Direct and Indirect Adaptive Fuzzy Control for a Class of MIMO Nonlinear Systems

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1. Introduction

During the last two decades, there has been significant progress in the area of adaptive control design of nonlinear systems (Krstic et al., 1995; Sastry & Isidori, 1989; Slotine & Li 1991; Spooner et al., 2002). Most of the developed adaptive control schemes assume that an accurate model of the system is available and the unknown parameters appear linearly with respect to known nonlinear functions. However, this assumption is not sufficient for many practical situations, because it is difficult to precisely describe a nonlinear system by known nonlinear functions and, therefore, the problem of controlling nonlinear systems with incomplete model knowledge remains a challenging task.

As a model free design method, fuzzy control has found extensive applications for complex and ill-defined plants (Passino & Yurkovich, 1998; Wang, 1994). Basically, fuzzy control is a human knowledge-based design methodology which is driven accordingly by fuzzy membership functions and fuzzy rules. However, it is sometimes difficult to find the matched membership functions and fuzzy rules for some plants, or the need may arise to tune the controller parameters if the plant dynamics change. In the hope to overcome this problem, based on the universal approximation theorem and on-line learning ability of fuzzy systems, several stable adaptive fuzzy control schemes have been developed to incorporate the expert knowledge systematically (Spooner & Passino, 1996; Spooner et al., 2002; Su & Stepanenko, 1994; Wang, 1994). The stability analysis in such schemes is performed by using the Lyapunov approach. Conceptually, there are two distinct approaches that have been formulated in the design of a fuzzy adaptive control system: direct and indirect schemes. The direct scheme uses fuzzy systems to approximate unknown ideal controllers (Chang, 2000; Chang, 2001; Labiod & Boucherit, 2003; Li & Tong, 2003; Ordonez & Passino, 1999; Spooner & Passino 1996; Wang, 1994), while the indirect scheme uses fuzzy systems to estimate the plant dynamics and then synthesizes a control law based

on these estimates (Boulkroune et al., 2008a; Boulkroune et al., 2008b; Chang, 2000; Chang, 2001; Chekireb et al., 2003; Chiu, 2005; Golea et al., 2003; Labiod et al., 2005; Ordonez & Passino, 1999; Spooner & Passino 1996; Su & Stepanenko, 1994; Wang, 1994).

For uncertain single-input single-output (SISO) nonlinear systems, fuzzy adaptive control schemes were proposed in (Chang, 2001; Essounbouli & Hamzaoui, 2006; Labiod & Boucherit, 2003; Spooner & Passino, 1996; Su & Stepanenko, 1994; Wang, 1994). The problem of adaptive fuzzy control of uncertain multi-input multi-output (MIMO) nonlinear systems is more difficult because of the coupling that exists between the control inputs and the outputs. This problem was studied in (Boulkroune et al., 2008a; Boulkroune et al., 2008b; Chang, 2000; Chekireb et al., 2003; Chiu, 2005; Golea et al., 2003; Labiod et al., 2005; Li & Tong, 2003; Ordonez & Passino, 1999; Tlemcani et al., 2007 ; Tong et al., 2000; Zhang & YI, 2007). We note that the direct adaptive approach turns out to require more restrictive assumptions than the indirect case, but is perhaps of more interest because it does not present any possible controller singularity problem.

In the aforementioned papers, the adjustable parameters of the fuzzy systems are updated by an adaptive law based on a Lyapunov approach, i.e., the parameter adaptive laws are designed in such a way to ensure the convergence of a Lyapunov function. However, for an effective adaptation, it is more judicious to directly base the parameter adaptation process on the identification error between the unknown function and its adaptive fuzzy approximation (Labiod & Guerra, 2007a; Labiod & Guerra, 2007b).

This chapter presents direct and indirect adaptive fuzzy control schemes for a class of continuous-time uncertain MIMO nonlinear dynamic systems. The proposed schemes are based on the results in (Labiod & Guerra, 2007a). In the direct approach, since fuzzy systems are used to approximate unknown ideal controllers, the adjustable parameters of the used fuzzy systems are updated using a gradient descent algorithm that is designed to minimize the error between the unknown ideal controllers and fuzzy controllers. On the other hand, in the indirect approach, since fuzzy systems are used to approximate the system's unknown nonlinearities, the adjustable parameters of the used fuzzy systems are updated using a gradient descent algorithm that is designed to minimize the error between the system's unknown nonlinearities and the used fuzzy systems. In both approaches, the stability analysis of the closed-loop system is performed using a Lyapunov approach. In particular, it is shown that the tracking errors are uniformly ultimately bounded and converge to a neighbourhood of the origin.

The organization of this chapter is as follows. The problem formulation and fuzzy systems description are given in section 2. The MIMO direct adaptive fuzzy controller with a proof of the stability results are presented in section 3. The MIMO indirect adaptive fuzzy controller with its stability analysis is given in section 4. Section 5 presents simulation results of the proposed direct adaptive control scheme applied to a two-link robot manipulator. Finally, section 6 concludes the chapter.

2. Problem formulation

We consider a class of uncertain MIMO nonlinear systems modeled by

$$y_1^{(r_1)} = f_1(x) + \sum_{j=1}^p g_{1j}(x) u_j$$
$$\vdots \tag{1}$$
$$y_p^{(r_p)} = f_p(x) + \sum_{j=1}^p g_{pj}(x) u_j$$

where $x = \left[y_1, \dot{y}_1, \dots, y_1^{(r_1-1)}, \dots, y_p, \dot{y}_p, \dots, y_p^{(r_p-1)} \right]^T \in \mathbb{R}^n$ with $n = \sum_{i=1}^p r_i$, is the overall state vector which is assumed available for measurement, $u = \left[u_1, \dots, u_p \right]^T \in \mathbb{R}^p$ is the control input vector, $y = \left[y_1, \dots, y_p \right]^T \in \mathbb{R}^p$ is the output vector, and $f_i(x)$, $g_{ij}(x)$, $i,j = 1, \dots, p$ are unknown smooth nonlinear functions.

Let us denote

$$y^{(r)} = \left[y_1^{(r_1)}, \dots, y_p^{(r_p)} \right]^T ; \quad f(x) = \left[f_1(x), \dots, f_p(x) \right]^T ; \quad G(x) = \begin{bmatrix} g_{11}(x) & \cdots & g_{1p}(x) \\ \vdots & \ddots & \vdots \\ g_{p1}(x) & \cdots & g_{pp}(x) \end{bmatrix}$$

Then, dynamic system (1) can be written in the following compact form

$$y^{(r)} = f(x) + G(x) u \tag{2}$$

The control objective is to design adaptive control $u_i(t)$ for system (1) such that the output $y_i(t)$ follows a specified desired trajectory $y_{di}(t)$ under boundedness of all signals.

Throughout this study we need the following assumptions.

A1: The matrix $G(x)$ is symmetric positive definite and bounded as $0 < \underline{g} I_p \le G(x) \le \overline{g} I_p$, where I_p is the $p \times p$ identity matrix, $\underline{g}$ and $\overline{g}$ are some positive constants.

A2: The desired trajectory $y_{di}(t)$ is a known bounded function of time with bounded known derivatives up to the r_i order.

Remark 1: Notice that Assumption A1 is a sufficient condition ensuring that the matrix $G(x)$ is always regular and, therefore, system (1) is feedback linearizable by a static state feedback. Although this assumption restricts the considered class of MIMO nonlinear systems, many physical systems, such as robotic systems (Slotine & Li, 1991), fulfill such a property.

Define the tracking errors as

$$e_1(t) = y_{d1}(t) - y_1(t)$$
$$\vdots \tag{3}$$
$$e_p(t) = y_{dp}(t) - y_p(t)$$

and the filtered tracking errors as

$$s_1(t) = \left(\frac{d}{dt} + \lambda_1\right)^{r_1 - 1} e_1(t) \ , \ \lambda_1 > 0$$

$$\vdots \tag{4}$$

$$s_p(t) = \left(\frac{d}{dt} + \lambda_p\right)^{r_p - 1} e_p(t) \ , \ \lambda_p > 0$$

From (4), $s_i(t) = 0$ represents a linear differential equation whose solution implies that the tracking error $e_i(t)$ and its derivatives up to $r_i - 1$ converge to zero (Slotine & Li, 1991). Thus, the control objective becomes the design of a controller to keep $s_i(t)$ at zero, $i = 1,\ldots,p$. Moreover, bounds on $s_i(t)$ can be directly translated into bounds on the tracking error. Specifically, if $\left|s_i(t)\right| \le \Phi_i$ where Φ_i is a positive constant, one can conclude that (Slotine & Li, 1991): $\left|e_i^{(j)}(t)\right| \le 2^j \lambda_i^{j - r_i + 1} \Phi_i$, $j = 0,\ldots,r_i - 1$, $i = 1,\ldots,p$. These bounds can be reduced by increasing the parameters λ_i.

The time derivatives of the filtered errors (4) can be written as

$$\dot{s}_1 = v_1 - f_1(\boldsymbol{x}) - \sum_{j=1}^{p} g_{1j}(\boldsymbol{x}) u_j$$

$$\vdots \tag{5}$$

$$\dot{s}_p = v_p - f_p(\boldsymbol{x}) - \sum_{j=1}^{p} g_{pj}(\boldsymbol{x}) u_j$$

where $v_1,\ldots,v_p$, are given as follows

$$v_1 = y_{d1}^{(r_1)} + \beta_{1,r_1 - 1} e_1^{(r_1 - 1)} + \ldots + \beta_{1,1} \dot{e}_1$$

$$\vdots \tag{6}$$

$$v_p = y_{dp}^{(r_p)} + \beta_{p,r_p - 1} e_p^{(r_p - 1)} + \ldots + \beta_{p,1} \dot{e}_p$$

with

$$\beta_{i,j} = C_{r_i - 1}^{j - 1} \lambda_i^{r_i - j} \ , \ i = 1,\ldots p, \ \ j = 1,\ldots,r_i - 1$$

Denote

$$\boldsymbol{s} = \left[s_1,\ldots,s_p\right]^T ; \ \boldsymbol{v} = \left[v_1,\ldots,v_p\right]^T$$

Then equation (5) can be written in matrix form as

$$\dot{\boldsymbol{s}} = \boldsymbol{v} - \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} \tag{7}$$

If the nonlinear functions $f(x)$ and $G(x)$ are known, to achieve the control objectives, one can use the following ideal nonlinear control law (Labiod & Guerra, 2007b)

$$u^* = G^{-1}(x)\left(-f(x) + v + Ks + K_0\tanh(s/\varepsilon_0)\right) \tag{8}$$

where, $K = diag[k_1,\ldots,k_p]$, $K_0 = diag[k_{01},\ldots,k_{0p}]$, with $k_i > 0$ and $k_{0i} > 0$, for $i = 1,\ldots,p$, ε_0 is a small positive constant, and $\tanh(\cdot)$ is the hyperbolic tangent function defined for the vector $s = [s_1,\ldots,s_p]^T$ as $\tanh(s/\varepsilon_0) = [\tanh(s_1/\varepsilon_0),\ldots,\tanh(s_p/\varepsilon_0)]^T$.

Effectively, when we select the control input as $u = u^*$, equation (7) simplifies to

$$\dot{s} = -Ks - K_0\tanh(s/\varepsilon_0) \tag{9}$$

or, equivalently

$$\dot{s}_i = -k_is_i - k_{0i}\tanh(s_i/\varepsilon_0), \quad i = 1,\ldots,p \tag{10}$$

From which we can conclude that $s_i(t) \to 0$ as $t \to \infty$ and, therefore, $e_i(t)$ and all its derivatives up to $r_i - 1$ converge to zero.

It is clear that if $f(x)$ and $G(x)$ are completely unknown, the proposed nonlinear control law (8) is not feasible. In this case, in order to overcome this design difficulty, we propose to use fuzzy systems to construct adaptively the unknown functions. The idea is to use fuzzy systems to identity the entire unknown control function (8) in the direct approach, and to identify the unknown nonlinear functions $f(x)$ and $G(x)$ in (8) in the indirect approach.

In this chapter we use the zero-order Takagi-Sugeno fuzzy system that performs a mapping from an input vector $z = [z_1,\ldots,z_m]^T \in \Omega_z \subset \mathbb{R}^m$ to a scalar output variable $y_f \in \mathbb{R}$, where $\Omega_z = \Omega_{z_1} \times \cdots \times \Omega_{z_m}$ and $\Omega_{z_i} \subset \mathbb{R}$. If we define M_i fuzzy sets F_i^j , $j = 1,\ldots,M_i$, for each input z_i , then the fuzzy system will be characterized by a set of if-then rules of the form (Wang, 1994; Jang & Sun, 1995; Passino & Yurkovich, 1998)

$$R^k : \text{If } z_1 \text{ is } G_1^k \text{ and}\ldots\text{and } z_m \text{ is } G_m^k \text{ Then } y_f \text{ is } y_f^k \quad (k = 1,\ldots,N)$$

where $G_i^k \in \{F_i^1,\ldots,F_i^{M_i}\}$, $i = 1,\ldots,n$, y_f^k is the crisp output of the k-th rule, and N is the total number of rules.

By using the singleton fuzzifier and the product inference engine, the final output of the fuzzy system is given as follows (Wang, 1994; Jang & Sun, 1995; Passino & Yurkovich, 1998)

$$y_f(z) = \frac{\sum_{k=1}^{N} \mu_k(z) y_f^k}{\sum_{k=1}^{N} \mu_k(z)} \tag{11}$$

where

$$\mu_k(z) = \prod_{i=1}^{m} \mu_{G_i^k}(z_i), \text{ with } \mu_{G_i^k} \in \left\{ \mu_{F_i^1}, \ldots, \mu_{F_i^{M_i}} \right\}$$

where $\mu_{F_i^j}(x_i)$ is the membership function of the fuzzy set F_i^j.

By introducing the concept of fuzzy basis functions (Wang, 1994), the output given by (11) can be rewritten in the following compact form

$$y_f(z) = w^T(z)\theta \tag{12}$$

where $\theta = \left[y_f^1, \ldots, y_f^N \right]^T$ is a vector grouping all consequent parameters, and $w(z) = \left[w_1(z), \ldots, w_N(z) \right]^T$ is a set of fuzzy basis functions defined as

$$w_k(z) = \frac{\mu_k(z)}{\sum_{j=1}^{N} \mu_j(z)}, \ k = 1, \ldots, N \tag{13}$$

The fuzzy system (12) is assumed to be well-defined so that $\sum_{j=1}^{N} \mu_j(z) \neq 0$ for all $z \in \Omega_z$.

It has been proved in (Wang, 1994) that fuzzy systems in the form of (12) with Gaussian membership functions can approximate continuous functions over a compact set to an arbitrary degree of accuracy provided that enough number of rules are considered. So, for a general smooth nonlinear function $f(z)$ defined from $\mathbb{R}^n$ to $\mathbb{R}$, there exists a fuzzy system in the form of (12) with some optimal parameters θ^* such that

$$\sup_{z \in \Omega_z} \left| f(z) - w^T(z)\theta^* \right| \leq \bar{\varepsilon} \ ),$$

where $\bar{\varepsilon}$ is a positive constant. Thus, one can express $f(z)$ as

$$f(z) = w^T(z)\theta^* + \varepsilon(z),$$

where $\varepsilon(z)$ is the fuzzy approximation error satisfying $\left| \varepsilon(z) \right| \leq \bar{\varepsilon}$ for $z \in \Omega_z$.

In this chapter, it is assumed that the structure of the fuzzy system and the fuzzy basis function parameters are properly specified in advance by the designer. This means that the designer decision is needed to determine the structure of the fuzzy system (that is, determine relevant inputs, number of membership functions for each input, membership

function parameters, number of rules), and the consequent parameters should be calculated by learning algorithms.

It should be noticed that fuzzy systems can be replaced by any other linearly parameterized universal function approximator without any technical difficulty such as neural networks and wavelet networks. However, only fuzzy logic systems can make use of linguistic information in a systematic way.

3. Direct adaptive fuzzy control

In section 2 we have established that there exists an ideal control law u^* given by (8) that can achieve the control objective. However, this nonlinear controller cannot be used since it depends on unknown functions. In this section, to circumvent this problem, we propose to use adaptive fuzzy systems for approximating this ideal controller, and the error between the fuzzy controller and the ideal controller will be used to update the free parameters of the fuzzy controller.

To develop the control law, we represent each component of the ideal input control vector $u^* = \left[u_1^*, \ldots, u_p^* \right]$ by a fuzzy system in the form of (12) as the following

$$u_i^*(z) = w_i^T(z)\theta_i^* + \varepsilon_i(z); \quad i = 1, \ldots, p \tag{14}$$

where $z = \left[x^T, s^T \right]^T$, $\varepsilon_i(z)$ is the fuzzy approximation error, θ_i^* is an unknown ideal parameter vector that minimizes the function $\left| \varepsilon_i(z) \right|$ over an operating compact set Ω_z, and $w_i(z)$ is a fuzzy basis function vector assumed suitably specified by the designer. In this study, we assume that the used fuzzy systems do not violate the universal approximation property on the compact set Ω_z, which is assumed large enough so that the variable z remains inside it under closed-loop control. So it is reasonable to assume that the fuzzy approximation error is bounded for all $z \in \Omega_z$.

Let us denote

$$\varepsilon(z) = \left[\varepsilon_1(z), \ldots, \varepsilon_p(z) \right]^T; \ \theta^* = \left[\theta_1^{*T}, \ldots, \theta_p^{*T} \right]^T, \ w(z) = diag\left[w_1(z), \ldots, w_p(z) \right]$$

Therefore, one can write (8) as

$$u^* = w^T(z)\theta^* + \varepsilon(z) \tag{15}$$

Since the ideal parameter vector θ^* is unknown, let us use its estimate θ instead to form the adaptive control

$$u(z) = w^T(z)\theta \tag{16}$$

The next step should be the design of an adaptive law for the free parameters θ such that the control law u approximates, as best as possible, the ideal controller u^*. To this end, let us define the error between the controllers u^* and u as:

$$e_u = u^* - u \tag{17}$$

The error e_u represents the actual deviation between the unknown function u^* and the online fuzzy approximator (16), while the fuzzy approximation error $\varepsilon(z)$ represents the minimum possible deviation between the unknown function u^* and the online fuzzy approximator, i.e. $\varepsilon(z)$ represents the minimum possible value of e_u.

Using (15) and (16), (17) becomes

$$e_u = u^* - w^T(z)\theta = w^T(z)\tilde{\theta} + \varepsilon(z) \tag{18}$$

where $\tilde{\theta} = \theta^* - \theta$ is the parameter estimation error vector.

Adding and subtracting $G(x)u^*$ to the right-hand side of (7), we obtain the error equation governing the closed-loop system

$$\dot{s} = v - f(x) - G(x)u + G(x)u^* - G(x)u^* \tag{19}$$

With (8) and (18), (19) becomes

$$\dot{s} = -K s - K_0 \tanh(s/\varepsilon_0) + G(x)e_u \tag{20}$$

Now, consider a quadratic cost function; that measures the discrepancy between the ideal controller and the actual fuzzy controller, defined as

$$J(\theta) = \frac{1}{2}e_u^T G(x)e_u = \frac{1}{2}\left(u^* - w^T(z)\theta\right)^T G(x)\left(u^* - w^T(z)\theta\right) \tag{21}$$

We use the gradient descent method to minimize the cost function (21) with respect to the adjustable parameters θ. Consequently, applying the gradient method (Slotine & Li, 1991; Ioannou & Sun, 1996), the minimizing trajectory $\theta(t)$ is generated by the following differential equation

$$\dot{\theta} = -\eta \nabla_\theta J(\theta) \tag{22}$$

where η is a positive constant parameter.

From (21), the gradient of $J(\theta)$ with respect to θ is

$$\frac{\partial J(\theta)}{\partial \theta} = -w(z)G(x)e_u$$

Therefore, the gradient descent algorithm becomes

$$\dot{\theta} = \eta\, w(z)G(x)e_u \qquad (23)$$

We recall here that the ideal controller u^* is unknown, so the error signal e_u defined in (17) is not available. Equation (20) will be used to overcome this design difficulty. Indeed, from (20), we see that even if the error vector e_u is not available, the vector $G(x)e_u$ is available, and it is given by

$$G(x)e_u = \dot{s} + K\,s + K_0 \tanh\left(s/\varepsilon_0\right)$$

Therefore, (23) becomes

$$\dot{\theta} = \eta\, w(z)\left\{\dot{s} + K\,s + K_0 \tanh\left(s/\varepsilon_0\right)\right\} \qquad (24)$$

As shown by (Ioannou & Sun, 1996), an adaptive law in the form of (24) cannot guarantee the boundedness of the parameters $\tilde{\theta}$ in the presence of approximation errors, which are unavoidable in such adaptive schemes. So, to improve the robustness of the adaptive law (24) in the presence of approximation errors, we modify it by introducing a σ-modification term as follows (Ioannou & Sun, 1996)

$$\dot{\theta} = \eta\, w(z)\left\{\dot{s} + K\,s + K_0 \tanh\left(s/\varepsilon_0\right)\right\} - \eta\,\sigma\,\theta \qquad (25)$$

where σ is a small positive constant.

The following theorem summarizes the stability result for the proposed direct adaptive control scheme.

Theorem 1: *Consider the system in (1) with the control law defined by (17). Suppose that Assumptions A1 and A2 hold, the approximation error $\varepsilon(z)$ in (18) is bounded as $\|\varepsilon(z)\| \le \bar{\varepsilon}$ where $\bar{\varepsilon}$ is a positive constant, and that the free parameters θ are updated according to (25). Then, all the closed-loop signals are uniformly ultimately bounded, and the tracking errors are attracted to a neighbourhood of the origin whose size can be adjusted by control parameters.*

Proof: Let us consider the following Lyapunov function candidate

$$V = \frac{1}{2}s^T s + \frac{1}{2\eta}\tilde{\theta}^T\tilde{\theta} \tag{26}$$

Differentiating (26) with respect to time and using (20) and (25), we get

$$\dot{V} = -s^T K s - s^T K_0 \tanh(s/\varepsilon_0) + s^T G(x)e_u - \tilde{\theta}^T\left(w(z)G(x)e_u - \sigma\theta\right) \tag{27}$$

With (18), (27) becomes

$$\dot{V} = -s^T K s - s^T K_0 \tanh(s/\varepsilon_0) + s^T G(x)e_u - e_u^{\ T}G(x)e_u + \varepsilon^T(z)G(x)e_u + \sigma\tilde{\theta}^T\theta \tag{28}$$

Using the following inequalities

$$\sigma\tilde{\theta}^T\theta \leq -\frac{\sigma}{2}\left\|\tilde{\theta}\right\|^2 + \frac{\sigma}{2}\left\|\theta^*\right\|^2$$

$$\varepsilon^T(z)G(x)e_u \leq \frac{1}{4}e_u^{\ T}G(x)e_u + \varepsilon^T(z)G(x)\varepsilon(z)$$

$$s^T G(x)e_u \leq \frac{1}{2}e_u^{\ T}G(x)e_u + \frac{1}{2}s^T G(x)s$$

we have

$$\dot{V} \leq -s^T\left(K - \frac{1}{2}G(x)\right)s - s^T K_0 \tanh(s/\varepsilon_0) - \frac{1}{4}e_u^{\ T}G(x)e_u - \frac{\sigma}{2}\left\|\tilde{\theta}\right\|^2 + \varepsilon(z)^T G(x)\varepsilon(z) + \frac{\sigma}{2}\left\|\theta^*\right\|^2 \tag{29}$$

Since $\varepsilon(z)$ and $G(x)$ are assumed bounded in this study and θ^* is a constant vector, we can define a positive constant bound ψ_1 as

$$\psi_1 = \sup_t\left(\varepsilon(z)^T G(x)\varepsilon(z)\right) + \frac{1}{2}\sigma\left\|\theta^*\right\|^2$$

Then, (29) can be simplified to

$$\dot{V} \leq -s^T\left(K - \frac{1}{2}\overline{g}I_p\right)s - \frac{\sigma}{2}\left\|\tilde{\theta}\right\|^2 - s^T K_0 \tanh(s/\varepsilon_0) - \frac{1}{4}e_u^{\ T}G(x)e_u + \psi_1 \tag{30}$$

We assume here that each design parameter k_i is chosen such that $k_i > \overline{g}/2$ and we let $\kappa = 2\min_{1 \leq i \leq p}(k_i - \overline{g}/2)$. Consequently, (30) can be bounded by

$$\dot{V} \le -\alpha_1 V - s^T K_0 \tanh\left(s/\varepsilon_0\right) - \frac{1}{4} e_u^{\ T} G(x) e_u + \psi_1 \ \le -\alpha_1 V + \psi_1 \tag{31}$$

where $\alpha_1 = \min(\kappa, \sigma\eta)$.

From (31), one can establish that the Lyapunov function candidate satisfies the following condition

$$0 \le V(t) \le \left(V(0) - \frac{\psi_1}{\alpha_1}\right) e^{-\alpha_1 t} + \frac{\psi_1}{\alpha_1} \tag{32}$$

This last condition implies that $s(t)$ and $\tilde{\theta}(t)$ are uniformly bounded, and that $s(t)$ is uniformly ultimately bounded with respect to the set $\Omega_s = \left\{s : \|s\| \le \sqrt{2\psi_1/\alpha_1}\right\}$. This consequently leads to uniform boundedness of the tracking errors $\left|e_i^{(j)}(t)\right| \le 2^j \lambda_i^{j-r_i+1} \sqrt{2\psi_1/\alpha_1}$, $j = 0,\ldots,r_i - 1$, $i = 1,\ldots,p$ (Slotine & Li, 1991).

Remark 2: In the absence of the approximation error, i.e., $\varepsilon(x) = 0$ in (18), by setting $\sigma = 0$ in (25), one can show that the tracking errors are asymptotically stable, i.e., $e_i(t) \to 0$ as $t \to \infty$, for $i = 1,\ldots,p$.

Remark 3: It is worth noticing that the parameter updating law (25) is not implementable in case the derivative of $s(t)$ is not available. However, a discrete implementable version of (25) can be obtained. Rewriting (25) as

$$\theta(t) = \theta(t - \Delta t) + \int_{t-\Delta t}^{t} \phi_1(\tau)\dot{s}(\tau)d\tau + \int_{t-\Delta t}^{t} \phi_2(\tau)d\tau,$$

where Δt is a small positive constant, $\phi_1 = \eta\, w(z)$ and $\phi_2 = \eta\, w(z)\left(K s + K_0 \tanh\left(s/\varepsilon_0\right)\right) - \eta\,\sigma\theta$. Using the fact that $\dot{s} = ds/dt$, the expression of $\theta(t)$ becomes

$$\theta(t) = \theta(t - \Delta t) + \int_{s(t-\Delta t)}^{s(t)} \phi_1(\tau)ds + \int_{t-\Delta t}^{t} \phi_2(\tau)d\tau.$$

By assuming that $\phi_1(t)$, $\phi_2(t)$ and $s(t)$ are continuous time functions and that Δt is small enough, a discrete implementable version of (25) is given by: $\theta(t) = \theta(t - \Delta t) + \phi_1(t - \Delta t)\left(s(t) - s(t - \Delta t)\right) + \phi_2(t - \Delta t)\Delta t$, which represents a good discrete approximation of the parameter update law (25) if Δt is chosen sufficiently small.

4. Indirect adaptive fuzzy control

In this section we propose to indirectly approximate the unknown ideal controller (8) by identifying the unknown functions $f_i(x)$ and $g_{ij}(x)$ using fuzzy systems. First, let us

assume that the nonlinear functions $f_i(x)$ and $g_{ij}(x)$ can be approximated, over a compact set D_x, by fuzzy systems of the form of (12) as follows

$$f_i(x) = f_i^*(x) + \varepsilon_{f_i}(x); \quad f_i^*(x) = w_{f_i}^T(x)\theta_{f_i}^*, \quad i = 1,\ldots,p \tag{33}$$

$$g_{ij}(x) = g_{ij}^*(x) + \varepsilon_{g_{ij}}(x); \quad g_{ij}^*(x) = w_{g_{ij}}^T(x)\theta_{g_{ij}}^*, \quad i,j = 1,\ldots,p \tag{34}$$

where $\varepsilon_{f_i}(x)$ and $\varepsilon_{g_{ij}}(x)$ are fuzzy approximation errors, $\theta_{f_i}^*$ and $\theta_{g_{ij}}^*$ are optimal parameter vectors that minimize functions $\left|\varepsilon_{f_i}(x)\right|$ and $\left|\varepsilon_{g_{ij}}(x)\right|$, respectively, and $w_{f_i}(x)$ and $w_{g_{ij}}(x)$ are fuzzy basis function vectors assumed suitably specified by the designer.

In this study, we assume that the used fuzzy systems do not violate the universal approximation property on the operating compact set D_x, which is assumed large enough so that state variables remain within D_x under closed-loop control. So it is reasonable to assume that the minimum approximation errors are bounded for all $x \in D_x$.

Since the ideal parameter vectors $\theta_{f_i}^*$ and $\theta_{g_{ij}}^*$ is unknown, let us use their estimates θ_{f_i} and $\theta_{g_{ij}}$ instead to form the adaptive approximations

$$\hat{f}_i(x) = w_{f_i}^T(x)\theta_{f_i}, \quad i = 1,\ldots,p \tag{35}$$

$$\hat{g}_{ij}(x) = w_{g_{ij}}^T(x)\theta_{g_{ij}}, \quad i,j = 1,\ldots,p \tag{36}$$

Denote

$$\hat{f}(x) = \left[\hat{f}_1(x),\ldots,\hat{f}_p(x)\right]^T, \quad \varepsilon_f(x) = \left[\varepsilon_{f1}(x),\ldots,\varepsilon_{fp}(x)\right]^T$$

$$\hat{G}(x) = \begin{bmatrix} \hat{g}_{11}(x) & \cdots & \hat{g}_{1p}(x) \\ \vdots & \ddots & \vdots \\ \hat{g}_{p1}(x) & \cdots & \hat{g}_{pp}(x) \end{bmatrix}, \quad \varepsilon_G(x) = \begin{bmatrix} \varepsilon_{g11}(x) & \cdots & \varepsilon_{g1p}(x) \\ \vdots & \ddots & \vdots \\ \varepsilon_{gp1}(x) & \cdots & \varepsilon_{gpp}(x) \end{bmatrix}$$

Now we can write an expression for the adaptive control law

$$u = \hat{G}^{-1}(x)\left(-\hat{f}(x) + v + Ks + K_0 \tanh(s/\varepsilon_0)\right) \tag{37}$$

This control term results from (8) by using the adaptive fuzzy approximations $\hat{f}(x)$ and $\hat{G}(x)$ instead of actual functions $f(x)$ and $G(x)$, respectively.

Adding and subtracting $\hat{f}(x)$ and $\hat{G}(x)u$ to the right-hand side of (7), we get

$$\dot{s} = v - \left(f(x) - \hat{f}(x)\right) - \left(G(x) - \hat{G}(x)\right)u - \hat{G}(x)u - \hat{f}(x) \tag{38}$$

Using the control law (37), (38) becomes

$$\dot{s} = -K s - K_0 \tanh(s/\varepsilon_0) - \left(f(x) - \hat{f}(x)\right) - \left(G(x) - \hat{G}(x)\right)u \tag{39}$$

where each element of the vector $\dot{s}$ is given by

$$\dot{s}_i = -k_i s_i - k_{0i} \tanh(s_i/\varepsilon_0) - \left(f_i(x) - \hat{f}_i(x)\right) - \sum_{j=1}^{p}\left(g_{ij}(x) - \hat{g}_{ij}(x)\right)u_j \tag{40}$$

The next task should be the design of adaptive laws for the free parameters θ_{fi} and θ_{gij} such that $\hat{f}(x) + \hat{G}(x)u$ approximates, as best as possible, the unknown nonlinear function $f(x) + G(x)u$. To this end, let us define the modelling error e_m between $f(x) + G(x)u$ and $\hat{f}(x) + \hat{G}(x)u$ as:

$$e_m = \left(f(x) + G(x)u\right) - \left(\hat{f}(x) + \hat{G}(x)u\right) = \left(f(x) - \hat{f}(x)\right) + \left(G(x) - \hat{G}(x)\right)u \tag{41}$$

where each component of the vector e_m is given by

$$e_{mi} = \left(f_i(x) - \hat{f}_i(x)\right) - \sum_{j=1}^{p}\left(g_{ij}(x) - \hat{g}_{ij}(x)\right)u_j = \left(f_i^*(x) - \hat{f}_i(x)\right) + \sum_{j=1}^{p}\left(g_{ij}^*(x) - \hat{g}_{ij}(x)\right)u_j + \varepsilon_i(x) \tag{42}$$

$$e_{mi} = w_{f_i}^T(x)\tilde{\theta}_{f_i} + \sum_{j=1}^{p}w_{g_{ij}}^T(x)\tilde{\theta}_{g_{ij}}u_j + \varepsilon_i(x) \tag{43}$$

with $\varepsilon_i(x) = \varepsilon_{f_i}(x) + \sum_{j=1}^{p}\varepsilon_{g_{ij}}(x)u_j$, and $\tilde{\theta}_{f_i} = \theta_{f_i}^* - \theta_{f_i}$ and $\tilde{\theta}_{g_{ij}} = \theta_{g_{ij}}^* - \theta_{g_{ij}}$.

Then, from (39) and (41) we have

$$e_m = -\left(\dot{s} + K s + K_0 \tanh(s/\varepsilon_0)\right) \tag{44}$$

Now, consider a quadratic cost function; that measures the discrepancy between the unknown nonlinearities and their adaptive fuzzy approximations, defined as

$$J(\theta) = \frac{1}{2}e_m^T e_m = \frac{1}{2}\sum_{i=1}^{p}\left[\left(f_i(x) - \hat{f}_i(x)\right) + \sum_{j=1}^{p}\left(g_{ij}(x) - \hat{g}_{ij}(x)\right)u_j\right]^2 \tag{45}$$

Applying the gradient method, the minimizing trajectories $\theta_{fi}(t)$ and $\theta_{gij}(t)$ are generated by the following differential equations

$$\begin{cases} \dot{\theta}_{fi} = -\eta \, \nabla_{\theta_{fi}} J(\theta) \\ \dot{\theta}_{gij} = -\eta \, \nabla_{\theta_{gij}} J(\theta) \end{cases} \tag{46}$$

where η is a positive constant parameter.

Therefore, the gradient descent algorithm becomes

$$\begin{cases} \dot{\theta}_{fi} = \eta \, w_{fi}(x) e_{mi} \\ \dot{\theta}_{gij} = \eta \, w_{gij}(x) u_j e_{mi} \end{cases} \tag{47}$$

Since the modelling error e_m is not available, equation (44) will be used to overcome this design difficulty. Then, we obtain

$$\begin{cases} \dot{\theta}_{fi} = -\eta \, w_{fi}(x)\big(\dot{s} + k_i s_i + k_{0i} \tanh(s_i/\varepsilon_0)\big) \\ \dot{\theta}_{gij} = -\eta \, w_{gij}(x) u_j \big(\dot{s} + k_i s_i + k_{0i} \tanh(s_i/\varepsilon_0)\big) \end{cases} \tag{48}$$

In order to improve the robustness of the adaptive law (48) in the presence of approximation errors, we modify it by introducing a σ-modification term as follows (Ioannou & Sun, 1996)

$$\begin{cases} \dot{\theta}_{fi} = -\eta \, w_{fi}(x)\big(\dot{s} + k_i s_i + k_{0i} \tanh(s_i/\varepsilon_0)\big) - \eta \, \sigma \theta_{fi} \\ \dot{\theta}_{gij} = -\eta \, w_{gij}(x) u_j \big(\dot{s} + k_i s_i + k_{0i} \tanh(s_i/\varepsilon_0)\big) - \eta \, \sigma \theta_{gij} \end{cases} \tag{49}$$

where σ is a small positive constant.

Before proceeding we need to introduce an assumption about the approximation errors $\varepsilon(x) = \varepsilon_f(x) + \varepsilon_G(x)u$. Since $\varepsilon(x)$ depends upon the control input u, $\varepsilon(x)$ is not assumed bounded by constant bounds but it is assumed bounded by functional bounds.

A3: The function $\varepsilon(x) = \varepsilon_f(x) + \varepsilon_G(x)u$ is bounded as follows

$$\|\varepsilon(x)\| \leq \sqrt{\delta_0 + \sum_{i=1}^{p} \delta_i s_i^2} \quad ; \quad \delta_i > 0, \, i = 0, \ldots, p.$$

The following theorem summarizes the stability result for the proposed indirect adaptive control scheme.

Theorem 2: *Consider the system in (1) with the control law defined by (37). Suppose that Assumptions A1-A3 hold and that the free parameters of the used fuzzy systems are updated according to (49). Then, all the closed-loop signals are uniformly ultimately bounded, and the*

tracking errors are attracted to a neighbourhood of the origin whose size can be adjusted by control parameters.

Proof: Let us consider the following Lyapunov function candidate

$$V = \frac{1}{2}\sum_{i=1}^{p}\left(s_i^2 + \frac{1}{\eta}\tilde{\theta}_{f_i}^T\tilde{\theta}_{f_i} + \frac{1}{\eta}\sum_{j=1}^{p}\tilde{\theta}_{g_{ij}}^T\tilde{\theta}_{g_{ij}} \right) \tag{50}$$

Differentiating (26) with respect to time and using (40), (43), (44) and (49), we get

$$\dot{V} = \sum_{i=1}^{p}\left(-k_i s_i^2 - k_{0i} s_i \tanh\left(s_i/\varepsilon_0\right) - s_i e_{mi} - e_{mi}\left(e_{mi} - \varepsilon_i(x)\right) + \sigma\tilde{\theta}_{f_i}^T\theta_{f_i} + \sigma\sum_{j=1}^{p}\tilde{\theta}_{g_{ij}}^T\theta_{g_{ij}} \right) \tag{51}$$

Using the following inequalities

$$\sigma\tilde{\theta}_{f_i}^T\theta_{f_i} \leq -\frac{\sigma}{2}\left\|\tilde{\theta}_{f_i}\right\|^2 + \frac{\sigma}{2}\left\|\theta_{f_i}^*\right\|^2, \quad \sigma\tilde{\theta}_{g_{ij}}^T\theta_{g_{ij}} \leq -\frac{\sigma}{2}\left\|\tilde{\theta}_{g_{ij}}\right\|^2 + \frac{\sigma}{2}\left\|\theta_{g_{ij}}^*\right\|^2$$

$$\varepsilon_i(x)e_{mi} \leq \frac{1}{4}e_{mi}^2 + \varepsilon_i^2(x), \quad s_i e_{mi} \leq \frac{1}{4}e_{mi}^2 + s_i^2$$

we obtain

$$\dot{V} = \sum_{i=1}^{p}\left(-(k_i-1)s_i^2 - k_{0i}s_i\tanh\left(\frac{s_i}{\varepsilon_0}\right) - \frac{1}{2}e_{mi}^2 - \frac{\sigma}{2}\left\|\tilde{\theta}_{f_i}\right\|^2 + \frac{\sigma}{2}\left\|\theta_{f_i}^*\right\|^2 - \frac{\sigma}{2}\sum_{j=1}^{p}\left\|\tilde{\theta}_{g_{ij}}\right\|^2 + \frac{\sigma}{2}\sum_{j=1}^{p}\left\|\theta_{g_{ij}}^*\right\|^2 + \varepsilon_i^2(x) \right) \tag{52}$$

Since $\left\|\varepsilon(x)\right\|^2 = \sum_{i=1}^{p}\varepsilon_i^2(x)$ and using assumption A3, we have

$$\dot{V} = \sum_{i=1}^{p}\left(-(k_i-\delta_i-1)s_i^2 - k_{0i}s_i\tanh\left(\frac{s_i}{\varepsilon_0}\right) - \frac{1}{2}e_{mi}^2 \right) + \psi_2 \tag{53}$$

with

$$\psi_2 = \delta_0 + \sum_{i=1}^{p}\left(\frac{\sigma}{2}\left\|\theta_{f_i}^*\right\|^2 + \frac{\sigma}{2}\sum_{j=1}^{p}\left\|\theta_{g_{ij}}^*\right\|^2 \right).$$

We assume here that each design parameter k_i is chosen such that $k_i > (\delta_i - 1)$ and we let $\kappa = 2\min_{1\leq i\leq p}(k_i - \delta_i - 1)$. Consequently, (53) can be bounded by

$$\dot{V} \le -\alpha_2 V - \boldsymbol{s}^T \boldsymbol{K}_0 \tanh\left(\boldsymbol{s}/\varepsilon_0\right) - \frac{1}{2} \boldsymbol{e}_m{}^T \boldsymbol{e}_m + \psi_2 \ \le -\alpha_2 V + \psi_2 \tag{54}$$

where $\alpha_2 = \min\left(\kappa, \sigma\eta\right)$.

From (54), one can establish that the Lyapunov function candidate satisfies the following condition

$$0 \le V\left(t\right) \le \left(V\left(0\right) - \frac{\psi_2}{\alpha_2}\right)e^{-\alpha_2 t} + \frac{\psi_2}{\alpha_2} \tag{55}$$

This last condition implies that $\boldsymbol{s}(t)$, $\tilde{\theta}_{f_i}(t)$ and $\tilde{\theta}_{g_{ij}}(t)$ are uniformly bounded, and that $\boldsymbol{s}(t)$ is uniformly ultimately bounded with respect to the set $\Omega_s = \left\{\boldsymbol{s} : \|\boldsymbol{s}\| \le \sqrt{2\psi_2/\alpha_2}\right\}$. This consequently leads to uniform boundedness of the tracking errors $\left|e_i^{(j)}\left(t\right)\right| \le 2^j \lambda_i^{j-r_i+1} \sqrt{2\psi_2/\alpha_2}$, $j = 0,\ldots,r_i - 1$, $i = 1,\ldots,p$ (Slotine & Li, 1991).

Remark 4: Since the matrix $\hat{G}(x)$ is generated on line, the control law (37) is not well-defined if $\hat{G}(x)$ becomes not regular. To overcome this singularity problem, we use a regularized inverse as in (Labiod et al., 2005) given by $\hat{G}^{-1}(x) \approx \hat{G}^T(x)\left[\gamma_0 I_p + \hat{G}(x)\hat{G}^T(x)\right]^{-1}$, where γ_0 is a small positive constant.

Remark 5: In the absence of the approximation error, i.e., $\varepsilon(x) = 0$, by setting $\sigma = 0$ in (49), one can show that the tracking errors are asymptotically stable, i.e., $e_i(t) \to 0$ as $t \to \infty$, for $i = 1,\ldots,p$.

5. Simulation results

In this section, we test the proposed direct adaptive fuzzy control scheme on the tracking control of a two-link rigid robot manipulator with the following dynamics (Labiod et al., 2005; Slotine & Li, 1991; Tong et al., 2000):

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} -h\dot{q}_2 & -h\left(\dot{q}_1 + \dot{q}_2\right) \\ h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \right\} \tag{56}$$

where

$$M_{11} = a_1 + 2a_3 \cos\left(q_2\right) + 2a_4 \sin\left(q_2\right), \ M_{22} = a_2, M_{21} = M_{12} = a_2 + a_3 \cos\left(q_2\right) + a_4 \sin\left(q_2\right),$$

$$h = a_3 \sin\left(q_2\right) - a_4 \cos\left(q_2\right),$$

with

$$a_1 = I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2, \ a_2 = I_e + m_e l_{ce}^2, \ a_3 = m_e l_1 l_{ce} \cos\delta_e, \ a_4 = m_e l_1 l_{ce} \sin\delta_e.$$

In the simulation, the following parameter values are used

$$m_1 = 1, m_e = 2, l_1 = 1, l_{c1} = 0.5, l_{ce} = 0.6, I_1 = 0.12, I_e = 0.25, \delta_e = 30°.$$

Let $y = \begin{bmatrix} q_1, & q_2 \end{bmatrix}^T$, $u = \begin{bmatrix} u_1, & u_2 \end{bmatrix}^T$, $x = \begin{bmatrix} q_1, & \dot{q}_1, & q_2, & \dot{q}_2 \end{bmatrix}^T$, and

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} = -M^{-1} \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix},$$

$$G(x) = \begin{bmatrix} g_{11}(x) & g_{12}(x) \\ g_{21}(x) & g_{22}(x) \end{bmatrix} = M^{-1} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1}.$$

Then, the robot system given by (56) can be expressed as

$$\ddot{y} = f(x) + G(x)u \tag{57}$$

which is in the input-output form given by (2). Since the matrix M is positive definite (Slotine & Li, 1991), then it is always regular and $G(x) = M^{-1}$ is also positive definite.

The control objective is to force the system outputs q_1 and q_2 to follow the desired trajectories $y_{d1}(t) = \sin(t)$ and $y_{d2}(t) = \cos(t)$, respectively.

To synthesize the direct adaptive fuzzy controller, two fuzzy systems in the form of (12) are used to generate the control signals u_1 and u_2. Each fuzzy system has $z = \begin{bmatrix} e_1(t), \dot{e}_1(t), e_2(t), \dot{e}_2(t) \end{bmatrix}^T$ as input, and for each input variable z_j, $j = 1, \ldots, 4$, three Gaussian membership functions are defined as

$$\mu_{F_j^1}(z_j) = \exp\left(-\frac{1}{2}\left(\frac{z_j + 1.25}{0.6}\right)^2\right), \ \mu_{F_j^2}(z_j) = \exp\left(-\frac{1}{2}\left(\frac{z_j}{0.6}\right)^2\right), \ \mu_{F_j^3}(z_j) = \exp\left(-\frac{1}{2}\left(\frac{z_j - 1.25}{0.6}\right)^2\right).$$

The robot initial conditions are $x(0) = \begin{bmatrix} 0.25 & 0 & 0.5 & 0 \end{bmatrix}^T$, and the initial values of the parameter estimates $\theta_1(0)$ and $\theta_2(0)$ are set equal to zero. The design parameters used in this simulation are chosen as follows:

$$\lambda_1 = 1, \ \lambda_2 = 1 \ , \ K = diag[1,1], \ K_0 = diag[5,5], \ \varepsilon_0 = 0.01, \ \eta = 5, \text{ and } \sigma = 0.001.$$

The simulation results for the first link are shown in Fig. 1, those for the second link are shown in Fig. 2, and the control input signals are shown in Fig. 3. We can note that the actual trajectories converge to the desired trajectories and the control signals are almost smooth. These simulation results demonstrate the tracking capability of the proposed direct

adaptive controller and its effectiveness for control tracking of uncertain multivariable nonlinear systems.
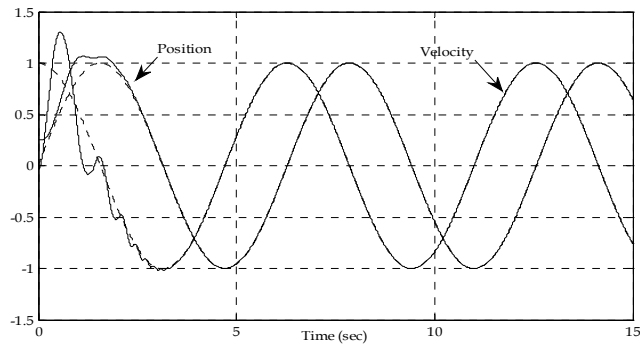

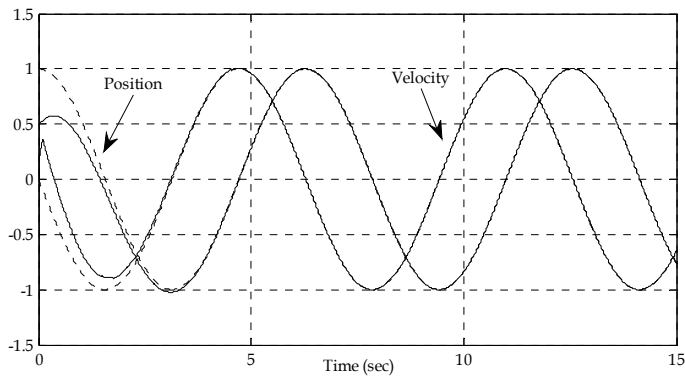Fig. 1. Tracking curves of link 1: actual (solid lines); desired (dotted lines).

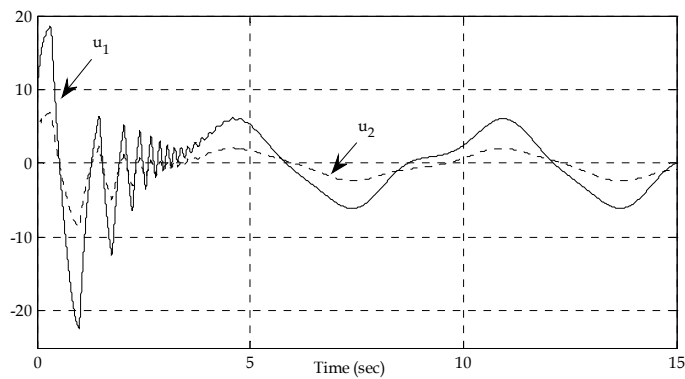Fig. 2. Tracking curves of link 2: actual (solid lines); desired (dotted lines).

Fig. 3. Control input signals: u_1 (solid line); u_2 (dotted line).

6. Conclusion

In this chapter, stable direct and indirect adaptive fuzzy controllers for a class of MIMO nonlinear systems with uncertain model dynamics are presented. In the direct scheme, fuzzy systems are used to construct adaptively an unknown ideal controller and their adjustable parameters are updated by using the gradient descent method in order to minimize the error between the unknown controller and the fuzzy controller. In the indirect scheme, the controller design is based on the approximation of the system's unknown nonlinearities by using fuzzy systems. The free parameters of the used fuzzy systems in this case are updated using a gradient descent algorithm that is designed to minimize the identification error between the unknown nonlinearities and their adaptive fuzzy approximations. Both approaches do not require the knowledge of the mathematical model of the plant, guarantee the uniform boundedness of all the signals in the closed-loop system, and ensure the convergence of the tracking errors to a neighbourhood of the origin. Simulation results for direct adaptive control scheme performed on a two-link robot manipulator illustrate the method. Future works will focus on extension of the approach to more general MIMO nonlinear systems and its improvement by introducing a state observer to provide an estimate of the state vector.

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