# Empirical Performance Investigation of a Büchi Complementation Construction

Daniel Weibel

August 3, 2015

**Abstract**

This will be the abstract.

**Acknowledgements**

# Contents

# Chapter 1

# The Fribourg Construction

## Contents

In this chapter we present the Fribourg construction. The Fribourg constructio is the Büchi complementation consruction whose empirical investigation of performance makes up the core of this thesis (Chapters **??** and **??**). The Fribourg construction belongs to the *slice-based* complementation approach that we reviewed in Section **??**. This means that it is based on reduced split trees, which we explained in Section **??**. Of the two versions of reduced split trees, left-to-right and right-to-left, our description of the Fribourg construction in this chapter uses right-to-left reduced split trees.

The Fribourg construction is being developed at the University of Fribourg by Joel Allred and Ulrich Ultes-Nitsche[1]. A detailed description of the construction has been published internally in 2014 as a technical report entitled "Complementing Büchi Automata with a Subset-tuple Construction" [1].

The aim of this chapter is not to give a formally precise description of the Fribourg construction. This has been done already by its authors in [1]. The same applies for proofs of correctness and calculations of complexity. Rather, the aim of this chapter is to describe the construction in an intuitive and practically oriented way. We want to convey the concrete steps to take when, figuratively, standing with a marker in front of a whiteboard with an automaton to complement. For the formal background we refer to [1].

## 1.1 The Construction

In this section we explain the basic Fribourg construction without any applied optimisations. The construction consists of two sub-constructions that we call the upper-part construction and the lower-part construction. In the following we first describe the parts of the construction that are common to these two sub-construction (Section 1.1.1). Then, in Section 1.1.2 and 1.1.3, we describe the points that are specific to the upper-part construction and the lower-part construction, respectively. In these two subsections, we also illustrate the application of the construction with a simple example that we work through step by step from the initial input automaton to the final complement automaton.

### 1.1.1 Basics

Below we first give a high-level view of the construction, and then explain how the construction generates new states. The generation of states is the heart of the construction, and common to the two sub-constructions that we explain in detail in the subsequent two subsections.

#### High-Level View

As mentioned, the Fribourg construction consists of two sub-constructions, the *upper-part construction* and the *lower-part construction*. These two sub-constructions are chained together, such that the output of the upper-part construction becomes the input of the lower-part construction. Figure 1.1 illustrates this in a functional view of the Fribourg construction.

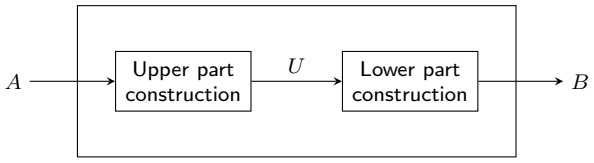



Figure 1.1: Functional view of the Fribourg construction with an automaton $A$ as input and its complement $B$ as output.

The automaton $A$ to be complemented is the input for the upper-part construction. The upper-part construction builds up a new automaton by starting with an initial state and adding state by state. The output of the upper part constructio is an intermediate automaton $U$. This intermediate automaton $U$ will be the *upper part* of the final complement automaton. The lower-part construction takes $U$ as

[1] As mentioned, the authors call their construction *subset-tuple construction*, in this thesis we will however use the name *Fribourg construction*.

input and adds further states and transitions to it until finally outputting an automaton $B$ which is the complement of the input automaton $A$. We say that the lower-part construction attaches the *lower part* of the final complement automaton to the upper part. This terminology comes from the fact that it is convenient to draw the states of the lower part below the states of the upper part [1]. This also explains why the two sub-constructions are calleed upper-part construction and lower-part construction.

**Generation of States**

The two sub-constructions can be seen as modified subset constructions. Like the subset construction, they work on a set of existing states and add to each state one sucessor for each symbol of the alphabet. The principle of how successor states are generated is the same for both sub-construction. The difference between the two is that the lower-part construction adds additional information to the states (the colours), so that the states generated by the lower-part construction are different from the states generated by the upper-part construction. Below we explain this basic principle of how new states are generated.

A state of the Fribourg consists of a tuple (that is, an ordered sequence) of subsets of states of the input automaton (note that we will from now on refer to the states of the input autmaton as the *input-states*, and to the states of the output automaton as *output-states*). This contrasts with the subset construction in which the output-states consist of a single subset of input-states. The states of the Fribourg construction are thus more structured than the states of the subset construction. We refer to the subsets of a tuple of a state as the *components* of this state.

The sequence of components of each state of the Fribourg construction is imposed by a slice (that is, a level) of a reduced split tree. We will explain the process of going from an existing state over the slices of a reduced split tree to a successor state further below. For now, we look at how a slice defines the structure of a state. Figure 1.2 shows a reduced split tree (on the left), and for each slice (framed by a shaded box), the state that this slice would define (on the right).

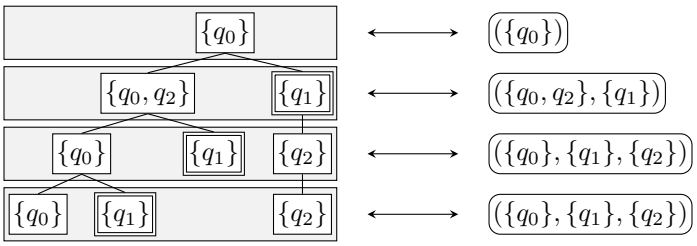



Figure 1.2: Relation between slices of a reduced split tree (left) and states of the output automaton of the Fribourg construction (right). The sequence of nodes in a slice determines the sequence of components in a state.

As can be seen in Figure 1.2, each node of a slice gives rise to a component in the corresponding state. To this end, it does not matter whether the nodes in the slices are siblings or whather their familial relations are. It is important to note, however, that the order of the nodes in a slice must be preserved in the order of the components in a state. It is similarly possible to interpret a state as a slice of a reduced split tree (indicated by the double-ended arrows in Figure 1.2). In this case, it is not possible to deduce the ancestors of the resulting nodes, however, this is not necessary for the construction.

The procedure of determing the successor of a given existing state on a certain alphabet symbol is as follows:

- Interpret the existing state as a slice of a reduced split tree
- Create the subsequent slice for the appropriate alphabet symbol
- Interpret the new slice as a state

The state that results from thew new slice is the successor of the existing state for the given alphabet symbol. If, for example, we want to define the successor on the symbol $a$ ($a$-successor) of the state ($\{q_0\}$), then as the first step we would create a slice representation of it that looks like the first slice from the top in Figure 1.2. Then, we create based on the input automaton the subsequent slice for symbol $a$, that

might look like the second slice from the top in Figure 1.2. Interpreting this slice as a state results in $(\{q_0\}2, \{q_1\})$, which thus is the $a$-successor of $\{q_0\}$.

Note that the translation of states to and from slices is only a mental aid and not a necessary ingredient. It is possible to deduce $(\{q_0\}2, \{q_1\})$ from $\{q_0\}$, in our previous example, directly without the detour over slices of a reduced split tree. However, by conceptually using reduced split trees, we can cover the central points of the state generation process by re-using our knowledge of the previously known and independent concept of reduced split trees. This includes the splitting of accepting and non-accepting input-states, the placement of the accepting subset to the right of the non-accepting subset, and the retention of only the first occurrence of an input-state from right to left.

### 1.1.2 Upper-Part Construction

In this subsection, we describe the upper-part construction, and demontrate its application with an example. The used example is based on the input automaton in Figure 1.3, and we will continue the same example in the subsequent subsection about the lower-part construction.

#### Description

The upper-part construction consists only of the generation of new states without any further features. It starts with an inital state containing a single component with the inital state of the input automaton. This first output-state will be the initial state of the final complement automaton. Similarly to the subset construction, each unprocessed state is then processed, which means that, for each symbol of the alphabet, a successor state is determined. This works according to the state generation process that we described in the last section. If the automaton already contains a state that is identical to a newly determined successor state, then only a transition to this existing state is added. If a state has no successor state for a given alphabet symbol, then nothing is added to the automaton. This process is repeated until all states have been processed. At this stage, a deterministic intermediate automaton without any accepting states has been produced. This intermediate automaton is the upper part of the final complement automaton.

A peculiarity is that the upper part must be complete. Completeness of an automaton means that every state has at least one outgoing transition for every symbol of the alphabet. If at the end of the upper-part construction the resulting automaton is not complete, then it must be made complete by adding a sink state. A sink state is a state with a loop transition for every alphabet symbol, and for every incomplete state, the missing transitions are added from these states to the sink state. This sink state that is added to the upper part must furthermore be accepting.

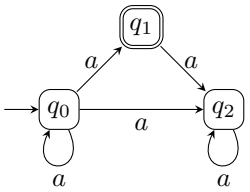



Figure 1.3: Non-deterministic Büchi automaton used for the examples in this section.

#### Example

We demonstrate the application of the upper part construction with the example automaton $A$ in Figure 1.3. Note that this automaton has an alphabet size of one, and thus can run only on a single $\omega$-word, namely $a^\omega$. Furthermore, the automaton does not accept this word, that is, it does not accept *any* word, which means that it is empty. We selected this specific automaton to keep our example simple and small. However, the construction works exactly similar for non-empty automata with larger alphabets.

The upper-part construction for the input automaotn $A$ from Figure 1.3 takes four steps. We show all these steps in Figure 1.4 (a) to (d). Below, we walk through them one by one.

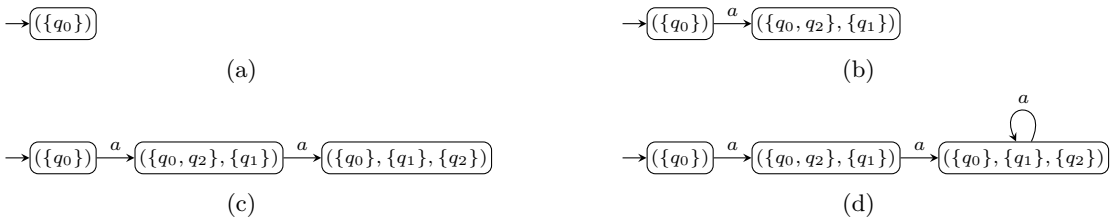
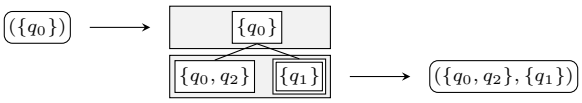



Figure 1.4: Steps of the upper-part construction based on the input automaton in Figure 1.3.

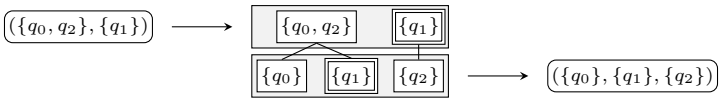
The first step in Figure 1.4 (a) is to start with the initial state. As can be seen, this state ($\{q_0\}$) contains only one component with the initial state of the input automaton $A$. This state will be the initial state of the final complement automaton $B$.

The next step Figure 1.4 (b) is to add the $a$-successor of the inital state ($\{q_0\}$). This is done with the state generation process that we described in the last section. In particular, this means: (1) interpret the state ($\{q_0\}$) as a slice of a reduced split tree, (2) based on the input automaton $A$, determine the slice of the next level of the tree, and (3) interpret this new slice as a state, which is the $a$-successor of ($\{q_0\}$). We can illustrate this steps in the following way:



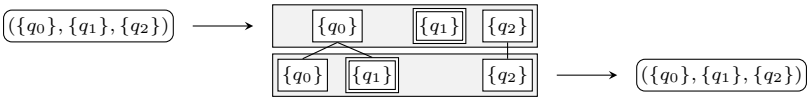
The state ($\{q_0\}$) results in a slice with only one node containing $\{q_0\}$. For creating the next slice, we determine the set of states of $A$ that can be reached from $\{q_0\}$ on symbol $a$, which is $\{q_0, q_1, q_2\}$. According to the rules of reduced split trees, since $q_1$ is an accepting state, this set is split into a non-accepting $\{q_0, q_2\}$, and an accepting $\{q_1\}$, and the non-accepting set becomes the left child, and the accepting set becomes the right child of the current node. Translating the resulting new slice to a state results in ($\{q_0, q_2\}, \{q_1\}$), which is thus the $a$-successor of ($\{q_0\}$).

The next step in Figure 1.4 (c) is to determine the $a$-successor the newly created state ($\{q_0, q_2\}, \{q_1\}$). Applying the same procedure as above, results in the following picture:



The state ($\{q_0, q_2\}, \{q_1\}$) corresponds to a slice with two nodes. For creating the next slice, these nodes must be processed from right to left. Regarding the first node from the right, the set of states that is reachable from $\{q_1\}$ in $A$ is $\{q_2\}$, which becomes thus the only child of this node. Regarding the second node, the state set reachable from $\{q_0, q_2\}$ is $\{q_0, q_1, q_2\}$. However, $q_2$ already exists to the right in the new slice, thus it is removed from the set. The resulting set $\{q_0, q_1\}$ is split into a non-accepting $\{q_0\}$ and an accepting $\{q_1\}$, which become the left child and right child of this node, respectively. The resulting slice corresponds to the state ($\{q_0\}, \{q_1\}, \{q_2\}$), which consequently becomes the $a$-successor of ($\{q_0, q_2\}, \{q_1\}$).

The last step in Figure 1.4 (d) consists in determining the $a$-successor of ($\{q_0\}, \{q_1\}, \{q_2\}$). The state generation procedure in this case looks as follows:



The state results in a slice with three nodes. Again, we have to process these nodes from right to left. The node with $\{q_2\}$ has non-accepting $\{q_2\}$ as its only child. Regarding the node with $\{q_1\}$, the reachable set in $A$ would be $\{q_2\}$, but since $q_2$ already exists to the right in the slice, it is omitted, and since this makes the reachable set empty, no child of this node is added to the new slice. Finally, regarding the last node, the set of states reachable from $\{q_0\}$ is $\{q_0, q_1, q_2\}$, but again, due to the omission of $q_2$ and

the splitting into a non-accepting and an accepting set, this results in the accepting right child $\{q_1\}$ and the non-accepting left child $\{q_0\}$. The state corresponding to this slice is $(\{q_0\}, \{q_1\}, \{q_2\})$. It is identical to the state we are currently processing, and thus, we just add a transition looping back to itself to $(\{q_0\}, \{q_1\}, \{q_2\})$.

At this stage, all the states in the intermediate automaton have been processed. Since the resulting automaton is complete, we do not need to add a sink state to it. This means that the uppe-part construction is finished.

Note that in this example, we had to determine only one successor for each state, because there is only one symbol in the alphabet of our example automaton. For input automata with larger alphabets, the procedure that we did for each state must be repeated for each symbol of the alphabet. This increases the number of required steps, however, the basic principles stay exactly the same.

### 1.1.3 Lower-Part Construction

The lower-part construction takes the intermediate automaton resulting from the upper-part construction as input, and adds additional states to it (the lower part of the output automaton). The lower-part construction generates new states basically in the same way as the upper-part construction, however, it adds an additional piece of information to each component of a newly created state, namely its colour. There are three colours, that we denote by 0, 1, and 2, that are assigned to components by the lower-part construction. Additionally, there is the colour $-1$ that is preliminarily assigned to each component of each state of the upper part. This serves to conveniently handle the colour assignment function of the lower-part construction. These colours make states different, that is, if two states have the same sequence of components, but the components have different colours, then the two states are regarded as different states.

The addition of colour assignments is the only difference to the basic state generation procedure that is used in the upper-part construction. In the following, we first describe how the colours are assigned, and after that, give some insight into what the colours actually mean.

1. Whether the component is accepting or non-accepting
2. The colour of the predecessor component
3. Whether the state containing the predecessor component contains any 2-coloured components

The construction of the lower part takes the states of the previously constructed upper part as input. This means that these states are taken as the initial "states to be processed". Thus, the states of the upper part will get additional successors in the lower part, which makes the states of the upper part non-deterministic[2]. The states of the lower part, in turn, are deterministic, and do not have transitions back to the upper part. This means that once a run switches from the upper to the lower part, it stays there infinitely (or dies there).

The construction of the lower part proceeds principially in the same way as the construction of the upper part. However, it includes some additional "structure" in the form of a decoration of the components. In particular, every component of a lower-part state is assigned a colour. This colour is determined at creation time of the containing state, and is never changed[3]. These colours are distinguising features for the containing states. This means that if two states contain the same components in the same order, but the components have different colours, then the two states are different. Clearly, this makes the number of possible states of the lower part much larger than the number of possible states of the upper part.

We denote these three colours for the components of the lower part by 0, 1 and 2. In order to determine the accepting set at the end of the construction, it is necessary to ditinguish the states from the upper part from the states of the lower part. This problem is solved by previously assigning the special colour $-1$ to all components of upper part-states.

The assignment of a colour to a component of a lower-part state depends on three things:

[2] Two points about this non-determinism are interesting. First, the states of the upper part are the only non-deterministic states, all the other states are deterministic. Second, the degree of non-determinism of these states is at most 2, and thus the degree of non-determinism of the entire complement automaton is at most 2.

[3] The colour of a component may be changed with the optimisations described in Section 1.2.

1. Whether the component is accepting or non-accepting
2. The colour of the predecessor component
3. Whether the state containing the predecessor component contains any 2-coloured components

How much to describe the concept of predecessor component, as it is already described in Section 1.1.1?

The rules for assigning one of the colours 0, 1, and 2 to a component $c$ are shown in Figure 1.5. There are two different sets of rules for the cases that the state containing the predecessor component *does* (Figure 1.5 (b)), or *does not* ((Figure 1.5 (a)) contain components with colour 2. In each of these cases, the two remaining criteria, whether $c$ is accepting or non-accepting, and the colour of the predecessor component $c_{pred}$, determine a single colour that must be assigned to component $c$ (bold in Figure 1.5).

| Colour of $c_{pred}$ | $c$ is non-accepting | $c$ is accepting |
|---|---|---|
| $-1$ | **0** | **2** |
| 0 | **0** | **2** |
| 1 | **2** | **2** |

(a) Case A: the predecessor state has *no* 2-coloured components

| Colour of $c_{pred}$ | $c$ is non-accepting | $c$ is accepting |
|---|---|---|
| 0 | **0** | **1** |
| 1 | **1** | **1** |
| 2 | **2** | **2** |

(b) Case B: the predecessor state *has* 2-coloured components

Figure 1.5: Rules for determining the colour of a component $c$, based on (1) the colour of the predecessor component $c_{pred}$, and (2) whether $c$ is an accepting or non-accepting component. There are two set of rules that are shown in the two subfigures: (a) the predecessor state does not have any components with colour 2, and (b) the predecessor state does have one or more components with colour 2.

In the first case, that the predecessor state contains no 2-coloured components, the possible colours of the predecessor components are $-1$, 0, and 1. Naturally, the predecessor component cannot be 2-coloured, but on the other hand, it might have colour $-1$, if the predecessor state is a state of the upper part. For the other case, that the predecessor state contains 2-coloured components, the possible colours for the predecessor component naturally include colour 2, but do not include colour $-1$, because a state containing 2-coloured components cannot be a state of the upper part.

The purpose of the colours is to signalise the presence or absence of certain runs of the input automaton on a specific word. Note that the complement of a non-deterministic automaton must accept a word if and only if *all* the runs of the input automaton on this words are rejecting. If there is a single run of the input automaton that accepts the word, then the complement automaton must not accept the word. Thus, we need a way to be sure that there are *no* accepting runs of the input automaton on a specific word, and then we can accept this word with the complement automaton.

**Colour 2** Is used to signalise the presence of "dangerous" runs, that is, runs that have the potential to becom accepting. If a component has colour 2, it means that there are input-runs that made a "right-turn", in terms of slices of reduced split trees, that is, visited an accepting state (see rules in Figure 1.5 (a) line 1 and 2). Note that all the successor components of a 2-coloured component are also 2-coloured, no matter if they are accepting or non-accepting (Figure 1.5 (b) line 3). A "string" of 2-coloured components can only be cut if a 2-coloured component has no successor components (no children in terms of reduced split trees). In this case we say that the 2-coloured component "disappears".

**Colour 1** Means basically the same as colour 2, namely that there are "dangerous" runs. The reason that colour 1 exists is a caveat that could arise if we would assign colour 2 to *every* component that just made a right-turn. In this case, it could happen that we miss the disappearance of 2-coloured components, and thus keep the containing output-state non-accepting instead of accepting. For this

reason, the trick with colour 1 works as follows. If the predecessor state already contains 2-coloured components, then every component of the current state that deserves to be 2-coloured gets colour 1 instead of colour 2. This can be seen in Figure 1.5 (b) line 1 and 2. These are actually "dangerous" components kept "on hold", becaus then trick goes on as follows. As soon as all the 2-coloured components of a state disappear, the successors of all the 1-coloured components get 2-coloured. This can be seen in Figure 1.5 (a) line 3. At this point, these "dangerous" components "on hold" get the real "dangerous" components.

**Colour 0** Means the absence of "dangerous" runs. This is because the corresponding input-run did not make any right-turns, that is, they went only through non-accepting states. These runs are "safe" in the sense that so far they bear no risk of becoming accepting runs.

Wrapping up, for constructing the lower part of the output automaton, one just has to apply the same successor creation procedure as for the upper part, with the addition of assigning colours to the new states' components according to the rules in Figure 1.5. In the end, when every state has been processed and the procedure ends, the only thing that is missing is to determine the accepting states of the resulting automaton. The rule is that every state of the lower part that does *not* contain any 2-coloured components is an accepting states. That means that every state of the automaton that contains exclusively 0-coloured and/or 1-coloured components is an accepting state.

In the following, we demonstrate the application of the lower-part construction by an example.

### Example

We continue the complementation of the example automaton in Figure **??** with the construction of the lower part. Therefore, we start where we left off the example in the last section, namely with the upper part. For this example, we will use the following notation for specifying the colour of a component $\{q\}$:

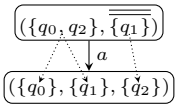
- $\widehat{\{q\}}$: colour $-1$
- $\{q\}$: colour 0
- $\overline{\{q\}}$: colour 1
- $\overline{\overline{\{q\}}}$: colour 2

Figure 1.6 shows some of the steps of the construction of the lower part. In Figure 1.6 (a), we start with the upper part that we previously constructed in the last section. The only difference is that we assigned colour $-1$ to all of the components of the upper part.

In Figure 1.6 (b), we created the $a$-successors of the three states of the upper part. The structure of these new states, apart from the colours, is determined by the same method that we used for the upper part. The only difference in the construction of the lower part is that each component is assigned a colour. For both new states, the predecessor states do not contain any 2-coloured components, thus we only need to consider the colour rules in Figure 1.5 (a). Regarding the colour of the predecessor components, they all have colour $-1$, thus we have to use the rule in Figure 1.5 (a) line 1 for all the new components. In this way, the components $\{q_0, q_2\}$, $\{q_0\}$, and $\{q_2\}$ are assigned colour 0, because they are non-accepting, and component $\{q_1\}$ gets colour 2, because it is accepting.

Note how we have to keep track for each component of the lower part whether it is accepting or non-accepting, and which is its predecessor component in the predecessor state.

In Figure 1.6 (c), we added the $a$-successor to the state $(\{q_0, q_2\}, \overline{\overline{\{q_1\}}})$. Disregarding the colours, this state has the form $(\{q_0\}, \{q_1\}, \{q_2\})$. Its predecessor state $(\{q_0, q_2\}, \overline{\overline{\{q_1\}}})$ contains a 2-coloured component, thus we have to use the colour rules in Figure 1.5 (b). Now we need to know which are the predecessor components of the components in $(\{q_0\}, \{q_1\}, \{q_2\})$. This information is contained in the two slices of the reduced split tree that were used to determine the structure of the new state. For the case of our two states, the successor relation of their components is as follows:



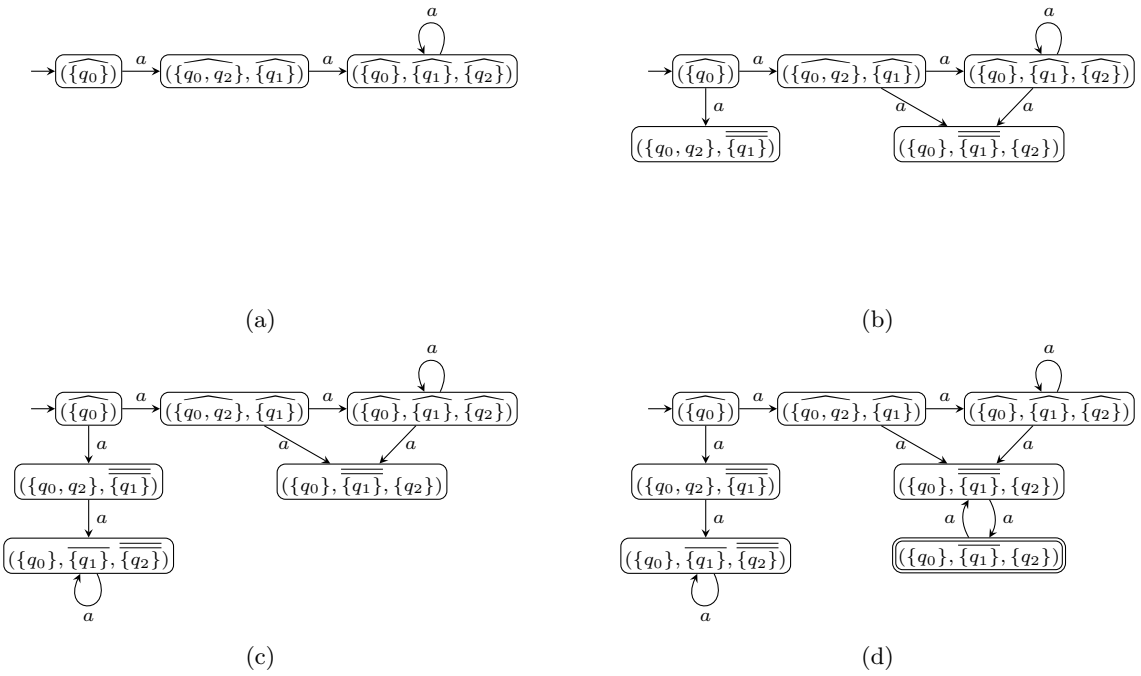
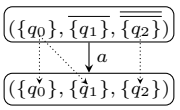



Figure 1.6: Selected steps of the construction of the lower part, starting with the upper part

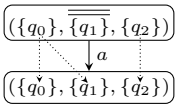
The predecessor component of $\{q_0\}$ is $\{q_0, q_2\}$, which has colour 0. Thus, we have to use the rule in Figure 1.5 (b) line 1, and $\{q_0\}$ gets the colour 0, because it is non-accepting. The predecessor component of $\{q_1\}$ is also $\{q_0, q_2\}$, and we have to use the same rule. However, since $\{q_1\}$ is accepting, it gets colour 1. Note how the use of colour 1 here prevents the introduction of a further 2-coloured component before an already existing 2-coloured component has disappeared. The predecesor component of $\{q_2\}$ is the 2-coloured $\overline{\overline{\{q_1\}}}$, and thus, according to the rule in Figure 1.5 (b) line 3, $\{q_2\}$ also gets colour 2.

Next, still in Figure 1.6 (c), we create in turn the successor state of $(\{q_0\}, \overline{\{q_1\}}, \overline{\overline{\{q_2\}}})$. The structure of the components stays the same for the successor. The succesor relation of the two states is as follows:



The result is that $\{q_0\}$ gets colour 0, $\{q_1\}$ gets colour 1, and $\{q_2\}$ gets colour 2. Thus, the successor state is identical to the current state, and we add loop.

Figure 1.6 (d) includes the remaining for arriving at the final complement automaton. First, we created the $a$-successor of the state $(\{q_0\}, \overline{\overline{\{q_1\}}}, \{q_2\})$. The complement successor relation of this state with its successor is as follows:



According to the rules in Figure 1.5 (b), component $\{q_0\}$ gets colour 0, $\{q_1\}$ gets colour 1, and $\{q_2\}$ gets colour 0. This results in a new state, since the colours are different from the ones in the current state. Interesting here is that we have the case that a 2-coloured component "disappears". This is because the component $\overline{\overline{\{q_1\}}}$ has no successor component in the successor state. For the $a$-successor of the new state $(\{q_0\}, \overline{\{q_1\}}, \{q_2\})$, this means in turn that we have to use the rules in Figure 1.5 (a), what results in the already existing state $(\{q_0\}, \overline{\overline{\{q_1\}}}, \{q_2\})$.

At this point, all the states in the automaton have been processed, and the construction is therefore completed. The only thing that remains to be done is to determine the accepting states of the automaton.

The rule is that each state of the lower part that does not contain any 2-coloured component is an accepting state. In our automaton this applies only to the state $(\{q_0\}, \overline{\{q_1\}}, \{q_2\})$, which is thus the only accepting state of the automaton.

It can be easily seen that the automaton in Figure 1.6 accepts the word $a^\omega$, which is not accepted by the input automaton in Figure **??**. Thus, the result of our construction is a correct complement of the input automaton.

A loose upper bound for the worst-case state complexity of the entire construction (including the construction of both, the upper and the lower part) has been calculated to be in $O((1.59n)^n)$, where $n$ is the number of states of the input automaton [1]. This result is subject to refinement in ongoing research by the authors of the Fribourg construction. As mentioned, this publication ([1]) also contains a formal desciption and a proof of correctness of the construction.

## 1.2 Optimisations

There are several optimisations to the basic Fribourg construction, which all have the goal to reduce the size of the output automaton. These optimisations are like "add-ons" and can be added to the basic construction and combined in different ways.

In this section, we describe three of these optimisations. For easier reference, we define an abbreviation for each optimisation: R2C, M1, and M2. These optimisations will also be subject to the empirical performance investigation of the Fribourg construction, starting from the next chapter, and there we will use the same abbreviations.

### 1.2.1 R2C: Remove States with Rightmost 2-Coloured Components

The R2C optimisation can be summarised as follows:

> If the input automaton is complete, then states of the lower part that have a rightmost component with colour 2 can be omitted.

This means that if during the construction of the lower part we determine a state that has a 2-coloured rightmost component, then we do not need to add this state to the automaton. This consequently also omits all the potential successors of this state from the automaton.

The reason for this is the following fact. If the input automaton is complete, then the rightmost component of an output-state *always* has at least one successor component (because of the completeness of the input automaton). However, this applies only to the rightmost component, as the successors of the other components might be omitted, due to the right-to-left precedence in right-to-left reduced split trees.

If the colour of the rightmost component is 2, then the successor component inherits this colour 2 (Figure 1.5 (b) line 3). This means inductively that all the successors of a state with a 2-coloured rightmost component will have a 2-coloured rightmost component. Or in other words, there is a 2-coloured component that will never disappear. Since states containing a 2-coloured component are non-accepting, these states form a cycle without a single accepting state, and this cycle can be removed without changing the language of the automaton.

Note, however, that the omission of these states can only be done if the input automaton is complete.

### 1.2.2 M1: Merge Components

The M1 optimisation allows the merging of adjacent components depending on their colour. With the merging of adjacent components, we mean the replacement of these components with their union (if regarding components as sets). The three merging rules are as follows.

1. Two adjacent 1-coloured components can be merged to a single 1-coloured component
2. Two adjacent 2-coloured components can be merged to a single 2-coloured component

3. A 2-coloured component adjacent to a 1-coloured component (in this order from left to right) can be merged to a single 2-coloured component

These mergings can be done recursively. This means that the result of a merging can again be subject to further merging, until nothing more can be merged. For example, the state $(\{q_0\}, \overline{\overline{\{q_1\}}}, \overline{\overline{\{q_2\}}}, \overline{\{q_3\}}, \overline{\{q_4\}}, \overline{\overline{\{q_5\}}})$ can be transformed to $(\{q_0\}, \overline{\overline{\{q_1, q_2, q_3, q_4, q_5\}}}$, by the recursively applying the above three merging rules.

Consequent applying of these mergings reduces the maximal number of states in the output automaton. An upper bound on the number of states in the lower part of the automaton has been calculated to be in $O((1.195n)^n)$, where $n$ is the number of states of the input automaton [1]. This is considerably lower than the $O((1.59n)^n)$ worst-case state growth of the Fribourg construction without the M1 optimisation. A formal description and a proof of correctness of the M1 optimisation can be found in [1] (Section 4.E).

### 1.2.3 M2:

The M2 optimisation is the most involved of the three optimisations. Furthermore, it can only be applied together with the M1 optimisation. The main point of the M2 optimisation can be summarised as follows:

> Every state of the lower part contains *at most* one 2-coloured component.

The purpose of this restriction is to further reduce the maximal number of states the construction can generate (that is, to reduce the worst-case state complexity of the construction).

- When determining the colours of the components of a new state, the components must be processed from right to left
- The first component that according to the rules in Figure 1.5 deserves colour 2 gets colour 2 as usual
  - If this component has a sibling[4] to its left, then this sibling gets colour 2 as well
- From the point on where colour 2 has been assigned to a component (and possibly its sibling), all the further components that according to Figure 1.5 deserve colour 2 get colour 1 instead of colour 2

This leaves the new state in a situation where at most two components have colour 2, namely the first one that deserves it from the right, and its possible sibling. In the case that there was a sibling, the application of the M1 optimisation (which is necessary for the application of the M2 optimisation) merges these two components to a single 2-coloured component. This makes the state in any case having at most one 2-coloured component.

This modification of the construction requires further care to be taken when the only 2-coloured component of a state "disappears", that is, has no successor components in the state under construction. In this case, one of the 1-coloured components of this state is made accepting. In more detail, the points to consider are as follows.

- Disappearance of a 2-coloured component: uf the predecessor of a new state has a 2-coloured component, and if after the assignment of colours to the components of this new state, as described above, the new state has no 2-coloured component, then we say that the 2-coloured component of the predecesor state disappeared.
- When the disappearance of a 2-coloured component is detected, and if the state under construction contains at least one 1-coloured component, then the following is done:
  - The position where the successor component of the disappeared component *would* be is determined
  - Select the first 1-coloured component that is to the left of the first a 0-coloured component that is to the left of this position
    - If the search arrives at the leftmost component of the state, then continue it starting from the rightmost component
    - If there is exactly one 1-coloured component in the state, then directly select this component

[4] With sibling we mean having the same predecessor component.

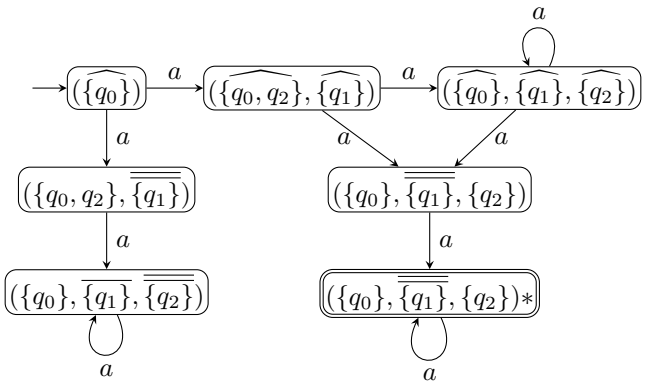



Figure 1.7: Result of applying the M2 optimisation to the complementation of the example automaton from Figure **??**.

- Change the colour of the selected component from 1 to 2
- Mark the state with a special mark (for example a $*$)
  - This mark distinguished states. That is, if two states have the same components with the same colours, but one has a mark and the other not, then they are two different states.

The result of this procedure is that the state has a single 2-coloured component again. However, as mentioned, this state must be accepting, even though it contains a 2-coloured component. It is the purpose of the mark to distinguish this state from other states with a 2-coloured component. The last thing we have to do is to change the acceptance condition to include all states of the lower part containing no 2-coloured components, plus all states having a mark.

In this case, the difference is the marked state $(\{q_0\}, \overline{\overline{\{q_1\}}}, \{q_2\})*$. It exists, because the 2-coloured component $\overline{\overline{\{q_1\}}}$ of the predecessor state disappears. According to the colour rules, the $\overline{\{q_1\}}$ in the new state then gets colour 1. This is where the construction without the M2 optimisation would left off. With the M2 optimisation, however, we have to select one of the 1-coloured components of the new state and change its colour to colour 2. There is only one 1-coloured component in the state, $\overline{\{q_1\}}$, and so we change its colour from 1 to 2. We also mark this state with a star. The successor of this marked state happens to be identical with itself (because the component $\overline{\overline{\{q_1\}}}$ again disappears), so we add a loop. In the end, the marked state is made accepting.

In [1] (Section 4.H) a very loose upper bound on the number of states of the lower part of $O((0.86n)^n)$ is proposed. This is a significant reduction from the upper bound of $O((1.195n)^n)$ that s achieved with the M1 optimisation. Furthermore, concrete calculations on the M2 optimisation suggest that the real upper bound might be as low as $O((0.76n)^n)$. This coincides with the established lower bound for Büchi complementation by Yan [55] and would make the construction optimal in the sense of worst-case state complexity. The actual complexity of the M2 optimisation is subject to further research by the authors of the Fribourg construction.

[1] Section 4.H

## 1.3 General Remarks

# Appendix A

# Plugin Installation and Usage

Since between the 2014–08–08 and 2014–11–17 releases of GOAL certain parts of the plugin interfaces have changed, and we adapted our plugin accordingly, the currently maintained version of the plugin works only with GOAL versions 2014-11-17 or newer. It is thus essential for any GOAL user to update to this version in order to use our plugin.

# Appendix B

# Median Complement Sizes of the GOAL Test Set

Bla bla bla

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 269 | 308 | 254 | 236 | 238 | 297 | 266 | 156 | 207 | 68 |
| 1.2 | 960 | 1,407 | 1,479 | 2,150 | 1,152 | 1,090 | 942 | 1,206 | 718 | 104 |
| 1.4 | 3,426 | 2,915 | 2,752 | 3,393 | 2,693 | 3,265 | 2,263 | 2,425 | 1,844 | 154 |
| 1.6 | 3,799 | 3,698 | 4,901 | 3,926 | 3,960 | 3,655 | 2,580 | 1,905 | 2,124 | 155 |
| 1.8 | 3,375 | 3,169 | 3,420 | 3,967 | 3,943 | 3,132 | 2,246 | 1,144 | 971 | 114 |
| 2.0 | 1,906 | 2,261 | 2,383 | 2,884 | 2,354 | 2,096 | 1,169 | 932 | 568 | 98 |
| 2.2 | 1,467 | 1,633 | 1,795 | 1,942 | 1,611 | 1,640 | 569 | 499 | 330 | 78 |
| 2.4 | 924 | 1,232 | 1,319 | 1,317 | 1,056 | 886 | 514 | 314 | 182 | 59 |
| 2.6 | 625 | 763 | 880 | 945 | 828 | 684 | 316 | 175 | 132 | 44 |
| 2.8 | 483 | 584 | 836 | 690 | 575 | 395 | 240 | 151 | 103 | 41 |
| 3.0 | 319 | 450 | 557 | 523 | 367 | 313 | 155 | 116 | 84 | 32 |

(a) Fribourg

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 269 | 308 | 254 | 236 | 238 | 297 | 266 | 156 | 207 | 68 |
| 1.2 | 960 | 1,407 | 1,479 | 2,150 | 1,152 | 1,090 | 942 | 1,206 | 718 | 104 |
| 1.4 | 3,426 | 2,915 | 2,752 | 3,393 | 2,693 | 3,265 | 2,263 | 2,425 | 1,844 | 154 |
| 1.6 | 3,799 | 3,698 | 4,901 | 3,926 | 3,960 | 3,655 | 2,580 | 1,905 | 2,124 | 155 |
| 1.8 | 3,375 | 3,169 | 3,420 | 3,967 | 3,943 | 3,093 | 2,246 | 1,144 | 971 | 114 |
| 2.0 | 1,906 | 2,184 | 2,383 | 2,818 | 2,354 | 1,989 | 1,127 | 885 | 568 | 97 |
| 2.2 | 1,410 | 1,561 | 1,639 | 1,884 | 1,609 | 1,588 | 496 | 464 | 284 | 78 |
| 2.4 | 884 | 1,200 | 1,234 | 1,184 | 939 | 806 | 373 | 256 | 165 | 55 |
| 2.6 | 575 | 731 | 815 | 860 | 751 | 575 | 246 | 162 | 114 | 43 |
| 2.8 | 431 | 530 | 672 | 466 | 371 | 274 | 174 | 120 | 85 | 36 |
| 3.0 | 232 | 325 | 344 | 360 | 269 | 169 | 91 | 85 | 53 | 27 |

(b) Fribourg+R2C

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 390 | 438 | 434 | 324 | 328 | 459 | 337 | 204 | 227 | 40 |
| 1.2 | 1,576 | 2,394 | 2,505 | 2,996 | 1,613 | 1,551 | 1,166 | 1,542 | 1,002 | 58 |
| 1.4 | 5,007 | 4,336 | 4,652 | 4,877 | 3,458 | 3,956 | 3,169 | 3,380 | 1,868 | 86 |
| 1.6 | 5,067 | 5,032 | 6,444 | 4,868 | 4,575 | 3,864 | 3,211 | 1,731 | 1,892 | 85 |
| 1.8 | 4,016 | 3,701 | 3,647 | 4,523 | 3,548 | 3,009 | 1,808 | 451 | 336 | 62 |
| 2.0 | 1,663 | 2,276 | 2,676 | 3,035 | 1,925 | 1,932 | 464 | 307 | 150 | 54 |
| 2.2 | 989 | 1,514 | 1,621 | 1,826 | 1,121 | 846 | 155 | 127 | 93 | 45 |
| 2.4 | 560 | 821 | 919 | 771 | 529 | 267 | 133 | 87 | 55 | 32 |
| 2.6 | 388 | 519 | 524 | 441 | 259 | 219 | 84 | 50 | 41 | 26 |
| 2.8 | 311 | 317 | 396 | 242 | 165 | 95 | 64 | 44 | 33 | 22 |
| 3.0 | 173 | 224 | 211 | 169 | 102 | 72 | 41 | 34 | 27 | 18 |

(c) Fribourg+R2C+C

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 225 | 223 | 195 | 181 | 187 | 199 | 189 | 124 | 161 | 68 |
| 1.2 | 731 | 971 | 946 | 1,071 | 629 | 562 | 488 | 568 | 388 | 104 |
| 1.4 | 2,228 | 1,701 | 1,543 | 1,732 | 1,241 | 1,287 | 945 | 944 | 727 | 154 |
| 1.6 | 2,489 | 2,263 | 2,331 | 2,133 | 1,777 | 1,443 | 964 | 757 | 889 | 155 |
| 1.8 | 2,381 | 2,027 | 2,009 | 2,075 | 1,618 | 1,243 | 1,005 | 592 | 515 | 114 |
| 2.0 | 1,390 | 1,569 | 1,416 | 1,573 | 1,093 | 1,008 | 594 | 464 | 330 | 98 |
| 2.2 | 1,118 | 1,197 | 1,150 | 1,151 | 879 | 809 | 317 | 330 | 241 | 78 |
| 2.4 | 712 | 885 | 836 | 809 | 580 | 535 | 316 | 231 | 145 | 59 |
| 2.6 | 498 | 569 | 601 | 627 | 497 | 412 | 217 | 137 | 113 | 44 |
| 2.8 | 391 | 455 | 578 | 456 | 374 | 263 | 173 | 119 | 90 | 41 |
| 3.0 | 258 | 350 | 392 | 354 | 253 | 208 | 119 | 97 | 74 | 32 |

(d) Fribourg+M1

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 215 | 213 | 189 | 174 | 175 | 192 | 186 | 121 | 156 | 68 |
| 1.2 | 712 | 914 | 913 | 1,075 | 619 | 563 | 526 | 620 | 416 | 104 |
| 1.4 | 2,075 | 1,620 | 1,503 | 1,650 | 1,254 | 1,339 | 1,003 | 1,006 | 848 | 154 |
| 1.6 | 2,344 | 2,062 | 2,340 | 2,016 | 1,755 | 1,520 | 1,053 | 858 | 986 | 155 |
| 1.8 | 2,205 | 1,873 | 1,920 | 2,040 | 1,689 | 1,315 | 1,080 | 664 | 598 | 114 |
| 2.0 | 1,290 | 1,485 | 1,405 | 1,522 | 1,134 | 1,044 | 652 | 531 | 392 | 98 |
| 2.2 | 1,023 | 1,119 | 1,092 | 1,127 | 868 | 875 | 376 | 359 | 262 | 78 |
| 2.4 | 674 | 849 | 790 | 807 | 617 | 544 | 355 | 251 | 156 | 59 |
| 2.6 | 478 | 549 | 594 | 597 | 510 | 431 | 231 | 147 | 116 | 44 |
| 2.8 | 370 | 439 | 559 | 455 | 382 | 283 | 182 | 124 | 93 | 41 |
| 3.0 | 249 | 341 | 388 | 348 | 260 | 225 | 123 | 101 | 77 | 32 |

(e) Fribourg+M1+M2

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 225 | 223 | 195 | 181 | 187 | 199 | 189 | 124 | 161 | 68 |
| 1.2 | 731 | 971 | 946 | 1,071 | 629 | 562 | 488 | 568 | 388 | 104 |
| 1.4 | 2,228 | 1,701 | 1,543 | 1,732 | 1,241 | 1,287 | 945 | 944 | 727 | 154 |
| 1.6 | 2,489 | 2,263 | 2,331 | 2,133 | 1,777 | 1,443 | 964 | 757 | 889 | 155 |
| 1.8 | 2,381 | 2,027 | 2,009 | 2,075 | 1,618 | 1,215 | 1,005 | 592 | 515 | 114 |
| 2.0 | 1,390 | 1,513 | 1,416 | 1,542 | 1,093 | 1,003 | 594 | 441 | 330 | 97 |
| 2.2 | 1,019 | 1,156 | 1,064 | 1,104 | 859 | 785 | 304 | 303 | 221 | 78 |
| 2.4 | 672 | 867 | 789 | 772 | 544 | 478 | 269 | 191 | 139 | 55 |
| 2.6 | 466 | 542 | 572 | 568 | 452 | 348 | 183 | 129 | 99 | 43 |
| 2.8 | 368 | 407 | 480 | 337 | 260 | 197 | 129 | 96 | 75 | 36 |
| 3.0 | 201 | 261 | 266 | 272 | 199 | 136 | 83 | 74 | 50 | 27 |

(f) Fribourg+M1+R2C

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 329 | 303 | 279 | 240 | 229 | 288 | 230 | 157 | 160 | 40 |
| 1.2 | 988 | 1,392 | 1,356 | 1,352 | 751 | 741 | 608 | 704 | 516 | 58 |
| 1.4 | 2,939 | 2,581 | 2,066 | 2,190 | 1,351 | 1,622 | 1,132 | 1,261 | 932 | 86 |
| 1.6 | 3,150 | 2,900 | 2,842 | 2,218 | 1,885 | 1,563 | 1,177 | 821 | 896 | 85 |
| 1.8 | 2,782 | 2,485 | 2,047 | 2,180 | 1,625 | 1,269 | 855 | 395 | 309 | 62 |
| 2.0 | 1,338 | 1,638 | 1,544 | 1,566 | 979 | 957 | 349 | 261 | 147 | 54 |
| 2.2 | 838 | 1,125 | 993 | 1,027 | 667 | 521 | 153 | 125 | 93 | 45 |
| 2.4 | 494 | 700 | 624 | 524 | 296 | 214 | 126 | 87 | 55 | 32 |
| 2.6 | 327 | 434 | 383 | 334 | 212 | 163 | 82 | 50 | 41 | 26 |
| 2.8 | 283 | 273 | 305 | 202 | 144 | 95 | 60 | 44 | 33 | 22 |
| 3.0 | 164 | 200 | 173 | 142 | 92 | 72 | 41 | 34 | 27 | 18 |

(g) Fribourg+M1+R2C+C

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 126 | 118 | 97 | 60 | 51 | 52 | 62 | 36 | 48 | 30 |
| 1.2 | 432 | 517 | 345 | 262 | 160 | 126 | 92 | 120 | 109 | 40 |
| 1.4 | 1,044 | 331 | 133 | 89 | 45 | 22 | 19 | 31 | 27 | 20 |
| 1.6 | 358 | 24 | 11 | 5 | 4 | 6 | 5 | 3 | 3 | 4 |
| 1.8 | 19 | 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2.0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2.2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2.4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2.6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2.8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3.0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

(h) Fribourg+R

Figure B.1: Median complement sizes of the 10,939 effective samples of the internal tests on the GOAL test set. The rows (1.0 to 3.0) are the transition densities, and the columns (0.1 to 1.0) are the acceptance densities.

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 130 | 117 | 109 | 77 | 69 | 61 | 56 | 40 | 40 | 29 |
| 1.2 | 387 | 456 | 352 | 281 | 155 | 136 | 101 | 105 | 75 | 45 |
| 1.4 | 822 | 683 | 394 | 376 | 230 | 204 | 151 | 120 | 105 | 63 |
| 1.6 | 890 | 594 | 458 | 321 | 237 | 178 | 134 | 114 | 113 | 61 |
| 1.8 | 624 | 507 | 324 | 275 | 196 | 136 | 110 | 92 | 89 | 41 |
| 2.0 | 362 | 286 | 211 | 176 | 117 | 103 | 79 | 64 | 59 | 34 |
| 2.2 | 248 | 222 | 124 | 116 | 82 | 73 | 56 | 52 | 50 | 28 |
| 2.4 | 147 | 145 | 114 | 87 | 56 | 48 | 43 | 39 | 35 | 19 |
| 2.6 | 115 | 117 | 67 | 61 | 47 | 42 | 32 | 29 | 29 | 15 |
| 2.8 | 95 | 71 | 52 | 45 | 38 | 29 | 27 | 25 | 23 | 13 |
| 3.0 | 59 | 60 | 47 | 35 | 32 | 27 | 22 | 21 | 20 | 10 |

(a) Piterman+EQ+RO

| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 171 | 174 | 166 | 124 | 118 | 117 | 100 | 67 | 84 | 35 |
| 1.2 | 622 | 833 | 803 | 877 | 529 | 398 | 320 | 372 | 215 | 53 |
| 1.4 | 2,086 | 1,618 | 1,367 | 1,676 | 1,065 | 967 | 664 | 682 | 494 | 78 |
| 1.6 | 2,465 | 2,073 | 2,182 | 1,959 | 1,518 | 1,259 | 767 | 545 | 623 | 78 |
| 1.8 | 2,310 | 1,963 | 1,950 | 1,988 | 1,485 | 1,095 | 746 | 418 | 346 | 57 |
| 2.0 | 1,318 | 1,482 | 1,393 | 1,461 | 981 | 871 | 434 | 338 | 228 | 50 |
| 2.2 | 1,068 | 1,145 | 1,085 | 1,067 | 772 | 747 | 263 | 235 | 158 | 40 |
| 2.4 | 689 | 838 | 809 | 751 | 524 | 466 | 240 | 159 | 93 | 30 |
| 2.6 | 469 | 531 | 555 | 565 | 437 | 360 | 169 | 94 | 71 | 23 |
| 2.8 | 369 | 421 | 536 | 405 | 329 | 224 | 130 | 81 | 58 | 21 |
| 3.0 | 244 | 327 | 360 | 322 | 219 | 176 | 85 | 64 | 49 | 16 |

(b) Slice+P+RO+MADJ+EG

Figure B.2: Median complement sizes of the 10,998 effective samples of the external tests without the Rank construction. The rows (1.0 to 3.0) are the transition densities, and the columns (0.1 to 1.0) are the acceptance densities.

# Appendix C

# Execution Times

| Construction | Mean | Min. | P25 | Median | P75 | Max. | Total | ≈ hours |
|---|---|---|---|---|---|---|---|---|
| Fribourg | 8.5 | 2.5 | 3.3 | 4.9 | 7.3 | 586.0 | 93,351.2 | 259 |
| Fribourg+R2C | 6.6 | 2.2 | 2.9 | 4.2 | 6.4 | 219.7 | 72,545.7 | 202 |
| Fribourg+R2C+C | 8.5 | 2.2 | 2.6 | 3.5 | 6.4 | 582.9 | 93,396.2 | 259 |
| Fribourg+M1 | 4.9 | 2.5 | 3.2 | 4.1 | 5.9 | 55.1 | 54,061.3 | 150 |
| Fribourg+M1+M2 | 4.6 | 2.2 | 2.9 | 3.8 | 5.1 | 38.4 | 49,848.0 | 138 |
| Fribourg+M1+R2C | 4.4 | 2.2 | 2.8 | 3.6 | 5.3 | 42.5 | 48,572.0 | 135 |
| Fribourg+M1+R2C+C | 5.6 | 2.5 | 3.2 | 4.0 | 6.5 | 147.4 | 60,918.9 | 169 |
| Fribourg+R | 7.5 | 2.2 | 3.0 | 3.9 | 6.3 | 470.5 | 82,387.3 | 229 |

Table C.1: Execution times in CPU time seconds for the 10,939 effectvie samples of the GOAL test set.

| Construction | Mean | Min. | P25 | Median | P75 | Max. | Total | ≈ hours |
|---|---|---|---|---|---|---|---|---|
| Piterman+EQ+RO | 3.0 | 2.2 | 2.6 | 2.8 | 3.0 | 42.9 | 21,410.6 | 59 |
| Slice+P+RO+MADJ+EG | 3.7 | 2.2 | 2.7 | 3.2 | 4.1 | 36.7 | 26,398.9 | 73 |
| Rank+TR+RO | 16.0 | 2.3 | 2.8 | 3.7 | 9.3 | 443.3 | 115,563.9 | 321 |
| Fribourg+M1+R2C | 4.0 | 2.2 | 2.7 | 3.1 | 4.4 | 410.4 | 28,970.8 | 80 |

Table C.2: Execution times in CPU time seconds for the 7,204 effectvie samples of the GOAL test set.

| Construction | Mean | Min. | P25 | Median | P75 | Max. | Total | ≈ hours |
|---|---|---|---|---|---|---|---|---|
| Piterman+EQ+RO | 3.6 | 2.2 | 2.7 | 2.9 | 3.4 | 365.7 | 39,663.4 | 110 |
| Slice+P+RO+MADJ+EG | 4.3 | 2.2 | 2.9 | 3.7 | 5.0 | 42.4 | 47,418.2 | 132 |
| Fribourg+M1+R2C | 4.7 | 2.2 | 2.8 | 3.6 | 5.3 | 410.4 | 52,149.0 | 145 |

Table C.3: Execution times in CPU time seconds for the 10,998 effectvie samples of the GOAL test set without the Rank construction.

| Construction | Michel 1 | Michel 2 | Michel 3 | Michel 4 | Fitted curve | Std. error |
|---|---|---|---|---|---|---|
| Fribourg | 2.3 | 4.0 | 88.8 | 100,976.0 | $(1.14n)^n$ | 0.64% |
| Fribourg+R2C | 2.3 | 3.4 | 27.4 | 27,938.3 | $(0.92n)^n$ | 0.64% |
| Fribourg+M1 | 2.2 | 3.6 | 17.9 | 6,508.4 | $(0.72n)^n$ | 0.63% |
| Fribourg+M1+M2 | 2.3 | 3.5 | 13.8 | 2,707.4 | $(0.62n)^n$ | 0.62% |
| Fribourg+M1+M2+R2C | 2.5 | 3.5 | 10.8 | 2,332.6 | $(0.61n)^n$ | 0.62% |
| Fribourg+R | 2.4 | 3.7 | 86.0 | 101,809.6 | $(1.14n)^n$ | 0.64% |

Table C.4: Execution times in CPU time seconds for the four Michel automata.

| Construction | Michel 1 | Michel 2 | Michel 3 | Michel 4 | Fitted curve | Std. error |
|---|---|---|---|---|---|---|
| Piterman+EQ+RO | 2.5 | 3.8 | 42.6 | 75,917.4 | $(1.08n)^n$ | 0.64% |
| Slice+P+RO+MADJ+EG | 2.3 | 3.6 | 11.4 | 159.5 | $(0.39n)^n$ | 0.38% |
| Rank+TR+RO | 2.2 | 3.0 | 6.4 | 30.0 | $(0.29n)^n$ | 0.18% |
| Fribourg+M1+M2+R2C | 2.5 | 3.5 | 10.8 | 2,332.6 | $(0.61n)^n$ | 0.62% |

Table C.5: Execution times in CPU time seconds for the four Michel automata.

# Bibliography


[1] J. Allred, U. Ultes-Nitsche. Complementing Büchi Automata with a Subset-Tuple Construction. Tech. rep.. University of Fribourg, Switzerland. 2014.

[2] C. Althoff, W. Thomas, N. Wallmeier. Observations on Determinization of Büchi Automata. In J. Farré, I. Litovsky, S. Schmitz, eds., *Implementation and Application of Automata*. vol. 3845 of *Lecture Notes in Computer Science*. pp. 262–272. Springer Berlin Heidelberg. 2006.

[3] S. Breuers, C. Löding, J. Olschewski. Improved Ramsey-Based Büchi Complementation. In L. Birkedal, ed., *Foundations of Software Science and Computational Structures*. vol. 7213 of *Lecture Notes in Computer Science*. pp. 150–164. Springer Berlin Heidelberg. 2012.

[4] J. R. Büchi. On a Decision Method in Restricted Second Order Arithmetic. In *Proc. International Congress on Logic, Method, and Philosophy of Science, 1960*. Stanford University Press. 1962.

[5] S. J. Fogarty, O. Kupferman, T. Wilke, et al. Unifying Büchi Complementation Constructions. *Logical Methods in Computer Science*. 9(1). 2013.

[6] E. Friedgut, O. Kupferman, M. Vardi. Büchi Complementation Made Tighter. In F. Wang, ed., *Automated Technology for Verification and Analysis*. vol. 3299 of *Lecture Notes in Computer Science*. pp. 64–78. Springer Berlin Heidelberg. 2004.

[7] E. Friedgut, O. Kupferman, M. Y. Vardi. Büchi Complementation Made Tighter. *International Journal of Foundations of Computer Science*. 17(04):pp. 851–867. 2006.

[8] C. Göttel. Implementation of an Algorithm for Büchi Complementation. BSc Thesis, University of Fribourg, Switzerland. November 2013.

[9] R. L. Graham, B. L. Rothschild, J. H. Spencer. *Ramsey theory*. vol. 20. John Wiley & Sons. 1990.

[10] J. E. Hopcroft, R. Motwani, J. D. Ullman. *Introduction to Automata Theory, Languages, and Computation*. Addison-Wesley. 2nd edition ed.. 2001.

[11] D. Kähler, T. Wilke. Complementation, Disambiguation, and Determinization of Büchi Automata Unified. In L. Aceto, I. Damgård, L. Goldberg, et al, eds., *Automata, Languages and Programming*. vol. 5125 of *Lecture Notes in Computer Science*. pp. 724–735. Springer Berlin Heidelberg. 2008.

[12] N. Klarlund. Progress measures for complementation of omega-automata with applications to temporal logic. In *Foundations of Computer Science, 1991. Proceedings., 32nd Annual Symposium on*. pp. 358–367. Oct 1991.

[13] J. Klein. Linear Time Logic and Deterministic Omega-Automata. *Master's thesis, Universität Bonn*. 2005.

[14] J. Klein, C. Baier. Experiments with Deterministic $\omega$-Automata for Formulas of Linear Temporal Logic. In J. Farré, I. Litovsky, S. Schmitz, eds., *Implementation and Application of Automata*. vol. 3845 of *Lecture Notes in Computer Science*. pp. 199–212. Springer Berlin Heidelberg. 2006.

[15] O. Kupferman, M. Y. Vardi. Weak Alternating Automata Are Not that Weak. In *Proceedings of the 5th Israeli Symposium on Theory of Computing and Systems*. pp. 147–158. IEEE Computer Society Press. 1997.

[16] O. Kupferman, M. Y. Vardi. Weak Alternating Automata Are Not that Weak. *ACM Trans. Comput. Logic*. 2(3):pp. 408–429. Jul. 2001.

[17] R. Kurshan. Complementing Deterministic Büchi Automata in Polynomial Time. *Journal of Computer and System Sciences*. 35(1):pp. 59 – 71. 1987.

[18] C. Löding. Optimal Bounds for Transformations of ω-Automata. In C. Rangan, V. Raman, R. Ramanujam, eds., *Foundations of Software Technology and Theoretical Computer Science*. vol. 1738 of *Lecture Notes in Computer Science*. pp. 97–109. Springer Berlin Heidelberg. 1999.

[19] R. McNaughton. Testing and generating infinite sequences by a finite automaton. *Information and Control*. 9(5):pp. 521 – 530. 1966.

[20] M. Michel. Complementation is more difficult with automata on infinite words. *CNET, Paris*. 15. 1988.

[21] A. Mostowski. Regular expressions for infinite trees and a standard form of automata. In A. Skowron, ed., *Computation Theory*. vol. 208 of *Lecture Notes in Computer Science*. pp. 157–168. Springer Berlin Heidelberg. 1985.

[22] D. E. Muller. Infinite Sequences and Finite Machines. In *Switching Circuit Theory and Logical Design, Proceedings of the Fourth Annual Symposium on*. pp. 3–16. Oct 1963.

[23] D. E. Muller, A. Saoudi, P. E. Schupp. Alternating automata, the weak monadic theory of the tree, and its complexity. In L. Kott, ed., *Automata, Languages and Programming*. vol. 226 of *Lecture Notes in Computer Science*. pp. 275–283. Springer Berlin Heidelberg. 1986.

[24] D. E. Muller, P. E. Schupp. Simulating Alternating Tree Automata by Nondeterministic Automata: New Results and New Proofs of the Theorems of Rabin, McNaughton and Safra. *Theoretical Computer Science*. 141(1–2):pp. 69 – 107. 1995.

[25] F. Nießner, U. Nitsche, P. Ochsenschläger. Deterministic Omega-Regular Liveness Properties. In S. Bozapalidis, ed., *Preproceedings of the 3rd International Conference on Developments in Language Theory, DLT'97*. pp. 237–247. Citeseer. 1997.

[26] J.-P. Pecuchet. On the complementation of Büchi automata. *Theoretical Computer Science*. 47(0):pp. 95 – 98. 1986.

[27] N. Piterman. From Nondeterministic Buchi and Streett Automata to Deterministic Parity Automata. In *Logic in Computer Science, 2006 21st Annual IEEE Symposium on*. pp. 255–264. 2006.

[28] N. Piterman. From Nondeterministic Buchi and Streett Automata to Deterministic Parity Automata. *Logical Methods in Computer Science*. 3(5):pp. 1–21. 2007.

[29] M. Rabin, D. Scott. Finite Automata and Their Decision Problems. *IBM Journal of Research and Development*. 3(2):pp. 114–125. April 1959.

[30] M. O. Rabin. Decidability of second-order theories and automata on infinite trees. *Transactions of the American Mathematical Society*. 141:pp. 1–35. July 1969.

[31] F. P. Ramsey. On a Problem of Formal Logic. *Proceedings of the London Mathematical Society*. s2-30(1):pp. 264–286. 1930.

[32] M. Roggenbach. Determinization of Büchi-Automata. In E. Grädel, W. Thomas, T. Wilke, eds., *Automata Logics, and Infinite Games*. vol. 2500 of *Lecture Notes in Computer Science*. pp. 43–60. Springer Berlin Heidelberg. 2002.

[33] S. Safra. On the Complexity of Omega-Automata. *Journal of Computer and System Science*. 1988.

[34] S. Safra. On the Complexity of Omega-Automata. In *Foundations of Computer Science, 1988., 29th Annual Symposium on*. pp. 319–327. Oct 1988.

[35] S. Schewe. Büchi Complementation Made Tight. In *26th International Symposium on Theoretical Aspects of Computer Science-STACS 2009*. pp. 661–672. 2009.

[36] A. Sistla, M. Vardi, P. Wolper. The complementation problem for Büchi automata with applications to temporal logic. In W. Brauer, ed., *Automata, Languages and Programming*. vol. 194 of *Lecture Notes in Computer Science*. pp. 465–474. Springer Berlin Heidelberg. 1985.

[37] A. P. Sistla, M. Y. Vardi, P. Wolper. The Complementation Problem for Büchi Automata with Applications to Temporal Logic. *Theoretical Computer Science*. 49(2–3):pp. 217 – 237. 1987.

[38] R. S. Streett. Propositional dynamic logic of looping and converse is elementarily decidable. *Information and Control*. 54(1–2):pp. 121 – 141. 1982.

[39] W. Thomas. Automata on Infinite Objects. In J. van Leeuwen, ed., *Handbook of Theoretical Computer Science (Vol. B)*. chap. Automata on Infinite Objects, pp. 133–191. MIT Press, Cambridge, MA, USA. 1990.

[40] W. Thomas. Languages, Automata, and Logic. In G. Rozenberg, A. Salomaa, eds., *Handbook of Formal Languages*. pp. 389–455. Springer Berlin Heidelberg. 1997.

[41] W. Thomas. Complementation of Büchi Automata Revisited. In J. Karhumäki, H. Maurer, G. Păun, et al, eds., *Jewels are Forever*. pp. 109–120. Springer Berlin Heidelberg. 1999.

[42] M.-H. Tsai, S. Fogarty, M. Vardi, et al. State of Büchi Complementation. In M. Domaratzki, K. Salomaa, eds., *Implementation and Application of Automata*. vol. 6482 of *Lecture Notes in Computer Science*. pp. 261–271. Springer Berlin Heidelberg. 2011.

[43] M.-H. Tsai, Y.-K. Tsay, Y.-S. Hwang. GOAL for Games, Omega-Automata, and Logics. In N. Sharygina, H. Veith, eds., *Computer Aided Verification*. vol. 8044 of *Lecture Notes in Computer Science*. pp. 883–889. Springer Berlin Heidelberg. 2013.

[44] Y.-K. Tsay, Y.-F. Chen, M.-H. Tsai, et al. Goal: A Graphical Tool for Manipulating Büchi Automata and Temporal Formulae. In O. Grumberg, M. Huth, eds., *Tools and Algorithms for the Construction and Analysis of Systems*. vol. 4424 of *Lecture Notes in Computer Science*. pp. 466–471. Springer Berlin Heidelberg. 2007.

[45] Y.-K. Tsay, Y.-F. Chen, M.-H. Tsai, et al. Goal Extended: Towards a Research Tool for Omega Automata and Temporal Logic. In C. Ramakrishnan, J. Rehof, eds., *Tools and Algorithms for the Construction and Analysis of Systems*. vol. 4963 of *Lecture Notes in Computer Science*. pp. 346–350. Springer Berlin Heidelberg. 2008.

[46] Y.-K. Tsay, Y.-F. Chen, M.-H. Tsai, et al. Tool support for learning Büchi automata and linear temporal logic. *Formal Aspects of Computing*. 21(3):pp. 259–275. 2009.

[47] Y.-K. Tsay, M.-H. Tsai, J.-S. Chang, et al. Büchi Store: An Open Repository of Büchi Automata. In P. Abdulla, K. Leino, eds., *Tools and Algorithms for the Construction and Analysis of Systems*. vol. 6605 of *Lecture Notes in Computer Science*. pp. 262–266. Springer Berlin Heidelberg. 2011.

[48] U. Ultes-Nitsche. A Power-Set Construction for Reducing Büchi Automata to Non-Determinism Degree Two. *Information Processing Letters*. 101(3):pp. 107 – 111. 2007.

[49] M. Vardi. An automata-theoretic approach to linear temporal logic. In F. Moller, G. Birtwistle, eds., *Logics for Concurrency*. vol. 1043 of *Lecture Notes in Computer Science*. pp. 238–266. Springer Berlin Heidelberg. 1996.

[50] M. Vardi. The Büchi Complementation Saga. In W. Thomas, P. Weil, eds., *STACS 2007*. vol. 4393 of *Lecture Notes in Computer Science*. pp. 12–22. Springer Berlin Heidelberg. 2007.

[51] M. Y. Vardi. Buchi Complementation: A Forty-Year Saga. In *5th symposium on Atomic Level Characterizations (ALC'05)*. 2005.

[52] M. Y. Vardi, T. Wilke. Automata: From Logics to Algorithms. In J. Flum, E. Grädel, T. Wilke, eds., *Logic and Automata: History and Perspectives*. vol. 2 of *Texts in Logic and Games*. pp. 629–736. Amsterdam University Press. 2007.

[53] T. Wilke. $\omega$-Automata. In J.-E. Pin, ed., *Handbook of Automata Theory*. European Mathematical Society. To appear, 2015.

[54] Q. Yan. Lower Bounds for Complementation of $\omega$-Automata Via the Full Automata Technique. In M. Bugliesi, B. Preneel, V. Sassone, et al, eds., *Automata, Languages and Programming*. vol. 4052 of *Lecture Notes in Computer Science*. pp. 589–600. Springer Berlin Heidelberg. 2006.

[55] Q. Yan. Lower Bounds for Complementation of omega-Automata Via the Full Automata Technique. *CoRR*. abs/0802.1226. 2008.