Progress Measures for Complementation of ω-Automata with Applications to Temporal Logic

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Abstract

We give a new approach to complementing ω-automata, which are finite-state automata defining languages of infinite words. Instead of using usual combinatorial or algebraic properties of transition relations, we show that a graph-theoretic approach based on the notion of *progress measures* is a potent tool for complementing ω-automata.

We apply progress measures to the classical problem of complementing *Büchi automata* and obtain a new simple method, which is optimal.

Our technique applies to *Streett automata* for which we also obtain an optimal complementation method. As a consequence, the powerful temporal logic ETL_S introduced by Safra and Vardi is much more tractable than previously thought.

1 Introduction

This paper presents a novel technique based on *progress measures* [Kla90, Kla91, KK91] for the complementation of ω-automata, which are finite-state automata defining languages of infinite words. Using this technique we obtain an elementary proof of the classic result that the class of languages defined by *Büchi automata* [Büc62] is closed under complementation. Applied to *Streett automata* [Str82], which are especially useful for describing fair computations, our technique yields a complementation method that achieves the lower bound for this type of problem. Our method relies on a compact representation of the Kleene-Brouwer ordering, which lies behind verification with disjunctions [KK91]. Our results imply a significant improvement in the complexity of deciding the powerful temporal logic ETL_S [SV89].

1.1 ω-Automata

In 1962 Büchi presented a natural acceptance condition allowing nondeterministic finite-state automata to define languages of infinite words [Büc62]. According to Büchi's condition, an automaton accepts an infinite word if there is a run that passes through a final state infinitely often. A language defined by this condition is called *ω-regular* in analogy with the usual regular languages of finite words defined by finite-state automata.

Büchi presented a proof that his automata are closed under complementation, i.e. for any Büchi automaton defining a language L of infinite words, there is a Büchi automaton defining the complement of L. This property is a crucial step of the paper, whose main result is that the second order theory of successor[1] is decidable.

Because of its significance to decision procedures for second order logics and temporal logics, the complementation property of Büchi automata has been proven in several ways [Cho74, Kur87, McN66, Pec86, SVW87, Sie70, Tho81]. These proofs involve rather sophisticated combinatorial or algebraic properties of the transition relation and a $2^{O(n^2)}$ or worse blow-up of the state space. Safra's acclaimed determinization construction [Saf88] (or the construction by Emerson and Jutla [EJ89]) shows that Büchi automata can be complemented with a blow-up of only $2^{O(n \cdot \log n)}$, which is also the lower bound [Mic88]. In this paper we show that the complementation result is obtainable in an elementary fashion without the need for a sophisticated determinization construction and without relying on Ramsey's Lemma or other advanced

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[1] The second order theory of successor is a logical language that contains the successor function $S(n) = n+1$, usual boolean connectives, equality, and quantification over numbers and sets of numbers.

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combinatorial tools used in the past.

1.2 Temporal logics and ω-automata

In the early eighties research focused again on properties of Büchi automata and other types of automata following the interest in temporal logic, which is a central topic in the study of properties of concurrent and distributed programs. Wolper pointed out [Wol83] that usual *Propositional Temporal Logic*, *TL*, based on operators $\Diamond$ ("it will be the case that"), $\Box$ ("it is always the case that"), $\mathcal{U}$ ("until") and $\bigcirc$ ("next time"), is deficient in expressive power and he proposed to boost *TL* with temporal operators defined in terms of linear context-free grammars. Wolper called this logic *ETL*, *Extended Temporal Logic*. This formalism defines all ω-regular languages, in contrast to *TL*. Thus *ETL* is in a much stronger category of specification languages. For example, it was shown that the power of ω-regular languages is sufficient and necessary to perform compositional specification of programs [LPZ85, Pnu86]. Significantly, deciding *ETL* is no more complex than deciding *TL*; in fact, the satisfiability problem for both logics is PSPACE-complete, see [SC85]. Thus it is natural to look for the most succinct formalism that expresses the class of ω-regular languages and is still in PSPACE.

This goal was pursued in the paper [WVS83], which introduced temporal operators corresponding to finite-state automata with simple acceptance conditions. Then in [SVW87] it was shown that the logic ETL_B based on automata with Büchi acceptance conditions is in PSPACE; this decidability result hinges on an exponential blow-up complementation method for Büchi automata. Since the Büchi condition is natural for program specification, this result provides an attractive logic that incorporates the concept of state and the temporal concept of eventuality. Applications of Büchi automata to verification were given in [AS87, AS89, MP87, Var87].

Safra and Vardi investigated whether it is possible to get "more bang for the buck" [SV89] by finding still more succinct ways of defining temporal operators while staying within PSPACE for the decision problem. One obvious approach is to look for more sophisticated automata acceptance conditions. Emerson and Lei [EL87] suggested using temporal logic formulas on the state space of the automata for defining accepting runs. However, such automata appear to be too general: they cannot be complemented with only an exponential space blow-up [SV89]. Thus a logic using such automata is likely to be much harder to decide than *TL*. Instead Safra and Vardi [SV89] proposed using automata with *Streett acceptance conditions*, which are especially useful since they can directly describe fair computations. Safra and Vardi showed that Streett automata are exponentially more succinct than Büchi automata and that the temporal logic ETL_S based on such connectives is in PSPACE. Their main result is a complementation method with a $2^{O(n^5)}$ blow-up in size, and it yields a decision procedure which runs in time $2^{O(n^5)}$. The decision procedure for ETL_B, however, runs in time $2^{O(n \cdot \log n)}$. This discrepancy "is significant enough to put a question mark on the practicality of ETL_S" [SV89]. The results in this paper show that this reservation is probably unwarranted. We obtain a complementation method for Streett automata with only a $2^{O(n \cdot \log n)}$ blow-up. This yields a decision procedure for ETL_S that runs in time $2^{O(n \cdot \log n)}$.

1.3 Our approach

An ω-automaton A*Cond* consists of a finite state automaton A and an acceptance condition *Cond*. An infinite input word α is accepted if there is a run over α satisfying *Cond*; here a *run* is an infinite sequence of states that the finite state automaton A may go through when reading α.

A simple type of acceptance condition is that of Büchi; it is given as a set R of *reconfirming states*. A run satisfies R if the run goes through a reconfirming state infinitely often. A *Streett condition* is a particular kind of temporal logic condition, which can be used to describe fair computations.

When complementing an automaton A*Cond*, we would like to use the classic subset construction [RS59] (see [HU79]). This gives us a deterministic automaton that piecemeal describes the *run graph* of all possible runs over an infinite input word α; in particular, the state of this automaton designates the *current subset* of states that A can reach by the prefix of α read so far. Thus the blow-up in the size of the state space is $2^{O(n)}$, where n is the number of states of A. To verify that A*Cond* does not accept α, we must check that along all paths of the run graph of α, the negated condition $\neg$*Cond* holds.

At this point we use the theory of *progress measures* [Kla90, Kla91, KK91], which are mappings that allow to verify *locally*, i.e. in terms of single transitions, that a *global* property, such as $\neg$*Cond*, holds about the infinite paths of a graph. Intuitively, the value of a progress measure at a vertex v quantifies how close any path through v is to satisfying the global property.

The idea pursued in this paper is to let the subset automaton guess the value of the progress measure for each state in the current subset. An obstacle to this approach is that a progress measure usually takes on infinitely many different values on a run graph and thus cannot be represented by a finite-state automaton. To overcome this problem we introduce the concept of a *quasi progress measure*, which can express temporary suspension of progress. A quasi progress measure ensures that $\neg \mathit{Cond}$ holds along any path if a well-foundedness criterion is satisfied, namely that progress never becomes permanently suspended.

We prove that the quasi progress measures considered need to take on only $O(n)$ different values on a run graph. This allows the subset automaton to be modified into a nondeterministic automaton, called a *quasi measure automaton*, that guesses the at most n progress values of the current subset. The state space of the quasi measure automaton needs to hold only $n \cdot \log O(n)$ bits of information; thus it has $2^{O(n \cdot \log n)}$ states.

A straightforward construction shows that the quasi measure automaton can be changed into a Büchi automaton $\overline{\mathbf{A}}\,\overline{R}$ such that an input word is accepted if and only if the well-foundedness condition of the quasi progress measure guessed is satisfied. Thus the nondeterministic automaton $\overline{\mathbf{A}}\,\overline{R}$ accepts exactly the complement of the language of A.

The quasi progress measures for *Büchi* conditions are just mappings into ordinals, and a simple complementation method results. Our method for Streett automata is substantially more complicated, although it is based on the same basic approach. A Streett condition is a conjunction of particularly simple disjunctions. Thus the complementation problem involves the dual condition, which is that of Rabin [Rab69], and we use the *Rabin measures* of [KK91] to quantify progress. Rabin measures are based on the Kleene-Brouwer ordering of recursion theory. A main obstacle is that the Kleene-Brouwer ordering and the Rabin measures must be represented in a compact form—analogous to the quasi progress measures above—to yield an optimal complementation method.

2 Automata, Subset Construction, and Quasi Progress Measures

The set of natural numbers $0, 1, \ldots$ is denoted ω, and the set of countable ordinals is denoted ω_1. The number of elements in a set X is denoted $|X|$. A *graph* $G = (V, E)$ consists of a countable set of *vertices* V and a set of directed *edges* $E \subseteq V \times V$. A *path* $v_0 v_1 \cdots$ in G is a finite or infinite sequence of vertices such that $(v_i, v_{i+1}) \in E$ for $i = 0, 1, \ldots$ The *length* of a finite path $v_0 \cdots v_n$ is n.

2.1 Automata and Subset Construction

An *automaton* $\mathbf{A} = (\Sigma, V, \rightarrow, V^0)$ consists of a countable alphabet Σ, a finite set of *states* V, a *transition relation* $\rightarrow \subseteq V \times \Sigma \times V$, and a set of *initial states* $V^0 \subseteq V$. The *size* $|\mathbf{A}|$ of A is defined as $|V|$.

A *behavior* α is an infinite word $a_0 a_1 \ldots$ of letters $a_i \in \Sigma$. A *run* of A over a behavior α is an infinite sequence of states $v_0 \xrightarrow{a_0} v_1 \xrightarrow{a_1} \cdots$ with $v_0 \in V^0$. The *language* $L(\mathbf{A})$ *accepted by* A is the set of behaviors that allow a run.

Given an automaton A and an $\alpha \in \Sigma^\omega$, we define the *run graph* $\mathcal{G}(\mathbf{A}, \alpha) = (V(G, \alpha), E(G, \alpha))$ describing all possible runs of A over α by:

$$\begin{aligned} V(G,\alpha) &= \{(v,i) \in V \times \omega \mid \exists v_0, \ldots, v_i : \\ &\qquad v_0 \in V^0, v_i = v, \text{ and } v_0 \xrightarrow{a_0} \cdots \xrightarrow{a_{i-1}} v_i\} \\ E(G,\alpha) &= \{((v,i),(v',i')) \in V(G,\alpha) \times V(G,\alpha) \mid \\ &\qquad i+1 = i' \text{ and } v \xrightarrow{a_i} v'\} \end{aligned}$$

Note that $\mathcal{G}(\mathbf{A}, \alpha)$ has at most width $|V|$, that is, for any i, G has at most $|V|$ vertices of the form (v, i).

An automaton can construct piecemeal the run graph in the following sense.

Definition 1 A *subset automaton* $\widetilde{\mathbf{A}}(P)$ *of* A is an automaton $\widetilde{\mathbf{A}} = (\Sigma, \widetilde{V}, \rightarrow_\sim, \widetilde{V}^0)$ with a mapping $P : \widetilde{V} \rightarrow \mathcal{P}V$ such that $L(\widetilde{\mathbf{A}}) = L(\mathbf{A})$ and for all runs $\rho = \tilde{v}_0 \xrightarrow{a_0} \tilde{v}_1 \xrightarrow{a_1} \cdots$ of $\widetilde{A}$

$$P(\tilde{v}_i) = \{v \mid (v,i) \in \mathcal{G}(\mathbf{A}, \alpha)\}.$$

Note that the classical subset construction [RS59] yields the deterministic subset automaton $\widetilde{\mathbf{A}} = (\Sigma, \mathcal{P}V, \rightarrow_\sim, \{V^0\})$ of size $2^{|\mathbf{A}|}$, where $\tilde{v} \xrightarrow{a}_\sim \tilde{v}'$ iff $\tilde{v}' = \{v' \mid \exists v \in \tilde{v} : v \xrightarrow{a} v'\}$, and P is the identity function.

2.2 Büchi Conditions

A *Büchi condition* $R \subseteq V$ is a set of *reconfirming states*. An infinite sequence $v_0 v_1 \cdots$ satisfies R, and we write $v_0 v_1 \cdots \models R$, if v_k is infinitely often reconfirming.

A *Büchi automaton* $\mathbf{A}R$ is an automaton A equipped with a Büchi condition R. A behavior α is *accepted* by $\mathbf{A}R$ if there is a run $v_0 \xrightarrow{a_0} v_1 \xrightarrow{a_1} \cdots$ of A over α such that $v_0 v_1 \cdots \models R$. $L(\mathbf{A}R)$ is the set of behaviors accepted by $\mathbf{A}R$.

A *co-Büchi condition* $\neg R$ is the dual of a Büchi condition: an infinite sequence $v_0 v_1 \cdots$ *satisfies* $\neg R$, and we write $v_0 v_1 \cdots \models \neg R$, if $v_0 v_1 \cdots \not\models R$, that is, if v_k is reconfirming only finitely many times.

2.3 Quasi Progress Measures

Definition 2 A *quasi progress measure* (μ, Q) on V is a mapping $\mu : V \rightarrow D \cup Q$, where D is a set of progress values, Q is a set of *quiescent values*, and $D \cap Q = \emptyset$. A quasi progress measure (μ, Q) is *well-founded* if there is no infinite path $v_0 v_1 \cdots$ in G such that $\mu(v_0) = \mu(v_1) = \cdots \in Q$.

Definition 3 A *quasi measure automaton* $\widetilde{\mathbf{A}}(P, \mu, Q)$ of $\mathbf{A}$ consists of a subset automaton $\widetilde{\mathbf{A}}(P)$ of $\mathbf{A}$ and a function $\mu : \widetilde{V} \rightarrow (V \hookrightarrow D \cup Q)$, for some D, such that $\mu(\tilde{v})(v)$ is defined for all $\tilde{v} \in \widetilde{V}$ and $v \in P(\tilde{v})$.[2] A run $\rho = \tilde{v}_0 \xrightarrow{a_0}_{\sim} \tilde{v}_1 \xrightarrow{a_1}_{\sim} \cdots$ over $\alpha = a_0 a_1 \cdots$ induces a quasi measure (μ_ρ, Q) on $\mathcal{G}(\mathbf{A}, \alpha)$ defined by $\mu_\rho(v, i) = \mu(\widetilde{v_i})(v)$.

Proposition 1 Given a quasi measure automaton $\widetilde{\mathbf{A}}(P, \mu, Q)$ of $\mathbf{A}$. There is a Büchi automaton $\mathcal{W}\widetilde{\mathbf{A}}(P, \mu, Q)$ such that

$$\alpha \in L(\mathcal{W}\mathbf{A}(P, \mu, Q))$$

iff

there is a run ρ over α such that (μ_ρ, Q) is a well-founded quasi measure for $\mathcal{G}(\mathbf{A}, \alpha)$.

Moreover, $\mathcal{W}\mathbf{A}(P, \mu, Q)$ has $|\widetilde{\mathbf{A}}| \cdot 2^{|\mathbf{A}|}$ states.

Proof The automaton $\widetilde{\mathbf{A}}$ is extended with a state component M that tracks the paths along which the value of μ remains constant and in Q. Formally, define $\widehat{\mathbf{A}} = (\Sigma, \widetilde{V} \times \mathcal{P}V, \rightarrow_\wedge, \widetilde{V}^0 \times \{\emptyset\})$, where

$$(\tilde{v}, M) \xrightarrow{a}_\wedge (\tilde{v}', M')$$

iff

$$\tilde{v} \xrightarrow{a}_\sim \tilde{v}' \text{ and}$$
$$M' = \begin{cases} P(\tilde{v}') & \text{if } M = \emptyset \\ \{v' \mid \exists v \in M : v \xrightarrow{a} v' \text{ and} & \text{if } M \neq \emptyset \\ \qquad \mu(\tilde{v})(v) = \mu(\tilde{v}')(v') \in Q \,\} & \end{cases}$$

Note that a run ρ of $\widetilde{\mathbf{A}}$ induces a unique run of $\widehat{\mathbf{A}}$. The component M denotes the subset of $P(\tilde{v})$ along which there is a path with a constant μ value in Q from the last time M was empty. Define $\widehat{R} = \mathcal{P}V \times \{\emptyset\}$, and define $\mathcal{W}\widetilde{\mathbf{A}}(P, \mu, Q) = \widehat{\mathbf{A}}\widehat{R}$. The size of $\mathcal{W}\widetilde{\mathbf{A}}(P, \mu, Q)$ is then $|\widetilde{\mathbf{A}}| \cdot 2^{|\mathbf{A}|}$. Let us prove the two directions of the "iff" in the proposition.

"$\Leftarrow$" Let $\rho = (\tilde{v}_0, M_0) \xrightarrow{a_0}_\wedge (\tilde{v}_1, M_1) \xrightarrow{a_1}_\wedge \cdots$ be a run such that (μ_ρ, Q) is a well-founded quasi measure on $\mathcal{G}(A, \alpha)$. If ρ is not an accepting run of $\widehat{\mathbf{A}}\widehat{R}$, then there is an i such that for all $j \geq i$, $M_j \neq \emptyset$. We may construct a forest as follows. The roots are $\langle (i, v_i) \rangle$, where $v_i \in P(\tilde{v}_i)$. The children of the roots are nodes on the form $\langle (i, v_i), (i+1, v_{i+1}) \rangle$, where $v_i \xrightarrow{a_i} v_{i+1}$, $v_{i+1} \in P(\tilde{v}_{i+1})$, and $\mu(\tilde{v}_i)(v_i) = \mu(\tilde{v}_{i+1})(v_{i+1}) \in Q$. The grandchildren are constructed similarly by appending states that are reachable from the children and for which the value of the measure is constant and in Q. Since for all $j \geq i$, $M_j \neq \emptyset$, this construction can be repeated infinitely often resulting in an infinite forest. Because the forest is finitely branching, it contains an infinite path by König's Lemma. This path corresponds to an infinite path in $\mathcal{G}(\mathbf{A}, \alpha)$ that is not well-founded with respect to (μ_ρ, Q). This is a contradiction.

"$\Rightarrow$" If ρ is an accepting run of $\widehat{\mathbf{A}}\widehat{R}$, then M is reset to $\emptyset$ infinitely many times, and it follows that there can be no infinite path in $\mathcal{G}(\mathbf{A}, \alpha)$ that is not well-founded with respect to (μ_ρ, Q). □

3 Complementation of Büchi automata

We start by observing that non-acceptance of α can be defined in terms of $\mathcal{G}(\mathbf{A}, \alpha)$:

Proposition 2 $\alpha \notin L(\mathbf{A}R)$ iff $\mathcal{G}(\mathbf{A}, \alpha) \models \neg(R \times \omega)$.

Proof This follows from the observation that any infinite path $(v_0, 0), (v_1, 1), \ldots$ in $\mathcal{G}(\mathbf{A}, \alpha)$ corresponds to a run of $\mathbf{A}$ over α, and vice versa. □

We use a progress measure to express locally—i.e. in terms of single edges—the global property that the co-Büchi condition holds for all paths:

Definition 4 A *co-Büchi progress measure* for $(G, \neg R)$, where $G = (V, E)$ and $R \subseteq V$, is a mapping $\mu : V \rightarrow \omega_1$ such that $(v, v') \in E$ implies that $v \overset{\mu}{\triangleright}_{\mathrm{cB}} v'$, where $v \overset{\mu}{\triangleright}_{\mathrm{cB}} v'$ if either $\mu(v) > \mu(v')$ or $\mu(v) = \mu(v')$ and $v' \notin R$.

[2] $f : A \hookrightarrow B$ means that f is a partial function from A to B. The domain of f is denoted $\mathrm{dom} f$.

Proposition 3 $G \models \neg R$ iff $(G, \neg R)$ has a co-Büchi measure.

Proof "$\Leftarrow$" Let μ be a co-Büchi progress measure and let $v_0 v_1 \cdots$ be an infinite path in G. Since $\mu(v_0) \geq \mu(v_1) \geq \cdots$, there is a K such that $\mu(v_K) = \mu(v_{K+1}) = \cdots$. By definition of a co-Büchi measure, $v_{K+1}, v_{K+2}, \ldots \notin R$. Thus $v_0 v_1 \cdots \models \neg R$.

"$\Rightarrow$" Let (G, R) be such that $G \models \neg R$. Define $\mathcal{R}(v) = \{v' \mid \exists \text{path of length} > 0 \text{ in } G \text{ from } v \text{ to } v'\}$.

Claim 1 If $V \neq \emptyset$, then there is a $v \in V$ such that either

$(\star)$ $\mathcal{R}(v) = \emptyset$, or
$(\star\star)$ $\mathcal{R}(v) \neq \emptyset$ and $\mathcal{R}(v) \cap R = \emptyset$

Proof Suppose for a contradiction that for all $v \in V$ neither $(\star)$ nor $(\star\star)$ holds. We construct a path satisfying R as follows. Let $u_0 \in V$. Since $(\star)$ and $(\star\star)$ do not hold for $v = u_0$, there is a $u_1 \in R$ and a path from u_0 to u_1. Similarly, there is a $u_2 \in R$ and a path from u_1 to u_2. Continuing in this fashion, we obtain an infinite path that passes through $u_1, u_2, \ldots$ This path satisfies R, which contradicts that $G \models \neg R$. □

For a vertex v satisfying Claim 1, define

$$W(v) = \begin{cases} \{v\} & \text{if } (\star) \text{ holds} \\ \mathcal{R}(v) & \text{if } (\star\star) \text{ holds} \end{cases}$$

Apply Claim 1 to obtain a vertex v_0. Define $W_0 = W(v_0)$ and remove W_0 from G. If G is not empty, then obtain a vertex v_1 by applying Claim 1 again. Define $W_1 = W(v_1)$ and remove W_1 from G. By continuing transfinitely, we obtain a countable ordinal $\widehat{\theta}$ and a partition $\{W_\theta\}_{\theta<\widehat{\theta}}$ Define $\mu(v) = \theta$, where $v \in W_\theta$. To see that μ is a co-Büchi measure, let $(v, v') \in E$. Since v' is removed at latest when v is removed, $\mu(v) \geq \mu(v')$. If $\mu(v) = \mu(v')$, then for some θ, v and $v' \in W_\theta$. If W_θ is of type $(\star)$, then $v = v'$, but (v, v) contradicts that $\mathcal{R}(v) = \emptyset$ for v of type $(\star)$. Thus W_θ is of type $(\star\star)$ and $v' \notin R$. □

The notion of a co-Büchi quasi progress measure is derived naturally from the notion of a co-Büchi progress measure:

Definition 5 A *quasi co-Büchi measure* (μ, Q) for $(G, \neg R)$ is a quasi progress measure $\mu : V \rightarrow \omega_1 \cup Q$, where $Q \subseteq \omega_1$, such that $(v, v') \in E$ implies that $v \overset{\mu}{\triangleright}_{\mathrm{cB}} v'$ or $\mu(v) = \mu(v') \in Q$. The *size* of μ is $|\mu(V)|$.

The existence of a well-founded quasi co-Büchi measure—one for which progress is only suspended temporarily—ensures that the co-Büchi condition holds:

Proposition 4 $G \models \neg R$ iff $(G, \neg R)$ has a well-founded quasi co-Büchi measure.

Proof "$\Leftarrow$" Let μ be a well-founded quasi co-Büchi measure and let $v_0 v_1 \cdots$ be a path in G. Then $\mu(v_0) \geq \mu(v_1) \geq \cdots$. Thus for some K, $\theta = \mu(v_K) = \mu(v_{K+1}) = \cdots$. Since μ is a well-founded quasi progress measure, $\theta \notin Q$. Thus by definition of a quasi co-Büchi measure, $v_{K+1}, v_{K+2}, \ldots \notin R$.

"$\Rightarrow$" By Proposition 3 and the fact that any co-Büchi measure is a well-founded quasi co-Büchi measure. □

Because the width of a run graph is at most $|\mathbf{A}|$, we can show that only $|\mathbf{A}|$ proper progress values and $|\mathbf{A}| + 1$ quiescent values are needed:

Proposition 5 Let $n = |\mathbf{A}|$. $\mathcal{G}(\mathbf{A}, \alpha) \models \neg R \times \omega$ iff $(\mathcal{G}(\mathbf{A}, \alpha), \neg R \times \omega)$ has a well-founded quasi co-Büchi measure $(\tilde{\mu}, Q)$ of size $2 \cdot n + 1$, where $\tilde{\mu} : V(\mathbf{A}, \alpha) \rightarrow \{0, \ldots, 2 \cdot n\}$ and $Q = \{0, 2, \ldots, 2 \cdot n\}$.

Proof "$\Leftarrow$" By Proposition 4.

"$\Rightarrow$" Assume that $\mathcal{G}(\mathbf{A}, \alpha) \models \neg R \times \omega$. Then by Proposition 3, $(\mathcal{G}(\mathbf{A}, \alpha), \neg R \times \omega)$ has a co-Büchi measure μ. By adding 1 to the value of μ everywhere, we can assume that $\mu(v) > 0$, for all $v \in V(\mathbf{A}, \alpha)$. Define the predicate *Const*(θ) to be true if there is a path $v_0 v_1 \cdots$ in $\mathcal{G}(\mathbf{A}, \alpha)$ such that $\theta = \mu(v_0) = \mu(v_1) = \cdots$. Since the graph $\mathcal{G}(\mathbf{A}, \alpha)$ has width at most n, it follows from the Pigeon Hole Principle that there are at most n different θ such that *Const*(θ). Let these values be $\theta_{n'} > \theta_{n'-1} > \cdots > \theta_1$, where $n' \leq n$. Define $\theta_0 = 0$ and $\theta_{n'+1} = 1 + \sup_{v \in V(\mathbf{A}, \alpha)} \mu(v)$. Define $\tilde{\mu}$ by

$$\tilde{\mu}(v) = \begin{cases} 2 \cdot i - 1 & \text{if } \mu(v) = \theta_i \\ 2 \cdot i & \text{if } \theta_{i+1} > \mu(v) > \theta_i \end{cases}$$

Then it is not hard to see that $(\tilde{\mu}, \{0, 2, \ldots, 2 \cdot n\})$ is a quasi co-Büchi progress measure. Moreover, $\tilde{\mu}$ is a well-founded quasi progress measure, because if

$\alpha \notin L(\mathbf{A}R)$ iff (Proposition 2)
$\mathcal{G}(\mathbf{A},\alpha) \models \neg R$ iff (Proposition 5)
$\exists \tilde{\mu} : V(\mathbf{A},\alpha) \rightarrow \{0,\ldots,2\cdot n\} : (\tilde{\mu},\{0,2,\ldots,2\cdot n\})$ is a w-f quasi co-Büchi measure for $(\mathcal{G}(\mathbf{A},\alpha),\neg R)$ iff (Claim)
$\exists \rho : \rho$ is a run of $\widetilde{A}$ over α such that μ_ρ is a w-f quasi co-Büchi measure for $\mathcal{G}(\mathbf{A},\alpha)$ iff (Proposition 1)
$\alpha \in L(\mathcal{W}\widetilde{\mathbf{A}}(P,\mu,Q))$

Figure 1: Complementation proof for Büchi automata

$v_0 v_1 \cdots$ is an infinite path in $\mathcal{G}(\mathbf{A},\alpha)$ such that for some i, $2\cdot i = \tilde{\mu}(v_0) = \tilde{\mu}(v_1) = \cdots$, then there is a K and some $\widehat{\theta}$, $\theta_{i+1} > \widehat{\theta} > \theta_i$ such that $\widehat{\theta} = \mu(v_K) = \mu(v_{K+1}) = \cdots$. But then *Const*$(\widehat{\theta})$ must hold, which contradicts the definition of *Const*. □

From Propositions 1, 2, and 5, we obtain:

Theorem 1 Given a Büchi automaton $\mathbf{A}R$, there is a Büchi automaton $\overline{\mathbf{A}\,R}$—with $2^{O(|\mathbf{A}|\cdot|\log \mathbf{A}|)}$ states—accepting the complement of $L(\mathbf{A}R)$.

Proof Define the quasi measure automaton $\widetilde{\mathbf{A}}(P,\mu,Q)$ by

$$\begin{aligned}
&\widetilde{\mathbf{A}} = (\Sigma, V \hookrightarrow \{0,\ldots,2\cdot n\}, \rightarrow_\sim, \{\hat{v} \mid \mathrm{dom}\hat{v} = V^0\})\\
&P(\hat{v}) = \mathrm{dom}\hat{v}\\
&\mu(\hat{v})(v) = \hat{v}(v)\\
&Q = \{0,2\ldots,2\cdot n\}\\
&\hat{v} \xrightarrow{a}_\sim \hat{v}' \text{ iff}\\
&\quad \mathrm{dom}\hat{v}' = \{v' \mid \exists v \in \mathrm{dom}\hat{v} : v \xrightarrow{a} v'\} \text{ and}\\
&\quad \text{if } v \in \mathrm{dom}\tilde{v}, v' \in \mathrm{dom}\tilde{v}', \text{ and } v \xrightarrow{a} v', \text{ then}\\
&\quad \begin{cases} \tilde{v}(v) \geq \tilde{v}'(v'), \text{ and} \\ v' \in R \text{ implies } \tilde{v}(v) > \tilde{v}'(v') \text{ or } \tilde{v}(v) = \tilde{v}'(v') \in Q \end{cases}
\end{aligned}$$

Claim 2 There is a bijective correspondance between runs of $\widetilde{\mathbf{A}}$ over α and quasi co-Büchi measures of the form $(\mu, \{0,2,\ldots,2\cdot n\})$ for $(\mathcal{G}(\mathbf{A},\alpha),\neg R)$.

Proof By construction of $\widetilde{\mathbf{A}}$. □

The automaton $\mathcal{W}\widetilde{\mathbf{A}}(P,\mu,Q)$ accepts the complement of $L(\mathbf{A}R)$ as shown in Figure 1. Thus let $\overline{\mathbf{A}\,R} = \mathcal{W}\widetilde{\mathbf{A}}(P,\mu,Q)$. Then $\overline{\mathbf{A}\,R}$ has $2^{O(|\mathbf{A}|\cdot\log|\mathbf{A}|)}$ states. □

4 Rabin and Street Conditions

A *Rabin condition* C on V is a set $\{(R_\chi, I_\chi) \mid \chi \in X\}$ of pairs (R_χ, I_χ), where X is a finite set of *colors* and where the pair (R_χ, I_χ) of color χ consists of $R_\chi \subseteq V$, which is the set of *χ-reconfirming states*, and $I_\chi \subseteq V$, which is the set of *χ-invalidating states*. We say that an infinite sequence $v_0 v_1 \cdots$ of states *satisfies* C, and we write $v_0 v_1 \cdots \models C$, if there is a color χ such that for infinitely many k, $v_k \in R_\chi$, and for only finitely many k, $v_k \in I_\chi$.

A *Streett condition* $\neg C$ is the complement of a Rabin condition, i.e. $v_0 v_1 \cdots \models \neg C$ iff $v_0 v_1 \cdots \not\models C$.

For technical reasons, we always assume without loss of generality that $0 \in X$ and that $R_0 = I_0 = \emptyset$ (one can always add the pair $(\emptyset,\emptyset)$ without changing the semantics of satisfaction). The *size* $|C|$ of C is the number of pairs in C. Note that $|C| \geq 1$.

A *Streett automaton* $\mathbf{A}\neg C$ is an automaton $\mathbf{A}$ with a Streett condition $\neg C$. The *size* of $\mathbf{A}\neg C$ is $|\mathbf{A}| \cdot |C|$. This definition captures that each set $|C|$ can be represented using $|\mathbf{A}|$ bits.

A Streett automaton accepts α if there is a run over α satisfying $\neg C$. Let $C \times \omega$ denote the Rabin condition

$$\{(R\times\omega, I\times\omega) \mid (R,I) \in C\}.$$

In analogy with Büchi automata, we have:

Proposition 6 $\alpha \notin L(\mathbf{A}\neg C)$ iff $\mathcal{G}(\mathbf{A},\alpha) \models C\times\omega$.

5 Rabin Relation and Measure

To complement Street automata in the way we complement Büchi automata, we use the progress measure for Rabin conditions proposed in [KK91]. This measure is based on pointer trees, also called direction trees. A *pointer tree* T is a countable prefix-closed subset of ω_1^*, the set of finite sequences of countable ordinals. Each sequence $t = \langle t^1, \ldots, t^\ell \rangle$ in T represents a *node*, which has *children* $t \cdot \langle d \rangle \in T$, where "$\cdot$" denotes concatenation. Here $d \in \omega_1$ is the *pointer* to $t \cdot \langle d \rangle$ from t. If t' is a prefix of $t \in T$, denoted $t' \lesssim t$, then t' is called an *ancestor* of t. We visualize pointer trees as growing upwards; see Figure 2, where children are depicted from left to right in descending order. The *level* $|t|$ of a node $t = \langle t^1, \ldots, t^\ell \rangle$ is the number ℓ; the level of $\langle\rangle$ is 0. The *prefix up to level* λ

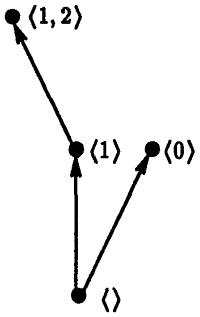


Figure 2: A pointer tree.

of $t = \langle t^1, \ldots, t^n \rangle$ is $\langle t^1, \ldots, t^{\min\{n,\lambda\}} \rangle$, denoted $t{\downarrow}\lambda$. The *height* of T is the maximum node level (if it exists).

Definition 6 (Kleene-Brouwer Ordering) The ordering $\succ$ on T is defined by: $t \succ t'$ if there is a λ such that $t{\downarrow}\lambda = t'{\downarrow}\lambda$ and either $\lambda = |t| < |t'|$ or $\lambda < |t|, |t'|$ and $t^{\lambda+1} > t'^{\lambda+1}$. The ordering $\succeq$ is defined as $t \succeq t'$ if $t \succ t'$ or $t = t'$.

This is a total order on T.

Lemma 1 (Kleene-Brouwer Ordering) If T is finite-path, then $\succ$ is well-ordered.

Rabin measures are based on colored pointer trees, which are defined by:

Definition 7 A *colored pointer tree* (T, ξ) is a pointer tree T with a partial mapping $\xi : T \hookrightarrow X$, where X is a set of *colors*, assigning a color $\xi(t) \in X$ to each node t in domξ. Let

$$\xi(\underset{\cdot}{\leq} t) = \{\xi(t') \mid t' \underset{\cdot}{\leq} t \text{ and } t' \in \mathrm{dom}\xi\}.$$

The following definition is slightly modified from [KK91]:

Definition 8 A *Rabin measure* (μ, T, ξ) for (G, C) is a mapping $\mu : V \to T$, where (T, ξ) is finite-path colored pointer tree, such that

(I) for all $v \in V$ and all $\chi \in \xi(\underset{\cdot}{\leq} \mu(v))$, $v \notin I_\chi$, and

(R) for all $(u, v) \in E$, $u \overset{\mu}{\underset{\mathrm{R}}{\rhd}} v$,

where

($\underset{\mathrm{R}}{\rhd}$) $u \overset{\mu}{\underset{\mathrm{R}}{\rhd}} v$ if $\mu(u) \succ \mu(v)$, or if there exists $\chi \in \xi(\underset{\cdot}{\leq} \mu(u) {\uparrow} \mu(v))$ such that $v \in R_\chi$.

Thus the value $\mu(v)$ of a Rabin progress measure μ denotes a list of colors such that (I) none of them are invalidating and such that (R) progress according to $\underset{\mathrm{R}}{\rhd}$ takes place across every edge.

Theorem 2 ([KK91]) $G \models C$ iff there is a Rabin progress measure for (G, C)

Proof "$\Leftarrow$" Let $v_0 v_1 \cdots$ be an infinite path in G. Then by (R), $v_0 \overset{\mu}{\underset{\mathrm{R}}{\rhd}} v_1 \overset{\mu}{\underset{\mathrm{R}}{\rhd}} \cdots$. It is not hard to see that there is a unique node $t \in T$ such that almost always $t \underset{\cdot}{\leq} \mu(v_k)$ and infinitely often $t = \mu(v_k) {\uparrow} \mu(v_{k+1})$. Let $\ell = |t|$. Suppose for a contradiction that there is no $\hat{t} \leq t$ such that $v_k \in R_{\xi(\hat{t})}$ infinitely often. Then by (R) and the definition of t, it can be seen that almost always, it holds that $\mu(v_k){\downarrow}(\ell + 1) \succeq \mu(v_{k+1}){\downarrow}(\ell + 1)$. Moreover, since $t = t_k \uparrow t_{k+1}$ holds infinitely often, $\mu(v_k){\downarrow}(\ell + 1) \succ \mu(v_{k+1}){\downarrow}(\ell + 1)$ holds infinitely often. Thus there exists a K such that $\mu(v_K) \downarrow (\ell + 1) \succeq \mu(v_{K+1}) \downarrow (\ell + 1) \succeq$, where infinitely many of the inequalities are strict. This contradicts the Kleene-Brouwer Ordering Lemma. Thus there is a $\hat{t} \underset{\cdot}{\leq} t$ such that $v_k \in R_{\xi(\hat{t})}$ infinitely often. By (I) and definition of t, it holds that $v_k \in I_{\xi(\hat{t})}$ only finitely often. Thus $v_0 v_1 \cdots \models (R_{\xi(\hat{t})}, I_{\xi(\hat{t})})$.

"$\Rightarrow$" See [KK91]. □

6 Complementation of Street automata

The constructions in this section are analogous to those of Section 3. The main difficulties of representing a Rabin progress measure and the underlying Kleene-Brouwer ordering in a compact way are addressed in the following.

Definition 9 A *quasi Rabin measure* $(\mu, T, \xi, T \cdot \langle Q \rangle)$ for (G, C) consists of a colored pointer tree (T, ξ), a set of *quiescent pointers* Q, and a mapping $\mu : V \rightarrow T \cup T \cdot \langle Q \rangle$ such that[3]

- $v \notin I_\chi$, for all $v \in V$ and $\chi \in \xi(\underset{\cdot}{\leq} \mu(v))$; and
- (u, v) implies either $u \overset{\mu}{\underset{\mathrm{R}}{\rhd}} v$ or $\mu(u) = \mu(v) = t \cdot \langle q \rangle$, where $t \in T$ and $q \in Q$.

Note that a quasi Rabin measure μ is well-founded only if there is no infinite path $v_0 v_1 \cdots$ in G such that $\mu(v_0) = \mu(v_1) = \cdots \in T \cdot \langle Q \rangle$.

Proposition 7 $G \models C$ iff (G, C) has a well-founded quasi Rabin measure.

Definition 10 Two pointer trees T and T' are *isomorphic* if there is a mapping $f : T \rightarrow T'$ that is a tree isomorphism (a root-preserving undirected graph isomorphism) and that is congruent with the Kleene-Brouwer ordering, i.e. $t \succ t'$ implies $f(t) \succ f(t')$.

Proposition 8 If $\mathcal{G}(\mathbf{A}, \alpha) \models C \times \omega$, then $(\mathcal{G}(\mathbf{A}, \alpha), C \times \omega)$ has a well-founded Rabin quasi measure $(\tilde{\mu}, T, \xi, T \cdot \langle Q \rangle)$, where T has at most $|\mathbf{A}|$ leaves and $|Q|$ is at most $|\mathbf{A}| + 1$.

Proof Assume that $\mathcal{G}(\mathbf{A}, \alpha) \models C \times \omega$. By Theorem 2 there is a Rabin measure (μ, T, ξ) for $(\mathcal{G}(\mathbf{A}, \alpha), C \times \omega)$. Define the predicate *Const*(t) to be true if there is a path $v_0 v_1 \cdots$ in $\mathcal{G}(\mathbf{A}, \alpha)$ such that t is almost always an ancestor of $\mu(v_i)$ and there is no $t' \underset{\cdot}{>} t$ with this property. Let $T_{\mathrm{fix}} = \{t_1, t_2, \ldots\}$, $t_1 \prec t_2 \prec \cdots$, be the set of t such that *Const*(t). Two paths corresponding to t_i and t_j, $i \neq j$, must be disjoint because t_i and t_j are not comparable with respect to the ancestor ordering and the values of μ are descendants of t_i and t_j, respectively. Thus we can use the Pigeon Hole Principle to show that $|T_{\mathrm{fix}}| \leq n$.

We can view T_{fix} as a tree that form the basis for approximating progress values. To do this approximation we map progress values into a canonical image T of T_{fix}. Define

$$d(t, \ell) = 2 \cdot \min\{i \mid t_i {\downarrow} \ell = t {\downarrow} \ell\} - 1$$
$$h(t) = \langle d(t, 1), \ldots, d(t, |t|) \rangle$$

This defines a mapping $h : T_{\mathrm{fix}} \rightarrow \{1, 3, \ldots, 2 \cdot n - 1\}^*$, which can be seen to be a pointer tree isomorphism.

For an example consider $T_{\mathrm{fix}} = \{t_1, t_2\}$, where $t_2 \succ t_1$, $|t_2| = 5$, $|t_1| = 4$, and $|t_2 \uparrow t_1| = 2$. The image of T_{fix} under h is shown in Figure 3 (thick arrows and solid nodes). The canonical representation is constructed so that nodes of the form $\tilde{t} \cdot \langle q \rangle$ can approximate values in between the leaves of T_{fix}; here $Q = \{0, 2, \ldots, 2 \cdot n\}$ is the set of quiescent pointers. Define $\tilde{\mu}$ by

$$\tilde{\mu}(v) = \begin{cases} h(\mu(v)) & \text{if } \mu(v) \in T_{\mathrm{fix}} \\ h(t_i) \cdot \langle 2 \cdot i \rangle & \text{if } \mu(v) \succ t_i \\ h(t_i) \cdot \langle 2 \cdot (i-1) \rangle & \text{if } t_i \succ \mu(v) \end{cases}$$

where i is the least value such that $\ell = |t_i \uparrow \mu(v)|$ is maximal.

Claim 3 $\tilde{\mu}(v) \neq \tilde{\mu}(v')$ implies

$\tilde{\mu}(v) \succ \tilde{\mu}(v')$ iff $\mu(v) \succ \mu(v')$, and
$|\tilde{\mu}(v) \uparrow \tilde{\mu}(v')| = |\mu(v) \uparrow \mu(v')|$

From Claim 3 it can be seen that $\tilde{\mu}$ is a well-founded quasi Rabin measure.

In the example above, the quiescent pointer values are gotten by appending 0, 2, or 4 to a node in the canonical tree. Figure 3 depicts these values by light arrows and circles. □

Proposition 9 A canonic pointer tree can be represented in $O(|\mathbf{A}| \cdot |C|)$ bits.

Our main result is:

Theorem 3 Given a Street automaton $\mathbf{A}\neg C$, there is a Rabin automaton $\overline{\mathbf{A}}\,\overline{C}$ accepting the complement of $L(\mathbf{A}\neg C)$. Moreover,

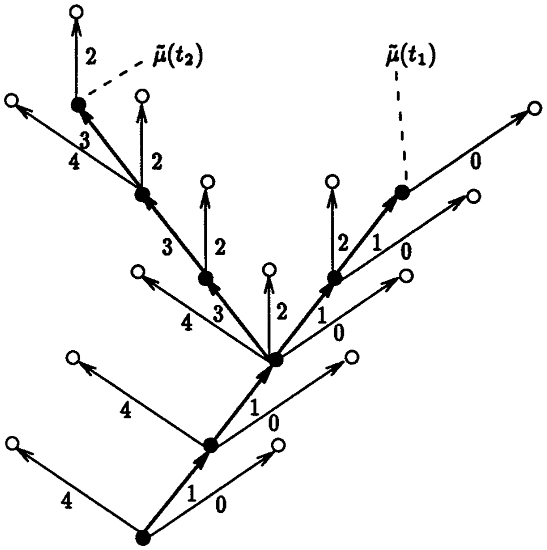
$$|\overline{\mathbf{A}}| = 2^{O(n \cdot (\log n + \log c))} \leq 2^{O(|\mathbf{A}\neg C| \cdot \log |\mathbf{A}\neg C|)} \text{ and}$$
$$|\overline{C}| = 2^{O(n \cdot c \cdot \log c)} \leq 2^{O(|\mathbf{A}\neg C| \cdot \log |\mathbf{A}\neg C|)} ,$$

where $n = |\mathbf{A}|$ and $c = |C|$. Thus, $|\overline{\mathbf{A}\neg C}| = 2^{O(|\mathbf{A}\neg \mathbf{C}| \cdot \log |\mathbf{A}\neg \mathbf{C}|)}$.

Proof Automaton $\overline{\mathbf{A}}\neg\overline{C}$ is the result of applying Proposition 1 to an automaton that guesses the values of the progress measures. $\overline{\mathbf{A}}\neg\overline{C}$ works as follows.

1. Initially $\overline{\mathbf{A}}\neg\overline{C}$ guesses the shape of T_{fix}. By Propositions 8 and 9, this choice can be represented in $O(n \cdot \log c)$ bits.

[3] $T \cdot \langle Q \rangle$ is the set of $t \cdot \langle q \rangle$, where $t \in T$ and $q \in Q$.



$\tilde{\mu}(t_1) = \langle 1, 1, 1, 1 \rangle$
$\tilde{\mu}(t_2) = \langle 1, 1, 3, 3, 3 \rangle$
$Q = \{0, 2, 4\}$

Figure 3: Canonic image T of a pointer tree T_{ex}.

2. At any point $\overline{\mathrm{A}\neg C}$ guesses the values of the progress measure for each state of A in the current subset. By Proposition 8 each progress value $\tilde{\mu}(v)$ in T_{fix} can be specified by a level and by a leaf of which $\tilde{\mu}(v)$ is an ancestor; in addition, there may be one out of $n+1$ quiescent values appended. Thus to specify $\tilde{\mu}$ for all states in the current subset requires $O(n \cdot (\log c + \log n))$ bits.

3. By guesswork $\overline{\mathrm{A}\neg C}$ also identifies the common ancestors that are needed to satisfy $\underset{\mathrm{R}}{\trianglerighteq}$ for all transitions between an old subset and a new subset. This also requires $O(n \cdot (\log c + \log n))$ bits.

4. The acceptance condition contains a Rabin pair $(\overline{R}, \overline{I}_{\chi})$ for each possible coloring χ of T_{fix}. $\overline{R}$ is the Büchi condition that arises from Proposition 1. $\overline{I}_{\chi}$ is a condition that is false only if

   - the $\underset{\mathrm{R}}{\trianglerighteq}$ relation is satisfied according to coloring χ of T_{fix} and the common ancestors identified in Step 3; and
   - for all v in current subset and for all χ described by $\tilde{\mu}(v)$ according to ξ, $v \notin I_{\chi}$.

□

7 Applications to Temporal Logic

In the full paper we discuss temporal logics based on automata connectives. We show how our complementation result for Street automata gives an exponential time decision procedure for the logic ETL_S. Our exponent is only $n \cdot \log n$, which is a significant improvement over the n^5 exponent obtained by Safra and Vardi [SV89].

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