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Fuzzy Model Reference Adaptive Control

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***Abstract*—This paper investigates a fuzzy model reference adaptive controller (FMRAC) for continuous-time multiple-input–multiple-output (MIMO) nonlinear systems. The proposed adaptive scheme uses a Takagi–Seguno (TS) fuzzy adaptive system, which allows for the inclusion of *a priori* information in terms of qualitative knowledge about the plant operating points or analytical regulators (e.g., state feedback) for those operating points. A proportional-integral update law is used to obtain a fast parameters adaptation. Stability and robustness of this adaptive scheme are established using Lyapunov stability tools. The simulation results, for a two-link robot, confirm the performance of the proposed approach.**

***Index Terms*—Adaptive control, fuzzy control, model reference, robustness, stability, Takagi–Seguno (TS) fuzzy system.**

I. INTRODUCTION

SINCE the Procyk and Mamdani self-organizing controller [1], many fuzzy adaptive systems were designed and some practical results were reported [2], [3]. Fuzzy adaptive systems provide the advantage that both numerical and qualitative information are used in the construction and the training stages. Furthermore, fuzzy systems are proven to be applied to approximate any continuous nonlinear function on a compact space (i.e., they are universal approximators) [3]–[6]. Recently, various fuzzy, direct and indirect, adaptive systems were proposed, and their stability is achieved using the Lyapunov theory (see, e.g., [4] and [7]–[12] for single-input–single-output (SISO) approaches, and [13]–[15] for multiple-input–multiple-output (MIMO) approaches). The common points between the above cited adaptive schemes are i) the fuzzy controller seeks to learn an unknown optimal feedback linearizing controller, which restricts the range of plants that can be controlled, since the feedback linearizing control is not suited for nonminimum phase plants [16]; ii) the controller parameters are updated using an integral law. However, other possibilities exist, and a quicker adaptation can be obtained using proportional-integral (PI) law [17], [18]; iii) apart from a few works, that investigate the Takagi–Seguno (TS) fuzzy system (see, e.g., [9] and [13]), the Mamdani fuzzy system is the mostly used in the fuzzy adaptive schemes developed up to now. Compared to the Mamdani fuzzy system, the TS fuzzy system provides more powerful representation, i.e., it is capable of describing a highly nonlinear plants using few rules. Moreover, since the output of the model has an explicit analytical form, it is possible to incorporate mathematical knowledge about the plant control, and its behavior can be analyzed using conventional control theory tools.

In this work, a new stable FMRAC for MIMO nonlinear continuous-time systems is introduced. A TS fuzzy controller is directly tuned to achieve the reference model tracking performance. The proposed fuzzy adaptive controller does not search to emulate any *a priori* prescribed optimal control law as in previous works. The use of the TS fuzzy system allows for the inclusion of the qualitative information about the plant operating points in the design of the controller structure (i.e., fuzzy sets definition and rules selection). If for some operating points linear regulators (e.g., state feedback) are available, they can be directly incorporated in the rules consequences. The controller parameters are updated using a PI law, which provides fast parameters update and hence for fast convergence of the tracking error. The stability and robustness properties of the proposed adaptive scheme are established in the Lyapunov theory framework. It is shown that the proposed FMRAC can learn how to control the nonlinear plant, provides for bounded internal signals, and achieves asymptotic tracking of a stable reference model, even when the plant is subject to external disturbances and parameters variations. The FMRAC performance is evaluated by a simulation study on two-link robot subject to friction disturbance and variable payload. The simulation results demonstrate the computational simplicity, the tracking performance and the robustness compared to well-known methods such as robust adaptive control.

The rest of the paper is organized as follows. Section II formulates the model reference adaptive control problem of MIMO nonlinear systems. Section III develops the FMRAC approach using the TS fuzzy adaptive controller. Section IV presents a detailed analysis of the stability and robustness of the proposed fuzzy adaptive scheme. Section V provides a comparative simulation study, and Section VI concludes the paper.

II. PROBLEM FORMULATION

Consider the MIMO nonlinear systems class described by

$$\begin{aligned} \dot{x}_{1_i} &= x_{2_i} \\ &\cdots \\ \dot{x}_{(n-1)_i} &= x_{n_i} \\ \dot{x}_{n_i} &= a_i(x)x + \sum_{j=1}^{p} b_{ij}(x)u_j + \eta_i(t) \end{aligned} \tag{1}$$

where $i = 1, \ldots, p$, $x = [\,x_1^T \quad x_2^T \quad \cdots \quad x_p^T\,]^T \in R^n$ is the state vector of the system assumed to be available, with $x_i^T = [\,x_{1_i} \quad x_{2_i} \quad \cdots \quad x_{n_i}\,] \in R^{n_i}$ and $n = n_1 + n_2 + \cdots + n_p$. $a_i(x) \in R^n$ are unknown smooth vector functions, $b_{ij}(x)$ are

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smooth unknown functions, η_i are unknown bounded external disturbances, and $u_j \in R$ is the jth input of the system. In addition, it is assumed that $b_{ii}(x) > 0$ in the relevant operating region [the case of $b_{ii}(x)$ negative can be also handled]. The nonlinear system (1) may be the model of a nonlinear plant [e.g., manipulator or electrical actuator models can be formulated as in (1)], or an estimated TS fuzzy model [19].

The stable, controllable, and linear time-invariant (LTI) reference models are given by the set of decoupled state equations

$$\dot{x}_{m_i} = A_{m_i} x_{m_i} + b_{m_i} r_i \tag{2}$$

where $x_{m_i} \in R^{n_i}$ is the state vector of the ith reference model, r_i is a bounded reference input, and A_{m_i}, b_{m_i} are given by

$$A_{m_i} = \begin{bmatrix} 0 & I_{n_i-1} \\ & -a_{m_i} \end{bmatrix}, \qquad b_{m_i}^T = [0 \quad \cdots \quad 0 \quad b_{n_i m}]$$

where $a_{m_i} = [a_{1_i m} \quad a_{2_i m} \quad \cdots \quad a_{n_i m}] \in R^{n_i}$.

The control problem can be stated as follows: design the control inputs u_j, $j = 1, \ldots, p$ such that the states of the plant (1) follow those of the reference models (2), under the condition that all involved signals in the closed loop remain bounded.

III. Fuzzy Adaptive Control Approach

The adaptive controller to be designed is a multiple-input–single-output (MISO) TS fuzzy system [19] constituted by a set of If–Then fuzzy rules of the form

$$R_k^i\text{: If } v^i \text{ is } V_k^i \text{ Then } u_{f_i} = c_{k1}^i x_1 + \cdots + c_{kn}^i x_n + c_{kn+1}^i r_i \tag{3}$$

where $k = 1, \ldots, m_i$, m_i is the number of rules, $v^i \in R^{q_i}$ is the fuzzy controller input vector, and the fuzzy sets V_k^i operate a fuzzy partition of the fuzzy controller input space (i.e., the fuzzification operators).

The output of the ith fuzzy controller is inferred as follows:

$$u_{f_i} = \frac{\sum_{k=1}^{m_i} \mu_k^i(v^i) \left(\sum_{j=1}^{n} c_{kj}^i x_j + c_{kn+1}^i r_i \right)}{\sum_{k=1}^{m_i} \mu_k^i(v^i)} \tag{4}$$

where $\mu_k^i(v^i)$ is the grade of membership of v^i in V_k^i (i.e., the firing strength of the rule k). In this paper, it is assumed that there exist always at least one active rule, i.e., $\sum_{k=1}^{m_i} \mu_k^i(v^i) > 0$.

Equation (4) can also be written in the following compact form:

$$u_{f_i} = \xi_i \Theta_i z_i \tag{5}$$

where

$$z_i = [x^T \quad r_i]^T$$

$$\Theta_i = \begin{bmatrix} c_{11}^i & \cdots & c_{1n+1}^i \\ \vdots & \cdots & \vdots \\ c_{m_i 1}^i & \cdots & c_{m_i n+1}^i \end{bmatrix}$$

and ξ_i is the vector of the normalized firing strengths, given by

$$\xi_i = \frac{1}{\sum_{k=1}^{m_i} \mu_k^i} [\mu_1^i \quad \mu_2^i \quad \cdots \quad \mu_{m_i}^i]. \tag{6}$$

In this work, the fuzzy controllers parameters are updated using the following PI law

$$\Theta_i = \psi_i + \phi_i \tag{7}$$

where the quantities ψ_i and ϕ_i are the proportional and the integral term respectively, given by

$$\psi_i = \gamma_{i1} b_{c_i}^T P_i e_i \xi_i^T z_i^T \tag{8}$$

and

$$\dot{\phi}_i = \gamma_{i2} b_{c_i}^T P_i e_i \xi_i^T z_i^T \tag{9}$$

where $b_{c_i}^T = [0 \quad \cdots \quad 0 \quad 1] \in R^{n_i}$, $\gamma_{i1}, \gamma_{i2} > 0$ are design constants, and the symmetric positive definite matrices P_i are the solutions (for $i = 1, \ldots, p$) of the following Lyapunov equations:

$$A_{m_i}^T P_i + P_i A_{m_i} = -Q_i. \tag{10}$$

Since A_{m_i} are Hurwitz matrices, positive definite matrices P_i and Q_i are guaranteed to exist [20].

To achieve the control objective, the control inputs are defined as

$$u_i = u_{f_i} + u_{s_i} \tag{11}$$

where u_{s_i} is an additional control term used to overcome the uncertainties.

The tracking error dynamic equation for the ith subsystem, using (1) and (2), (11) and (5), is given by

$$\dot{e}_i = A_{m_i} e_i - b_{c_i} \Bigg[a_i(x)x + a_{m_i} x_i - b_{n_i m} r_i + b_{ii}(x) \xi_i \Theta_i z_i + b_{ii}(x) u_{s_i} + \sum_{j=1,\, j \neq i}^{p} b_{ij}(x) u_j + \eta_i(t) \Bigg] \tag{12}$$

where $e_i = x_{m_i} - x_i$ is the ith subsystem tracking error.

Introducing (8) and (9), (12) can be arranged as

$$\dot{e}_i = A_{m_i} e_i - b_{c_i} \Bigg[-a_{c_i}(x) z_i + b_{ii}(x) \xi_i \phi_i z_i + b_{ii}(x) \xi_i \psi_i z_i + b_{ii}(x) u_{s_i} + \sum_{j=1,\, j \neq i}^{p} b_{ij}(x) u + \eta_i \Bigg] \tag{13}$$

where

$$a_{c_i}(x) = [-(a_i(x) + a_{m_i}) \quad b_{n_i m}]. \tag{14}$$

Theoretical results [3]–[6] have shown that fuzzy systems are universal approximators, i.e., they can approximate any smooth function on a compact space. Due to this approximation capability, we can assume that the nonlinear term $a_{c_i}(x) z_i / b_{ii}(x)$

can be completely described by TS fuzzy system plus a modeling error ω_i (ω_i is called the minimum approximation error). This means that there exists parameters ϕ^* such that

$$\frac{a_{c_i}(x)}{b_{ii}(x)}\, z_i = \xi_i \phi_i^* z_i + \omega_i. \tag{15}$$

Since the optimal parameters ϕ_i^* are unknown, we will make use of their estimates ϕ_i. Then, (15) can be rewritten as

$$\frac{a_{c_i}(x)}{b_{ii}(x)}\, z_i = \xi_i \phi_i z_i + \xi_i \tilde{\phi}_i z_i + \omega_i \tag{16}$$

where $\tilde{\phi}_i = \phi_i^* - \phi_i$ are the parameter estimation errors.

Introducing (16), (13) becomes

$$\dot{e}_i = A_{m_i} e_i - b_{c_i} b_{ii}(x) \Bigg[-\xi_i \tilde{\phi}_i z_i + \xi_i \psi_i z_i + u_{s_i} + \frac{1}{b_{ii}(x)} \sum_{j=1,\, j\neq i}^{p} b_{ij}(x) u_j + d_i \Bigg] \tag{17}$$

where $d_i = (\eta_i / b_{ii}(x)) - \omega_i$ are the uncertainties terms.

IV. Stability and Robustness

To establish the stability of the proposed fuzzy adaptive system, the following usual assumptions are introduced.

Assumption 1: The input gains are bounded by $0 < \underline{b}_{ii} \leq b_{ii}(x) \leq \overline{b}_{ii}$, their variations are upper bounded by $|\dot{b}_{ii}(x)| \leq \beta_{ii}(x)$, and the external disturbances are upper bounded by $|\eta_i| \leq \overline{\eta}_i$, where $\overline{\eta}_i$, $\underline{b}_{ii}$ and $\overline{b}_{ii}$ are some positive constants and $\beta_{ii}(x)$ are some known functions.

Assumption 2: The approximation errors are upper bounded by $|\omega_i| \leq \overline{\omega}_i$ where $\overline{\omega}_i$ are known positive constants.

The stability of the proposed FMRAC scheme is summarized by the following theorem.

Theorem 1: The feedback system composed by the nonlinear system (1) that satisfies Assumptions 1–2, the reference models (2) and the control inputs (11) with the update law (8) and (9) is globally asymptotically stable and the tracking error converges to zero.

Proof: Consider the Lyapunov function candidate

$$V = \sum_{i=1}^{p} V_i \tag{18}$$

with

$$V_i = \frac{\gamma_{i2}}{2b_{ii}(x)}\, e_i^T P_i e_i + \frac{1}{2}\, \mathrm{tr}\left(\tilde{\phi}_i^T \tilde{\phi}_i\right). \tag{19}$$

The differentiation of (19) along the trajectory of (17) yields

$$\begin{aligned} \dot{V}_i = & -\frac{\gamma_{i2}}{2b_{ii}(x)}\, e_i^T Q_i e_i - \frac{\gamma_{i2}\dot{b}_{ii}(x)}{2b_{ii}(x)^2}\, e_i^T P_i e_i \\ & + \mathrm{tr}\left(\tilde{\phi}_i^T \dot{\tilde{\phi}}_i\right) - \gamma_{i2} e_i^T P_i b_{c_i} \\ & \cdot \Bigg[-\xi_i \tilde{\phi}_i z_i + \xi_i \psi_i z_i + u_{si} \\ & + \frac{1}{b_{ii}(x)} \sum_{j=1,\, j\neq i}^{p} b_{ij}(x) u_j + d_i \Bigg]. \end{aligned} \tag{20}$$

Further, (20) can be arranged as

$$\begin{aligned} \dot{V}_i = & -\frac{\gamma_{i2}}{2b_{ii}(x)}\, e_i^T Q_i e_i - \frac{\gamma_{i2}\dot{b}_{ii}(x)}{2b_{ii}^2(x)}\, e_i^T P_i e_i \\ & + \mathrm{tr}\left[\tilde{\phi}_i^T \left(\dot{\tilde{\phi}}_i + \gamma_{i2} b_{c_i}^T P_i e_i \xi_i^T z_i^T\right)\right] \\ & - \gamma_{i2} e_i^T P_i b_{c_i} \xi_i \psi_i z_i - \gamma_{i2} e_i^T P_i b_{c_i} \\ & \cdot \left(u_{si} + \frac{1}{b_{ii}(x)} \sum_{j=1,\, j\neq i}^{p} b_{ij}(x) u_j + d_i \right). \end{aligned} \tag{21}$$

Observing from (8) that

$$\gamma_{i2} e_i^T P_i b_{c_i} \xi_i \psi_i z_i = \frac{\gamma_{i2}}{\gamma_{i1}}\, \mathrm{tr}[\psi_i^T \psi_i]. \tag{22}$$

Since (22) is positive–semidefinite, (21) becomes

$$\begin{aligned} \dot{V}_i \leq & -\frac{\gamma_{i2}}{2b_{ii}(x)}\, e_i^T Q_i e_i - \frac{\gamma_{i2}\dot{b}_{ii}(x)}{2b_{ii}^2(x)}\, e_i^T P_i e_i \\ & + \mathrm{tr}\left[\tilde{\phi}_i^T \left(\dot{\tilde{\phi}}_i + \gamma_{i2} b_{c_i}^T P_i e_i \xi_i^T z_i^T\right)\right] - \gamma_{i2} e_i^T P_i b_{c_i} \\ & \cdot \left(u_{si} + \frac{1}{b_{ii}(x)} \sum_{j=1,\, j\neq i}^{p} b_{ij}(x) u_j + d_i \right). \end{aligned} \tag{23}$$

Then substituting with (9) in (23) and using the fact that $\dot{\tilde{\phi}} = -\dot{\phi}$ leads to

$$\begin{aligned} \dot{V}_i \leq & -\frac{\gamma_{i2}}{2b_{ii}(x)}\, e_i^T Q_i e_i - \frac{\gamma_{i2}\dot{b}_{ii}(x)}{2b_{ii}^2(x)}\, e_i^T P_i e_i - \gamma_{i2} e_i^T P_i b_{c_i} \\ & \cdot \left(u_{s_i} + \frac{1}{b_{ii}(x)} \sum_{j=1,\, j\neq i}^{p} b_{ij}(x) u_j + d_i \right). \end{aligned} \tag{24}$$

At this point, the additional control terms u_{s_i} are chosen as

$$\begin{aligned} u_{s_i} = & \left(\frac{1}{\underline{b}_{ii}} \sum_{j=1,\, j\neq i}^{p} \overline{b}_{ij} |u_j| + \overline{d}_i \right) \\ & \cdot \mathrm{sgn}(e_i^T P_i b_{c_i}) + \frac{\beta_{ii}(x)}{2\underline{b}_{ii}^2}\, e_i^T P_i e_i \end{aligned} \tag{25}$$

where $\overline{d}_i = (\overline{\eta}_i / \underline{b}_{ii}) + \overline{\omega}_i$ is the upper bound on the ith uncertainty term.

Then, introducing (25) in (24) yields

$$\dot{V}_i \leq -\frac{\gamma_{2i}}{2b_{ii}(x)}\, e_i^T Q_{ii} e_i. \tag{26}$$

It follows immediately from (18) and (26) that:

$$\dot{V} \leq -\sum_{i=1}^{p} \frac{\gamma_{i2}}{2b_{ii}(x)}\, e_i^T Q_i e_i. \tag{27}$$

That is

$$\dot{V} \leq -\sum_{i=1}^{p} \frac{\gamma_{i2}}{2\overline{b}_{ii}}\, e_i^T Q_i e_i. \tag{28}$$

Thus, $\dot{V}$ is always negative in the e_i space if $e_i \neq 0$, then $V_i \in L_\infty$. Therefore, $e_i, \tilde{\phi}_i \in L_\infty$ for $i = 1, \ldots, p$. Since all variables in the right-hand side of (17) are bounded, $\dot{e}_i$ are bounded, i.e., $\dot{e}_i \in L_\infty$ for $i = 1, \ldots, p$. Integrating both sides of (28) yields

$$\sum_{i=1}^{p} \int_0^\infty \|e_i\|^2 \, dt \le \sum_{i=1}^{p} \frac{2\overline{b}_{ii}}{\gamma_{i2}\lambda_{\min}(Q_i)} V_i(0) \qquad (29)$$

where $\lambda_{\min}(Q_i)$ is the minimum eigenvalue of Q_i. Since the right side of (29) is bounded, $e_i \in L_2$ for $i = 1, \ldots, p$. Using Barbalat's lemma [20], we have that the errors converge asymptotically to zero, i.e., $\lim_t \to \infty \, e_i = 0$ for $i = 1, \ldots, p$. □

Remark 1: Since the control inputs are computed simultaneously, the inputs u_j are not available when the input u_i is designed, and therefore, (25) cannot be computed. As proposed in [13], one can observe that

$$|u_j| \le |\xi_j \Theta_j z_j| + k_{sj} + \frac{\beta_{jj}(x)}{2\underline{b}_{jj}^2} e_j^T P_j e_j, \quad j = 1, \ldots, p \quad (30)$$

where k_{sj} is an upper bound on the jth switching term. Thus, (25) can written as

$$u_{s_i} = \left(\frac{1}{\underline{b}_{ii}} \left[\sum_{j=1,\, j\ne i}^{p} \overline{b}_{ij} \cdot \left(|\xi_j \Theta_j z_j| + k_{sj} + \frac{\beta_{jj}(x)}{2\underline{b}_{jj}^2} e_j^T P_j e_j \right) \right] + \overline{d}_i \right) \cdot \mathrm{sgn}(e_i^T P_i b_{c_i}) + \frac{\beta_{ii}(x)}{2\underline{b}_{ii}^2} e_i^T P_i e_i. \qquad (31)$$

It follows from (30) and (31) that the ith switching term should be bounded by:

$$\frac{1}{\underline{b}_{ii}} \left[\sum_{j=1,\, j\ne i}^{p} \overline{b}_{ij} \left(|\xi_j \Theta_j z_j| + k_{sj} + \frac{\beta_{jj}(x)}{2\underline{b}_{jj}^2} e_j^T P_{ij} e_j \right) \right] + \overline{d}_i + \frac{\beta_{ii}(x)}{2\underline{b}_{ii}^2} e_i^T P_i e_i \le k_{si} \quad (32)$$

which can be rewritten as

$$\underline{b}_{ii} k_{si} - \sum_{j=1,\, j\ne i}^{p} \overline{b}_{ij} k_{sj} \ge \sum_{j=1,\, j\ne i}^{p} \overline{b}_{ij} \left(|\xi_j \Theta_j z_j| + \frac{\beta_{jj}(x)}{2\underline{b}_{jj}^2} e_j^T P_j e_j \right) + \underline{b}_{ii} \overline{d}_i + \frac{\beta_{ii}(x)}{2\underline{b}_{ii}} e_i^T P_i e_i. \qquad (33)$$

For $i = 1, \ldots, p$, (33) can be arranged in the following matrix form:

$$\begin{bmatrix} \underline{b}_{11} & -\overline{b}_{12} & \cdots & -\overline{b}_{1p} \\ -\overline{b}_{21} & \underline{b}_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\overline{b}_{p-1p} \\ -\overline{b}_{p1} & \cdots & -\overline{b}_{pp-1} & \underline{b}_{pp} \end{bmatrix} \begin{bmatrix} k_{s1} \\ k_{s2} \\ \vdots \\ k_{sp} \end{bmatrix} \ge \begin{bmatrix} \upsilon_1 \\ \upsilon_2 \\ \vdots \\ \upsilon_p \end{bmatrix} \quad (34)$$

where

$$\upsilon_i = \sum_{j=1,\, j\ne i}^{p} \overline{b}_{ij} \left(|\xi_j \Theta_j z_j| + \frac{\beta_{jj}(x)}{2\underline{b}_{jj}^2} e_j^T P_j e_j \right) + \underline{b}_{ii} \overline{d}_i + \frac{\beta_{ii}(x)}{2\underline{b}_{ii}} e_i^T P_i e_i. \quad (35)$$

As pointed out in [13], the condition for (34) to be well posed is that

$$\underline{b}_{ii} > \sum_{j=1,\, j\ne i}^{p} \overline{b}_{ij}, \qquad i, j = 1, \ldots, p, \quad j \ne i. \qquad (36)$$

Although the result derived here is similar to the one in [13], there is a fundamental difference. All the terms appearing in (35) are available for computation, whereas in [13] the errors between the control inputs and the optimal ones are used, which are unavailable. Another difference is that, in [13] it is considered that there is only one upper bound for all switching terms, which may result in an unnecessary effort for subsystems subject to small disturbance or with weak coupling parameters.

Remark 2: Considering (7)–(9), the parameters of the fuzzy adaptive controller are adjusted using the PI law

$$\Theta_i = \gamma_{i1} b_{c_i}^T P_i e_i \xi_i^T z_i^T + \gamma_{i2} \int_0^t b_{c_i}^T P_i e_i \xi_i^T z_i^T \, dt. \qquad (37)$$

This update law provides a faster parameters update compared to the integral-like laws, and then high tracking accuracy can be achieved. Hence, the transient error size (29) can be reduced by designing a good initial fuzzy controller, or by increasing the update gains γ_{i1} and γ_{i2} in (37).

Remark 3: In some cases it can be shown that the compensation for the input gain variation is not required and the second term in (25) can be removed. To show this fact, let us remove the last term in (25), then replacing in (24) yields

$$\dot{V}_i \le -\frac{\gamma_{i2}}{2b_{ii}(x)} e_i^T Q_i e_i - \frac{\gamma_{i2} \dot{b}_{ii}(x)}{2b_{ii}^2(x)} e_i^T P_i e_i \qquad (38)$$

or, equivalently

$$\dot{V}_i \le \frac{\gamma_{i2}}{2b_{ii}^2(x)} \left[-b_{ii}(x) e_i^T Q_i e_i - \dot{b}_{ii}(x) e_i^T P_i e_i \right] \qquad (39)$$

then using norms on (39) gives

$$\dot{V}_i \le \frac{\gamma_{i2}}{2b_{ii}^2(x)} \left[-\underline{b}_{ii} \lambda_{\min}(Q_i) \|e_i\|^2 + \beta_{ii}(x) \lambda_{\max}(P_i) \|e_i\|^2 \right] \qquad (40)$$

where $\|\cdot\|$ is the Euclidean norm and $\lambda_{\max}(P_i)$ is the maximum eigenvalue of P_i. From (40), it can be observed that if the following condition holds:

$$\underline{b}_{ii} \lambda_{\min}(Q_i) > \beta_{ii}(x) \lambda_{\max}(P_i) \qquad \forall\, x \qquad (41)$$

then $\dot{V}_i$ is always negative and the stability is guaranteed. In the special case where $\dot{b}_{ii}(x)$ is constant, the matrices P_i and Q_i can be always chosen appropriately to fulfill (41).

Remark 4: Due to the discontinuous nature of the switching control, chattering may occur. Practically, chattering is undesirable, since high control activity is involved, and unmodeled dynamics may be excited. To overcome this problem, the switching control (25) can smoothed as [22]

$$u_{s_i} = \left(\frac{1}{\underline{b}_{ii}} \sum_{j=1,\, j\ne i}^{p} \overline{b}_{ij} |u_j| + \overline{d}_i \right) \left(1 + \frac{\sigma_i}{\beta_i} \right) \cdot \frac{e_i^T P_i b_{c_i}}{|e_i^T P_i b_{c_i}| + \sigma_i} + \frac{\beta_{ii}(x)}{2\underline{b}_{ii}^2} e_i^T P_i e_i \quad (42)$$

where σ_i and β_i are positive constants. The constant σ_i is chosen based on engineering considerations to achieve an admissible tracking error. The ratio σ_i/β_i determines the control gain magnitude. Replacing (42) in (24) yields

$$\dot{V}_i \le -\frac{\gamma_{i2}}{2b_{ii}(x)} e_i^T Q_i e_i - \gamma_{i2} e_i^T P_i b_{c_i} \cdot \left[\left(\frac{1}{\underline{b}_{ii}} \sum_{j=1,\, j\neq i}^{p} \overline{b}_{ij}|u_j| + \overline{d}_i \right) \left(1+\frac{\sigma_i}{\beta_i}\right) \frac{e_i^T P_i b_{c_i}}{|e_i^T P_i b_{c_i}| + \sigma_i} + \left(\frac{1}{b_{ii}(x)} \sum_{j=1,\, j\neq i}^{p} b_{ij}(x) u_j + d_i \right) \right] \tag{43}$$

or, equivalently

$$\dot{V}_i \le -\frac{\gamma_{i2}}{2b_{ii}(x)} e_i^T Q_i e_i - \gamma_{i2} |e_i^T P_i b_{c_i}| \cdot \left[\left(\frac{1}{\underline{b}_{ii}} \sum_{j=1,\, j\neq i}^{p} \overline{b}_{ij}|u_j| + \overline{d}_i \right) \left(1+\frac{\sigma_i}{\beta_i}\right) \frac{|e_i^T P_i b_{c_i}|}{|e_i^T P_i b_{c_i}| + \sigma_i} + \left(\frac{1}{b_{ii}(x)} \sum_{j=1,\, j\neq i}^{p} b_{ij}(x) u_j + d_i \right) \mathrm{sgn}(e_i^T P_i b_{c_i}) \right]. \tag{44}$$

Then, if $|e_i^T P_i b_{c_i}| \ge \beta_i$ we get $\dot{V}_i \le 0$. On the other hand, if $|e_i^T P_i b_{c_i}| < \beta_i$ then

$$\dot{V}_i \le -\frac{\gamma_{i2}}{2b_{ii}(x)} e_i^T Q_i e_i + \gamma_{i2}\sigma_i \cdot \left| \frac{1}{b_{ii}(x)} \sum_{j=1,\, j\neq i}^{p} b_{ij}(x) u_j + d_i \right|. \tag{45}$$

Hence, the tracking error converges to the bounded region given by

$$\Omega(e_i) = \left\{ e_i / \|e_i\| \le \left(\frac{2u_0\sigma_i}{\lambda_{\min}(Q_i)} \right)^{1/2} \right\} \tag{46}$$

where

$$u_0 = \sum_{j=1,\, j\neq i}^{p} \overline{b}_{ij}\overline{u}_j + \overline{b}_{ii}\overline{d}_i \tag{47}$$

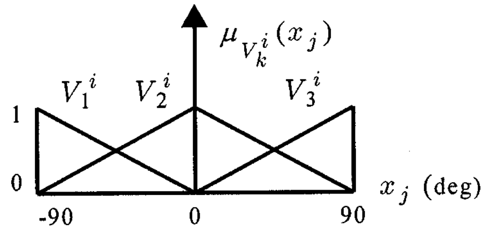

Fig. 1. Membership functions for the fuzzy controllers design.

with $\overline{u}_j$ is the bound given by (30). However, in this case the updating law (9) does not ensure the boundedness of the controller parameters (i.e., the integral term). To avoid the parameter drift, various techniques were developed [21]. One possible rule is to modify the update law (9) such as

$$\dot{\phi}_i = \begin{cases} \gamma_{i2}\xi_i^T b_{c_i}^T P_i e_i z_i^T, & \text{if } \|e_i\| \notin \Omega(e_i) \\ 0, & \text{if } \|e_i\| \in \Omega(e_i) \end{cases} \tag{48}$$

where $\Omega(e_i)$ is a bounded region containing the origin of the error space. Using the Lyapunov function (19), it can be shown that: if $\|e_i\| \notin \Omega(e_i)$, then $\dot{V}_i$ is negative. When the error enters $\Omega(e_i)$, then $\|\phi_i\| = \phi_{i0}$ for some $\phi_{i0} \ge 0$. Hence, V_i, e_i, and ϕ_i are ensured to remain bounded.

Remark 5: Although this study was conducted for nonlinear systems of the form (1), the results can be extended, without any modification, to the more general class of nonlinear systems where $a_i(x)x = \alpha_i(x)$, with $\alpha_i(x)$ a smooth nonlinear function. In this case, the reference inputs r_i should be smooth.

V. Simulation

The proposed FMRAC is applied to the two-link robot described by

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F_v(\dot{q}) + F_c(\dot{q}) = u \tag{49}$$

where q, $\dot{q}$ and $\ddot{q}$ are the 2 × 1 vectors of joint angles, velocities, and accelerations, respectively. $D(q)$ is the 2 × 2 inertia matrix, $C(q)$ is the 2 × 1 Coriolis and centrifugal vector, $G(q)$ is the 2 × 1 gravitational torque vector, $F_v(\dot{q})$ and $F_c(\dot{q})$ are the viscous and coulomb friction torques respectively, and u is the 2 × 1 vector of control torques, as shown at the bottom of the page. The physical parameters values are $l = 1$ m, $m_1 = m_2 = 1$ kg and g $= 9.41$ m/s^2. The frictions coefficients are as follows: $k_{v2} = k_{c2} = 0.5$, $k_{v1} = 0.3$ and $k_{c1} = 0.2$.

$$D = \begin{bmatrix} \frac{1}{3} m_1 l^2 + \frac{4}{3} m_2 l^2 + m_2 l^2 \cos(q_2) & \frac{1}{3} m_2 l^2 + \frac{1}{2} m_2 l^2 \cos(q_2) \\ \frac{1}{3} m_2 l^2 + \frac{1}{2} m_2 l^2 \cos(q_2) & \frac{1}{3} m_2 l^2 \end{bmatrix}$$

$$C = \begin{bmatrix} -m_2 l^2 \sin(q_2)\dot{q}_2 & -\frac{1}{2} m_2 l^2 \sin(q_2)\dot{q}_2 \\ \frac{1}{2} m_2 l^2 \sin(q_2)\dot{q}_1 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} \frac{1}{2} m_1 \mathrm{g} l \cos(q_1) + \frac{1}{2} m_2 \mathrm{g} l \cos(q_1+q_2) + m_2 \mathrm{g} l \cos(q_1) \\ \frac{1}{2} m_2 \mathrm{g} l \cos(q_1+q_2) \end{bmatrix}$$

$$F_v = \begin{bmatrix} k_{v1} & \dot{q}_1 \\ k_{v2} & \dot{q}_2 \end{bmatrix} \quad F_c = \begin{bmatrix} k_{c1}\,\mathrm{sign}(\dot{q}_1) \\ k_{c2}\,\mathrm{sign}(\dot{q}_2) \end{bmatrix}.$$

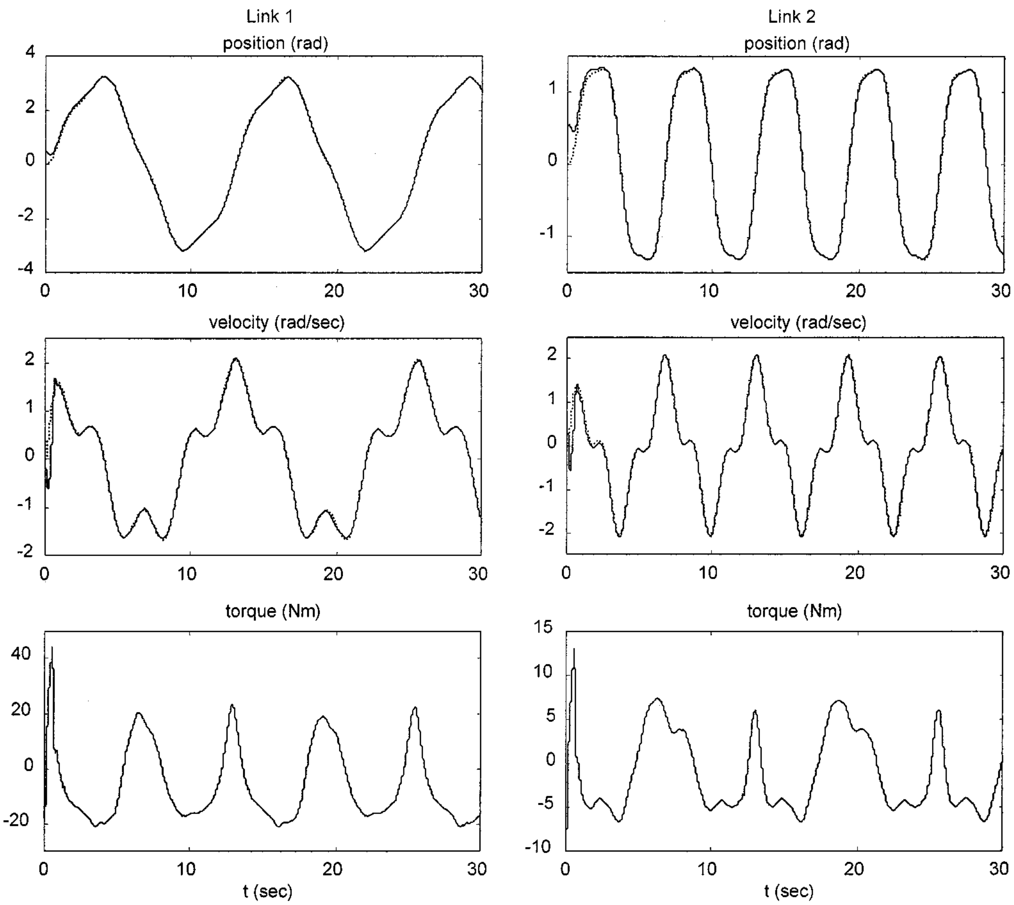

Fig. 2. Robot response under the FMRAC (— plant; . . . reference).

Taking the state vector $x^T = [\,x_1^T \quad x_2^T\,]$ with $x_1^T = [\,q_1 \quad \dot{q}_1\,]$ and $x_2^T = [\,q_2 \quad \dot{q}_2\,]$, yields

$$\begin{bmatrix} a_1(x) \\ a_2(x) \end{bmatrix} = D^{-1}[-C\dot{q} - G - F_v]$$

$$\begin{bmatrix} b_{11}(x) & b_{12}(x) \\ b_{21}(x) & b_{22}(x) \end{bmatrix} = D^{-1}, \qquad \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = D^{-1}F_c.$$

The reference models are

$$A_{m_{1,2}} = \begin{bmatrix} 0 & 1 \\ -16 & -8 \end{bmatrix} \quad b_{m_{1,2}} = \begin{bmatrix} 0 \\ 16 \end{bmatrix}.$$

The solution of (10), for $Q_{1,2} = \mathrm{diag}\{15,\ 5\}$, yields

$$P_{1,2} = \begin{bmatrix} 9.7 & 0.47 \\ 0.47 & 0.37 \end{bmatrix}.$$

The fuzzy controllers are defined by the following set of rules:

FMRAC1:

If x_1 is V_k^1

then $u_{f_1} = c_{k1}^1 x_1 + c_{k2}^1 x_2 + c_{k3}^1 x_3 + c_{k4}^1 x_4 + c_{k5}^1 r_1$

FMRAC2:

If x_3 is V_k^2

then $u_{f_2} = c_{k1}^2 x_1 + c_{k2}^2 x_2 + c_{k3}^2 x_3 + c_{k4}^2 x_4 + c_{k5}^2 r_2$

where V_k^1 and V_k^2, $k = 1, \ldots, 3$ are fuzzy sets constructed as in Fig. 1. The fuzzy controllers outputs are given by

$$u_{f_1} = \xi_1 \Theta_1 z_1 \tag{50}$$
$$u_{f_2} = \xi_2 \Theta_2 z_2 \tag{51}$$

where ξ_1 (ξ_2) is the 1×3 vector of the normalized strengths, Θ_1 (Θ_2) is the 3×5 parameters matrix, $z_1 = [\,x^T \quad r_1\,]^T$ and $z_2 = [\,x^T \quad r_2\,]^T$. The parameters of both the fuzzy controllers are updated using the PI law (37), with the update gains $\gamma_{11} = \gamma_{12} = 7$ and $\gamma_{21} = \gamma_{22} = 300$. The switching terms are selected as in (42) with $\delta_1 = \delta_2 = 0.1$.

Considering the variation of q_2 in the interval $[-\pi/2,\ \pi/2]$ and the physical parameters of the robot the bounds on the matrix B elements are found to be $\underline{b}_{11} = 0.75$, $\underline{b}_{22} = 3.75$, $\overline{b}_{11} = 1.72$, $\overline{b}_{22} = 13.7$ and $\overline{b}_{12} = \overline{b}_{21} = 4.3$. Since those values violate (36), the implemented values are tuned to $\underline{b}_{11} = 1.7$, $\underline{b}_{22} = 2$ and $\overline{b}_{12} = \overline{b}_{21} = 1.3$. The bounding functions for the variations of b_{11} and b_{22} are $\beta_{11} = 0.8|x_4|$ and $\beta_{22} = 6.5|x_4|$, respectively. The bounds on the approximation error are tuned through the simulation to the following values: $\overline{\omega}_1 = \overline{\omega}_2 = 0.9$. Using the knowledge at hand on the robot dynamic and physical parameters, the bounds on the disturbance terms are determined to be: $\overline{d}_1 = \overline{\omega}_1 + \overline{\eta}_1/\underline{b}_{11} = 1.36$ and $\overline{d}_2 = \overline{\omega}_2 + \overline{\eta}_2/\underline{b}_{22} = 2.46$, with $\overline{\eta}_1 = \overline{b}_{11}k_{c1} + \overline{b}_{12}k_{c2} = 0.78$ and $\overline{\eta}_2 = \overline{b}_{21}k_{c1} + \overline{b}_{22}k_{c2} = 3.13$.

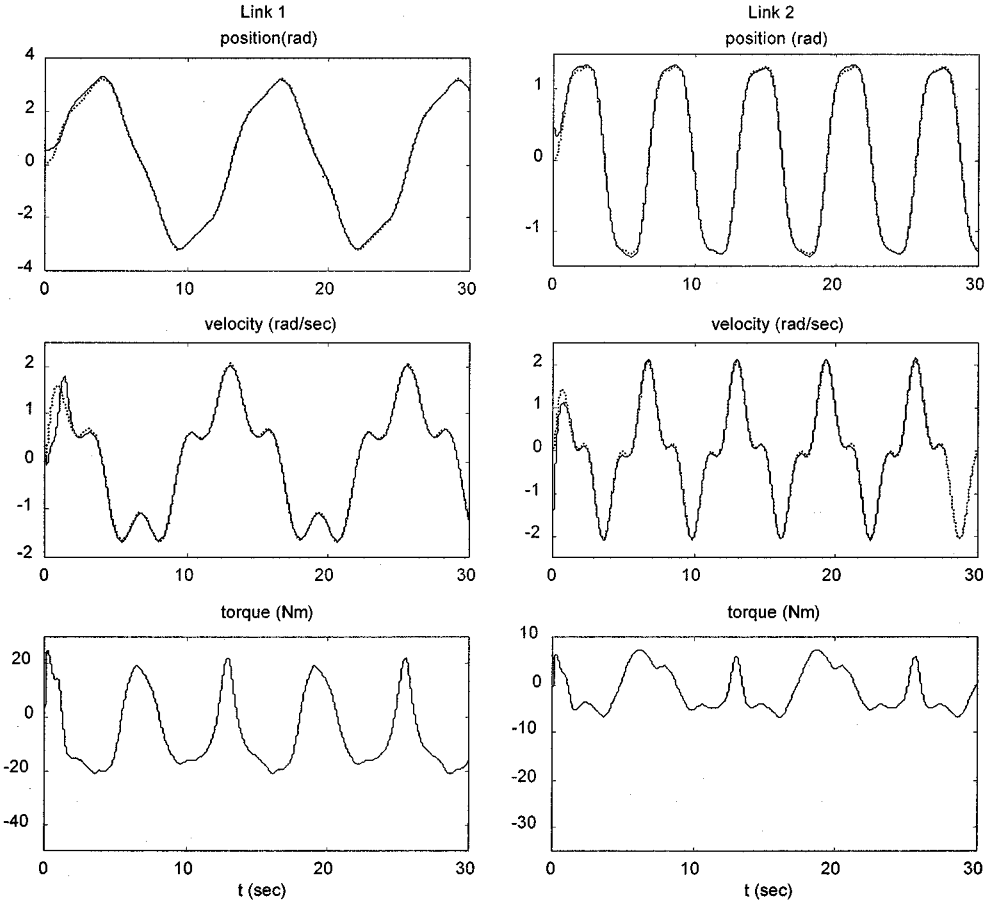

Fig. 3. Robot response under the RAC (— plant; . . . reference).

For the comparison purpose, we consider also the robust adaptive controller (RAC) of [23], which is shown to perform superiorly compared to the adaptive control [24] and the robust computed torque [25]. The control torque is given by

$$u = D\left(q, \hat{\theta}\right)\ddot{q}_r + C\left(q, \dot{q}, \hat{\theta}\right)\dot{q}_r + G\left(q, \hat{\theta}\right) + F_v\left(\dot{q}, \hat{\theta}\right) - K_d S - d_0\,\mathrm{sat}(S/\delta) \quad (52)$$

$$\dot{\hat{\theta}}_j = -\Gamma W(q, \dot{q}, \dot{q}_r, \ddot{q}_r)S - \sigma\Gamma\hat{\theta} \quad (53)$$

where $K_d > 0$, $\Gamma > 0$ are design parameter matrices, $W(\cdot)$ is a regressors matrix derived from the robot dynamics, $\hat{\theta}$ is the vector of unknown parameters and σ is the σ-switching modification. The reference variable q_r and S are defined as

$$q_r = q_d - \Lambda\int e(t)\,dt$$

and

$$S = \dot{q} - \dot{q}_r$$

where $q_d^T = [\,x_{1m_1} \quad x_{1m_2}\,]$ is the reference, Λ is diagonal positive–definite matrix, and $e = q_d - q$ is the tracking error, d_0 are the upper bounds on the disturbance terms, $\mathrm{sat}(.)$ is the saturation function and δ are the boundary layer control parameters.

The RAC assumes the knowledge of the detailed structure of robot dynamics and the upper bounds on the unknown parameters. The RAC design parameters are chosen as in [23] that is, $\delta_1 = \delta_2 = 0.1$, $K_d = \mathrm{diag}\{2, 2\}$ and $\Lambda = \mathrm{diag}\{20, 15\}$, $\Gamma = \mathrm{diag}\{0.05, 0.05\}$ and $d_0 = \mathrm{diag}\{k_{c1}, k_{c2}\}$. The link's masses, considered to be the only unknown parameters, are initialized as $m_1(0) = 0.8$ and $m_2(0) = 1.2$. The convergence of the link's masses is guaranteed, since the persistence of excitation is assured for the chosen reference inputs, $r_1 = \pi(\sin(t/2) + 0.1\sin(2t))$ and $r_2 = \pi(0.5\sin(t) + 0.1\sin(3t))$.

First, the transient performance, under the FMRAC and the RAC, is evaluated. The position and velocity responses and the developed torques are depicted in Fig. 2 for the FMRAC and in Fig. 3 for the RAC. It can be seen from those figures that both the controllers develop practically a similar efforts to realize the tracking objective. The transient dynamic is short for the two controllers, which turns as an advantage for the FMRAC since no initial knowledge was incorporated into the design phase.

Second, to examine the two controllers robustness against the parameters variation, a payload is introduced at the time 50 s and m_2 becomes 4 kg when the robot pickup a 3-kg mass object. The influence of this event on the tracking precision is depicted in Fig. 4. For the FMRAC (solid line), this variation has a little effect on the tracking performance, and small instantaneous oscillation is observed on the velocity response which is rapidly compensated. For the RAC (dashed line), the mass variation causes high tracking errors, especially for the link 2, as can be seen from the large transient position and velocity errors.

The final test concerns the step response to the reference inputs $r_1 = r_2 = \pi/2$. Fig. 5 shows the positions errors for

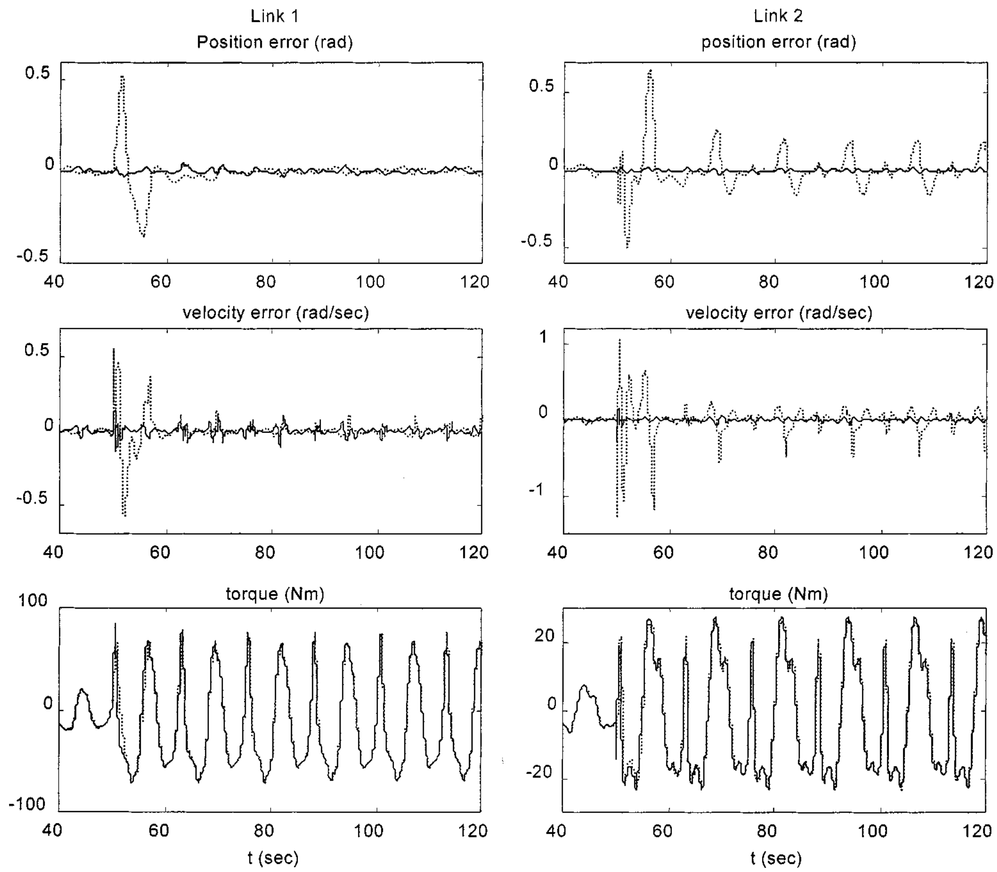

Fig. 4. Tracking errors for mass change at $t = 50$ s (— FMRAC; . . . RAC).

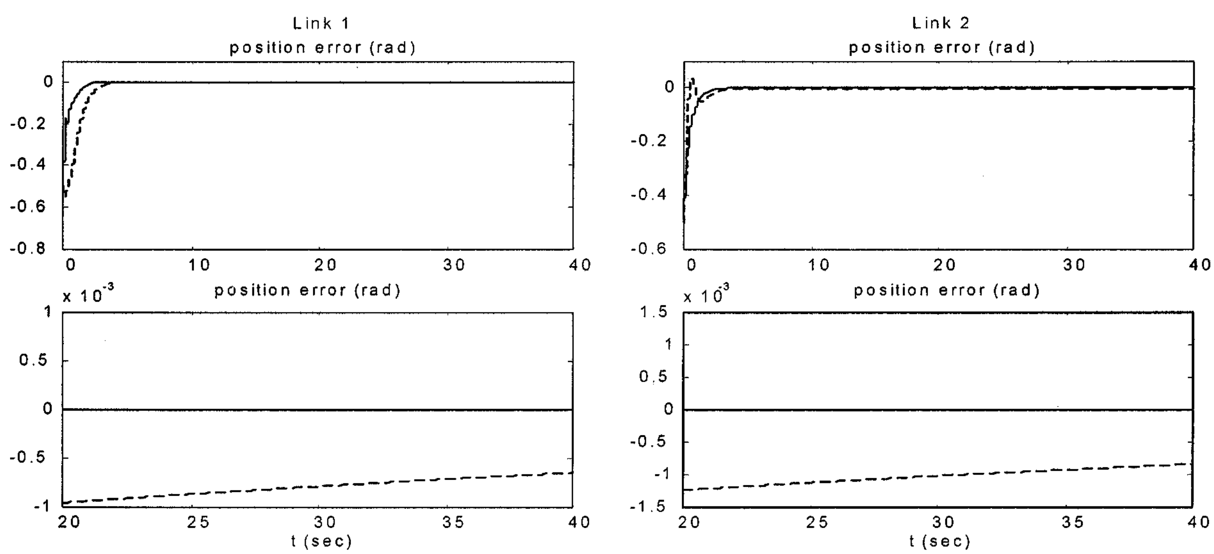

Fig. 5. Step response positions errors (— FMRAC; - - - RAC).

both the FMRAC (solid line) and the RAC (dashed line). As can be seen from the bottom figures (time scale from 20 to 40 s), the FMRAC achieves better steady-state errors compared to the RAC.

VI. Conclusion

A new fuzzy model reference adaptive controller, for MIMO nonlinear systems, is presented. Its main characteristics are: First, a TS fuzzy controller is used (in a different way from the cited works), which allows the inclusion of *a priori* analytical information (e.g., state feedback). Second, the controller converges to the best possible controller in its operating space, and does not search to approximate any predefined regulator. Third, a PI update law is used to obtain fast adaptation rate. Simulation results have shown that the proposed approach is applicable to a broader class of nonlinear systems. Moreover, it was shown that the implementation is very simple, since a three rule controller was sufficient to achieve the control of the two-link robot, which permits possible real-time applications.

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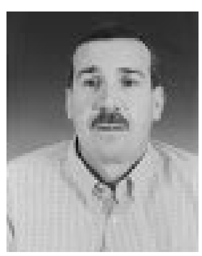

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