A Robust Stabilization Problem of Fuzzy Control Systems and Its Application to Backing up Control of a Truck-Trailer

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***Abstract*—A robust stabilization problem for fuzzy systems is discussed in accordance with the definition of stability in the sense of Lyapunov. We consider two design problems: nonrobust controller design and robust controller design. The former is a design problem for fuzzy systems with no premise parameter uncertainty. The latter is a design problem for fuzzy systems with premise parameter uncertainty. To realize two design problems, we derive four stability conditions from a basic stability condition proposed by Tanaka and Sugeno: nonrobust condition, weak nonrobust condition, robust condition, and weak robust condition. We introduce concept of robust stability for fuzzy control systems with premise parameter uncertainty from the weak robust condition. To introduce robust stability, admissible region and variation region, which correspond to stability margin in the ordinary control theory, are defined. Furthermore, we develop a control system for backing up a computer simulated truck-trailer which is nonlinear and unstable. By approximating the truck-trailer by a fuzzy system with premise parameter uncertainty and by using concept of robust stability, we design a fuzzy controller which guarantees stability of the control system under a condition. The simulation results show that the designed fuzzy controller smoothly achieves backing up control of the truck-trailer from all initial positions.**

I. Introduction

One of the most important concepts concerning the properties of control systems is stability. Stability analysis of fuzzy control systems has been difficult because fuzzy systems are essentially nonlinear systems. Recently, some useful stability techniques [1]–[4], which are based on nonlinear stability theory, have been reported. Therefore, stability analysis of fuzzy control systems became easy. In order to develop fuzzy control theory in the future, we will have to seek more advanced stability theory. An approach to construct more advanced stability theory is to develop an analysis technique for robust stability.

This paper deals with a robust stablization problem for fuzzy control systems with premise parameter uncertainty as shown in Fig. 1. In Fig. 1, $W+\Delta W$ denotes a fuzzy controlled object (fuzzy model) with premise parameter uncertainty (PPU), where ΔW is unknown PPU. C denotes a fuzzy controller. In this paper, the fuzzy model and the fuzzy controller are represented by Takagi and Sugeno's model whose consequent parts are described by linear equations. The purpose of the robust stabilization problem is to find out an admissible region of ΔW such that the fuzzy control system is asymptotically stable in the large when W and C are given. We mainly consider the robust stabilization problem for PPU in this paper because it is a peculiar problem in fuzzy control systems. On the other hand, we can also consider consequent parameter uncertainty. However, consequent parameter uncertainty is not a peculiar problem because each consequent part is represented by a linear equation. We can easily analyze a robust stabilization problem for consequent parameter uncertainty by using the ordinary robust control theory such as H_∞ control. Therefore, we will not consider consequent parameter uncertainty in this paper.

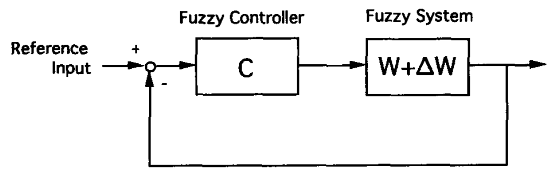

Fig. 1. A robust stabilization problem.

Next, we design a control system for backing up a computer simulated truck-trailer by applying the robust stabilization problem. It is shown that the designed fuzzy controller guarantees stability of the control systems under a condition. It is well known that backing up control of truck-trailers is a difficult exercise for all but the most skilled truck drivers since its dynamics is nonlinear and unstable. In practice, in order to succeed backing up the desired position, drivers often have to attempt backing, going forward, backing again, going forward again, etc., and can finally realize backing up to the desired position. Thus, the forward and backward movements help to position the truck-trailer for successful backing up to the desired position. A more difficult backing up sequence would only allow backing, with no forward movements permitted. The specific problem in this simulation is to back up a truck-trailer along the desired trajectory from an arbitrary initial position by manipulating the steering. Of course, only backing up is allowed. Some papers have reported that learning controls such as fuzzy control, neural control, and both of them [5]–[8] realize backing up control of a computer simulated truck-trailer. However, as far as we know, these studies have not analyzed stability of the control system. It is, in practice, important to guarantee stability of control systems. Our goal

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in this simulation is to design a fuzzy controller such that the control system is asymptotically stable in the large, that is, such that backing up control can be perfectly achieved from all initial positions.

Recently, Tanaka, and Sugeno [3], [9] have derived some conditions for ensuring stability of fuzzy dynamic systems. In Section II, we derive a stability condition by weakening the basic stability condition (BSC) proposed by Tanaka and Sugeno. This condition is named weak stability condition (WSC). Section III shows two design problems: nonrobust controller design and robust controller design. The former is a design problem for fuzzy systems with no PPU. The latter is a design problem for fuzzy systems with PPU. To realize two design problems, we derive four stability conditions: nonrobust stability condition (NSC), weak nonrobust stability condition (WNSC), robust stability condition (RSC), and weak robust stability condition (WRSC). In particular, WRSC is the most important condition. Because, we can introduce concept of robust stability for fuzzy control systems with PPU by using the WRSC. To introduce robust stability, admissible region and variation region, which correspond to stability margin in the ordinary control theory, are defined. In Section IV, we develop a control system for backing up a computer simulated truck-trailer model. We show that the designed fuzzy controller guarantees stability of the control system under a condition.

II. Basic Stability Condition and Weak Stability Condition

The fuzzy system, proposed by Takagi and Sugeno [10], is described by fuzzy IF-THEN rules which locally represent linear input–output relations of a system. This fuzzy system is of the following form:

Rule: IF $x(k)$ is $\mathcal{A}_{1i}$ and $\cdots$ and $x(k-n+1)$ is $\mathcal{A}_{ni}$
THEN $\mathbf{x}_i(k+1) = \mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k)$,

where

$$\mathbf{x}^T(k) = [x(k), x(k-1), \cdots, x(k-n+1)],$$
$$\mathbf{u}^T(k) = [u(k), u(k-1), \cdots, u(k-m+1)],$$

$i = 1, 2, \cdots, r$ and r is the number of IF-THEN rules. $\mathbf{x}_i(k+1)$ is the output from the i-th IF-THEN rule, and $\mathcal{A}_{ij}$ is a fuzzy set. Given a pair of $(\mathbf{x}(k), \mathbf{u}(k))$, the final output of the fuzzy system is inferred as follows.

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{r} w_i(k)\{\mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k)\}}{\sum_{i=1}^{r} w_i(k)} \tag{1}$$

where

$$w_i(k) = \prod_{j=1}^{n} \mathcal{A}_{ij}(x(k-j+1))$$

$\mathcal{A}_{ij}(x(k-j+1))$ is the grade of membership of $x((k-j+1))$ in $\mathcal{A}_{ij}$.

The free system of (1) is defined as

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{r} w_i(k)\mathbf{A}_i\mathbf{x}(k)}{\sum_{i=1}^{r} w_i(k)} \tag{2}$$

where it is assume that

$$\sum_{i=1}^{r} w_i(k) > 0$$
$$w_i(k) \geq 0 \qquad i = 1, 2, \cdots, r. \tag{3}$$

for all k. Each linear consequent equation represented by $\mathbf{A}_i\mathbf{x}(k)$ is called "subsystem."

The basic stability condition (BSC), proposed by Tanaka and Sugeno, for ensuring stability of (2) is given as follows.

Theorem 2.1 [3]: Basic stability condition (BSC): The equilibrium of a fuzzy system described by (2) is asymptotically stable in the large if there exists a common positive definite matrix $\mathbf{P}$ such that

$$\mathbf{A}_i^T\mathbf{P}\mathbf{A}_i - \mathbf{P} < \mathbf{0} \tag{4}$$

for $i = 1, 2, \cdots, r$, that is, for all the subsystems.

The proof of this theorem will be given in Appendix A. This theorem is reduced to the Lyapunov stability theorem for linear discrete systems when $r = 1$. Theorem 2.1 gives, of course, a sufficient condition for ensuring stability of (2). We may intuitively guess that a fuzzy system is asymptotically stable in the large if all its subsystems are stable, that is, if all its $\mathbf{A}_i$'s are stable matrices. However, this is not the case in general: we will discuss it in Example 2.1.

We should notice that the BSC of (4) depends only on $\mathbf{A}_i$. In other words, the BSC does not depend on $w_i(k)$. This fact becomes a key point when weakening the BSC.

[Example 2.1] Let us consider the following fuzzy free system;

Rule 1 : IF $x(k-1)$ is $\mathcal{A}_1$ THEN $\mathbf{x}_1(k+1) = \mathbf{A}_1\mathbf{x}(k)$,
Rule 2 : IF $x(k-1)$ is $\mathcal{A}_2$ THEN $\mathbf{x}_2(k+1) = \mathbf{A}_2\mathbf{x}(k)$,

where

$$\mathbf{x}(k)^T = [x(k) \quad x(k-1)],$$
$$\mathbf{A}_1 = \begin{bmatrix} 1.0 & -0.5 \\ 1.0 & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} -1.0 & -0.5 \\ 1.0 & 0 \end{bmatrix}.$$

Fig. 2 shows membership functions of $\mathcal{A}_1$ and $\mathcal{A}_2$. Fig. 3(a) and (b) illustrates the behavior of the following linear systems for the initial condition $x(0) = -0.70$ and $x(1) = 0.90$, respectively: $\mathbf{x}(k+1) = \mathbf{A}_1\mathbf{x}(k)$ and $\mathbf{x}(k+1) = \mathbf{A}_2\mathbf{x}(k)$. These linear systems are stable since $\mathbf{A}_1$ and $\mathbf{A}_2$ are stable matrices. However, the fuzzy system, which consists of these stable linear systems, is unstable as shown in Fig. 3(c).

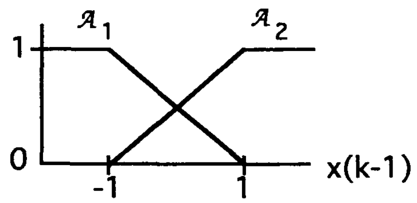

Fig. 2. Membership functions.

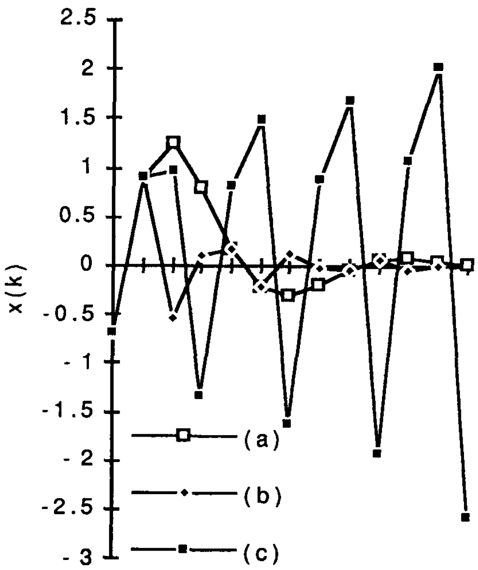

Fig. 3. (a) Behavior of $\mathbf{A}_1\mathbf{x}(k)$. (b) Behavior of $\mathbf{A}_2\mathbf{x}(k)$. (c) Behavior of fuzzy system.

Next, we weaken the BSC of (4). If it is possible, we may be able to find Lyapunov functions more easily. Theorem 2.2 gives a weakened condition for the BSC, that is, a WSC.

Theorem 2.2: Weak stability condition (WSC): The equilibrium of a fuzzy system described by (2) is asymptotically stable in the large if there exists a positive definite matrix $\mathbf{P}$ such that

$$\begin{aligned}\mathbf{S} = &\sum_{i=1}^{r} w_i(k)w_i(k)\{\mathbf{A}_i^T\mathbf{P}\mathbf{A}_i - \mathbf{P}\}\\ &+ \sum_{i<j} w_i(k)w_j(k)\{\mathbf{A}_i^T\mathbf{P}\mathbf{A}_i - \mathbf{P} + \mathbf{A}_j^T\mathbf{P}\mathbf{A}_j - \mathbf{P}\\ &- (\mathbf{A}_i - \mathbf{A}_j)^T\mathbf{P}(\mathbf{A}_i - \mathbf{A}_j)\} < \mathbf{0}. \qquad (5)\end{aligned}$$

Proof: It follows directly from the proof of Theorem 2.1.

As mentioned above, the BSC of (4) depends only on $\mathbf{A}_i$. The WBSC of (5) depends not only on $\mathbf{A}_i$ but also on $w_i(k)$. This means that the BSC is weakened. In other words, we may be able to find a positive definite matrix $\mathbf{P}$ which satisfies the WBSC even if there does not exist a common positive definite matrix $\mathbf{P}$ which satisfies the BSC. We will show it in Example 3.1. However, we should notice that Theorem 2.2 gives, of course, a sufficient condition for ensuring stability of (2).

III. Design Problems of Nonrobust Controller and Robust Controller

We have shown the stability conditions for fuzzy systems in the previous section. We apply the stability conditions to two design problems of fuzzy controllers in this section. We consider two cases.

Case 1 No premise parameter uncertainty.

Case 2 Premise parameter uncertainty.

Cases 1 and 2 are related to nonrobust controller design and robust controller design, respectively. As shown in Section II, the final output of a fuzzy system is calculated by (1) if it has no PPU, that is, (Case 1). Conversely, if it has PPU, that is, (Case 2), the final output is calculated by

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{r}(w_i(k) + \Delta w_i(k))\{\mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k)\}}{\sum_{i=1}^{r}(w_i(k) + \Delta w_i(k))} \qquad (6)$$

where $\Delta w_i(k)$ denotes unknown PPU. $\Delta w_i(k)$ is a value such that

$$-1 \le \Delta w_i(k) \le 1, \qquad \text{for all } i$$
$$w_i(k) + \Delta w_i(k) \ge 0, \qquad \text{for all } i$$
$$\sum_{i=1}^{r}(w_i(k) + \Delta w_i(k)) > 0$$

for all k. Equation (6) is reduced to (1) when $\Delta w_i(k) = 0$ for all k and i.

We stabilize the fuzzy system, (1) or (6), by the following controller.

Rule: IF $x(k)$ is $\mathcal{A}_{1i}$ and $\cdots$ and $x(k-n+1)$ is $\mathcal{A}_{ni}$
THEN $\mathbf{u}_i(k) = \mathbf{F}_i\mathbf{x}(k)$,

where $i = 1, 2, \cdots, r$. The final output of this fuzzy controller is calculated by

$$\mathbf{u}(k) = \frac{\sum_{i=1}^{r} w_i(k)\mathbf{F}_i\mathbf{x}(k)}{\sum_{i=1}^{r} w_i(k)} \qquad (7)$$

where we must use $w_i(k)$ instead of $w_i(k) + \Delta w_i(k)$ as a weight of i-th rule because $\Delta w_i(k)$'s are unknown. Of course, parameters of the controller are the consequent matrices $\mathbf{F}_i$.

We derive two nonrobust stability conditions and two robust stability conditions for ensuring stability of the fuzzy control system which consists of (1) [or (6)] and (7). The former is related to nonrobust controller design (Case 1). The latter is related to robust controller design (Case 2).

A. Nonrobust Stability Conditions (Case 1)

Assume that $\Delta w_i(k) = 0$ for all k and i. By substituting (7) into (1), we obtain

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k)\{\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_j\}\mathbf{x}(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k)}. \tag{8}$$

From (8),

$$\begin{aligned}\mathbf{x}(k+1) &= \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k)\{\mathbf{A}_j + \mathbf{B}_i\mathbf{F}_j\}\mathbf{x}(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k)} \\ &= \frac{1}{R}\left[\sum_{i=1}^{r} w_i(k)w_i(k)\mathbf{G}_{ii}\mathbf{x}(k)\right. \\ &\quad \left. + 2\sum_{i<j} w_i(k)w_j(k)\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\mathbf{x}(k)\right]\end{aligned} \tag{9}$$

where

$$\begin{aligned}\mathbf{G}_{ij} &= \mathbf{A}_i + \mathbf{B}_i\mathbf{F}_j, \\ R &= \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k).\end{aligned}$$

Without loss of generality, (9) can be rewritten as follows:

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{r(r+1)/2} v_i(k)\mathbf{H}_i\mathbf{x}(\mathbf{k})}{\sum_{i=1}^{r(r+1)/2} v_i(k)} \tag{10}$$

where

$$\begin{aligned}\mathbf{H}_{\Sigma_{t=1}^{i}(t-1)+i} &= \mathbf{G}_{ij}, && i=j \\ \mathbf{H}_{\Sigma_{t=1}^{i}(t-1)+i} &= (\mathbf{G}_{ij}+\mathbf{G}_{ji})/2, && i<j \\ v_{\Sigma_{t=1}^{j}(t-1)+i}(k) &= w_i(k)w_j(k), && i=j \\ v_{\Sigma_{t=1}^{j}(t-1)+i}(k) &= 2w_i(k)w_j(k). && i<j\end{aligned}$$

By applying Theorem 2.1 to (10), we derive a nonrobust stability condition (NSC) for the fuzzy control system, (10), with no PPU. Theorem 3.1 gives a NSC for (10).

Theorem 3.1: Nonrobust stability condition (NSC): The equilibrium of a fuzzy control system described by (10) is asymptotically stable in the large if there exists a common positive definite matrix $\mathbf{P}$ such that

$$\mathbf{H}_i^T\mathbf{P}\mathbf{H}_i - \mathbf{P} < 0 \tag{11}$$

for $i = 1, 2, \cdots, r(r+1)/2$.

Proof: It follows directly from Theorem 2.1.

The nonrobust design problem for Theorem 3.1 is to select $\mathbf{F}_j\,(j = 1, 2, \cdots, r)$ which satisfies the NSC of (11) for a common positive definite matrix $\mathbf{P}$ when $\mathbf{A}_j$ and $\mathbf{B}_j$ are given.

Next, we weaken the NSC of Theorem 3.1 by applying Theorem 2.2 to (10). Theorem 3.2 gives a weakened NSC, this is, a WNSC.

Theorem 3.2: Weak nonrobust stability condition (WNSC): The equilibrium of a fuzzy control system described by (10) is asymptotically stable in the large if there exists a positive definite matrix $\mathbf{P}$ such that

$$\begin{aligned}\mathbf{S} = &\sum_{i=1}^{r(r+1)/2} v_i(k)v_i(k)\{\mathbf{H}_i^T\mathbf{P}\mathbf{H}_i - \mathbf{P}\} \\ &+ \sum_{i<j} v_i(k)v_j(k)\{\mathbf{H}_i^T\mathbf{P}\mathbf{H}_i - \mathbf{P} + \mathbf{H}_j^T\mathbf{P}\mathbf{H}_j - \mathbf{P} \\ &- (\mathbf{H}_i - \mathbf{H}_j)^T\mathbf{P}(\mathbf{H}_i - \mathbf{H}_j)\} < \mathbf{0}.\end{aligned} \tag{12}$$

Proof: It follows directly from Theorem 2.2.

The nonrobust design problem for Theorem 3.2 is to select $\mathbf{F}_j\,(j = 1, 2, \cdots, r)$ which satisfies the WNSC of (12) for a common positive definite matrix $\mathbf{P}$ when $\mathbf{A}_j$ and $\mathbf{B}_j$ are given.

Example 3.1 shows that we can find a positive definite matrix $\mathbf{P}$ which satisfies the WNSC of (12) even if there does not exist a common positive definite matrix $\mathbf{P}$ which satifies the NSC of (11).

[Example 3.1] Let us consider the following fuzzy system and fuzzy controller;

$$\begin{aligned}&\text{Rule}: \ \text{IF } x(k) \text{ is } \mathcal{A}_1 \text{ THEN } \mathbf{x}_1(k+1) = \mathbf{A}_1\mathbf{x}(k) + \mathbf{B}_1u(k) \\ &\text{Rule}: \ \text{IF } x(k) \text{ is } \mathcal{A}_2 \text{ THEN } \mathbf{x}_2(k+1) = \mathbf{A}_2\mathbf{x}(k) + \mathbf{B}_2u(k)\end{aligned}$$

where

$$\mathbf{A}_1 = \begin{bmatrix}1.0 & 0.5\\ 1.0 & 0\end{bmatrix} \quad \mathbf{B}_1 = \begin{bmatrix}0.2\\ 0\end{bmatrix}$$
$$\mathbf{A}_2 = \begin{bmatrix}1.0 & -0.5\\ 1.0 & 0\end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix}0.58\\ 0\end{bmatrix}$$

$$\begin{aligned}&\text{Rule}: \ \text{IF } x(k) \text{ is } \mathcal{A}_1 \text{ THEN } u_1(k) = \mathbf{F}_1\mathbf{x}(k) \\ &\text{Rule}: \ \text{IF } x(k) \text{ is } \mathcal{A}_2 \text{ THEN } u_2(k) = \mathbf{F}_2\mathbf{x}(k)\end{aligned}$$

where

$$\begin{aligned}\mathbf{F}_1 &= [-5.0 \quad -3.75] \\ \mathbf{F}_2 &= [-1.72 \quad 0.43].\end{aligned}$$

From $\mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_1, \mathbf{B}_2, \mathbf{F}_1$, and $\mathbf{F}_2$, we obtain

$$\mathbf{H}_1 = \mathbf{H}_3 = \begin{bmatrix}0 & -0.25\\ 1 & 0\end{bmatrix}$$
$$\mathbf{H}_2 = \begin{bmatrix}-0.62 & -1.04\\ 1 & 0\end{bmatrix}.$$

It is found that there does not exist a common positive definite matrix $\mathbf{P}$ such that

$$\mathbf{H}_i^T\mathbf{P}\mathbf{H}_i - \mathbf{P} < 0 \qquad i = 1, 2, 3$$

since $\mathbf{H}_2$ is an unstable matrix. Therefore, the NSC of (11) is not satisfied.

Next, we show that the WNSC of (12) is satisfied if we select

$$\mathbf{P} = \begin{bmatrix} 2.2 & 0 \\ 0 & 1.2 \end{bmatrix}$$

as a common positive definite matrix $\mathbf{P}$. This matrix $\mathbf{P}$ was selected by the construction procedure of the literature [12]. By substituting $\mathbf{H}_1 \sim \mathbf{H}_3$ and $\mathbf{P}$ into (12), we obtain

$$\mathbf{S} = \begin{bmatrix} \alpha & \gamma \\ \gamma & \beta \end{bmatrix}$$

where

$$\begin{aligned}\alpha =& -v_1^2(k) - 0.15v_2^2(k) - v_3^2(k) - 2v_1(k)v_2(k) \\ & - 2v_1(k)v_3(k) - 2v_2(k)v_3(k) - 2v_2(k)v_4(k) \\ \beta =& -1.06v_1^2(k) - 1.25v_1(k)v_2(k) + 1.20v_2^2(k) \\ & - 2.13v_1(k)v_3(k) - 1.25v_2(k)v_3(k) - 1.06v_3^2(k) \\ \gamma =& \, 0.34v_1(k)v_2(k) + 1.43v_2^2(k) + 0.34v_2(k)v_3(k).\end{aligned}$$

In order that $\mathbf{S}$ is a negative definite matrix, it must be satisfied that

$$\alpha < 0 \quad \text{and} \quad \alpha \times \beta - \gamma \times \gamma > 0.$$

It is clear that $\alpha < 0$. On the other hand,

$$\begin{aligned}\alpha \times \beta - \gamma \times \gamma =& \, 1.06v_1^4(k) + 2.22v_2^4(k) + 1.06v_3^4(k) \\ & + 3.38v_1^3(k)v_2(k) + 1.34v_1^2(k)v_2^2(k) \\ & - 3.19v_1(k)v_2^3(k) \\ & + 4.25v_1^3(k)v_3(k) + 10.1v_1^2(k)v_2(k)v_3(k) \\ & + 2.68v_1(k)v_2^2(k)v_3(k) - 3.19v_2^3(k)v_3(k) \\ & + 6.38v_1^2(k)v_3^2(k) \\ & + 10.1v_1(k)v_2(k)v_3^2(k) + 1.34v_2^2(k)v_3^2(k) \\ & + 4.25v_1(k)v_3^3(k) \\ & + 3.38v_2(k)v_3^3(k).\end{aligned}$$

By substituting

$$\begin{aligned}v_1(k) &= w_1(k)w_1(k) \\ v_2(k) &= 2w_1(k)w_2(k) \\ v_3(k) &= w_2(k)w_2(k)\end{aligned}$$

into the above equation, we obtain

$$\begin{aligned}& f_1(w_1(k), w_2(k)) = \\ & 1.06w_1^8(k) + 6.75w_1^7(k)w_2(k) \\ & + 9.62w_1^6(k)w_2^2(k) \\ & - 5.29w_1^5(k)w_2^3(k) \\ & - 18.4w_1^4(k)w_2^4(k) - 5.29w_1^3(k)w_2^5(k) \\ & + 9.62w_1^2(k)w_2^6(k) \\ & + 6.75w_1(k)w_2^7(k) + 1.06w_2^8(k)\end{aligned}$$

where

$$f_1(w_1(k), w_2(k)) = \alpha \times \beta - \gamma \times \gamma.$$

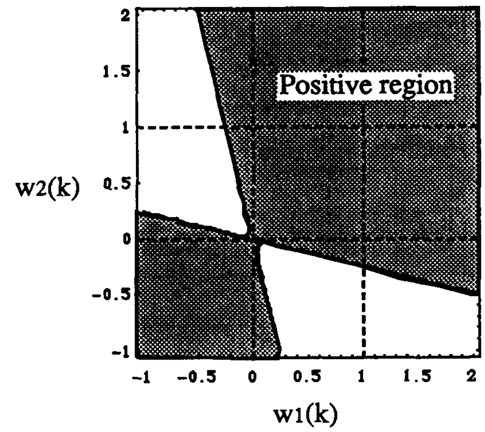

Fig. 4. Positive region of $f_1(w_1(k), w_2(k))$.

Fig. 4 shows a positive region such that $f_1(w_1(k), w_2(k)) > 0$. It is found from Fig. 4 that

$$f_1(w_1(k), w_2(k)) > 0$$

when $0 \le w_1(k) \le 1$ and $0 \le w_2(k) \le 1$. This means that the WNSC of (12) is satisfied for $0 \le w_1(k) \le 1$ and $0 \le w_2(k) \le 1$, that is, for any fuzzy sets $\mathcal{A}_1$ and $\mathcal{A}_2$.

B. Robust Stability Conditions (Case 2)

By substituting (7) into (6), we obtain

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r}(w_i(k) + \Delta w_i(k))w_j(k)\mathbf{G}_{ij}\mathbf{x}(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r}(w_i(k) + \Delta w_i(k))w_j(k)} \tag{13}$$

where

$$\mathbf{G}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{F}_j.$$

Without loss of generality, (13) can be rewritten as follows.

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{r\times r} v_i(k)\mathbf{H}_i\mathbf{x}(k)}{\sum_{i=1}^{r\times r} v_i(k)} \tag{14}$$

where

$$\begin{aligned}\mathbf{H}_{(i-1)\times r+j} &= \mathbf{G}_{ij} \\ v_{(i-1)\times r+j}(k) &= (w_i(k) + \Delta w_i(k))w_j(k)\end{aligned}$$

for all i and j. By appling Theorem 2.1 to (14), we derive a robust stability condition (RSC) for the fuzzy control system (14) with PPU. Theorem 3.3 gives a RSC for (14).

Theorem 3.3: Robust stability condition (RSC): The equilibrium of a fuzzy control system described by (14) is asymptotically stable in the large if there exists a common positive definite matrix $\mathbf{P}$ such that

$$\mathbf{H}_i^T\mathbf{P}\mathbf{H}_i - \mathbf{P} < 0 \tag{15}$$

for $i = 1, 2, \cdots, r \times r$.

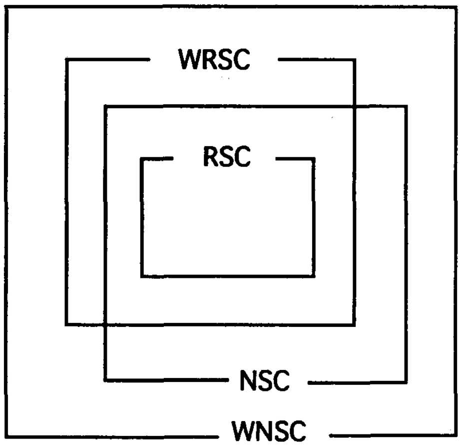

Fig. 5. Relation among NSC, WNSC, RSC, and WRSC.

Proof: It follows directly from Theorem 2.1.

The robust design problem for Theorem 3.3 is to select $\mathbf{F}_j\,(j = 1, 2, \cdots, r)$ which satisfies the RSC of (15) for a common positive definite matrix $\mathbf{P}$ when $\mathbf{A}_j$ and $\mathbf{B}_j$ are given. It is clear that (15) implies (11).

Next, we weaken the RSC of Theorem 3.3 by applying Theorem 2.2 to (14). Theorem 3.4 gives a weakened RSC, that is, a WRSC.

Theorem 3.4: Weak robust stability condition (WRSC): The equilibrium of a fuzzy control system described by (14) is asymptotically stable in the large if there exists a positive definite matrix $\mathbf{P}$ such that

$$\begin{aligned}\mathbf{S} = &\sum_{i=1}^{r\times r} v_i(k)v_i(k)\{\mathbf{H}_i^T\mathbf{P}\mathbf{H}_i - \mathbf{P}\}\\ &+ \sum_{i<j} v_i(k)v_j(k)\{\mathbf{H}_i^T\mathbf{P}\mathbf{H}_i - \mathbf{P} + \mathbf{H}_j^T\mathbf{P}\mathbf{H}_j - \mathbf{P}\\ &- (\mathbf{H}_i - \mathbf{H}_j)^T\mathbf{P}(\mathbf{H}_i - \mathbf{H}_j)\} < \mathbf{0}. \qquad (16)\end{aligned}$$

Proof: It follows directly from Theorem 2.2.

The robust design problem for Theorem 3.4 is to select $\mathbf{F}_j\,(j = 1, 2, \cdots, r)$ which satisfies the WRSC to (16) for a common positive definite matrix $\mathbf{P}$ when $\mathbf{A}_j$ and $\mathbf{B}_j$ are given. Of course, (16) implies (12).

Fig. 5 shows the relation among four conditions: NSC, WNSC, RSC, and WRSC.

C. Admissible Region and Variation Region

To introduce robust stability, we define admissible region and variation region, which correspond to stability margin in the ordinary control theory, from the WRSC of Theorem 3.4. When a positive definite matrix $\mathbf{P}$ is given, by substituting $\mathbf{H}_i$ and $v_i(k)$ into (16) and by solving (16) for $\Delta w_i(k)$, we can find an admissible region (AR) of $\Delta w_i(k)$'s which guarantee stability of the fuzzy system, (14), with PPU. On the other hand, we can find a variation region (VR) of $\Delta w_i(k)$'s, which shows model error between a real system and an approximated fuzzy model, if a mathematical model of the real system is given. By introducing AR and VR, robust stability for the fuzzy system with PPU can be defined. Definition 3.1 shows the definition of robust stability. We will concretely explain how to find AR and VR in Example 3.2.

[Definition 3.1] Assume that AR and VR denote an admissible region and a variation region for a fuzzy system with PPU, respectively. If the AR perfectly includes the VR, that is,

$$VR \subseteq AR \qquad (17)$$

it is said that the fuzzy system with PPU is robust stability.

[Definition 3.2] If

$$VR_1 \subset VR_2 \subseteq AR$$

it is said that a fuzzy system (FS_1) with VR_1 is more robust than a fuzzy system (FS_2) with VR_2, that is, a stability margin of the FS_1 is larger than that of the FS_2.

Example 3.2 gives a simple example for robust stability.

[Example 3.2] Let us consider the following fuzzy system and fuzzy controller:

$$\begin{aligned}&\text{Rule}: \ \text{IF}\ x(k)\ \text{is}\ \mathcal{A}_1\ \text{THEN}\ \mathbf{x}_1(k+1) = \mathbf{A}_1\mathbf{x}(k) + \mathbf{B}_1 u(k)\\ &\text{Rule}: \ \text{IF}\ x(k)\ \text{is}\ \mathcal{A}_2\ \text{THEN}\ \mathbf{x}_2(k+1) = \mathbf{A}_2\mathbf{x}(k) + \mathbf{B}_2 u(k)\end{aligned}$$

where

$$\mathbf{A}_1 = \begin{bmatrix}1.0 & 0.5\\ 1.0 & 0\end{bmatrix} \quad \mathbf{B}_1 = \begin{bmatrix}0.2\\ 0\end{bmatrix}$$
$$\mathbf{A}_2 = \begin{bmatrix}1.0 & -0.5\\ 1.0 & 0\end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix}0.4\\ 0\end{bmatrix}$$

$$\begin{aligned}&\text{Rule}: \ \text{IF}\ x(k)\ \text{is}\ \mathcal{A}_1\ \text{THEN}\ u_1(k) = \mathbf{F}_1\mathbf{x}(k),\\ &\text{Rule}: \ \text{IF}\ x(k)\ \text{is}\ \mathcal{A}_2\ \text{THEN}\ u_2(k) = \mathbf{F}_2\mathbf{x}(k),\end{aligned}$$

where

$$\begin{aligned}\mathbf{F}_1 &= [-5.0 \quad -3.75]\\ \mathbf{F}_2 &= [-2.5 \quad 0.63].\end{aligned}$$

Fig. 6 shows membership functions of the fuzzy sets $\mathcal{A}_1$ and $\mathcal{A}_2$. From $\mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_1, \mathbf{B}_2, \mathbf{F}_1$, and $\mathbf{F}_2$, we obtain

$$\mathbf{H}_1 = \mathbf{H}_4 = \begin{bmatrix}0 & -0.25\\ 1 & 0\end{bmatrix},$$
$$\mathbf{H}_2 = \begin{bmatrix}0.5 & 0.63\\ 1 & 0\end{bmatrix}, \quad \mathbf{H}_3 = \begin{bmatrix}-1 & -2\\ 1 & 0\end{bmatrix}.$$

It is found that there does not exist a common positive definite matrix $\mathbf{P}$ such that

$$\mathbf{H}_i^T\mathbf{P}\mathbf{H}_i - \mathbf{P} < \mathbf{0} \qquad i = 1, 2, 3, 4$$

since $\mathbf{H}_2$ and $\mathbf{H}_3$ are unstable matrices. Therefore, the RSC of (15) is not satisfied.

Next, let us find an admissible region of $\Delta w_i(k)$'s which satisfies the WRSC of (16). Assume that

$$\mathbf{P} = \begin{bmatrix}2.2 & 0\\ 0 & 1.2\end{bmatrix}.$$

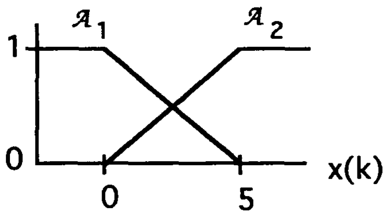

Fig. 6. Membership functions.

This matrix $\mathbf{P}$ was selected by the construction procedure of the literature [12]. By substituting $\mathbf{H}_1 \sim \mathbf{H}_4$ and $\mathbf{P}$ into (16), we obtain

$$\mathbf{S} = \begin{bmatrix} \alpha & \gamma \\ \gamma & \beta \end{bmatrix}$$

where

$$\begin{aligned}
\alpha =& -v_1^2(k) - 0.45v_2^2(k) + 1.2v_3^2(k) - v_4^2(k) \\
& - 2v_1(k)v_2(k) - 2v_1(k)v_3(k) - 4.2v_2(k)v_3(k) \\
& - 2v_1(k)v_4(k) - 2v_2(k)v_4(k) - 2v_3(k)v_4(k) \\
\beta =& -1.06v_1^2(k) - 0.34v_2^2(k) - 7.6v_3^2(k) - 1.06v_4^2(k) \\
& - 3.09v_1(k)v_2(k) - 0.20v_1(k)v_3(k) - 2.13v_1(k)v_4(k) \\
& - 7.90v_2(k)v_3(k) - 3.09v_2(k)v_4(k) - 0.20v_3(k)v_4(k), \\
\gamma =& 0.69v_2^2(k) + 4.40v_3^2(k) - 0.28v_1(k)v_2(k) \\
& + 0.55v_1(k)v_3(k) - 3.58v_2(k)v_3(k) \\
& - 0.28v_2(k)v_4(k) + 0.55v_3(k)v_4(k).
\end{aligned}$$

In order that $\mathbf{S}$ is negative definite matrix, it must be satisfied that

$$\alpha < 0 \quad \text{and} \quad \alpha \times \beta - \gamma \times \gamma > 0.$$

To obtain a graphical image of an admissible region which guarantees robust stability, we assume that

$$\begin{aligned}
& w_1(k) + w_2(k) = 1, \\
& (w_1(k) + \Delta w_1(k)) + (w_2(k) + \Delta w_2(k)) = 1.
\end{aligned}$$

By substituting

$$\begin{aligned}
v_1(k) &= (w_1(k) + \Delta w_1(k))w_1(k) \\
v_2(k) &= (w_1(k) + \Delta w_1(k))w_2(k) \\
v_3(k) &= (w_2(k) + \Delta w_2(k))w_1(k) \\
v_4(k) &= (w_2(k) + \Delta w_2(k))w_2(k)
\end{aligned}$$

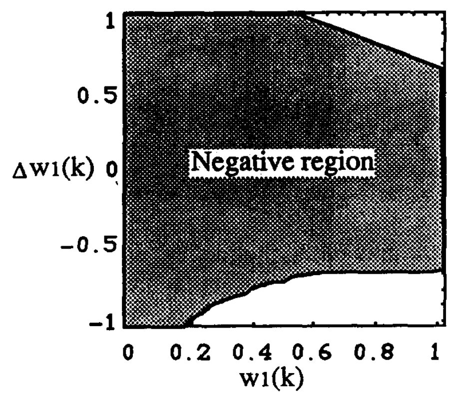

Fig. 7. Negative region of $g_1(w_1(k), w_2(k))$.

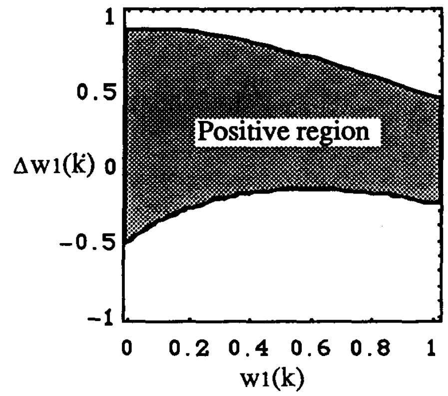

Fig. 8. Positive region of $g_2(w_1(k), w_2(k))$.

into the above equations with respect to α, β, γ and by eliminating $w_2(k)$ and $\Delta w_2(k)$, we can derive

$$\begin{aligned}
g_1(w_1(k), \Delta w_1(k)) =& -1.0 + 0.55\Delta w_1^2(k) \\
& - 1.1w_1(k)\Delta w_1(k) \\
& + 1.1w_1(k)\Delta w_1^2(k) + 0.55w_1^2(k) \\
& + 0.55w_1^2(k)\Delta w_1^2(k) - 1.1w_1^3(k) \\
& + 1.1w_1^3(k)\Delta w_1(k) + 0.55w_1^4(k) \\
g_2(w_1(k), \Delta w_1(k)) =& 1.06 + 0.96\Delta w_1(k) - 2.34\Delta w_1^2(k) \\
& - 0.96w_1(k) + 5.65w_1(k)\Delta w_1(k) \\
& - 4.69w_1(k)\Delta w_1^2(k) - 1.38w_1^2(k) \\
& - 2.34w_1^2(k)\Delta w_1^2(k) + 4.69w_1^3(k) \\
& - 4.69w_1^3(k)\Delta w_1(k) - 2.34w_1^4(k)
\end{aligned}$$

where

$$\begin{aligned}
& g_1(w_1(k), \Delta w_1(k)) = \alpha, \\
& g_2(w_1(k), \Delta w_1(k)), = \alpha \times \beta - \gamma \times \gamma.
\end{aligned}$$

Fig. 7 shows a negative region such that $g_1(w_1(k), \Delta w_1(k)) < 0$. Fig. 8 shows a positive region such that $g_2(w_1(k), \Delta w_1(k)) > 0$. The AR, which guarantees robust stability, is intersection region of the negative region and the positive region. Fig. 9 shows the AR.

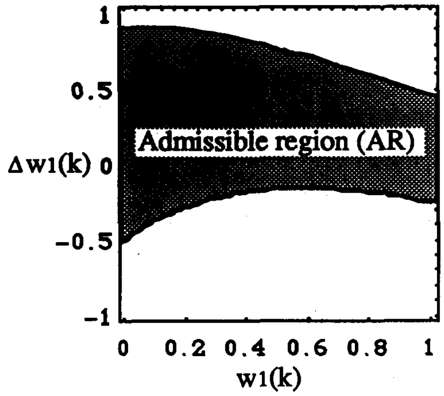

Fig. 9. Admissible region (AR) for robust stability.

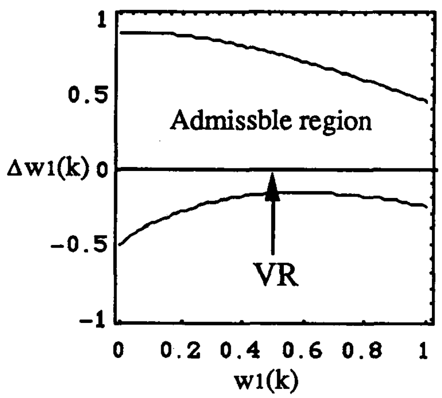

Fig. 10. Variation region (VR) for case a).

Let us consider robust stability for three cases of fuzzy systems with different premise parameter uncertainty;

$$\begin{aligned} &\text{Case a)} \quad \Delta w_1(k) = 0 \\ &\text{Case b)} \quad w_1(k) + \Delta w_1(k) = \mathcal{A}_1(x(k)+4) \\ &\text{Case c)} \quad w_1(k) + \Delta w_1(k) = \mathcal{A}_1(x(k)-4) \end{aligned}$$

for all k. Case a) means that the fuzzy system has no PPU. In order to check robust stability defined in Definition 3.1, we need to find the VR for each case. Fig. 10 shows the VR for the Case a). In this case, the VR becomes a straight line ($\Delta w_1(k) = 0$) because of no PPU. By comparing the AR in Fig. 9 with the VR in Fig. 10, $VR \subset AR$. Therefore, this control system is robust stability.

In Case b),

$$\begin{aligned} \Delta w_1(k) &= \mathcal{A}_1(x(k)+4) - \mathcal{A}_1(x(k)) \\ &= \mathcal{A}_1(x(k)+4) - w_1(k). \end{aligned}$$

From

$$\begin{aligned} &\mathcal{A}_1(x(k)+4) = 1, && x(k) < 4 \\ &\mathcal{A}_1(x(k)+4) = -\tfrac{1}{5}x(k) + \tfrac{9}{5}, && 4 \le x(k) \le 9 \\ &\mathcal{A}_1(x(k)+4) = 0, && 9 < x(k) \end{aligned}$$

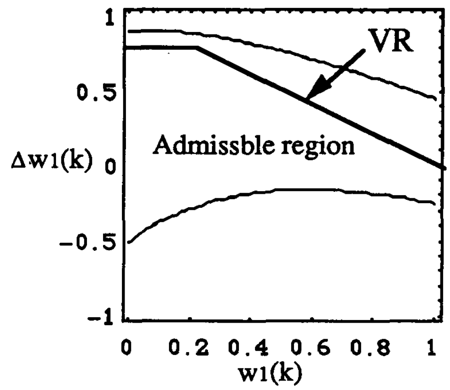

Fig. 11. Variation region (VR) for case b).

we obtain

$$\begin{aligned} &\Delta w_1(k) = 1 - w_1(k), && x(k) < 4 \\ &\Delta w_1(k) = -\tfrac{1}{5}x(k) + \tfrac{9}{5} - w_1(k), && 4 \le x(k) \le 9 \\ &\Delta w_1(k) = -w_1(k). && 9 < x(k) \end{aligned}$$

By substituting

$$\begin{aligned} &\mathcal{A}_1(x(k)) = w_1(k) = 1, && x(k) < 0 \\ &\mathcal{A}_1(x(k)) = w_1(k) = -\tfrac{1}{5}x(k) + 1, && 0 \le x(k) \le 5 \\ &\mathcal{A}_1(x(k)) = w_1(k) = 0, && 5 < x(k) \end{aligned}$$

into the above equations and by eliminating $x(k)$, we can derive

$$\begin{aligned} &\Delta w_1(k) = 1 - w_1(k) && 0.2 < w_1(k) \le 1 \\ &\Delta w_1(k) = 0.8, && 0 < w_1(k) \le 0.2 \\ &\Delta w_1(k) \in [0, 0.8] && w_1(k) = 0 \end{aligned}$$

From the above relation between $w_1(k)$ and $\Delta w_1(k)$, we obtain the VR for this case. Fig. 11 shows the VR. By comparing the AR in Fig. 9 with the VR in Fig. 11, $VR \subset AR$. Therefore, this control system is robust stability.

In Case c),

$$\begin{aligned} \Delta w_1(k) &= \mathcal{A}_1(x(k)-4) - \mathcal{A}_1(x(k)) \\ &= \mathcal{A}_1(x(k)-4) - w_1(k). \end{aligned}$$

From

$$\begin{aligned} &\mathcal{A}_1(x(k)-4) = 0, && x(k) < -4 \\ &\mathcal{A}_1(x(k)-4) = -\tfrac{1}{5}x(k) + \tfrac{1}{5}, && -4 \le x(k) \le 1 \\ &\mathcal{A}_1(x(k)-4) = 1, && 1 < x(k) \end{aligned}$$

in the same way as the Case b), we obtain

$$\begin{aligned} &\Delta w_1(k) = -w_1(k), && 0 \le w_1(k) \le 0.8 \\ &w_1(k) = -0.8 && 0.8 < w_1(k) < 1 \\ &\Delta w_1(k) \in [-0.8, 0]. && w_1(k) = 1. \end{aligned}$$

From the above relation, we obtain the VR for Case c). Fig. 12 shows the VR. By comparing the AR with the $VR, VR \not\subset AR$. For example, when $x(k) = 1, w_1(k) = 0.8$ and $\Delta w_1(k) = -0.8$, that is, $(0.8, -0.8)$ which corresponds to the point " P" in Fig. 12. This point "P" does not belong

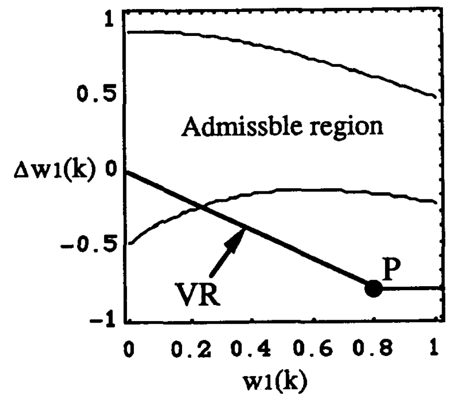

Fig. 12. Variation region (VR) for case c).

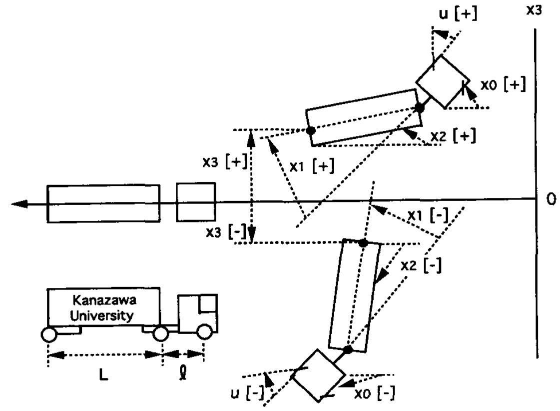

Fig. 14. Truck-trailer model and its coordinate system.

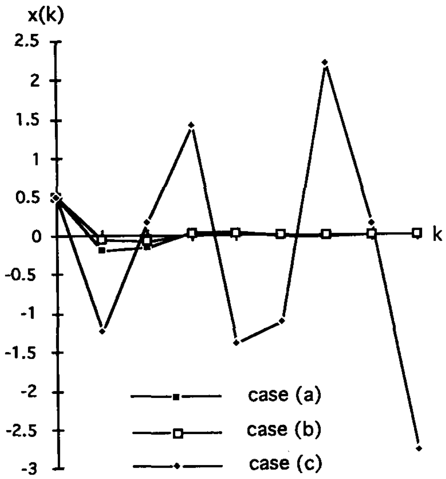

Fig. 13. Control results.

to the AR. Therefore, robust stability may not be guaranteed in this case.

Fig. 13 shows control results for $x(0) = x(1) = 0.5$. It is found from Fig. 13 that the control system of Case c) is not asymptotically stable in the large.

IV. Robust Control of a Truck-Trailer

We have shown an analysis technique of robust stability in Section III. In this section, we apply the analysis technique to backing up control of a computer simulated truck-trailer. It is well known that backing up control of a truck-trailer is very difficult since its dynamics is nonlinear and unstable. Some papers have reported that learning controls such as fuzzy control, neurtal control, and both of them [4]–[7] realize backing up control of a computer simulated truck-trailer. However, as far as we know, these studies have not analyzed stability of the control system. It is, in practice, important to guarantee stability of control system. Our final goal in this simulation is to design a fuzzy controller such that the control system is asymptotically stable in the large, that is, such that backing up control can be perfectly achieved from all initial positions.

A. Models of a Truck-Trailer

We use the truck-trailer model formulated by Ichihashi [7] in this simulation. After simplifying the truck trailer-model, we approximate the simplified model by a fuzzy model with PPU.

Ichihashi used the following truck-trailer model.

$$x_0(k+1) = x_0(k) + v \cdot t/l \cdot \tan[u(k)] \tag{18}$$

$$x_1(k) = x_0(k) - x_2(k) \tag{19}$$

$$x_2(k+1) = x_2(k) + v \cdot t/L \cdot \sin[x_1(k)] \tag{20}$$

$$x_3(k+1) = x_3(k) + v \cdot t \cdot \cos[x_1(k)] \cdot \sin[\{x_2(k+1) + x_2(k)\}/2] \tag{21}$$

$$x_4(k+1) = x_4(k) + v \cdot t \cdot \cos[x_1(k)] \cdot \cos[\{x_2(k+1) + x_2(k)\}/2] \tag{22}$$

where

$x_0(k)$:	angle of truck
$x_1(k)$:	angle difference between truck and trailer
$x_2(k)$:	angle of trailer
$x_3(k)$:	vertical position of rear end of trailer
$x_4(k)$:	horizontal position of rear end of trailer
$u(k)$:	steering angle

l is the length of truck, L is the length of trailer, t is sampling time, and v is the constant speed of backing up. In this paper, $l = 2.8$ [m], $L = 5.5$ [m], $v = -1.0$ [m/s], and $t = 2.0$ [s]. With respect to $x_1(k)$, 90 [deg] and -90 [deg] correspond to two "jackknife" positions. Fig. 14 shows the truck-trailer model and its coordinate system.

Let's simplify the Ichihashi's model. If $x_1(k)$ and $u(k)$ are always small values, the Ichihashi's truck-trailer model can be simplified as follows.

$$x_0(k+1) = x_0(k) + v \cdot t/l \cdot u(k) \tag{23}$$

$$x_1(k) = x_0(k) - x_2(k) \tag{24}$$

$$x_2(k+1) = x_2(k) + v \cdot t/L \cdot x_1(k) \tag{25}$$

$$x_3(k+1) = x_3(k) + v \cdot t \cdot \sin[\{x_2(k+1) + x_2(k)\}/2] \tag{26}$$

$$x_4(k+1) = x_4(k) + v \cdot t \cdot \cos[\{x_2(k+1) + x_2(k)\}/2]. \tag{27}$$

Of course, the dynamics of this simplified model may not perfectly agree with that of the Ichihashi's model when the value of $x_1(k)$ or $u(k)$ is a large value. We will consider influence of the model error in the simulation.

In the case of backing up control, the controlled variable $x_4(k)$ is not necessary, because the purpose of our control is to back up a truck-trailer along a desired trajectory (the straight line of $x_3(k) = 0$), that is, to regulate $x_1(k) \sim x_3(k)$ by manipulating the steering angle $u(k)$.

From (23), (24), and (25), we obtain

$$x_1(k+1) = (1 - v \cdot t/L)x_1(k) + v \cdot t/l \cdot u(k). \tag{28}$$

On the other hand, from (25) and (26),

$$x_3(k+1) = x_3(k) + v \cdot t \cdot \sin[x_2(k) + v \cdot t/\{2L\} \cdot x_1(k)]. \tag{29}$$

Therefore, the simplified model can be described by (28), (25), and (29). We should notice that (29) has nonlinearity.

Next, we approximate the simplified model by a fuzzy model. A fuzzy approximation method, proposed by Tanaka and Sano [11], is used to approximate the simplified model. As mentioned above, (29) has nonlinearity because its second term is represented by sine function whose value changes according to the value of $x_2(k) + v \cdot t/\{2L\} \cdot x_1(k)$. We should notice that any sine function can be exactly described by a fuzzy model.

$$0 \le \sin(x) \le x, \qquad -\pi \le x \le \pi$$

the sine function can be exactly described by

$$\sum_{i=1}^{2} \tilde{w}_i y_i \Big/ \sum_{i=1}^{2} \tilde{w}_i$$

where $y_1 = 0, y_2 = x$ and $\tilde{w}_1, \tilde{w}_2 \in [0, 1]$, that is,

$$\sin(x) = \sum_{i=1}^{2} \tilde{w}_i y_i \Big/ \sum_{i=1}^{2} \tilde{w}.$$

This means that the sine function can be interpolated by using y_1 and y_2. By solving the above equation for $\tilde{w}_i$, we can obtain the membership value for each x. Assume that $\tilde{w}_1 + \tilde{w}_2 = 1$. Then, since

$$\tilde{w}_1 y_1 + (1 - \tilde{w}_1)y_2 = \sin(x)$$

we obtain

$$\tilde{w}_1 = \frac{\sin(x) - y_2}{y_1 - y_2} = \frac{\sin(x) - x}{-x} = 1 - \frac{\sin(x)}{x}$$

$$\tilde{w}_2 = \frac{\sin(x)}{x}.$$

Kawamoto [13] has reported that nonlinear models can be exactly described by fuzzy models using this idea. A disadvantage of this approach is that fuzzy sets in premise parts generally become complex, that is, that fuzzy sets represented by $\tilde{w}_1$ and $\tilde{w}_2$ are not always unimodal and normal. As pointed out in Zadhe's papers [14]–[16], fuzzy sets can be regarded as linguistic variables. This is the most important feature when applying fuzzy sets to real systems. In general, normal and unimodal (simple) fuzzy sets are required to represent concept of lingustic variables such as big and very high, more or less old and so on. In this approach, the important feature is lost although it realizes an exact approximation. On the other hand, in Tanaka and Sano's approach [11], normal and unimodal fuzzy sets are used in premise parts although it does not always realize an exact approximation. We will show later that, by introducing concept of PPU, this approach also realizes an exact approximation, that is, non-linear systems can be exactly described by fuzzy models with PPU.

Let us approximate the simplified model by a fuzzy model. From the above discussion with respect to sine function,

$$0 \le \sin\left\{x_2(k) + \frac{\nu \cdot t}{2L}x_1(k)\right\} \le x_2(k) + \frac{\nu \cdot t}{2L}x_1(k). \tag{30}$$

Therefore,

$$\begin{aligned} x_3(k) &\le x_3(k) + \sin\left\{x_2(k) + \frac{\nu \cdot t}{2L}x_1(k)\right\} \\ &\le x_3(k) + x_2(k) + \frac{\nu \cdot t}{2L}x_1(k). \end{aligned}$$

The left-hand side and the right-hand side of the inequality correspond to $x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) = 0$ and $x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) = 180°$ or $-180°$ (π (rad) or $-\pi$ (rad), respectively. Therefore, when

$$x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) \cong 0[\text{deg}],$$

the state equation of the simplified model is approximated by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 - \frac{\nu \cdot t}{L} & 0 & 0 \\ \frac{\nu \cdot t}{L} & 1 & 0 \\ \frac{\nu^2 \cdot t^2}{2L} & \nu \cdot t & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} \frac{\nu \cdot t}{l} \\ 0 \\ 0 \end{bmatrix} u(k). \tag{31}$$

On the other hand, when

$$x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) = 180° \quad \text{or} \quad -180°(\pi\,(\text{rad})\,\text{or} \quad -\pi\,(\text{rad}))$$

the state equation of the simplified model is approximated by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 - \frac{\nu \cdot t}{L} & 0 & 0 \\ \frac{\nu \cdot t}{L} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} \frac{\nu \cdot t}{l} \\ 0 \\ 0 \end{bmatrix} u(k). \tag{32}$$

However, we must notice that (32) is uncontrollable. Therefore, a control law cannot be uniquely determined in this case. To avoid it, we approximate the simplified model for

$$x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) = 179.997[\deg] \quad \text{or} \quad -179.997[\deg] \tag{33}$$

instead of

$$x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) = 180\,[\deg] \quad \text{or} \quad -180[\deg].$$

Then, it can be described by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 - \frac{\nu \cdot t}{L} & 0 & 0 \\ \frac{\nu \cdot t}{L} & 1 & 0 \\ \frac{d \cdot \nu^2 \cdot t^2}{2L} & d \cdot \nu \cdot t & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} \frac{\nu \cdot t}{l} \\ 0 \\ 0 \end{bmatrix} u(k) \tag{34}$$

where $d = 10^{-2}/\pi$. Of course, this system is theoretically controllable.

From (31) and (34), a fuzzy model, which approximately represents the dynamics of the truck-trailer, can be derived as follows.

Rule : If $x_2(k) + v \cdot t/\{2L\} \cdot x_1(k)$ is about 0 (rad), then $\mathbf{x}(k+1) = \mathbf{A}_1\mathbf{x}(k) + \mathbf{b}_1 u(k)$

Rule : If $x_2(k) + v \cdot t/\{2L\} \cdot x_1(k)$ is about π (rad) or $-\pi$ (rad), then $\mathbf{x}(k+1) = \mathbf{A}_2\mathbf{x}(k) + \mathbf{b}_2 u(k)$

where $\mathbf{x}(k)^T = [x_1(k), x_2(k), x_3(k)]$. The consequent equations of Rule 1 and Rule 2 correspond to (31) and (34), respectively. After all, the dynamics of the approximated fuzzy model is represented by

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{2} w_i(k)\{\mathbf{A}_i\mathbf{x}(k) + \mathbf{b}_i u(k)\}}{\sum_{i=1}^{2} w_i(k)} \tag{35}$$

where $w_i(k)$ is membership value of the fuzzy set in Rule i. Fig. 15 shows the fuzzy sets of "about 0 (rad)" and "about π (rad) or $-\pi$ (rad)." We define the fuzzy sets as simple triangles. The fuzzy sets are unimodal and normal, however, the dynamics of the approximated fuzzy model does not exactly describe that of the simplified model. Let us show that an exact approximation can be realized by using a fuzzy model with PPU instead of (35).

The simplified model can be generally represented by

$$\mathbf{x}(k+1) = f(\mathbf{x}(k), u(k))$$

where f is a nonlinear function. Then, the condition,

$$\mathbf{A}_1\mathbf{x}(k) + \mathbf{b}_1 u(k) \le f(\mathbf{x}(k), u(k)) \le \mathbf{A}_2\mathbf{x}(k) + \mathbf{b}_2 u(k)$$

is satisfied under

$$179.997^\circ < x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) < -179.997^\circ$$

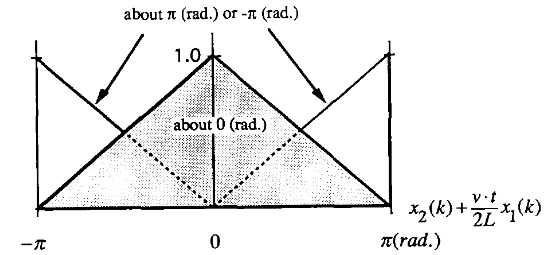

Fig. 15. Fuzzy sets.

because

$$d\left\{x_2(k) + \frac{\nu \cdot t}{2L}x_1(k)\right\} \le \sin\left(x_2(k) + \frac{\nu \cdot t}{2L}x_1(k)\right) \le x_2(k) + \frac{\nu \cdot t}{2L}x_1(k)$$

for

$$179.997^\circ < x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) < -179.997^\circ.$$

This means that the dynamics of the simplified model can be interpolated by two linear systems, $\mathbf{A}_1\mathbf{x}(k) + \mathbf{b}_1 u(k)$ and $\mathbf{A}_2 x(k) + \mathbf{b}_2 u(k)$, under

$$179.997^\circ < x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) < -179.997^\circ.$$

That is, for $^{\exists}\tilde{w}_i(k) \in [0, 1]$ and all k, (36) exactly describes the dynamics of the simplified model.

$$f(\mathbf{x}(k), u(k)) = \frac{\sum_{i=1}^{2} \tilde{w}_i(k)\{\mathbf{A}_i\mathbf{x}(k) + \mathbf{b}_i u(k)\}}{\sum_{i=1}^{2} \tilde{w}_i(k)}. \tag{36}$$

By replacing $\tilde{w}_i(k)$ with $w_i(k) + \Delta w_i(k)$, we can obtain

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{2} (w_i(k) + \Delta w_i(k))\{\mathbf{A}_i\mathbf{x}(k) + \mathbf{b}_i u(k)\}}{\sum_{i=1}^{2} (w_i(k) + \Delta w_i(k))} \tag{37}$$

where $\Delta w_i(k)$ is a value such that

$$-1 \le \Delta w_i(k) \le 1, \quad \text{for all } i$$
$$w_i(k) + \Delta w_i(k) \ge 0, \quad \text{for all } i$$
$$\sum_{i=1}^{r} (w_i(k) + \Delta w_i(k)) > 0$$

for all k. Equation (37), a fuzzy system with PPU, exactly describes the dynamics of the simplified model under

$$179.997^\circ < x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) < -179.997^\circ.$$

B. Controller Design and Robust Stability Analysis

We design a fuzzy controller for backing up the truck-trailer described by (37). The main idea of the controller design is to derive each control rule so as to compensate each rule of a fuzzy system. Fig. 16 shows concept of the controller design.

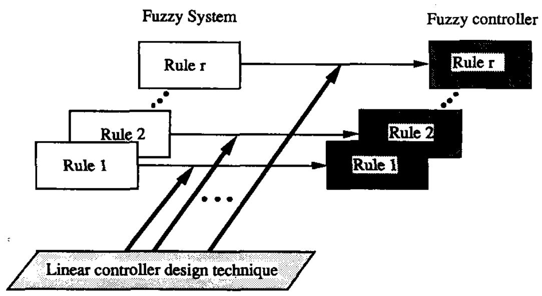

Fig. 16. Concept of controller design.

From Rule 1 and Rule 2 of the approximated fuzzy model, we derive Rule 1 and Rule 2 of fuzzy controller, respectively:

$$\begin{aligned}&\text{Rule}: \quad \text{If } x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) \text{ is about } 0 \text{ (rad)},\\ &\qquad \text{then } u(k) = \mathbf{f}_1\mathbf{x}(k),\\ &\text{Rule}: \quad \text{If } x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) \text{ is about } \pi \text{ (rad)}\\ &\qquad \text{or } -\pi \text{ (rad), then } u(k) = \mathbf{f}_2\mathbf{x}(k),\end{aligned}$$

where $\mathbf{f}_1$ and $\mathbf{f}_2$ are feedback gains. We use the exact same fuzzy sets in the premise part of the fuzzy controller. The purpose of controller design is to determine feedback gains of $\mathbf{f}_1$ and $\mathbf{f}_2$. The following feedback gains are used in the simulation:

$$\begin{aligned}f_{11} &= 1.2837, \quad f_{12} = -0.4139, \quad f_{13} = 0.0201\\ f_{21} &= 0.9773, \quad f_{22} = -0.0709, \quad f_{23} = 0.0005.\end{aligned}$$

Riccati equation for linear discrete systems was used to determine these feedback gains because each consequent part is represented by a linear state equation. The detailed derivation of feedback gains $\mathbf{f}_1$ and $\mathbf{f}_2$ will be given in Appendix B.

Next, we consider robust stability of the backing up control system which consists of the simplified model and the fuzzy controller. From $\mathbf{A}_1, \mathbf{A}_2, \mathbf{b}_1$, and $\mathbf{b}_2$ of the approximated fuzzy model and $\mathbf{f}_1$ and $\mathbf{f}_2$ of the fuzzy controller, we obtain

$$\mathbf{H}_1 = \begin{bmatrix} 0.448 & 0.296 & -0.014 \\ -0.364 & 1 & 0 \\ 0.364 & -2 & 1 \end{bmatrix},$$

$$\mathbf{H}_2 = \begin{bmatrix} 0.666 & 0.051 & -0.383 \times 10^{-3} \\ -0.364 & 1 & 0 \\ 0.364 & -2 & 1 \end{bmatrix},$$

$$\mathbf{H}_3 = \begin{bmatrix} 0.448 & 0.296 & -0.014 \\ -0.364 & 1 & 0 \\ 0.116 \times 10^{-2} & -0.637 \times 10^{-2} & 1 \end{bmatrix},$$

$$\mathbf{H}_4 = \begin{bmatrix} 0.666 & 0.051 & -0.383 \times 10^{-3} \\ -0.364 & 1 & 0 \\ 0.116 \times 10^{-2} & -0.637 \times 10^{-2} & 1 \end{bmatrix}.$$

We can find the AR of $\Delta w_i(k)$'s which satisfies the WRSC of (16) if we select

$$\mathbf{P} = \begin{bmatrix} 113.9 & -92.61 & 2.540 \\ -92.61 & 110.7 & -3.038 \\ 2.540 & -3.038 & 0.5503 \end{bmatrix}$$

as a common positive definite matrix $\mathbf{P}$. This matrix $\mathbf{P}$ was selected by the construction procedure of the literature [12].

By substituting $\mathbf{H}_1 \sim \mathbf{H}_4, \mathbf{P}$ and

$$\begin{aligned}v_1(k) &= (w_1(k) + \Delta w_1(k))w_1(k),\\ v_2(k) &= (w_1(k) + \Delta w_1(k))w_2(k),\\ v_3(k) &= (w_2(k) + \Delta w_2(k))w_1(k),\\ v_4(k) &= (w_2(k) + \Delta w_2(k))w_2(k)\end{aligned}$$

into (16) by eliminating $w_2(k)$ and $\Delta w_2(k)$, we can find an AR of $\Delta w_1(k)$ which satisfies the WRSC of (16). Fig. 25 shows the AR.

Next, we find a VR by comparing the simplified model described by (28), (25), and (29) with the approximated fuzzy model with PPU described by (37). Now, let us assume that

$$p(k) \equiv x_2(k) + \frac{\nu \cdot t}{2L} x_1(k). \tag{38}$$

It is sufficient to find a VR for

$$0^\circ \leq p(k) \leq 180^\circ$$

because each membership function is symmetric with respect to the ordinate ($p(k) = 0$) as shown in Fig. 15. Moreover, $\sin(p(k))$, is also symmetric with respect to the ordinate ($p(k) = 0$). In other words, the same VR is obtained also for $p(k) < 0$. The membership function of the fuzzy set $\mathcal{A}_1$ defined in Fig. 15 is represented by

$$w_1(k) = -\frac{1}{\pi}p(k) + 1 \tag{39}$$

when

$$0^\circ \leq p(k) \leq 180^\circ. \tag{40}$$

From (30), we can derive the following equation.

$$(w_1(k) + \Delta w_1(k)) \cdot p(k) = \sin p(k). \tag{41}$$

By solving the above equation for $\Delta w_1(k)$, we obtain

$$\Delta w_1(k) = \frac{\sin p(k)}{p(k)} - w_1(k). \tag{42}$$

On the other hand, by solving (39) for $p(k)$,

$$p(k) = \pi(1 - w_1(k)). \tag{43}$$

By substituting (43) into (42), we obtain

$$\Delta w_1(k) = -w_1(k) + \frac{\sin(\pi \cdot \{1 - w_1(k)\})}{\pi \cdot \{1 - w_1(k)\}}. \tag{44}$$

VR is a region which satisfies (44) for $0 \leq w_1(k) < 1$ and $-1 \leq \Delta w_1(k) \leq 1$. Fig. 26 shows the VR. The VR (the line) perfectly belongs to the AR when

$$0.0484 < w_1(k). \tag{45}$$

From (38) and (43), (45) corresponds to

$$x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) < 2.99 \text{ [rad]} = 171.29\text{[deg]}.$$

Therefore, the control system at least satisfies the WRSC when

$$-179.29\text{[deg]} < x_2(k) + v \cdot t/\{2L\} \cdot x_1(k) < 171.29\text{[deg]} \tag{46}$$

for all k. In other words, under the condition of (46), the designed fuzzy controller guarantees stability of the control

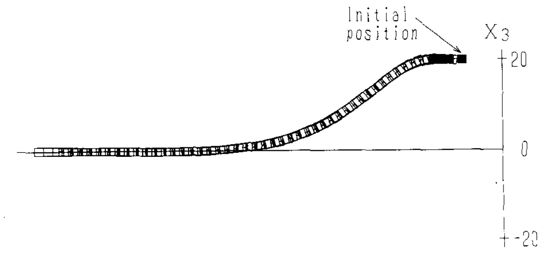

Fig. 17. Control result for Case 1 (simplified model).

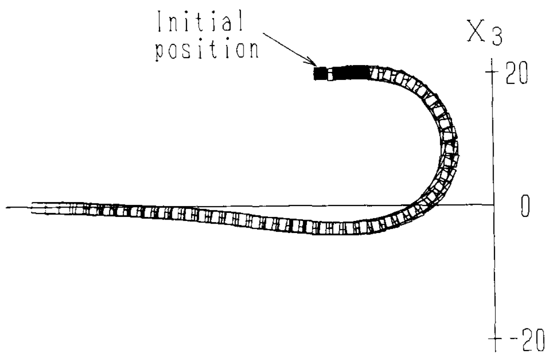

Fig. 18. Control result for Case 2 (simplified model).

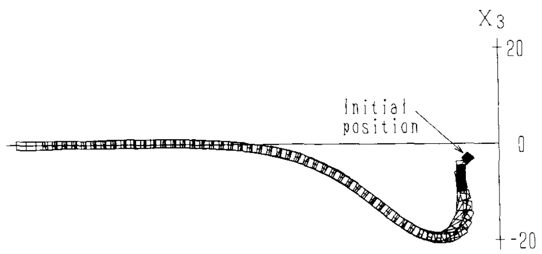

Fig. 19. Control result for Case 3 (simplified model).

system which consists of the simplified model and the fuzzy controller, that is, the backing up control can be perfectly achieved from all initial positions.

The line of VR is close to the straight line of $\Delta w_1(k) = 0$ in the all range of $w_1(k)$. This means that the dynamics of the approximated fuzzy system agree well that of the simplified model, that is, that a fairly good approximation is realized.

C. Simulation Results

In this simulation, we use two kinds of the controlled objects: the Ichihashi's model and the simplified model. Table I shows four cases of initial positions used in this simulation. Case 2 and Case 4 require to turn the truck-trailer in order to realize a perfect backing up control. In particular, Case 4 is a jackknife position.

Figs. 17–20 shows simulation results for four cases of initial positions in the simplified model. Figs. 21–24 show simulation results for four cases of initial positions in the Ichihashi's model. The following points can be pointed out from the simulation results.

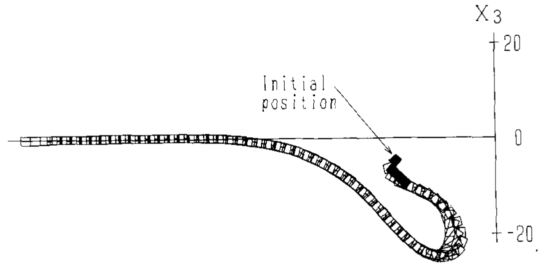

Fig. 20. Control result for Case 4 (simplified model).

TABLE I
INITIAL POSITIONS OF TRUCK-TRAILER

	$x_1(0)$ [deg]	$x_2(0)$ [deg]	$x_3(0)$ [m]
Case 1	0	0	20
Case 2	0	180	20
Case 3	−45	90	−10
Case 4	−90	135	−10

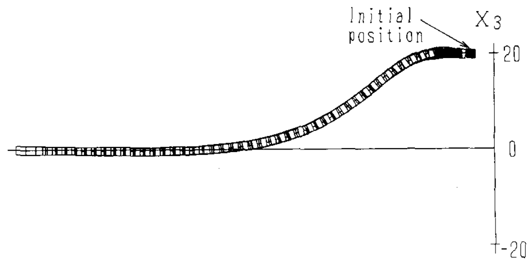

Fig. 21. Control result for Case 1 (Inchihashi's model).

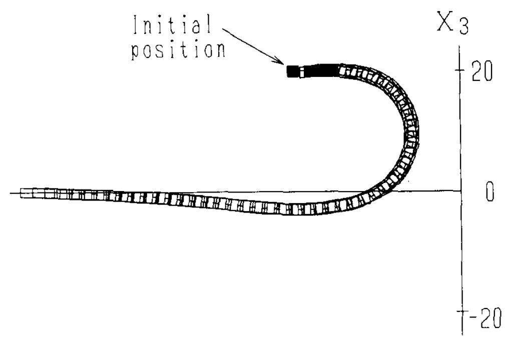

Fig. 22. Control result for Case 2 (Inchihashi's model).

1) The designed fuzzy controller can effectively achieve backing up control even from difficult initial positions such as Cases 2 and 4 in both of the Ichihashi's model and the simplified model.
2) The dynamics of the simplified model agree well with that of the Ichihashi's model.

Nobody can deny the first point, because the control system is asymptotically stable in the large under the condition of (46). The second point shows that a fairly good approximation is realized.

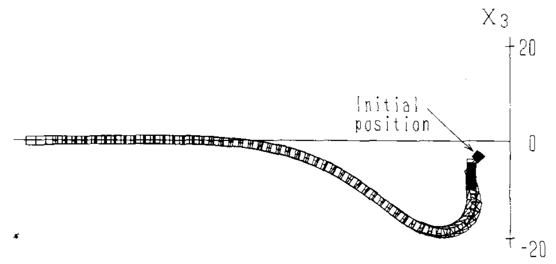

Fig. 23. Control result for Case 3 (Inchihashi's model).

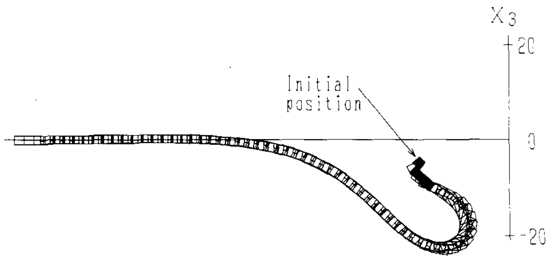

Fig. 24. Control result for Case 4 (Inchihashi's model).

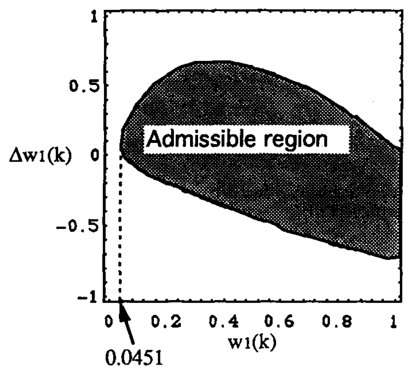

Fig. 25. Admissible region of the truck-trailer system.

V. Conclusion

A robust stabilization problem for fuzzy systems has been discussed in accordance with the definition of stability in the sense of Lyapunov. We have considered two design problems: nonrobust controller design and robust controller design. The former is a design problem for fuzzy systems with no premise parameter uncertainty. The latter is a design problem for fuzzy systems with premise parameter uncertainty. To realize two design problems, we have derived four stability conditions from a basic stability condition proposed by Tanaka and Sugeno: nonrobust condition, weak nonrobust condition, robust condition and weak robust condition. We have introduced concept of robust stability for fuzzy control systems with premise parameter uncertainty from the weak robust condition. To introduce robust stability, admissible region and variation region, which correspond to stability margin in the ordinary control theory, have been defined. Furthermore, we have developed a control system for backing up a computer simulated truck-trailer which is nonlinear and unstable. By approximating the truck-trailer by a fuzzy system with premise parameter uncertainty and by using concept of robust stability, we have designed a fuzzy controller which guarantees stability of the control system under a condition. The simulation results show that the designed fuzzy controller smoothly achieves backing up control of the truck-trailer from all initial positions.

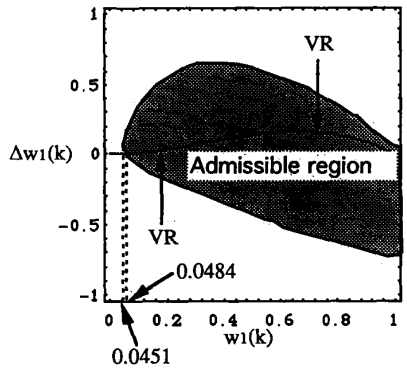

Fig. 26. Variation region of the truck-trailer system.

Appendix A
The Proof of Theorem 2.1

A lemma is necessary in order to prove the condition of Theorem 2.1. The proof of the lemma is given in the literature [3].

Lemma [3] If $\mathbf{P}$ is a positive definite matrix such that

$$\mathbf{A}^T\mathbf{PA} - \mathbf{P} < 0 \quad \text{and} \quad \mathbf{B}^T\mathbf{PB} - \mathbf{P} < 0$$

then

$$\mathbf{A}^T\mathbf{PB} + \mathbf{B}^T\mathbf{PA} - 2\mathbf{P} < 0.$$

The proof of Theorem 2.1 Consider the scale function $V(\mathbf{x}(k))$ such that

$$V(x(k)) = \mathbf{x}^T(k)\mathbf{P}\mathbf{x}(k)$$

where $\mathbf{P}$ is a positive definite matrix. This function satisfies the following properties:

a) $V(\mathbf{0}) = 0$
b) $V(\mathbf{x}(k)) > 0 \quad \text{for} \quad \mathbf{x}(k) \neq 0$
c) $V(\mathbf{x}(k))$ approaches infinity as $||\mathbf{x}(k)|| \to \infty$.

Next,

$$\begin{aligned}
&\Delta V(\mathbf{x}(k))\\
&= V(\mathbf{x}(k+1)) - V(\mathbf{x}(k))\\
&= \mathbf{x}^T(k+1)\mathbf{P}\mathbf{x}(k+1) - \mathbf{x}^T(k)\mathbf{P}\mathbf{x}(k)\\
&= \left(\frac{\sum_{i=1}^{r} w_i(k)\mathbf{A}_i\mathbf{x}(k)}{\sum_{i=1}^{r} w_i(k)}\right)^T \mathbf{P} \left(\frac{\sum_{i=1}^{r} w_i(k)\mathbf{A}_i\mathbf{x}(k)}{\sum_{i=1}^{r} w_i(k)}\right)\\
&\quad - \mathbf{x}^T(k)\mathbf{P}\mathbf{x}(k)\\
&= \frac{\sum_{i,j=1}^{r} w_i(k)w_j(k)\mathbf{x}^T(k)\{\mathbf{A}_i^T\mathbf{P}\mathbf{A}_j - \mathbf{P}\}\mathbf{x}(k)}{\sum_{i,j=1}^{r} w_i(k)w_j(k)}\\
&= \left[\sum_{i=1}^{r} w_i(k)w_i(k)\mathbf{x}^T(k)\{\mathbf{A}_i^T\mathbf{P}\mathbf{A}_i - \mathbf{P}\}\mathbf{x}(k)\right.\\
&\quad + \sum_{i<j}^{r} w_i(k)w_j(k)\mathbf{x}^T(k)\{\mathbf{A}_i^T\mathbf{P}\mathbf{A}_j + \mathbf{A}_j^T\mathbf{P}\mathbf{A}_i\\
&\quad \left. - 2\mathbf{P}\}\mathbf{x}(k)\right] \Big/ \sum_{i,j=1}^{r} w_i(k)w_j(k)
\end{aligned}$$

where $w_i(k) \geq 0$ for $i \in \{1, 2, \cdots, r\}$ and $\Sigma_{i=1}^{r} w_i(k) > 0$. From the above Lemma and (4), we obtain

$$\text{d)} \quad \Delta V(\mathbf{x}(k)) < 0.$$

From (a)–(d), $V(\mathbf{x}(k))$ is a Lyapunov function and the fuzzy system (2) is asymptotically stable in the large.

Appendix B
The Derivation of Feedback Gains

By solving the Riccati equation,

$$\mathbf{P} = \mathbf{A}^T\mathbf{P}\mathbf{A} + \mathbf{Q} - \mathbf{A}^T\mathbf{P}\mathbf{b}(r + \mathbf{b}^T\mathbf{P}\mathbf{b})^{-1}\mathbf{b}^T\mathbf{P}\mathbf{A}$$

for $\mathbf{P}$, we can obtain the optimal feedback control law

$$u(t) = -(r + \mathbf{b}^T\mathbf{P}\mathbf{b})^{-1}\mathbf{b}^T\mathbf{P}\mathbf{A}\mathbf{x}(t) = \mathbf{f}\mathbf{x}(t)$$

such that the quadratic performance function

$$J = \sum_{k=0}^{\infty}(\mathbf{x}(k)^T\mathbf{Q}\mathbf{x}(k) + u(k)^T r u(k))$$

is minimized, where $\mathbf{Q}$ and r are a weighting matrix and a weighting value, that is,

$$\mathbf{Q} = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \quad q_1 > 0, q_2 > 0, \quad \text{and} \quad q_3 > 0$$

$r > 0$.

In the case of the truck-trailer model, r is a scalar since it is a single input system. $(\mathbf{A}, \mathbf{b})$ must be at least controllable in order that the Riccati equation can be solved for $\mathbf{P}$. By solving

$$\mathbf{P}_1 = \mathbf{A}_1^T\mathbf{P}_1\mathbf{A}_1 + \mathbf{Q} - \mathbf{A}_1^T\mathbf{P}_1\mathbf{b}_1(r + \mathbf{b}_1^T\mathbf{P}_1\mathbf{b}_1)^{-1}\mathbf{b}_1^T\mathbf{P}_1\mathbf{A}_1$$

for $\mathbf{P}_1$ and by substituting the $\mathbf{P}_1$ into

$$\mathbf{f}_1 = -(r + \mathbf{b}_1^T\mathbf{P}_1\mathbf{b}_1)^{-1}\mathbf{b}_1^T\mathbf{P}_1\mathbf{A}_1$$

where $\mathbf{f}_1 = [f_{11}, f_{12}, f_{13}], q_1 = 10, q_2 = 1, q_3 = 1$, and $r = 1000$, we obtain

$$f_{11} = 1.2837, \quad f_{12} = -0.4139, \quad f_{13} = 0.0201.$$

Using the same manner, we can obtain the feedback gains, $\mathbf{f}_2$;

$$f_{21} = 0.9773, \quad f_{22} = -0.0709, \quad f_{23} = 0.0005$$

where $q_1 = 10, q_2 = 10, q_3 = 0.01$, and $r = 1000$.

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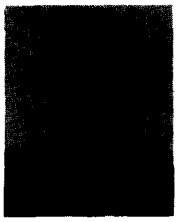

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