REGULAR EXPRESSIONS FOR INFINITE TREES AND A STANDARD FORM OF AUTOMATA

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Summary For Rabin pair automata [R1] a standard form is defined /def. 2/ i.e. such that an ordered subset $\{s_1,\dots,s_{2I-1}\}$ of states is distinguished in such a way that a path of a run is accepting /rejecting if for some i even/ odd, $1 \leq i \leq 2I-1$, the s_i appears infinitely often, and all s_j , $j < i$ only finitely many times. The class of standard automata is big enough to represent all f.a. representable sets /th.1/ but has many properties similar to special automata defined in [R1] . A standard reguler expression is defined /def. 6/ describing a process of forming of an infinite tree, as well as a process of building of an automaton /analysis and synthesis theorems 3,4/. The standard regular expressions are a generalisation of McNaughtons formula $\bigcup \alpha\beta^{\omega}$ /cf. [N] /.

1. Notions. We refer the reader to automata on infinite binary trees as defined in [R1] , and to automata on finite trees as given in [TW] . For simplicity reasons the theorems are stated and proved for a binary case, but some minor changes will allow the theorems to be true for mixed trees as given in [M3] .

Only in par. 3 the regular expressions theory is shown explicitely to be working for mixed trees.

An alphabet Σ is fixed. A /valued, infinite, binery/ tree $t = (T,v)$ is a tree $T = \{0,1\}^*$ with a valuation $v : T \longrightarrow \Sigma$. The relation $x \leqslant y$ is an order, where $x \leqslant y$ iff there exists $z \in T$ such that $xz = y$. Maximal chains for $\leqslant$ are called paths of T. For $A,B \in PS(S)$ the relation $A < B$ denotes proper inclusion.

A /partial/ table is a triple $\widetilde{J} = \langle S,M,s_{in} \rangle$, where S is a finite set of states $s_{in} \in S$, amd $M : S \times \Sigma \longrightarrow PS(S \times S)$ is a transition function. A run of $\widetilde{J}$ on t is a total function $r : T \longrightarrow S$ such that : $r(\Lambda) = s_{in}$, $\langle r(x0), r(x1) \rangle \in M(r(x), v(x))$ for $x \in T$.

A finite automaton on trees is a pair $\mathfrak{A} = \langle \widetilde{J}, W \rangle$ of a table $\widetilde{J}$ and a set $W \leqslant S^{\omega}$ of accepting paths. A tree t is accepted by the $\mathfrak{A}$ iff there exists a run r of $\widetilde{J}$ on t, such that for each path $\pi \leqslant T$, $r / \pi \in W$, i.e. each path of the run is accepting.

Let $\Omega = \{(U_i,L_i) : 0 \leqslant i < I, U_i,L_i \leqslant S\}$ be a collection of pairs and $\mathcal{F} \leqslant PS(S)$ a collection of subsets. Let $s : \omega \rightarrow S$ be a sequence. We shall define a set $[\Omega]$ of sequences on S as follows: $s \in [\Omega]$ iff card $(s^{-1}(U_i)) \geqslant \omega$ and card $(s^{-1}(L_i)) < \omega$, for some $i : 0 \leqslant i < I$. We speak of a Rabin automaton or a pair automaton if $W = [\Omega]$. The $I = I(\mathfrak{A})$ is called a Rabin pair index of the automaton. Let us define $s \in [\mathcal{F}]$ iff $\{x :$ card $(s^{-1}(x)) \geqslant \omega\} \in \mathcal{F}$. For $W = [\mathcal{F}]$ we shall say that $\mathfrak{A}$ is a Muller, or set, automaton.

<u>Definition 1.</u> A Rabin automaton is a chain form automaton /c.f.a./ iff $L_0 < L_1 < \ldots < L_{I-1}$ for the $L_0,\ldots,L_{I-1}$ appearing in the Ω.