
	GOAL test set	Michel test set
Internal	**IG**	**IM**
External	**EG**	**EM**

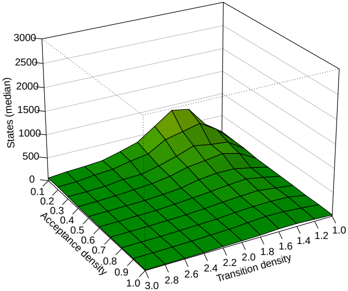
- Piterman+EQ+RO
- Rank+TR+RO
- Slice+P+RO+MADJ+EG
- Fribourg+M1+R2C

EG: Complement Sizes (7,204 Eff. Samples)

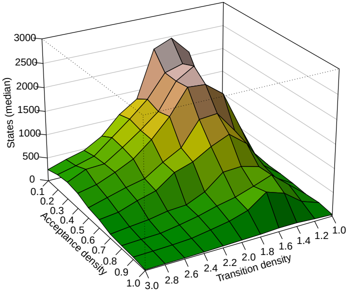


EG: Complement Sizes (10,998 Eff. Samples)

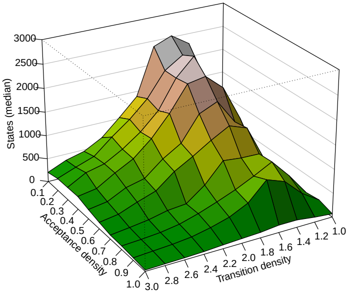




Piterman+EQ+RO



Slice+P+RO+MADJ+EG



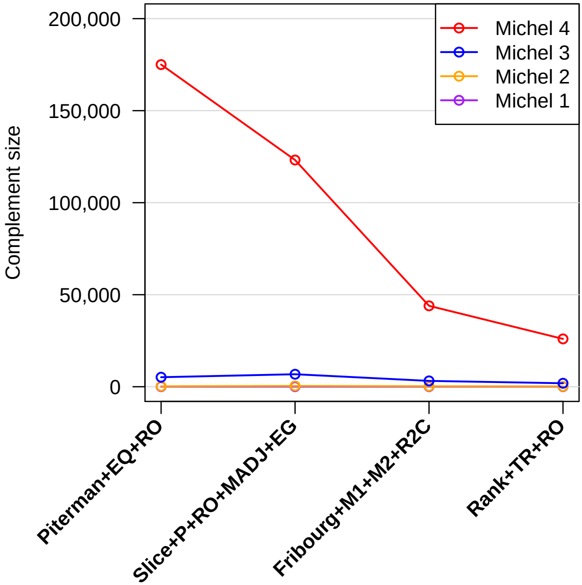
Fribourg+M1+R2C

Results: External Tests on Michel Test Set

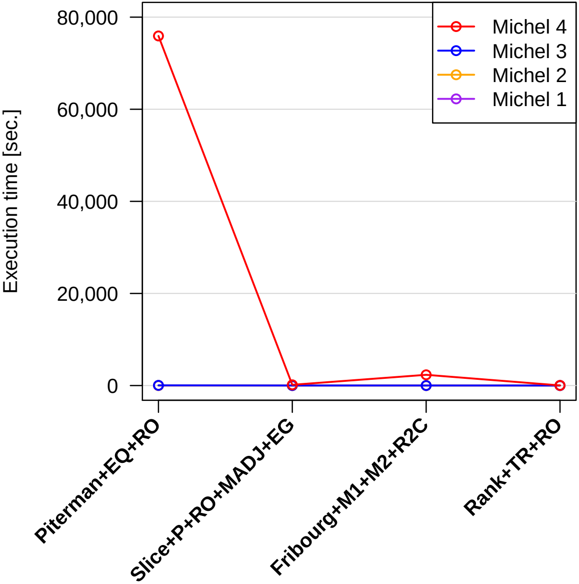
	GOAL test set	Michel test set
Internal	**IG**	**IM**
External	**EG**	**EM**

- Piterman+EQ+RO
- Rank+TR+RO
- Slice+P+RO+MADJ+EG
- Fribourg+M1+M2+R2C

EM: Complement Sizes



EM: Execution times



Conclusions

- Performance of Fribourg construction
  - Optimisations R2C and M1 have a very positive impact
  - M2 brings no overall improvement for GOAL test set
    - However, improves the performance on the Michel automata
  - R2C+C makes easy automata easier and hard automata harder
    - Mean of complement size increases, median decreases
- Büchi complementation in general
  - Worst-case complexities do not reflect actual performance
  - Actual performance is multifaceted and hard to predict
  - There is no "best" construction
    - All constructions have individual strengths and weaknesses
- Future work
  - Further statistical analyses of results

The End

Thank you for listening!

Thank you very much for listening!

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