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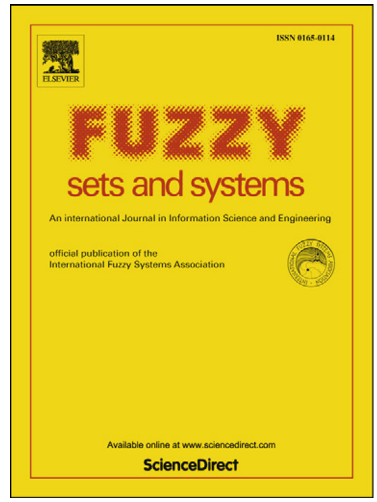

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Indirect adaptive fuzzy control for a nonholonomic/underactuated wheeled inverted pendulum vehicle based on a data-driven trajectory planner

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Abstract

In this study, we investigate an error data-based trajectory planner and indirect adaptive fuzzy control for a class of wheeled inverted pendulum vehicle systems. Based on the error dynamics, the closed-loop trajectory planner can generate the desired velocity values. Using the virtual acceleration input for the tilt angle subsystem, composite control for the rotational and longitudinal subsystems can be constructed via indirect adaptive fuzzy and sliding mode control approaches to achieve simultaneous velocity tracking and tilt angle stabilization. We rigorously prove the system stability and convergence of the tracking error signals using the Lyapunov theory and LaSalle's invariance theorem. The results of our numerical simulations demonstrated the efficiency of the proposed control strategies and the implementations of the algorithms.

Keywords: adaptive mechanism, data-driven, indirect fuzzy control, nonholonomic/underactuated, wheeled inverted pendulum.

1. Introduction

A wheeled inverted pendulum (WIP) vehicle possesses only two active driving wheels but it can implement nearly every mode of motion, such as advancing, steering, and stopping. Over the past few years, there has been extensive interest in the development of these types of vehicles (e.g., [1, 2, 3, 4]). Due to their advantageous features, such as compact construction, convenient operation, high maneuverability, and low fuel consumption, WIP vehicles can be employed in emerging applications in commercial, civilian, and military areas. However, the efficient control of the WIP platform is a challenging problem, mainly because: (*i*) it is difficult to use current modeling techniques to obtain an accurate vehicle dynamic model, which is generally highly nonlinear, time-varying, and coupled in nature; (*ii*) the system suffers from a nonholonomic constraint on the kinematic level, which traditional control methods are unable to use directly; and (*iii*) the dynamics of the suspension possess obvious underactuated characteristics, which means that the number of control inputs is less than the number of degree of freedom. All of these difficult issues are hindrances to the control community [5, 6, 7].

Many previous studies have focused on control methods to implement trajectory tracking for WIP vehicles, such as conventional proportional-integral-derivative (PID) control, feedback stabilization, sliding mode schemes, and intelligent control. It is well known that PID control is a simple and easy approach but due to the complexities of WIP vehicles, it is a difficult to select three appropriate gains [8, 9]. Thus, to improve the system control performance, feedback stabilization based on an approximate linear model is widely employed. For example, in [10], a two-level controller was proposed for tracking and stabilizing the vehicle's posture based on a partial linear model where the internal dynamics were isolated. Similar methods were also employed by [11, 12]. In addition, the variable structure method is another powerful approach for controlling WIP vehicles. For instance, Yue et al. employed a sliding mode scheme to effectively control a two-wheeled vehicle with a dropping pendulum-like suspension, where the mobile platform was fairly similar to that in a WIP vehicle [13]. However, most of these controllers rely on the system model information, which can rarely be obtained in practice. It should also be mentioned that many advanced modeling and identification techniques can be used for model-based control design (e.g., [14, 15, 16]), but it is still difficult to obtain an explicit description of the vehicle in real time. Intelligent control, including the use of fuzzy control and neural networks (NNs), has been shown to be a powerful technique for control design when dealing with complex dynamical systems (e.g., [17, 18, 19, 20]). In particular, Li et al. developed an output feedback adaptive NN controller with a linear dynamic compensator to achieve stable dynamic balance of the vehicle body, while tracking of the desired trajectories could also be achieved at the same time [21, 22]. Moreover, in [23], an adaptive fuzzy logic control system was explored based on updated laws for a WIP vehicle and the control performance was improved. Based on these studies, it is clear that intelligent control is an effective tool for controlling WIP vehicles, which have complex dynamics and nonholonomic/underactuated behaviors.

Furthermore, compared with the aforementioned control approaches, fuzzy control can usually achieve better control performance because it does not depend on an exact mathematical model and it can effectively synthesize the successful experiences of operators using linguistic instructions [24, 25, 26, 27, 28]. In general, fuzzy control comprises indirect fuzzy control and direct fuzzy control. In contrast to direct fuzzy control, which is based on the difference between the performance of the actual system and the ideal performance level, indirect fuzzy control applies the universal approximation theorem to obtain system models of the controlled object, which is appropriate for merging the other control techniques that comprise a composite controller in an entire underactuated system [29, 30]. Surprisingly, direct fuzzy control has been used widely to control WIP vehicles (e.g., [31, 32, 33]), but few studies have employed the indirect fuzzy scheme to control this type of wheeled mobile platform.

In addition to the underactuated behaviors of WIP vehicles, the nonholonomic constraint is another unavoidable problem when a vehicle tracks the reference trajectories. If the wheeled mobile platform moves while the wheel-terrain satisfies the pure rolling condition, then the constraint cannot be integrated into a linear form [34, 35]. According to Brockett's necessary condition for asymptotic stabilization, no continuous time-invariant state-feedback controller exists that can asymptotically stabilize the mobile platform [36]. For a WIP vehicle, the combination of underactuated suspension and the nonholonomic constraint represent a challenging task during control system synthesis. With the exception of a few previous studies, the tracking objective has focused on tracking the longitudinal and rotational velocities, whereas tracking of the position trajectory was described in the Cartesian frame. Therefore, simultaneously addressing the underactuated and nonholonomic problems for WIP vehicles is still an open problem, and thus the motivation of the present study.

In the present study, we propose a novel data-driven closed-loop trajectory planner that differs from the traditional path planning, which solves the nonholonomic constraint and underactuated control issues on kinematic and dynamic levels, respectively. With respect to the trajectory planner, we conceive a hypothesized vehicle with the desired posture values and we then establish the error dynamics based on the posture tracking error data so we can apply Lyapunov stability theory to guarantee the stability of the closed-loop planner. It should be noted that the closed-loop trajectory planner is similar to a close-loop system based on the vehicle kinematics, where the planner's outputs are regarded as the desired longitudinal and rotational velocities for the vehicle's dynamic system. This treatment allows the data-driven trajectory planner to generate the desired values in real time, where the closed-loop feedback characteristics can enhance the robustness of the planner system against unavoidable parameter variation and unpredictable external disturbances. In addition, we propose composite control by introducing a virtual control method to address the underactuated problem. To explore the underactuated behaviors, the overall system is divided into three subsystems, i.e., rotational velocity, longitudinal velocity, and tilt angle subsystems, where the latter two subsystems are considered together because they are controlled by only one input. In our further analysis, the acceleration of the mid-point of the vehicle axis is assumed to be a virtual control input that allows the transformation of the underactuated behaviors into actuated behaviors. Therefore, we present a composite controller that combines indirect fuzzy control and the variable structure technique, which can guarantee the simultaneous tracking of the desired longitudinal velocity as well as stabilizing the tilt angle of the suspension.

In summary, the main contributions of the present study are as follows.

1) A dynamic model is proposed via the Lagrangian methodology, which can clearly describe the coupling effects, time-varying behaviors, and nonlinear characteristics of WIP vehicles.

2) Based on the tracking errors, a closed-loop trajectory planner is presented, which can resolve the nonholonomic constraint and underactuated problems on the kinematic and dynamic levels, respectively.

3) A composite controller is constructed according to indirect fuzzy control and the variable structure technique, where the concept of virtual acceleration input is proposed to allow the simultaneous tracking of velocities and stabilization of the tilt angle.

The remainder of this paper is organized as follows. In Section 2, we introduce the mathematic model of a WIP vehicle and we state the problem formulation. In Section 3, we present the error data-driven theory and we describe the closed-loop trajectory planner system. The indirect adaptive fuzzy control method is explained in Section 4 and the procedures involved in the design process are also described in detail. In Section 5, we consider the specific design methods for the three main subsystems and the results of numerical simulations are presented that verify the effectiveness of the proposed control strategy. In Section 7, we give our conclusions.

2. Problem formulation and preliminaries

In this section, we describe the kinematics and dynamics of a WIP vehicle, as well as presenting some preliminary details and formulations. The vehicle parameters and variables used to describe the vehicle system are shown in Fig.1 and Table 1.

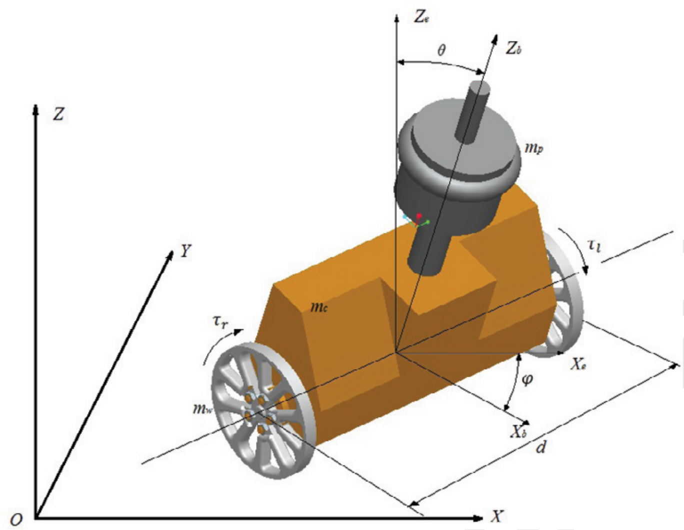

Figure 1. Schematic diagram showing the vehicle's characteristic parameters.

Table 1. Key parameters related to a WIP vehicle

Notation	Definition
$x,\ y$	Current position of the vehicle on the XOY plane
$\theta_l,\ \theta_r$	Rotational angles of the left and right wheels
θ	Tilt angle of the vehicle body
φ	Rotational angle of the vehicle
υ	Longitudinal velocity of the vehicle
ω	Rotational velocity of the vehicle
m_p	Mass of the inverted pendulum
m_c	Mass of the chassis
m_w	Mass of each wheel
J_v	Moment of inertia for the chassis and pendulum about the Z-axis
J_c	Moment of inertia for the chassis about the Y-axis
J_w	Moment of inertia for the wheel with respect to the Y-axis
r	Radius of the wheels
l	Distance from the body center of gravity to the wheel axis
g	Acceleration due to gravity
d	Distance between the two wheels along the axle center

2.1. General underactuated system

The dynamics of the WIP vehicle with an underactuated body subject to a nonholonomic constraint can be formulated by

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) = B(q)\tau + A^T\lambda, \tag{1}$$

where $q = [q_1\ q_2\ q_3\ q_4]^T = [x\ y\ \varphi\ \theta]^T \in \mathbb{R}^4$ is the vector of the generalized coordinates, the elements of which are independent, $M(q) \in \mathbb{R}^{4\times4}$ denotes the inertia matrix, $V_m(q,\dot{q})\dot{q} \in \mathbb{R}^4$ is the matrix of the Coriolis and centrifugal forces, $G(q) \in \mathbb{R}^4$ is the vector of gravitational forces, $B(q) \in \mathbb{R}^{4\times2}$ is the matrix relative to the control efforts, $\tau \in \mathbb{R}^2$ is the vector of control inputs, $A^T\lambda \in \mathbb{R}^4$ is the vector of constraint forces, where $A^T \in \mathbb{R}^4$ is a Jacobian matrix, and $\lambda \in \mathbb{R}$ comprise Lagrangian multipliers that correspond to the system constraints.

Furthermore, if this is an incomplete controllable system, then the entire system can be divided into two parts: actuated and underactuated subsystems. We define $q = [q_a\ q_u]^T$, where q_a and q_u denote the actuated and underactuated state variables, and the system (1) can be rewritten as

$$M(q) = \begin{bmatrix} M_a & M_{au} \\ M_{ua} & M_u \end{bmatrix};\ \ V_m(q,\dot{q}) = \begin{bmatrix} V_a & V_{au} \\ V_{ua} & V_u \end{bmatrix};\ \ G(q) = \begin{bmatrix} G_a \\ G_u \end{bmatrix};\ \ B(q) = \begin{bmatrix} B_a & B_u \\ 0 & 0 \end{bmatrix};\ \ \tau = \begin{bmatrix} \tau_a \\ 0 \end{bmatrix},$$

where M_a and M_u are the inertia submatrices relative to the actuated and underactuated parts, respectively. M_{au} and M_{ua} are the coupling inertia matrices of the two subsystems. Similarly, V_a and V_u denote the centripetal and Coriolis submatrices, and V_{au} and V_{ua} are related to their coupling effects. G_a and G_u are the corresponding gravitational torque terms for the actuated and underactuated subsystems, respectively. τ_a is the control input vector for the actuated subsystem [37, 38].

2.2. Reduced dynamics

In terms of the actuated subsystem, the system constraints, including the nonholonomic constraints, can be described by

$$S_a^T(q)\dot{q}_a = 0, \tag{2}$$

where S_a is the kinematic constraint matrix related to the system constraints. The effect of the constraints can be taken as a restriction of the dynamics on the manifold Ω_n as follows.

$$\Omega_n \triangleq \{(q_a, \dot{q}_a) | S_a^T \dot{q}_a = 0\} \tag{3}$$

Assume that the annihilator of the co-distribution spanned by the covector fields S_a is a smooth nonsingular distribution Δ. This distribution Δ is spanned by a set of smooth and linearly independent vector fields $A_1(q)$ and $A_2(q)$, i.e., $\Delta \triangleq \mathrm{span}\{A_1(q), A_2(q)\}$, for which the local coordinates satisfy the following relation:

$$S_a^T A^T = 0, \tag{4}$$

where $A \triangleq [A_1(q), A_2(q)]$, and we note that $A^T A$ is of full rank. Constraint (2) implies the existence of the vector ν such that

$$\dot{q}_a = S_a(q)\nu. \tag{5}$$

Moreover, let $z = [\nu\ \dot{q}_u]^T$ be a new coordinate of the system. By differentiating (5) with respect to time, substituting the results into the system (1), and multiplying $S(q) = \mathrm{diag}[S_a^T\ I]$ on both sides of the equation to eliminate the kinematic nonholonomic constraint $A^T\lambda$, we obtain the reduced order dynamics of the WIP vehicle as follows:

$$\bar{M}(z)\dot{z} + \bar{V}_m(z,\dot{z})z + \bar{G}(z) = \bar{B}(z)\tau, \tag{6}$$

where

$$\bar{M}(z) = \begin{bmatrix} S_a^T M_a S_a & S_a^T M_{au} \\ M_{ua} S_a & M_u \end{bmatrix};\ \ \bar{V}_m(z,\dot{z}) = \begin{bmatrix} S_a^T M_a \dot{S}_a + S_a^T V_a S_a & S_a^T V_{au} \\ M_{ua}\dot{S}_a + V_{ua} S_a & V_u \end{bmatrix};$$

$$\bar{G}(z) = \begin{bmatrix} S_a^T G_a \\ G_u \end{bmatrix};\ \ \bar{B}(z) = \begin{bmatrix} S_a^T B_a \\ 0 \end{bmatrix}.$$

2.3. Dynamics of the WIP vehicle

The facts that the stated variables for the WIP vehicle are coupled together and they exhibit strong nonlinear characteristics make it difficult to establish a dynamic model of this type on a mobile platform. In this subsection, using the Lagrangian-Eulerian method, we derive a dynamic model for to allow the synthesis of the system.

Assumption 1. *Sufficient friction between the wheels of the mobile platform and the ground can be guaranteed such that the assumption of nonholonomic constraints holds.*

Define $q = [q_a\ q_u]^T$, where $q_a = [x\ y\ \varphi]^T$ and $q_u = \theta$. If the pure rolling constraint is satisfied, then the velocities of the left and right wheels are governed by $\dot{\theta}_r = \frac{\upsilon}{r} + \frac{\omega d}{2r}$ and $\dot{\theta}_l = \frac{\upsilon}{r} - \frac{\omega d}{2r}$. The system energy can be divided into three parts: the kinetic energies for the wheels T_w, the vehicle body T_c, and the pendulum T_p, which are calculated, respectively, by

$$T_w = \frac{1}{2}J_w(\dot\theta_l^2 + \dot\theta_r^2) + \frac{1}{2}m_w(\upsilon_l^2 + \upsilon_r^2) = J_w\left(\frac{\upsilon}{r}\right)^2 + m_w\upsilon^2 + \frac{1}{4}\left(m_w + \frac{J_w}{r^2}\right)d^2\omega^2 \quad (7)$$

$$T_c = \frac{1}{2}m_c\upsilon^2 + \frac{1}{2}J_c\dot\theta^2 + \frac{1}{2}J_v\dot\varphi^2 \quad (8)$$

$$T_p = \frac{1}{2}m_p(\upsilon + l\dot\theta\cos\theta)^2 + \frac{1}{2}m_p(-l\dot\theta\sin\theta)^2 + \frac{1}{2}m_p(l\sin\theta)^2\omega^2. \quad (9)$$

In addition, the total potential energy is

$$U = -m_p gl(1-\cos\theta). \quad (10)$$

Therefore, the Lagrangian multiplier can be obtained by

$$\begin{aligned} L &= T_w + T_c + T_p - U \\ &= \frac{1}{2}J_c\dot\theta^2 + \frac{1}{2}m_p(\dot x\cos\varphi + \dot y\sin\varphi + l\dot\theta\cos\theta)^2 + \frac{1}{2}\left(m_c + 2m_w + \frac{2J_w}{r^2}\right)(\dot x\cos\varphi + \dot y\sin\varphi)^2 \\ &\quad + \frac{1}{4}\left(m_w + \frac{J_w}{r^2}\right)d^2\dot\varphi^2 + \frac{1}{2}m_p l^2(\dot\theta^2 + \dot\varphi^2)\sin^2\theta + \frac{1}{2}J_v\dot\varphi^2 + m_p gl(1-\cos\theta). \end{aligned} \quad (11)$$

Using the generalized coordinates as $q = [q_1\ q_2\ q_3\ q_4]^T = [x\ y\ \varphi\ \theta]^T$, a dynamic model of the WIP vehicle can be obtained using Lagrange's dynamical equation as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot q_i}\right) - \frac{\partial L}{\partial q_i} = u_i, \quad (i = 1, \ldots, 4), \quad (12)$$

where u_i represents the generalized force with respect to the corresponding general coordinate q_i. Furthermore, system (1) can be transformed into the reduced dynamics (6), where the transfer matrix between q and $z = [\omega\ \upsilon\ \dot\theta]^T$ can be formulated as follows.

$$S(q) = \begin{bmatrix} S_a(q) & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} \cos\varphi & 0 & 0 \\ \sin\varphi & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (13)$$

Therefore, the nonholonomic constraint can be eliminated from the dynamic model (1) since $S^TA^T = 0$. Using the aforementioned computational procedures, a reduced order dynamics with the same form as (6) can ultimately be obtained, where the specific matrices are as follows:

$$\bar M(q) = \begin{bmatrix} m_v & 0 & m_p l\cos\theta \\ 0 & J_\varphi + m_p l^2\sin^2\theta & 0 \\ m_p l\cos\theta & 0 & J_c + m_p l^2 \end{bmatrix}; \quad \bar B = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ -d & d \\ 0 & 0 \end{bmatrix}; \quad u_a = \begin{bmatrix} \tau_l \\ \tau_r \end{bmatrix};$$

$$\bar V_m(q,\dot q) = \begin{bmatrix} 0 & 0 & -m_p l\dot\theta\sin\theta \\ 0 & -\frac{1}{2}m_p l^2\dot\theta\sin 2\theta & -\frac{1}{2}m_p l^2\dot\varphi\sin 2\theta \\ 0 & -\frac{1}{2}m_p l^2\dot\varphi\sin 2\theta & 0 \end{bmatrix}; \quad G(q) = \begin{bmatrix} 0 \\ 0 \\ -m_p gl\sin\theta \end{bmatrix},$$

where $J_\varphi = J_v + \frac{1}{2}(m_w + \frac{J_w}{r^2})d^2$ and $m_v = m_p + m_c + 2m_w + \frac{2J_w}{r^2}$, both of which are introduced for simplicity. It is interesting to note that the nonholonomic constraint has been eliminated from the system.

Property 1. *The inertia matrix $\bar M$ is symmetric and positive definite.*

Property 2. *The matrix $\dot{\bar M} - 2\bar V_m$ is skew-symmetric.*

2.4. Three subsystems

Note that the longitudinal and rotational movements of the vehicle are driven by the summation and difference of the left and right wheel torques. Hence, we define the new control inputs as follows.

$$\tau_\upsilon = \tau_l + \tau_r \quad (14)$$

$$\tau_\omega = -\tau_l + \tau_r \quad (15)$$

According to (6), the vehicle dynamics are related to four variables: the vehicle rotational velocity ω, longitudinal velocity υ, the tilt angular θ, and its angular velocity $\dot{\theta}$. For the purposes of control, the reduced order system (6) can be partitioned into three subsystems as follows:

1) ω-subsystem:

$$\begin{aligned}\dot{\omega} &= f_{\omega}(\theta, \dot{\theta}, \dot{\varphi}) + g_{\omega}(\theta) \\ &= -\frac{m_p l^2 \dot{\varphi}\dot{\theta}\sin 2\theta}{J_{\varphi} + m_p l^2 \sin^2\theta} + \frac{d}{r(J_{\varphi} + m_p l^2 \sin^2\theta)}\tau_{\omega},\end{aligned} \tag{16}$$

2) υ-subsystem:

$$\begin{aligned}\dot{\upsilon} &= f_{\upsilon}(\theta, \dot{\theta}, \dot{\varphi}) + g_{\upsilon}(\theta) \\ &= \frac{1}{\Omega}\left(J_{\theta} m_p l \dot{\theta}^2 \sin\theta - m_p^2 l^3 \dot{\varphi}^2 \sin\theta\cos^2\theta - m_p^2 l^2 g \sin\theta\cos\theta\right) + \frac{J_{\theta}}{\Omega r}\tau_{\upsilon},\end{aligned} \tag{17}$$

3) θ-subsystem:

$$\begin{aligned}\ddot{\theta} &= f_{\theta}(\theta, \dot{\theta}, \dot{\varphi}) + g_{\theta}(\theta) \\ &= \frac{1}{2\Omega}\left(2m_v m_p g l \sin\theta + m_v m_p l^2 \dot{\varphi}^2 \sin 2\theta - m_p^2 l^2 \dot{\theta}^2 \sin 2\theta\right) - \frac{m_p l \cos\theta}{\Omega r}\tau_{\upsilon},\end{aligned} \tag{18}$$

where $J_{\theta} = J_c + m_p l^2$ and $\Omega = m_v J_{\theta} - m_p^2 l^2 \cos^2\theta$. Thus, we conclude that $J_{\varphi} + m_p l^2 \sin^2\theta > 0$, and $\Omega = m_v J_{\theta} - m_p^2 l^2 \cos^2\theta$ is also positive for all $\theta \in R$. Therefore, for a practical mechanical system, the three subsystems (16–18) have no singular point globally.

3. Dynamic trajectory planning

3.1. Error data-driven planner

The three subsystems are established essentially on the level of system dynamics, but in practice, the desired trajectories are usually given on Cartesian coordinates. Therefore, it is always a requirement that in addition to holding the vehicle body around the upright position, the actual configuration center of the WIP vehicle described by x, y, and φ should converge rapidly to the reference trajectory with the desired values of x_d, y_d, and φ_d, respectively. Thus, the trajectory tracking control objective for the WIP vehicle with the reduced order system can be summarized as: find the control inputs τ_{ω} and τ_{υ} such that $\omega \to \omega_d$ and $\upsilon \to \upsilon_d$ as $t \to \infty$, as well as the vehicle body that holds an upright position with the unstable equilibrium, i.e., $\theta \to 0$ as $t \to \infty$.

Assumption 2. *All of the desired trajectories, including ω_d and υ_d, and their time derivatives up to the third order are continuously differentiable and bounded for all $t \geq 0$. For the desired tilt angle of the WIP vehicle, it is expected that $\theta_d = 0$ and $\dot{\theta}_d = 0$.*

We propose a trajectory planner, which can be used to generate the desired signals for the inner dynamic systems, while the posture errors will converge to zero within a finite time. It should be noted that from another perspective, this planner can be taken as a closed-loop control system. The trajectory planner is illustrated in Fig.2.

Let $\bar{q}(t) = [x\ y\ \varphi]^T$ be a vector in the set of the actual vehicle's posture information and define $\bar{q}_d(t) = [x_d\ y_d\ \varphi_d]^T$ as the vector describing a hypothesized vehicle that needs to be tracked, then a tracking error space $\bar{q}_e = \bar{q}_d - \bar{q} = [x_e\ y_e\ \varphi_e]^T$ is deduced in the following manner. It is interesting to note that from the viewpoint of control, an appropriate input $\nu_c = [\upsilon_c\ \omega_c]^T$ should be found to achieve the objective $\lim_{t\to\infty} \bar{q}_e(t) = 0$, but from the planner's viewpoint, the planner should generate ν_c as the desired trajectories for the reduced order subsystems (16)–(18). By further calculation and analysis, the posture errors can be obtained as follows.

$$\bar{q}_e = \begin{bmatrix} x_e \\ y_e \\ \varphi_e \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d - x \\ y_d - y \\ \varphi_d - \varphi \end{bmatrix} \tag{19}$$

By differentiating (19) and considering the nonholonomic constraint (i.e., $\dot{x}\sin\varphi = \dot{y}\cos\varphi$), the posture error dynamics can be achieved as follows.

$$\dot{\bar{q}}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\varphi}_e \end{bmatrix} = \begin{bmatrix} \upsilon_d \cos\varphi_e + \omega y_e - \upsilon \\ \upsilon_d \sin\varphi_e - \omega x_e \\ \omega_d - \omega \end{bmatrix} \tag{20}$$

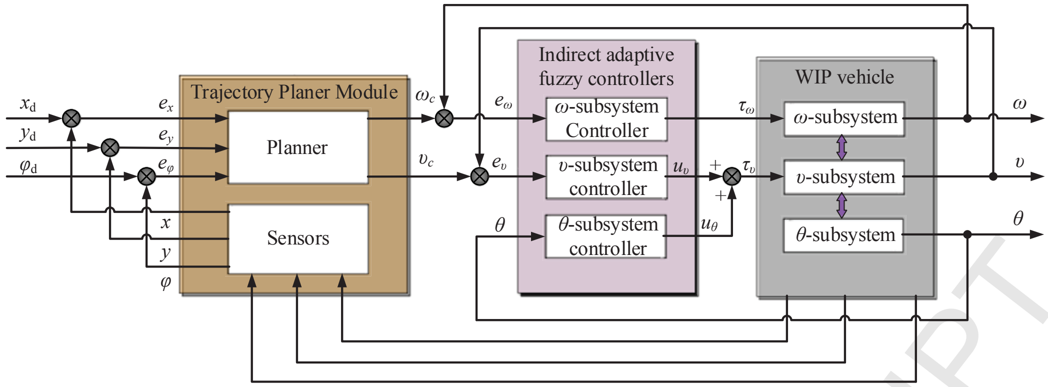

Figure 2. Schematic diagram of the indirect adaptive fuzzy control system.

Remark 1. *Note that ω_c and υ_c are different from the desired values of ω_d and υ_d. In practice, the desired trajectory for an actual vehicle is always given by the vehicle posture variables x_d, y_d and φ_d, which can further compute the desire velocities ω_d and υ_d in advance, but this transformation is irreversible. The trajectory planner will generate ω_c and υ_c to provide a connection between the reduced order subsystems and vehicle postures system.*

3.2. Implementation of the closed-loop planner

If the trajectory planner employs a closed-loop structure, then it will be highly robust to overcome the initial bias deviation as well as rejecting the outside disturbances. Based various studies, a controller derived by the direct Lyapunov methodology (e.g., [34, 35]) can be employed as a planner for the WIP vehicle, as follows:

$$\begin{bmatrix} \upsilon_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} \upsilon_d \cos\varphi_e + \lambda_3 x_e \\ \omega_d + \lambda_1 \upsilon_d y_e + \lambda_2 \sin\varphi_e \end{bmatrix}, \tag{21}$$

where λ_1, λ_2, and λ_3 are positive constants. Then, a theorem related to the trajectory planner can be summarized as follows.

Theorem 3.1. *As an error data-based driven system, (19) and (20), then by using the trajectory planner (21), all of the signals in the closed-loop planner system are bounded and the tracking errors will converge asymptotically to zero, i.e.,*

$$\lim_{t\to\infty} \left[|x_e(t)| + |y_e(t)| + |\varphi_e(t)| \right] = 0.$$

Proof. Define a candidate Lyapunov function as follows.

$$V_1 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{1}{\lambda_1}(1 - \cos\varphi_e) \tag{22}$$

Clearly, $V_1 \geq 0$. Differentiating V_1 with respect to time yields

$$\begin{aligned} \dot{V}_1 &= x_e\dot{x}_e + y_e\dot{y}_e + \frac{1}{\lambda_1}\dot{\varphi}_e \sin\varphi_e \\ &= x_e(\upsilon_d\cos\varphi_e + \omega y_e - \upsilon) + y_e(\upsilon_d \sin\varphi_e + \omega x_e) + \frac{1}{\lambda_1}(\omega_d - \omega)\sin\varphi_e \end{aligned}. \tag{23}$$

After substituting (21) into the equation above, it holds that

$$\dot{V}_1 = -\lambda_3 x_e^2 - \frac{\lambda_2}{\lambda_1}\sin^2\varphi_e \tag{24}$$

.

Obviously, $\dot{V}_1 \leq 0$. According to the Lyapunov stability criterion and the LaSalle invariance principle, for all initial conditions $\bar{q}(0)$, the corresponding solution q_e converges to zero. Thus, the theorem is proved. □

4. Indirect adaptive fuzzy control

Fuzzy control is used widely for sophisticated nonlinear systems because it is not dependent on an accurate model of the control object. In addition, the fuzzy logic can merge the experiences of experts or operators in the system controller, thereby making the closed-loop system much more intelligent and flexible under uncertain disturbances. Fuzzy control can be divided into direct and indirect control methods. In terms of the adaptability, indirect fuzzy control is better than the direct method because the system model information utilized in the controller can be updated online, thereby reducing the dependency on accurate mechanical parameters [39, 40].

4.1. Problem description

Consider a nonlinear system with n-order as follows:

$$\bar{x}^{(n)} = f\left(\bar{x}, \dot{\bar{x}}, \cdots, \bar{x}^{(n-1)}\right) + g\left(\bar{x}, \dot{\bar{x}}, \cdots, \bar{x}^{(n-1)}\right) u \tag{25}$$

, where $f(\cdot)$ and $g(\cdot)$ are unknown nonlinear functions, and $u \in \mathbb{R}^n$ is the control input.

Suppose that the desired value is $\bar{y}_m$. Then, the tracking error can be defined as $e = \bar{y}_m - \bar{y} = \bar{y}_m - \bar{x}$ and it has $\boldsymbol{e} = [e\ \dot{e}, \cdots, e^{(n-1)}]^T$. If the feedback gains are set as $\boldsymbol{k} = [\bar{k}_n, \cdots, \bar{k}_1]^T$, where $\bar{k}_i$ is the coefficient, which can make all the roots of the polynomial $\mathrm{s}^n + \bar{k}_1 \mathrm{s}^{n-1} + \cdots + \bar{k}_n$ on the left half of the complex plane, then the controller can be designed as follows.

$$u^* = \frac{1}{g(\boldsymbol{x})}\left[-f(\boldsymbol{x}) + \bar{y}_m^{(n)} + \boldsymbol{k}^T \boldsymbol{e}\right] \tag{26}$$

By substituting (26) into (25), we have

$$e^{(n)} + \bar{k}_1 e^{(n-1)} + \cdots + \bar{k}_n e = 0. \tag{27}$$

Considering the characteristic of $\boldsymbol{k}$, this means that $e(t) \to 0$ as $t \to \infty$, i.e., the system output $\bar{y}$ can converge asymptotically to its desired output $\bar{y}_m$.

4.2. Fuzzy logic mechanism

In practice, it is almost impossible to obtain accurate values of $f(\cdot)$ and $g(\cdot)$ in advance; therefore, the fuzzy logic mechanism can be employed to estimate the true values of $f(\cdot)$ and $g(\cdot)$ online based on their corresponding updated values $\hat{f}(\cdot)$ and $\hat{g}(\cdot)$. By the universal approximate theorem of the fuzzy logic mechanism, the estimated errors can be controlled to the desired boundedness as much as possible.

Taking $\hat{f}(\boldsymbol{x}|\Theta)$ as an example, the estimated value can be formulated according to three main steps, as follows.

Step 1 For the variables $\bar{x}_i$ $(i = 1, 2, \cdots, n)$, define the fuzzy set $\mathbb{A}_i^{l_i}$ $(l_i = 1, 2, \cdots, p_i)$, where p_i is the number of the fuzzy set.

Step 2 Construct the fuzzy logic system $\hat{f}(\boldsymbol{x}|\Theta)$ using the fuzzy rules with the number of $\prod_{i=1}^{n} p_i$, as follows:

$$\mathrm{R}^{(j)}:\ \text{IF}\ \bar{x}_1\ is\ \mathbb{A}_1^{l_1}\ and\ \cdots\ and\ \bar{x}_n\ is\ \mathbb{A}_1^{l_n}\ \text{THEN}\ \hat{f}\ is\ \mathbb{E}^{l_1 \cdots l_n}, \tag{28}$$

where $l_i = 1, 2, \cdots, p_i$ and $i = 1, 2, \cdots, n$.

Step 3 The outputs of this fuzzy logic system with the center-average defuzzifier, product inference, and singleton fuzzier are of the following form:

$$\hat{f}(\boldsymbol{x}|\Theta_f) = \frac{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} \bar{y}_f^{l_1 \cdots l_n} \left(\prod_{i=1}^{n} \mu_{\mathbb{A}_i^{l_i}}(\bar{x}_i)\right)}{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} \left(\prod_{i=1}^{n} \mu_{\mathbb{A}_i^{l_i}}(\bar{x}_i)\right)}, \tag{29}$$

where $\mu_{\mathbb{A}_i^j}(\bar{x}_i)$ is the membership function for $\bar{x}_i$. Let $\bar{y}_f^{l_1 \cdots l_n}$ be the free parameter and collect them in the set as $\Theta_f \in \mathbb{R}^{\prod_{i=1}^{n} p_i}$. Define vector the $\xi(x)$, and then (29) can be replaced by

$$\hat{f}(\boldsymbol{x}|\Theta_f) = \Theta_f^T \xi(x), \tag{30}$$

where $\xi(x)$ is a vector defined as

$$\xi_{l_1, \cdots, l_n}(x) = \frac{\prod_{i=1}^{n} \mu_{\mathbb{A}_i^{l_i}}(\bar{x}_i)}{\sum_{l_1=1}^{p_1} \cdots \sum_{l_n=1}^{p_n} \left(\prod_{i=1}^{n} \mu_{\mathbb{A}_i^{l_i}}(\bar{x}_i)\right)}. \tag{31}$$

4.3. Indirect adaptive fuzzy control

By selecting reasonable design parameters, the controllers with the estimated values can approach the appropriate controllers with accurate values to any arbitrary tolerance. The indirect fuzzy controller (Fig.3) can be outlined as:

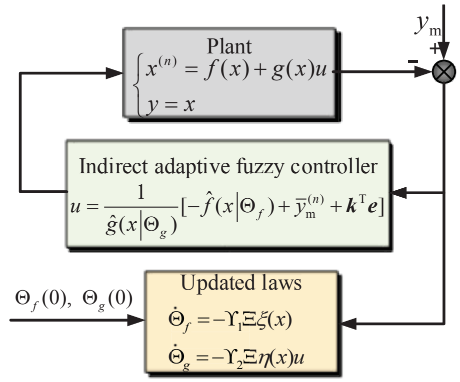

Figure 3. Schematic diagram of indirect adaptive fuzzy scheme

$$u = \frac{1}{\hat{g}(\boldsymbol{x}|\Theta_g)}\left[-\hat{f}(\boldsymbol{x}|\Theta_f) + \bar{y}_m^{(n)} + \boldsymbol{k}^T\boldsymbol{e}\right], \tag{32}$$

where $f(\boldsymbol{x}|\Theta_f)$ and $\hat{g}(\boldsymbol{x}|\Theta_g)$ are governed by

$$\hat{f}(\boldsymbol{x}|\Theta_f) = \hat{\Theta}_f^T \xi(x) \tag{33}$$

$$\hat{g}(\boldsymbol{x}|\Theta_g) = \hat{\Theta}_g^T \eta(x) \tag{34}$$

.

In addition, to enhance the adaptability of the indirect fuzzy logic mechanism, adaptive laws are employed to update the true values of the free parameters. The updated laws can be designed as:

$$\dot{\hat{\Theta}}_f = -\Upsilon_1 \Xi \xi(x) \tag{35}$$

$$\dot{\hat{\Theta}}_g = -\Upsilon_2 \Xi \eta(x) u \tag{36}$$

, where Υ_1 and Υ_2 are positive design parameters used to determine the control performance, $\eta(x)$ is the vector for approaching $g(\cdot)$, and Ξ represents the additional terms in the controllers, which are formulated during the control design process.

5. Synthesis of the three control subsystems

5.1. θ-subsystem

In practice, maintaining the upright position of the vehicle body is crucial for the WIP vehicle because after the pendulum-like vehicle body falls down, the vehicle cannot be recovered. Furthermore, it is a challenge to maintain the upright position in an unstable equilibrium for subsystem (18), so more control effects are needed to sustain the stability of the θ-subsystem. Therefore, a variable structure control method with adaptive laws is required to deal with this special underactuated subsystem.

Note that θ can be used directly to represent the tracking error of the tilt angle of the vehicle body since the desired tilt angle of the pendulum-like vehicle body is $\theta_d = 0$. In addition, θ-subsystem and υ-subsystem share the same control input τ_υ, so to clarify the explicit relationship of every variable, this control input can be calculated by (17), before substituting τ_υ into (18). As a result, an underactuated θ-subsystem without control effects can be obtained as follows.

$$\ddot{\theta} = \frac{m_p l^2}{2J_\theta}\omega^2 \sin 2\theta + \frac{m_p g l}{J_\theta}\sin\theta - \frac{m_p l \cos\theta}{J_\theta}\dot{\upsilon} \tag{37}$$

The longitudinal acceleration $\dot{\upsilon}$ from (37) can be taken as a virtual input, which allows the original underactuated dynamics to be converted into actuated dynamics. In particular, to decrease the dependency on the accurate dynamic model, the indirect fuzzy logic mechanism is employed to support the sliding mode control approach when constructing the controller, where the sliding mode control enhances the robustness to parameter variation and external disturbance, while the fuzzy logic mechanism provides adaptability to the dynamic environment [41, 42].

The sliding mode manifold is defined as

$$s = \dot{\theta} + c\theta, \tag{38}$$

where c is a positive design parameter.

According to the fuzzy logic mechanism, the estimated entries, i.e., $\hat{f}_\theta(x|\Theta_{f\theta})$ and $\hat{g}_\theta(x|\Theta_{g\theta})$, are designed to make the controller. The optimal parameters are assumed to be

$$\Theta^*_{f\theta} = \arg \min_{\Theta_{f\theta}\in\Omega_{f\theta}} \left[\sup_{x\in\mathbb{R}^n} |\hat{f}_\theta(x|\Theta_{f\theta}) - f_\theta(x)| \right] \tag{39}$$

$$\Theta^*_{g\theta} = \arg \min_{\Theta_{g\theta}\in\Omega_{g\theta}} \left[\sup_{x\in\mathbb{R}^n} |\hat{g}_\theta(x|\Theta_{g\theta}) - g_\theta(x)| \right], \tag{40}$$

where $\Omega_{f\theta}$ and $\Omega_{g\theta}$ are the sets for $\Theta_{f\theta}$ and $\Theta_{g\theta}$, respectively.

Furthermore, the minimum approximate errors can be defined as follows.

$$\epsilon_{f\theta} = \hat{f}_\theta(x|\Theta^*_{f\theta}) - f_\theta(x) \tag{41}$$

$$\epsilon_{g\theta} = \hat{g}_\theta(x|\Theta^*_{g\theta}) - g_\theta(x) \tag{42}$$

Define $\tilde{\Theta}_{f\theta} = \hat{\Theta}_{f\theta} - \Theta^*_{f\theta}$. Due to the universal approximation theorem of fuzzy system, we conclude that $\epsilon_{f\theta}$ and $\epsilon_{g\theta}$ can converge to a compact set, and thus it follows that $\dot{\tilde{\Theta}}_{f\theta} = \dot{\hat{\Theta}}_{f\theta}$. Therefore, the adaptive laws can be introduced to update the true values of the controllers, which enhances the adaptive capability of the closed-loop system.

Theorem 5.1. *For the θ-subsystem (18), if the sliding mode surface is selected as in (38), then all of the signals in the closed-loop θ-subsystem are uniformly ultimately bounded, and the tracking errors will converge to a compact set, with the indirect adaptive fuzzy logic-based sliding mode controllers as follows:*

$$u_\theta = \frac{1}{\hat{\Theta}^T_{g\theta}\eta(x)} \left[-\hat{\Theta}^T_{f\theta}\xi(x) + c\dot{\theta} - k_1 sgn(s) - k_2 s \right] \tag{43}$$

$$\dot{\hat{\Theta}}_{f\theta} = \gamma_1 s\xi(x) \tag{44}$$

$$\dot{\hat{\Theta}}_{g\theta} = \gamma_2 s\eta(x)u_\theta \tag{45}$$

, where $sgn(\cdot)$ is a signum function, and γ_1, γ_2 are positive design parameters. $\hat{\Theta}_{f\theta}$ and $\hat{\Theta}_{g\theta}$ are the weight values, and $\xi(x)$ and $\eta(x)$ are their membership functions, respectively.

Proof. Consider a positive Lyapunov candidate, as follows.

$$V_2 = \frac{1}{2}s^2 + \frac{1}{2\gamma_1}(\Theta_{f\theta} - \Theta^*_{f\theta})^T(\Theta_{f\theta} - \Theta^*_{f\theta}) + \frac{1}{2\gamma_2}(\Theta_{g\theta} - \Theta^*_{g\theta})^T(\Theta_{g\theta} - \Theta^*_{g\theta}) \tag{46}$$

By differentiating V_2 along the trajectory (37) with respect to time, we have

$$\begin{aligned}
\dot{V}_2 &= s\dot{s} + \frac{1}{\gamma_1}\tilde{\Theta}^T_{f\theta}\dot{\hat{\Theta}}_{f\theta} + \frac{1}{\gamma_2}\tilde{\Theta}^T_{g\theta}\dot{\hat{\Theta}}_{g\theta} \\
&= s\left[\Theta^{*T}_{f\theta}\xi(x) + \epsilon_{f\theta} + \Theta^{*T}_{g\theta}\eta(x)u_\theta + \epsilon_{g\theta}u_\theta - \lambda\dot{\theta}\right] + \frac{1}{\gamma_1}\tilde{\Theta}^T_{f\theta}\dot{\hat{\Theta}}_{f\theta} + \frac{1}{\gamma_2}\tilde{\Theta}^T_{g\theta}\dot{\hat{\Theta}}_{g\theta} \\
&= s\left[(\hat{\Theta}_{f\theta} - \tilde{\Theta}_{f\theta})^T\xi(x) + (\hat{\Theta}_{g\theta} - \tilde{\Theta}_{g\theta})^T\eta(x)u_\theta + \epsilon_{f\theta} + \epsilon_{g\theta}u_\theta - \lambda\dot{\theta}\right] + \frac{1}{\gamma_1}\tilde{\Theta}^T_{f\theta}\dot{\hat{\Theta}}_{f\theta} + \frac{1}{\gamma_2}\tilde{\Theta}^T_{g\theta}\dot{\hat{\Theta}}_{g\theta} \\
&= s\left[\hat{\Theta}^T_{f\theta}\xi(x) + \hat{\Theta}^T_{g\theta}\eta(x)u_\theta - \lambda\dot{\theta} + \epsilon_{f\theta} + \epsilon_{g\theta}u_\theta\right] - s\tilde{\Theta}^T_{f\theta}\xi(x) - \tilde{\Theta}^T_{g\theta}\eta(x)u_\theta + \frac{1}{\gamma_1}\tilde{\Theta}^T_{f\theta}\dot{\hat{\Theta}}_{f\theta} + \frac{1}{\gamma_2}\tilde{\Theta}^T_{g\theta}\dot{\hat{\Theta}}_{g\theta}
\end{aligned} . \tag{47}$$

After substituting (43)–(45) into (47), it holds that

$$\dot{V}_2 = -k_1|s| - k_2 s^2 + s\epsilon_{f\theta} + s\epsilon_{g\theta}u_\theta \tag{48}$$

.

According to the universal approximation theorem, $\epsilon_{f\theta} \to 0$ and $\epsilon_{g\theta} \to 0$ as $t \to \infty$, and thus $V_2 \leq 0$. Therefore, the theorem can be proved by the Lyapunov stability theory and LaSalle's invariance theorem. □

5.2. υ-subsystem

For the υ-subsystem, the control objective is to find an appropriate control input u_υ such that the vehicle's longitudinal velocity can track the desired value. It should be mentioned that the control input u_θ, which is employed to maintain the upright position of θ-subsystem, can be considered as a disturbance to the control τ_υ. We define the tracking errors as $e_\upsilon = \upsilon - \upsilon_c$ and $e_\omega = \omega - \omega_c$, where υ_c and ω_c are the desired velocities generated by the trajectory planner. From (16)–(17), the error dynamics are formulated as follows.

$$\dot{e}_\upsilon = \frac{1}{\Omega}\left[J_\theta m_p l\dot{\theta}^2 \sin\theta - m_p^2 l^3 (e_\omega + \omega_c)^2 \sin\theta\cos^2\theta - m_p^2 l^2 g\sin\theta\cos\theta\right] - \dot{\upsilon}_c + \frac{J_\theta}{\Omega r}u_\upsilon \tag{49}$$

Similarly, $\hat{f}_\upsilon(x|\Theta_{f\upsilon})$ and $\hat{g}_\upsilon(x|\Theta_{g\upsilon})$ can be used to make the system controller, and the optimal parameters can be hypothesized as

$$\Theta^*_{f\upsilon} = \arg\min_{\Theta_{f\upsilon}\in\Omega_{f\upsilon}}\left[\sup_{x\in\mathbb{R}^n}|\hat{f}_\upsilon(x|\Theta_{f\upsilon}) - f_\upsilon(x)|\right] \tag{50}$$

$$\Theta^*_{g\upsilon} = \arg\min_{\Theta_{g\upsilon}\in\Omega_{g\upsilon}}\left[\sup_{x\in\mathbb{R}^n}|\hat{g}_\upsilon(x|\Theta_{g\upsilon}) - g_\upsilon(x)|\right], \tag{51}$$

where $\Omega_{f\upsilon}$, $\Omega_{g\upsilon}$ are the sets for $\Theta_{f\upsilon}$ and $\Theta_{g\upsilon}$, respectively.

Moreover, the minimum approximate errors can be governed by

$$\epsilon_{f\upsilon} = \hat{f}_\upsilon(x|\Theta^*_{f\upsilon}) - f_\upsilon(x) \tag{52}$$

$$\epsilon_{g\upsilon} = \hat{g}_\upsilon(x|\Theta^*_{g\upsilon}) - g_\upsilon(x) \tag{53}$$

.

Define $\tilde{\Theta}_{f\upsilon} = \hat{\Theta}_{f\upsilon} - \Theta^*_{f\upsilon}$. By the universal approximation theorem, we conclude that $\epsilon_{f\upsilon}$ and $\epsilon_{g\upsilon}$ can converge to a compact set, and thus it holds that $\dot{\tilde{\Theta}}_{f\upsilon} = \dot{\hat{\Theta}}_{f\upsilon}$. By using the adaptive laws to update the true values employed in the controllers, a theorem can be stated as follows.

Theorem 5.2. *For the υ-subsystem (17), the signals in the closed-loop υ-subsystem are uniformly ultimately bounded and the tracking errors converge to a compact set, with the indirect adaptive fuzzy logic-based controllers as follows:*

$$u_\upsilon = \frac{1}{\hat{\Theta}^T_{g\upsilon}\eta(x)}\left(\hat{\Theta}^T_{f\upsilon}\xi(x) - k_3 e_\upsilon\right) \tag{54}$$

$$\dot{\hat{\Theta}}_{f\upsilon} = \gamma_3 e_\upsilon \xi(x) \tag{55}$$

$$\dot{\hat{\Theta}}_{g\upsilon} = \gamma_4 e_\upsilon \eta(x) u_\upsilon \tag{56}$$

, where $k_3,\ \gamma_3,\ \gamma_4$ are positive design parameters. $\hat{\Theta}_{f\upsilon}$ and $\hat{\Theta}_{g\upsilon}$ are the weight values, and $\xi(x)$ and $\eta(x)$ are their membership functions, respectively.

Proof. Consider a positive Lyapunov candidate as follows.

$$V_3 = \frac{1}{2}e_\upsilon^2 + \frac{1}{2\gamma_3}(\Theta_{f\upsilon} - \Theta^*_{f\upsilon})^T(\Theta_{f\upsilon} - \Theta^*_{f\upsilon}) + \frac{1}{2\gamma_4}(\Theta_{g\upsilon} - \Theta^*_{g\upsilon})^T(\Theta_{g\upsilon} - \Theta^*_{g\upsilon}) \tag{57}$$

Differentiating V_3 along the trajectory (17) with respect to time yields

$$\begin{aligned}
\dot{V}_3 &= e_\upsilon\dot{e}_\upsilon + \frac{1}{\gamma_3}\tilde{\Theta}^T_{f\upsilon}\dot{\hat{\Theta}}_{f\upsilon} + \frac{1}{\gamma_4}\tilde{\Theta}^T_{g\upsilon}\dot{\hat{\Theta}}_{g\upsilon}\\
&= e_\upsilon(f_\upsilon + g_\upsilon u_\upsilon) + \frac{1}{\gamma_3}\tilde{\Theta}^T_{f\upsilon}\dot{\hat{\Theta}}_{f\upsilon} + \frac{1}{\gamma_4}\tilde{\Theta}^T_{g\upsilon}\dot{\hat{\Theta}}_{g\upsilon}\\
&= e_\upsilon(\Theta^{*T}_{f\upsilon}\xi(x) + \epsilon_{f\upsilon} + \Theta^{*T}_{g\upsilon}\eta(x)u_\upsilon + \epsilon_{g\upsilon}u_\upsilon) + \frac{1}{\gamma_3}\tilde{\Theta}^T_{f\upsilon}\dot{\hat{\Theta}}_{f\upsilon} + \frac{1}{\gamma_4}\tilde{\Theta}^T_{g\omega}\dot{\hat{\Theta}}_{g\omega}\\
&= e_\upsilon\left[(\hat{\Theta}_{f\upsilon} - \tilde{\Theta}_{f\upsilon})^T\xi(x) + \epsilon_{f\upsilon} + (\hat{\Theta}_{g\upsilon} - \tilde{\Theta}_{g\upsilon})^T\eta(x)u_\upsilon + \epsilon_{g\upsilon}u_\upsilon\right] + \frac{1}{\gamma_3}\tilde{\Theta}^T_{f\upsilon}\dot{\hat{\Theta}}_{f\upsilon} + \frac{1}{\gamma_4}\tilde{\Theta}^T_{g\upsilon}\dot{\hat{\Theta}}_{g\upsilon}\\
&= e_\upsilon\left[\hat{\Theta}^T_{f\upsilon}\xi(x) + \hat{\Theta}^T_{g\upsilon}\eta(x)u_\upsilon + \epsilon_{f\upsilon} + \epsilon_{g\upsilon}u_\upsilon\right] - e_\upsilon\tilde{\Theta}^T_{f\upsilon}\xi(x) - e_\upsilon\tilde{\Theta}^T_{g\upsilon}\eta(x)u_\upsilon\\
&\quad + \frac{1}{\gamma_3}\tilde{\Theta}^T_{f\upsilon}\dot{\hat{\Theta}}_{f\upsilon} + \frac{1}{\gamma_4}\tilde{\Theta}^T_{g\upsilon}\dot{\hat{\Theta}}_{g\upsilon}.
\end{aligned} \tag{58}$$

After substituting (54)–(56) into (58), it follows that

$$\dot{V}_3 = -k_3 e_\upsilon^2 + e_\upsilon \epsilon_{f\upsilon} + e_\upsilon \epsilon_{g\upsilon} u_\upsilon \tag{59}$$

. Obviously, $V_3 \leq 0$ since $\epsilon_{f\upsilon} \to 0$ and $\epsilon_{g\upsilon} \to 0$ as $t \to \infty$. Hence, the theorem can be proved by the Lyapunov stability theory and LaSalle's invariance theorem. □

Using the previous results, we note that u_θ and u_υ are not the real control inputs for the system, so a composite controller for the WIP vehicle can be obtained as follows.

$$\tau_\upsilon = u_\theta + u_\upsilon \tag{60}$$

Remark 2. *The control input (60) is a composite controller where the control input u_θ, which is the acceleration required to maintain the balance of the vehicle body, is taken as a type of external dynamic disturbance with a high frequency for the υ-subsystem. Since u_θ and u_υ can guarantee the stability of the respective subsystem, then τ_υ can ensure the stability of the entire underactuated system.*

5.3. ω-subsystem

The control objective for ω-subsystem is to find a control input τ_ω such that the rotational tracking error, i.e., $e_\omega = \omega - \omega_c$, should converge to a compact set. For this purpose, the error dynamics of the ω-subsystem are formulated as

$$\dot{e}_\omega = -\frac{m_p l^2 \dot{\theta} \sin 2\theta}{J_\varphi + m_p l^2 \sin^2\theta}(e_\omega + \omega_c) - \dot{\omega}_c + \frac{d}{r(J_\varphi + m_p l^2 \sin^2\theta)}\tau_\omega \tag{61}$$

.

Likewise, $\hat{f}_\omega(x|\Theta_{f\omega})$ and $\hat{g}_\omega(x|\Theta_{g\omega})$ are used to design the system controller, and the optimal parameters can be hypothesized as

$$\Theta^*_{f\omega} = \arg\min_{\Theta_{f\omega}\in\Omega_{f\omega}} \left[\sup_{x\in\mathbb{R}^n} |\hat{f}_\omega(x|\Theta_{f\omega}) - f_\omega(x)|\right] \tag{62}$$

$$\Theta^*_{g\omega} = \arg\min_{\Theta_{g\omega}\in\Omega_{g\omega}} \left[\sup_{x\in\mathbb{R}^n} |\hat{g}_\omega(x|\Theta_{g\omega}) - g_\omega(x)|\right], \tag{63}$$

where $\Omega_{f\omega}$, $\Omega_{g\omega}$ are the sets for $\Theta_{f\omega}$ and $\Theta_{g\omega}$, respectively.

Similarly, the minimum approximate errors can be governed by

$$\epsilon_{f\omega} = \hat{f}_\omega(x|\Theta^*_{f\omega}) - f_\omega(x) \tag{64}$$

$$\epsilon_{g\omega} = \hat{g}_\omega(x|\Theta^*_{g\omega}) - g_\omega(x). \tag{65}$$

Define $\tilde{\Theta}_{f\omega} = \hat{\Theta}_{f\omega} - \Theta^*_{f\omega}$. With the universal approximation theorem, we find that $\epsilon_{f\omega}$ and $\epsilon_{g\omega}$ can converge to a compact set, and thus we obtain $\dot{\tilde{\Theta}}_{f\omega} = \dot{\hat{\Theta}}_{f\omega}$. By introducing the adaptive laws to update the true values in the controllers, a theorem can be stated as follows.

Theorem 5.3. *For the ω-subsystem (16), all of the signals in the closed-loop ω-subsystem are uniformly ultimately bounded and the tracking errors converge to a compact set, with the indirect adaptive fuzzy logic-based controllers as follows:*

$$\tau_\omega = \frac{1}{\hat{\Theta}^T_{g\omega}\eta(x)}\left(\hat{\Theta}^T_{f\omega}\xi(x) - k_4 e_\omega\right) \tag{66}$$

$$\dot{\hat{\Theta}}_{f\omega} = \gamma_5 e_\omega \xi(x) \tag{67}$$

$$\dot{\hat{\Theta}}_{g\omega} = \gamma_6 e_\omega \eta(x)\tau_\omega \tag{68}$$

, where $k_4,\ \gamma_5,\ \gamma_6$ are positive design parameters, $\hat{\Theta}_{f\omega}$ and $\hat{\Theta}_{g\omega}$ are the weight values, and $\xi(x)$ and $\eta(x)$ are their membership functions, respectively.

Proof. Consider a positive Lyapunov candidate as follows.

$$V_4 = \frac{1}{2}e_\omega^2 + \frac{1}{2\gamma_5}(\Theta_{f\omega} - \Theta^*_{f\omega})^T(\Theta_{f\omega} - \Theta^*_{f\omega}) + \frac{1}{2\gamma_6}(\Theta_{g\omega} - \Theta^*_{g\omega})^T(\Theta_{g\omega} - \Theta^*_{g\omega}) \tag{69}$$

By differentiating V_4 along the trajectory (16) with respect to time, it holds that

$$\begin{aligned}
\dot{V}_4 &= e_\omega \dot{e}_\omega + \frac{1}{\gamma_5}\tilde{\Theta}_{f\omega}^T \dot{\hat{\Theta}}_{f\omega} + \frac{1}{\gamma_6}\tilde{\Theta}_{g\omega}^T \dot{\hat{\Theta}}_{g\omega} \\
&= e_\omega (f_\omega + g_\omega \tau_\omega) + \frac{1}{\gamma_5}\tilde{\Theta}_{f\omega}^T \dot{\hat{\Theta}}_{f\omega} + \frac{1}{\gamma_6}\tilde{\Theta}_{g\omega}^T \dot{\hat{\Theta}}_{g\omega} \\
&= e_\omega (\Theta_{f\omega}^{*T}\xi(x) + \epsilon_{f\omega} + \Theta_{g\omega}^{*T}\eta(x)\tau_\omega + \epsilon_{g\omega}\tau_\omega) + \frac{1}{\gamma_5}\tilde{\Theta}_{f\omega}^T \dot{\hat{\Theta}}_{f\omega} + \frac{1}{\gamma_6}\tilde{\Theta}_{g\omega}^T \dot{\hat{\Theta}}_{g\omega} \\
&= e_\omega \left[(\hat{\Theta}_{f\omega} - \tilde{\Theta}_{f\omega})^T \xi(x) + \epsilon_{f\omega} + (\hat{\Theta}_{g\omega} - \tilde{\Theta}_{g\omega})^T \eta(x)\tau_\omega + \epsilon_{g\omega}\tau_\omega\right] + \frac{1}{\gamma_5}\tilde{\Theta}_{f\omega}^T \dot{\hat{\Theta}}_{f\omega} + \frac{1}{\gamma_6}\tilde{\Theta}_{g\omega}^T \dot{\hat{\Theta}}_{g\omega} \\
&= e_\omega \left[\hat{\Theta}_{f\omega}^T \xi(x) + \hat{\Theta}_{g\omega}^T \eta(x)\tau_\omega + \epsilon_{f\omega} + \epsilon_{g\omega}\tau_\omega\right] - e_\omega \tilde{\Theta}_{f\omega}^T \xi(x) - e_\omega \hat{\Theta}_{g\omega}^T \eta(x)\tau_\omega \\
&\quad + \frac{1}{\gamma_5}\tilde{\Theta}_{f\omega}^T \dot{\hat{\Theta}}_{f\omega} + \frac{1}{\gamma_6}\tilde{\Theta}_{g\omega}^T \dot{\hat{\Theta}}_{g\omega}.
\end{aligned} \tag{70}$$

After substituting (66)–(68) into (70), it follows that

$$\dot{V}_4 = -k_4 e_\omega^2 + e_\omega \epsilon_{f\omega} + e_\omega \epsilon_{g\omega}\tau_\omega \tag{71}$$

.

Obviously, $V_4 \le 0$ as $\epsilon_{f\omega} \to 0$ and $\epsilon_{g\omega} \to 0$ as $t \to \infty$. Therefore, the theorem can be proved by the Lyapunov stability theory and LaSalle's invariance theorem. □

Remark 3. *Singularities exist in the controllers for the indirect adaptive control scheme, and this is an unavoidable and troublesome issue for adaptive control. This problem can be addressed by rewriting the scalar* $\frac{1}{\Theta_{gi}^T \eta(x)}$, $(i = \theta, \upsilon, \omega)$, *with the following terms as*

$$\frac{\Theta_{gi}^T \eta(x)}{\left[\Theta_{gi}^T \eta(x)\right]^2 + \varrho}, \tag{72}$$

where $\varrho > 0$ *is a sufficiently small coefficient to avoid singularity.*

6. Simulations

In this section, we present the results of numerical simulations performed for a WIP vehicle to verify the feasibility and effectiveness of the proposed control strategy in the MATLAB environment. The physical parameters of the WIP vehicle were set as follows: m_p = 3kg, m_c = 5kg, m_w = 1kg, d = 1.0m, r = 0.5m, l = 1.0m, J_w = 1.5kgm^2, J_c = 5kgm^2, and J_φ = 1.5kgm^2. In addition, if the desired trajectory is assumed to be a circular path with a radius of R = 2m, then it can be described in Cartesian coordinates as follows.

$$\begin{cases} x_d &= R\cos\omega_d t \\ y_d &= R\sin\omega_d t \\ \varphi_d &= \omega_d t \end{cases}$$

It should be noted that the longitudinal and rotational velocities used in the proposed controllers can be computed by $\upsilon_d = \sqrt{x_d^2 + y_d^2}$ and $\omega_d = \dot{\varphi}_d$, respectively. The vehicle's initial conditions for the posture error vectors, velocity vector, and posture state variables were taken as $\bar{q}_e(0) = [x_e(0)\ y_e(0)\ \varphi_e(0)]^T = [-0.05\ 0\ \pi/24]^T$, $\dot{z}(0) = [\upsilon(0)\ \omega(0)\ \dot{\theta}(0)]^T = [0\ 0\ 0]^T$, and $z(0) = [x_\upsilon(0)\ \varphi(0)\ \theta(0)]^T = [0\ 0.01\ 0.15]^T$, respectively. For the indirect adaptive fuzzy system, two fuzzy controllers were employed to control the longitudinal and rotational subsystems separately. The fuzzy membership functions $\xi(x_i)$ and $\eta(x_i)$, as shown in Fig.4, were set as follows.

$$\begin{aligned}
\mu_{NM}(x_i) &= \exp\left[-((x_i + \pi/6)/(\pi/24))^2\right] \\
\mu_{NS}(x_i) &= \exp\left[-((x_i + \pi/12)/(\pi/24))^2\right] \\
\mu_{Z}(x_i) &= \exp\left[-(x_i/(\pi/24))^2\right] \\
\mu_{PS}(x_i) &= \exp\left[-((x_i - \pi/12)/(\pi/24))^2\right] \\
\mu_{PM}(x_i) &= \exp\left[-((x_i - \pi/6)/(\pi/24))^2\right]
\end{aligned}$$

To satisfy the control performance requirements, the design coefficients of the proposed control scheme were set as follows: $c = 2,\ k_1 = 4,\ k_2 = 2,\ k_3 = 2,\ k_4 = 1;\ \sigma_1 = \sigma_3 = 2,\ \sigma_2 = 1;\ \gamma_1 = \gamma_3 = 10,\ \gamma_2 = \gamma_4 = 1;\ \varrho =$

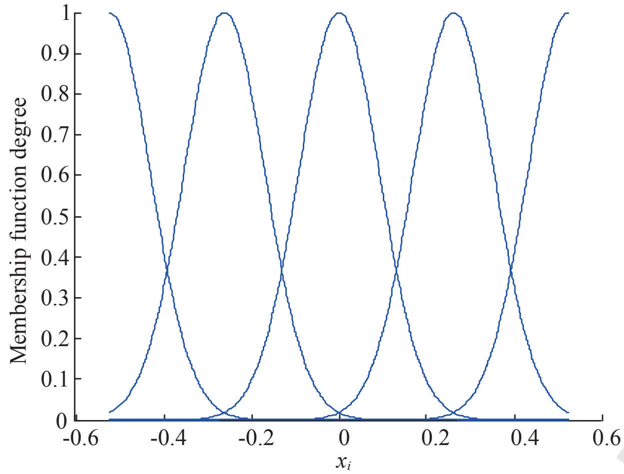

Figure 4. Membership function for the proposed fuzzy logic.

0.001. The sign functions in the sliding mode control schemes were replaced by hyperbolic tangent functions, i.e., $\tanh(\cdot)$, to reduce the inherent chattering caused by the variable structure of the control system.

Using these design parameters, the practical and desired tracking trajectories on the $x - y$ plane are plotted in Fig.5, while the tracking errors are shown in Fig.6. These results demonstrate that the posture of the WIP vehicle can track the reference trajectory very well using the proposed control methods. However, it should be noted that a dramatic regulation process occurred at the start point, which was because the underactuated vehicle body had to maintain an upright position while the vehicle's posture converged to the desired trajectories. In addition, the enlarged inset in Fig.6 clearly demonstrates that the tracking errors x_e, y_e and φ_e converged to zero within 15 s, thereby indicating the this dramatic regulation process could be suppressed rapidly.

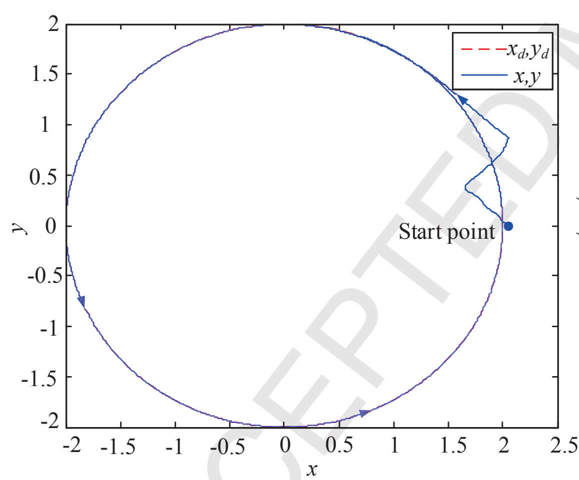

Figure 5. Practical and desired tracking trajectories.

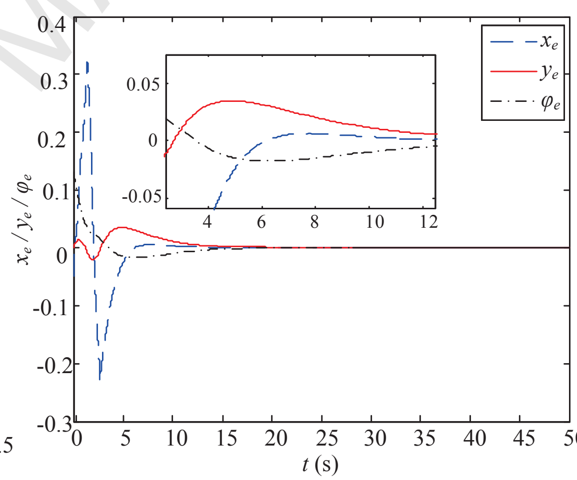

Figure 6. Posture tracking errors for the WIP vehicle.

The time responses of the velocity tracking results are shown in Fig.7, which indicate the effects of longitudinal and rotational velocity tracking. These results demonstrate that the practical velocity variables υ and ω can track the desired velocities, υ_c and ω_c, generated by the trajectory planner within finite time, thereby verifying the effectiveness of the indirect adaptive fuzzy controllers. In addition, it should be noted that an abrupt change occurred at about 2.5 s. This phenomenon indicates that control of the longitudinal velocity is more challenging because the longitudinal velocity controlled by the torque u_υ but it is also affected by the disturbance-like virtual control input u_θ.

In particular, the stabilities of the tilt angle and its angular velocity are of crucial importance for control purposes. Their time responses are shown in Fig.8, which indicate that θ and $\dot{\theta}$ can converge rapidly to zero. Thus, these results verified that the composite controller produced by combining indirect adaptive fuzzy control and the sliding mode technique achieved acceptable control performance for the underactuated θ-subsystem.

Figures 9 and 10 show the estimated processes for the fuzzy logic system. The simulation results demonstrate

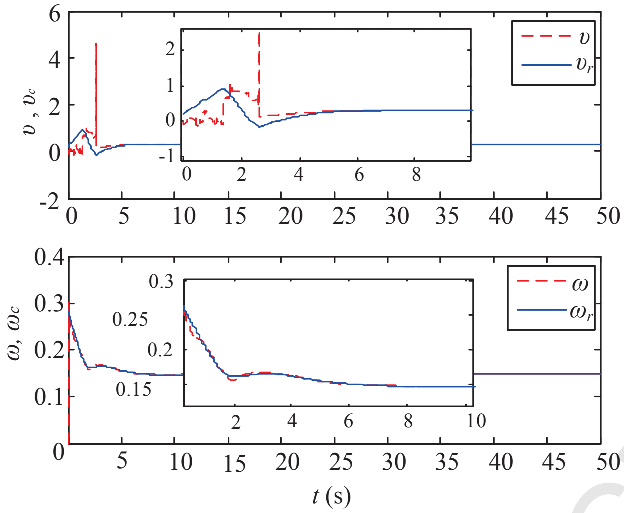

Figure 7. Response of velocity tracking over time.

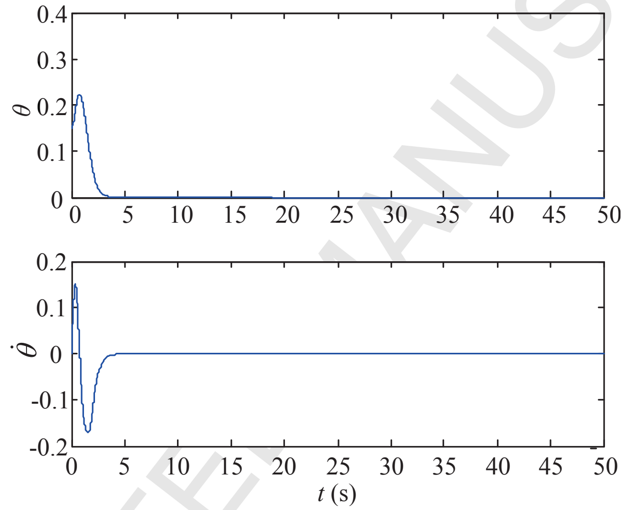

Figure 8. Responses of the tilt angle and angular velocity.

that the proposed controllers exhibited better adaptive features with the designed closed-loop system because of the indirect fuzzy logic mechanism. Our further analysis showed that $\hat{f}_{\upsilon}(x|\Theta_{f\upsilon})$, $\hat{g}_{\upsilon}(x|\Theta_{g\upsilon})$, and $\hat{f}_{\omega}(x|\Theta_{f\omega})$ could converge to their respective values within a compact set in 5 s despite the presence of elastic oscillations at the initial time. By contrast, the time response of $\hat{g}_{\omega}(x|\Theta_{g\omega})$ did not agree well with the true value of $g_{\omega}(x|\Theta_{g\omega})$, but its tracking error converged to a compact set, which implies that the proposed adaptive laws are effective and that the global stability is also guaranteed.

7. Conclusion

In this study, we proposed an error data-based trajectory planner and indirect adaptive fuzzy control for a WIP vehicle with an underactuated suspension, which is subject to a nonholonomic constraint. Using the closed-loop planner, the nonholonomic constraint and underactuated problems can be resolved on kinematic and dynamic levels, respectively. To address the underactuated problem of the tilt angle dynamics, we treat the longitudinal accelerated velocity as a virtual control input to ensure that the inverted pendulum-like vehicle body maintains an upright position, which is an unstable equilibrium for the system. The proposed control method is a type of composite control that synthesizes indirect fuzzy control and the variable structure technique in an effective manner. Using an indirect fuzzy mechanism, the controller can merge the successful experiences of operators. By employing the sliding mode control technique, the controller is robust to uncertain disturbance. In addition, adaptive laws are employed to ensure independence from the vehicle mechanical parameters in the controller. These advantages are crucial and significant for the performance of the WIP vehicle control system. It should

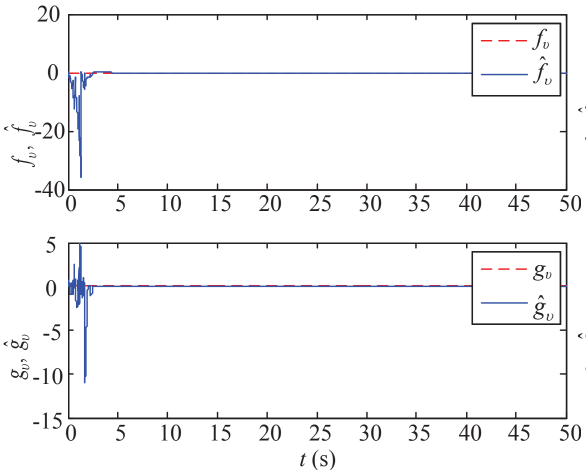

Figure 9. Responses of f_υ, $\hat{f}_\upsilon$ and g_υ, $\hat{g}_\upsilon$

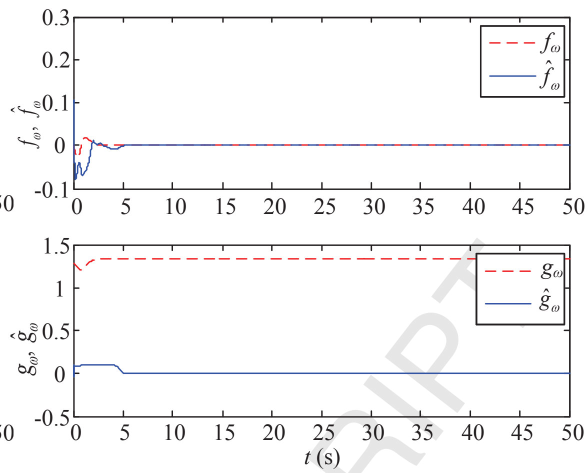

Figure 10. Responses of f_ω, $\hat{f}_\omega$ and g_ω, $\hat{g}_\omega$

also be noted that in addition to WIP vehicles, the proposed control strategy is equally applicable to any other underactuated mechanical system, even systems with an unstable equilibrium point. In our further research, we will perform experiments to verify the effectiveness of the proposed control approaches and strategies.

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