

*ETH*zürich

**GRAMAZIO
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ETHzürich

MAS Digital Fabrication

Implicit modeling

October 26th, 2020

ETH zürich

DARCH

Departement Architektur



Institute of Technology in Architecture
Faculty of Architecture / ETH Zurich

dbt

**GRAMAZIO
KOHLER
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E E A**

Week 1

Mesh Subdivision

Rule-based design

Week 2

Implicit Modeling

Distance functions

Implicit what?

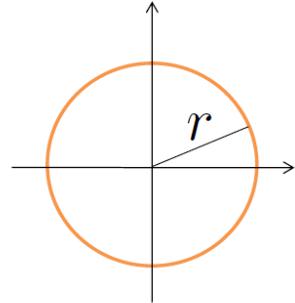
Implicit representation of geometry

Points aren't known directly, but satisfy some **relationship**

E.g., unit sphere is all points $p(x,y,z)$ such that $x^2 + y^2 + z^2 = 1$

$$f(x, y) = x^2 + y^2 - r^2$$

Frep Vs Brep



$$f(x, y, z) = x^2 + y^2 + z^2 - r^2$$



Implicit representation of geometry

More generally, $f(x,y,z) = 0$: on the surface

$f(x,y,z) < 0$: inside

$f(x,y,z) > 0$: outside

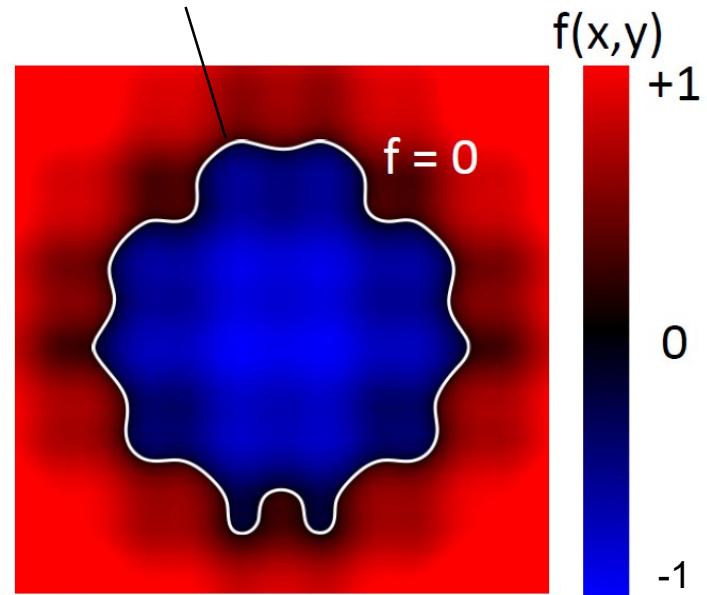
Volume representation

Easy inside / outside test

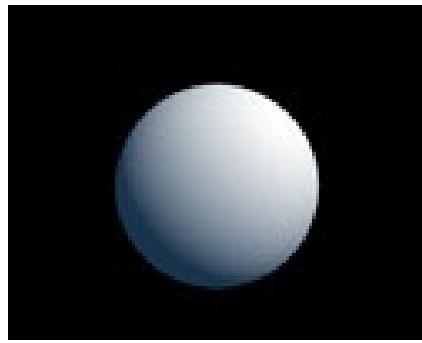
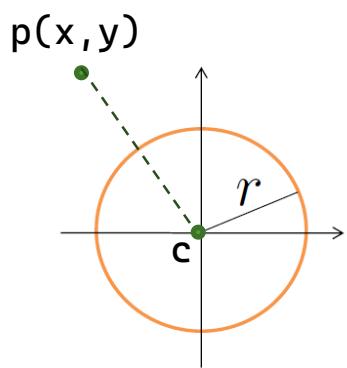
Unambiguous separation of inside/outside

Always ‘watertight’ shapes

isosurface (3D)
isoline (2D)



Signed distance function (SDF)



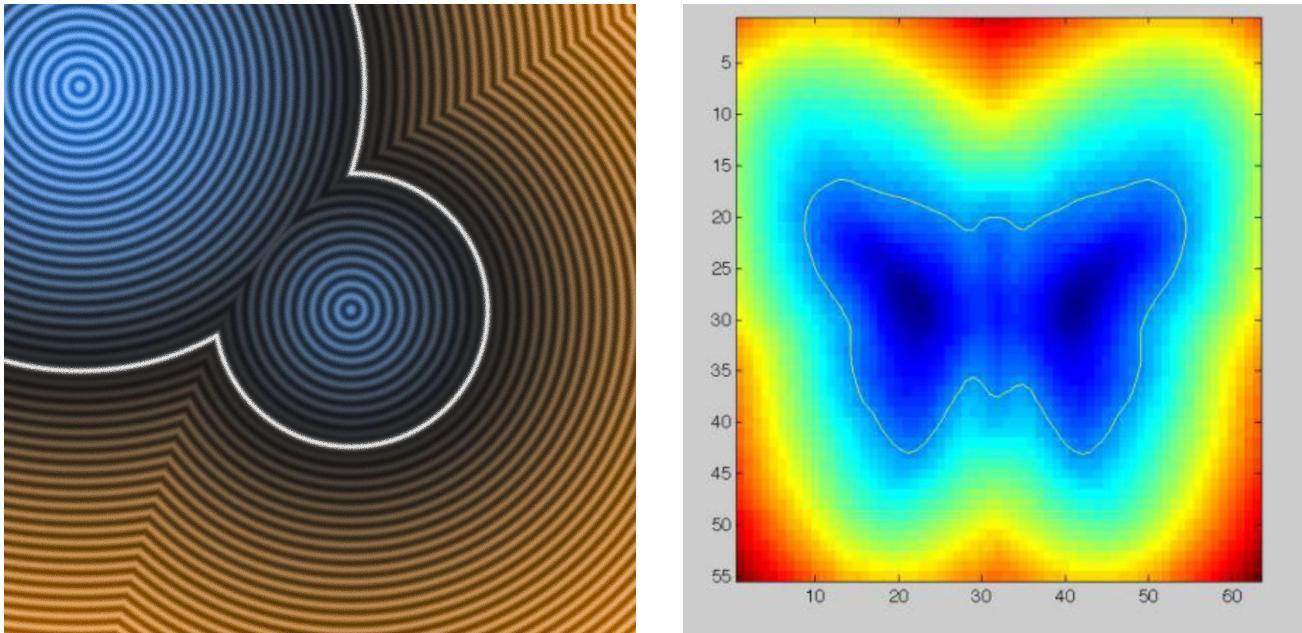
$$f(x, y) = \text{dist}(p(x, y), c) - r$$

$f(x, y) > 0 \rightarrow \text{outside}$

$f(x, y) = 0 \rightarrow \text{on the surface}$

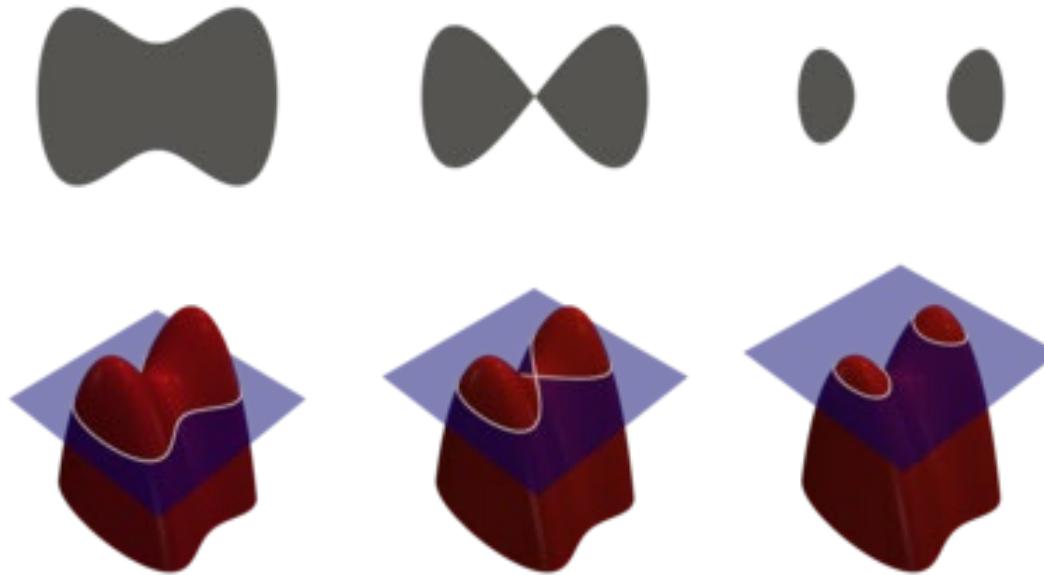
$f(x, y) < 0 \rightarrow \text{inside}$

Signed distance field (SDF)

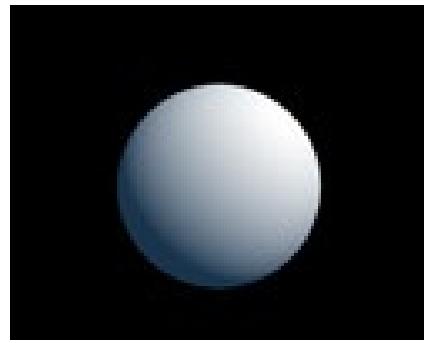


- **Offsetting** becomes very easy: to offset $f(x,y)$ just do $f(x,y) = f(x,y) \pm v$
- **Changes in topology** do not pose any problem

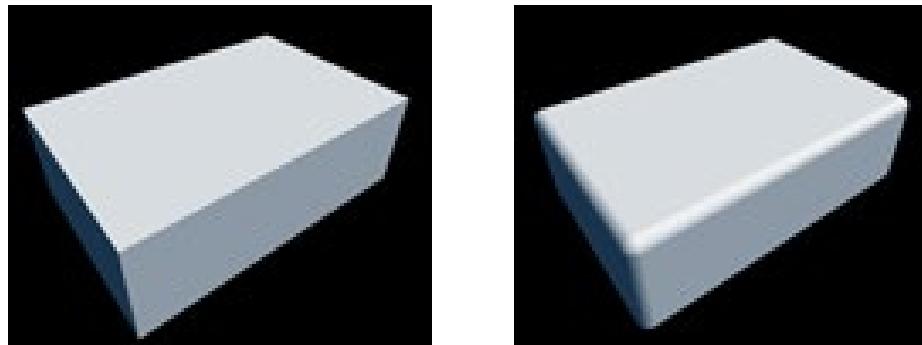
Level set



Primitives

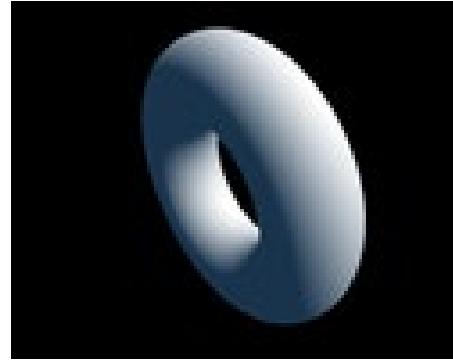

$$\text{dist}(\mathbf{p}(x,y,z), \mathbf{c}) - \text{radius} = 0$$

Primitives



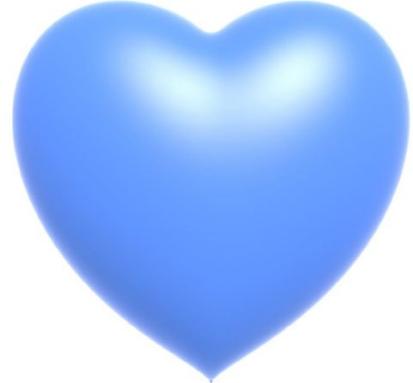
```
q = abs(p) - b  
dist(max(q,0.0)) + min(max(q.x,max(q.y,q.z)),0.0) - radius = 0
```

Primitives



```
q = vec2(length(p.xz)-t.x,p.y)
length(q)-t.y = 0
```

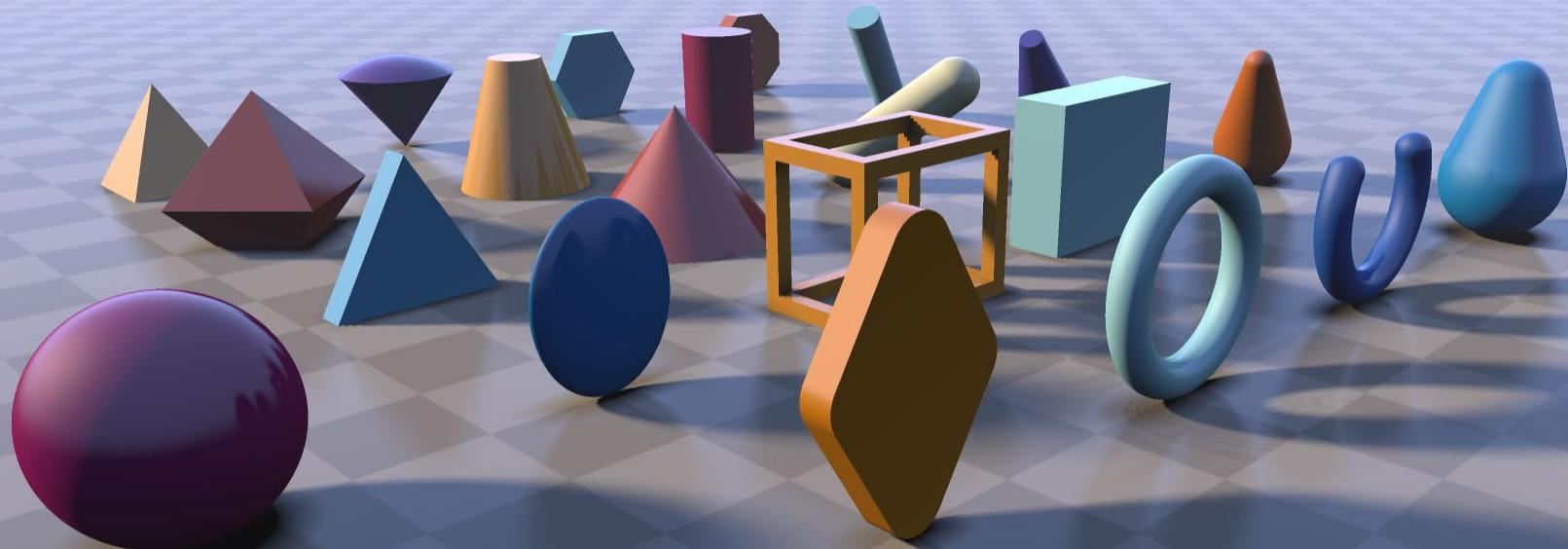
Primitives



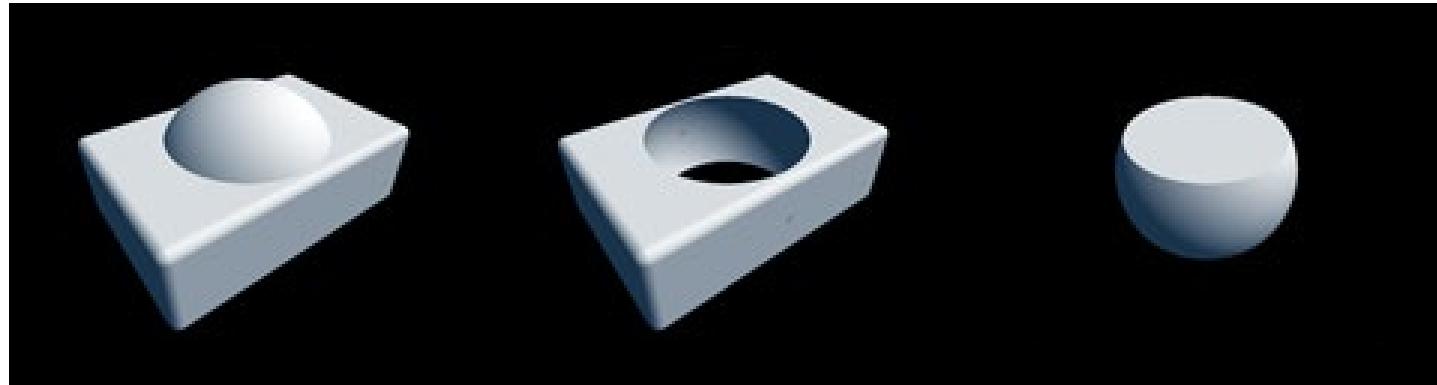
$$(x^2 + \frac{9y^2}{4} + z^2 - 1)^3 = \\ x^2z^3 + \frac{9y^2z^3}{80}$$

Making a heart with mathematics: <https://www.youtube.com/watch?v=aNR4n0i2ZIM>

Primitives



Boolean combinations



union

subtraction

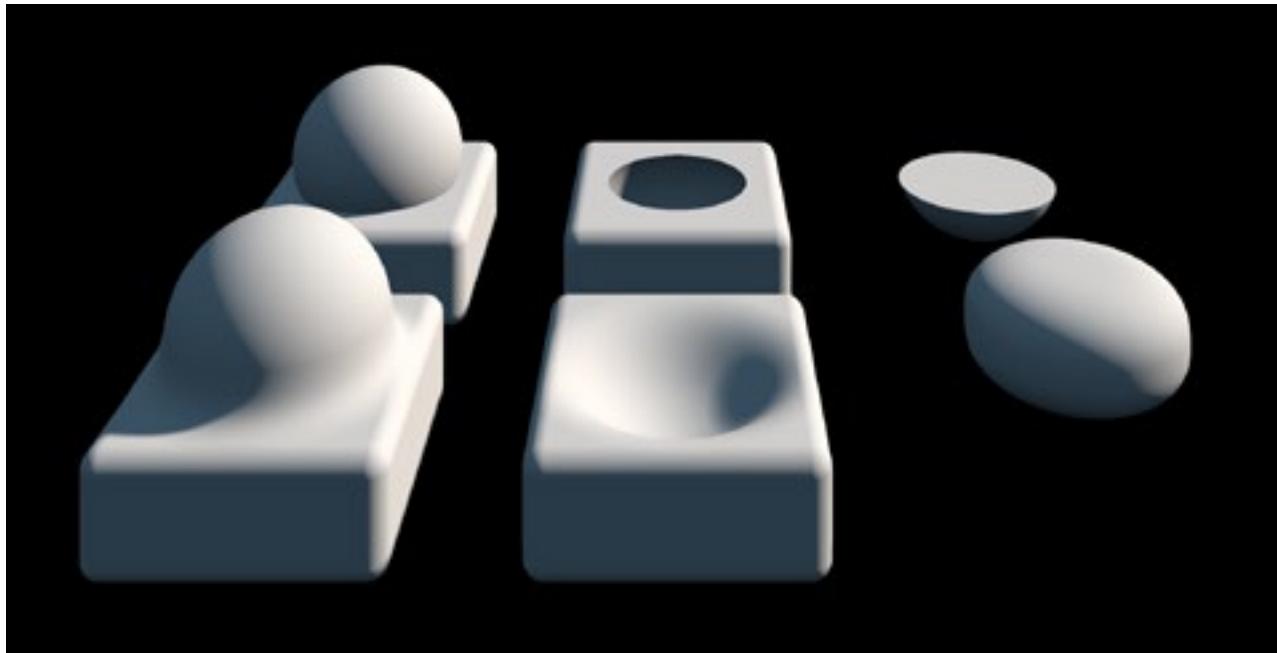
intersection

$$f(x) = \min(\text{sphere}(x), \text{box}(x))$$

$$f(x) = \max(-\text{sphere}(x), \text{box}(x))$$

$$f(x) = \max(\text{sphere}(x), \text{box}(x))$$

Boolean combinations

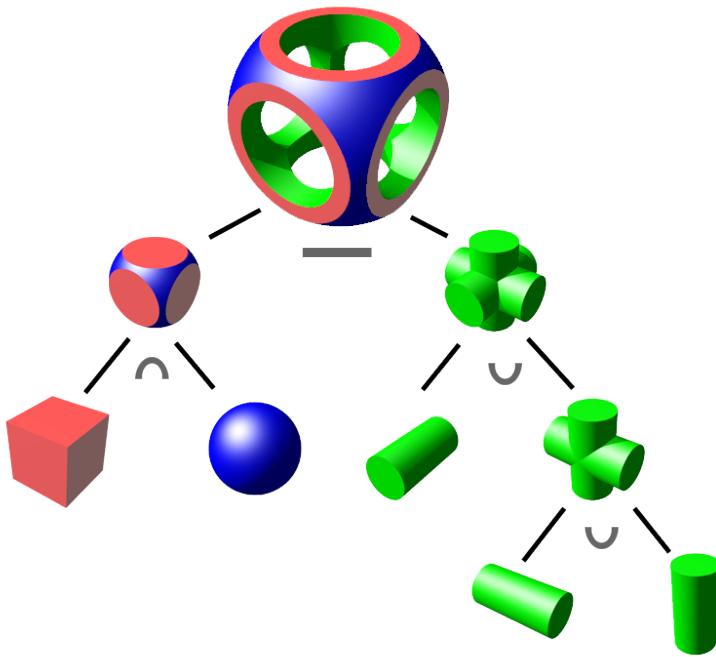


smooth union

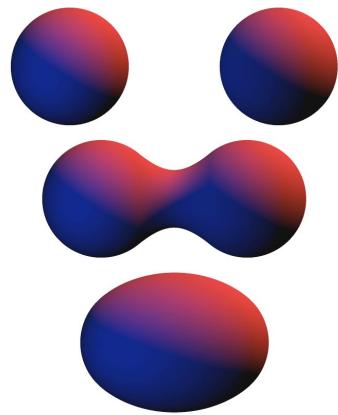
smooth subtraction

smooth intersection

Constructive solid geometry tree



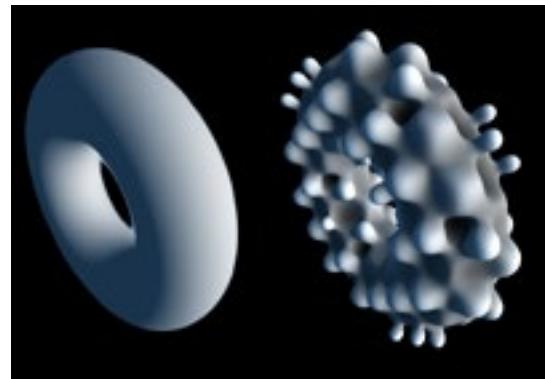
Metaballs, blobs



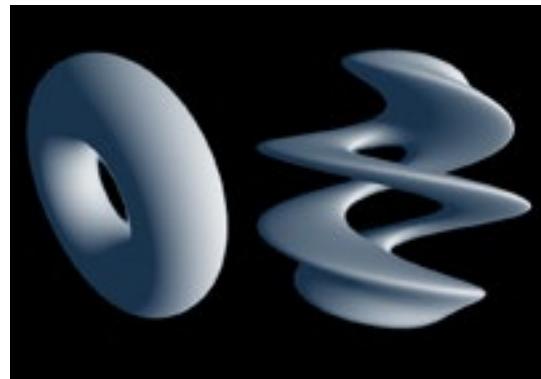
$$\sum_{i=0}^m \text{metaball}_i(x, y, z) \leq \text{threshold}$$



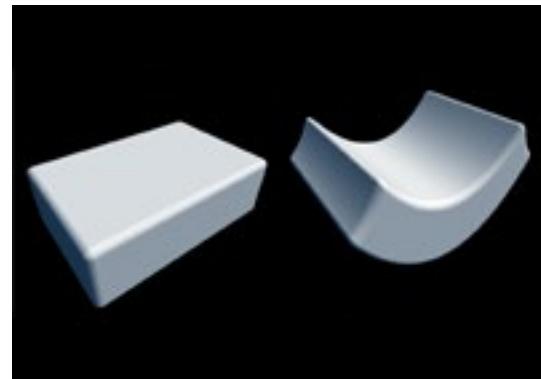
Deformations, distortions



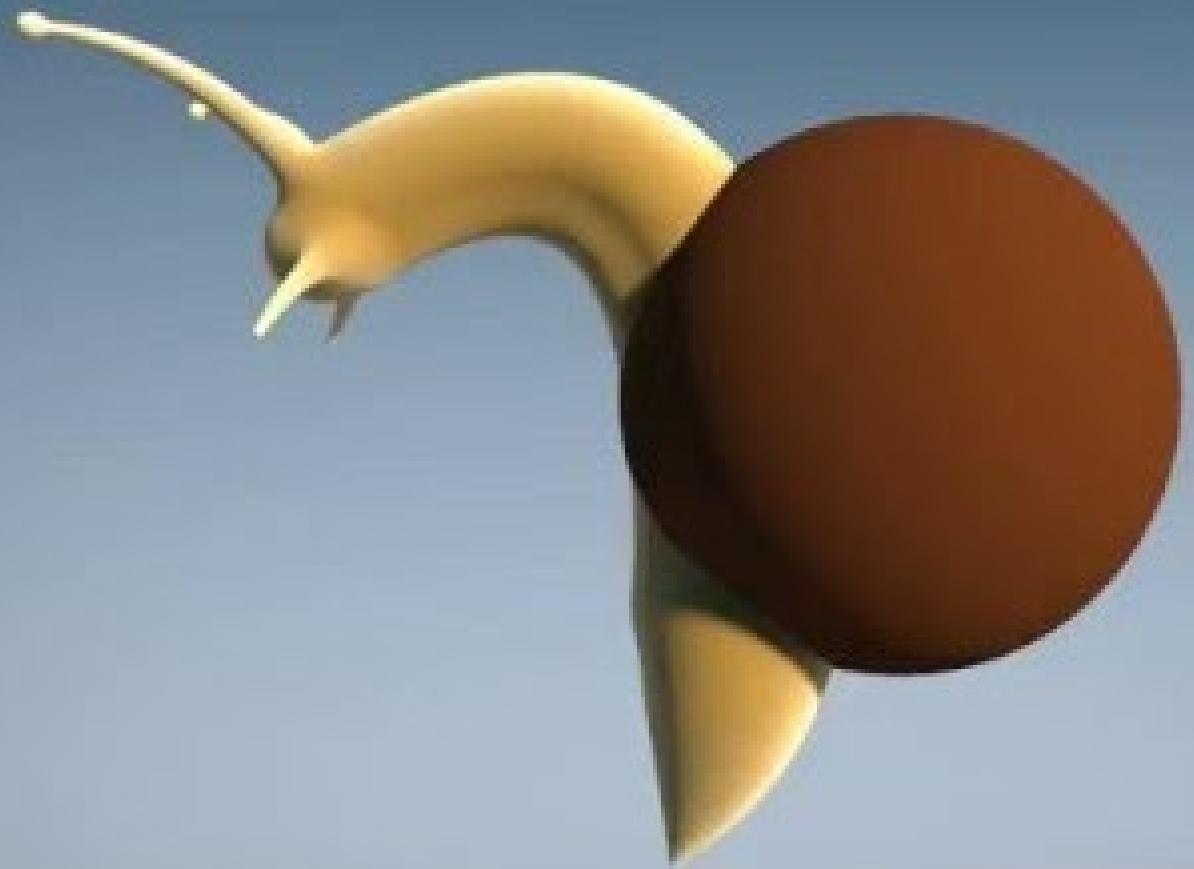
displacement



twist

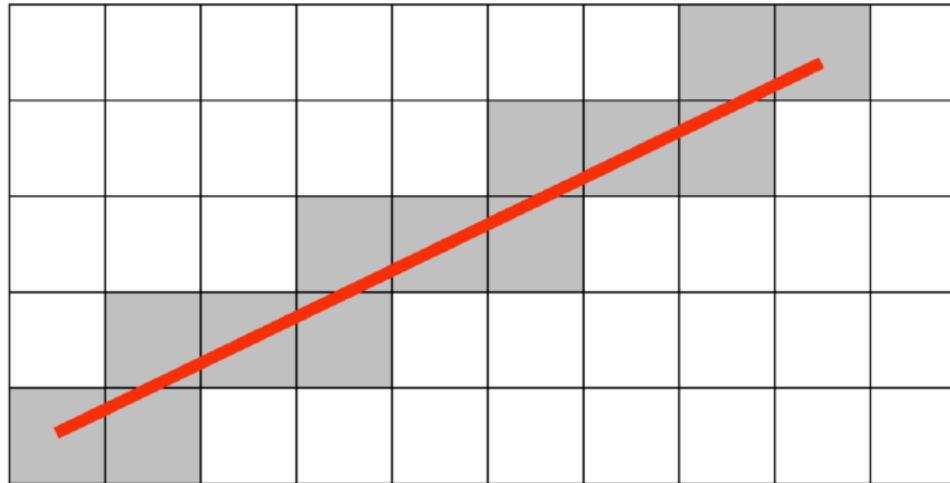


bending

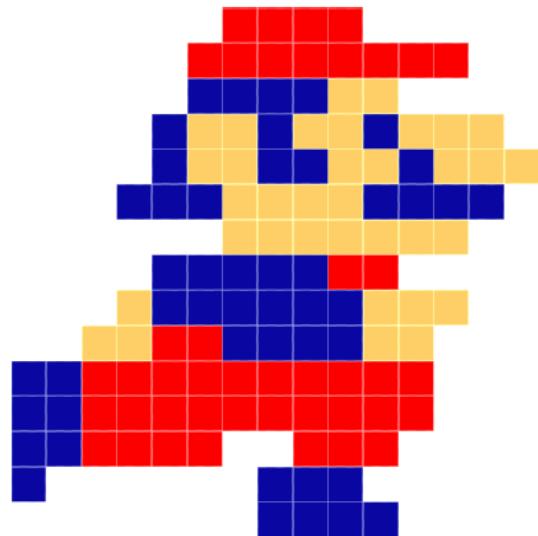


From the continuous to the discrete world

Discretization



Discretization



Regular grid, 2D

3.1	2.4	1.8	1.4	1.1	1.0	1.1	1.4	1.8	2.4	3.1	3.8	4.6	5.4	6.3
2.4	1.7	1.0	0.5	0.1	0.0	0.1	0.5	1.0	1.7	2.4	3.2	4.1	4.9	5.8
1.8	1.0	0.2	-0.4	-0.8	-1.0	-0.8	-0.4	0.2	1.0	1.8	2.7	3.6	4.5	5.5
1.4	0.5	-0.4	-1.2	-1.8	-2.0	-1.8	-1.2	-0.4	0.5	1.4	2.3	3.3	4.2	5.2
1.1	0.1	-0.8	-1.8	-2.6	-3.0	-2.6	-1.8	-0.8	0.1	1.1	2.1	3.1	4.1	5.1
1.0	0.0	-1.0	-2.0	-3.0	-4.0	-3.0	-2.0	-1.0	0.0	1.0	2.0	3.0	4.0	5.0
1.1	0.1	-0.8	-1.8	-2.6	-3.0	-2.6	-1.8	-0.8	0.1	1.1	2.1	3.1	4.1	5.1
1.4	0.5	-0.4	-1.2	-1.8	-2.0	-1.8	-1.2	-0.4	0.5	1.4	2.0	2.0	2.0	2.0
1.8	1.0	0.2	-0.4	-0.8	-1.0	-0.8	-0.4	0.2	1.0	1.8	2.7	3.6	4.5	5.5
2.4	1.7	1.0	0.5	0.1	0.0	0.1	0.5	1.0	1.7	2.4	3.2	4.1	4.9	5.8
3.1	2.4	1.8	1.4	1.1	1.0	1.1	1.4	1.8	2.4	3.1	3.8	4.6	5.4	6.3
3.8	3.2	2.7	2.3	2.1	2.0	2.1	2.3	2.7	3.2	3.8	4.5	5.2	6.0	6.8
4.6	4.1	3.6	3.3	3.1	3.0	3.1	3.3	3.6	4.1	4.6	5.2	5.9	6.6	7.4
5.4	4.9	4.5	4.2	4.1	4.0	4.1	4.2	4.5	4.9	5.4	6.0	6.6	7.3	8.0
6.3	5.8	5.5	5.2	5.1	5.0	5.1	5.2	5.5	5.8	6.3	6.8	7.4	8.0	8.7

3.1	2.4	1.8	1.4	1.1	1.0	1.1	1.4	1.8	2.4	3.1	3.8	4.6	5.4	6.3
2.4	1.7	1.0	0.5	0.1	0.0	0.1	0.5	1.0	1.7	2.4	3.2	4.1	4.9	5.8
1.8	1.0	0.2	-0.4	-0.8	-1.0	-0.8	-0.4	0.2	1.0	1.8	2.7	3.6	4.5	5.5
1.4	0.5	-0.4	-1.2	-1.8	-2.0	-1.8	-1.2	-0.4	0.5	1.4	2.0	2.0	2.0	2.0
1.1	0.1	-0.8	-1.8	-2.6	-3.0	-2.6	-1.8	-0.8	0.1	1.0	1.0	1.0	1.0	2.0
1.0	0.0	-1.0	-2.0	-3.0	-4.0	-3.0	-2.0	-1.0	0.0	0.0	0.0	0.0	1.0	2.0
1.1	0.1	-0.8	-1.8	-2.6	-3.0	-2.6	-1.8	-0.8	0.1	1.0	1.0	1.0	1.0	2.0
1.4	0.5	-0.4	-1.2	-1.8	-2.0	-2.0	-2.0	-2.0	0.0	1.0	2.0	2.0	2.0	2.0
1.8	1.0	0.2	-0.4	-0.8	-1.0	-0.8	-0.4	0.2	1.0	1.8	2.7	3.6	4.5	5.5
2.4	1.7	1.0	0.5	0.1	0.0	0.1	0.5	1.0	1.7	2.4	3.2	4.1	4.9	5.8
3.1	2.4	1.8	1.4	1.1	1.0	1.1	1.4	1.8	2.4	3.1	3.8	4.6	5.4	6.3
3.8	3.2	2.7	2.3	2.1	2.0	2.1	2.3	2.7	3.2	3.8	4.5	5.2	6.0	6.8
4.6	4.1	3.6	3.3	3.1	3.0	3.1	3.3	3.6	4.1	4.6	5.2	5.9	6.6	7.4
5.4	4.9	4.5	4.2	4.1	4.0	4.1	4.2	4.5	4.9	5.4	6.0	6.6	7.3	8.0
6.3	5.8	5.5	5.2	5.1	5.0	5.1	5.2	5.5	5.8	6.3	6.8	7.4	8.0	8.7

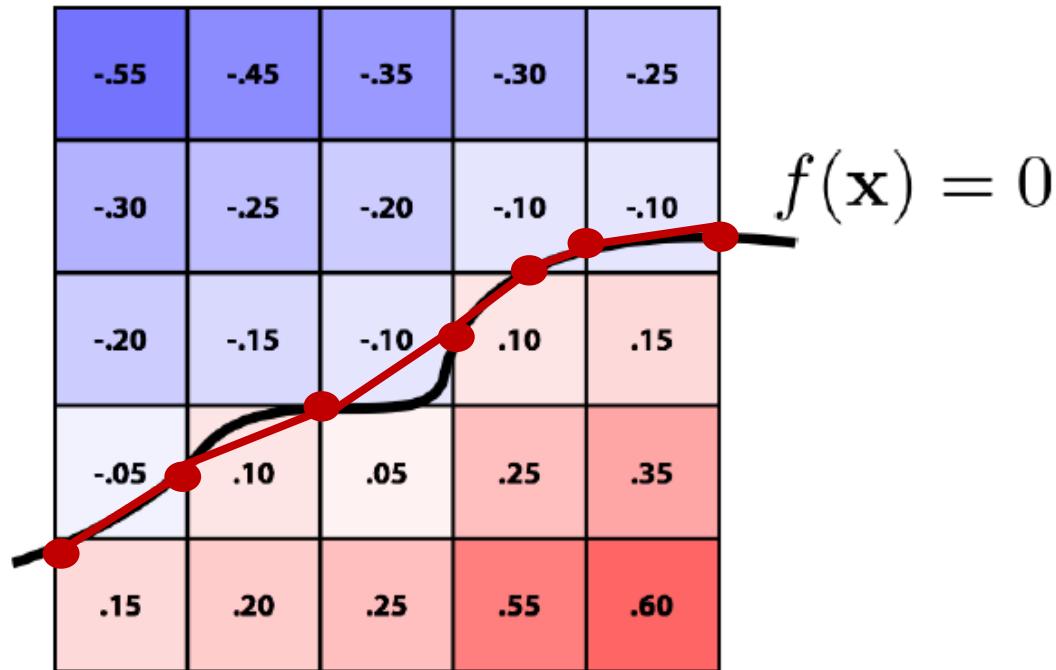
3.1	2.4	1.8	1.4	1.1	1.0	1.1	1.4	1.8	2.4	3.1	3.8	4.6	5.4	6.3
2.4	1.7	1.0	0.5	0.1	0.0	0.1	0.5	1.0	1.7	2.4	3.2	4.1	4.9	5.8
1.8	1.0	0.2	-0.4	-0.8	-1.0	-0.8	-0.4	0.2	1.0	1.8	2.7	3.6	4.5	5.5
1.4	0.5	-0.4	-1.2	-1.8	-2.0	-1.8	-1.2	-0.4	0.5	1.4	2.3	3.3	4.2	5.2
1.1	0.1	-0.8	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	1.1	2.1	3.1	4.1	5.1
1.0	0.0	-1.0	-2.0	-3.0	-4.0	-3.0	-2.0	-1.0	0.0	0.0	0.0	0.0	1.0	2.0
1.1	0.1	-0.8	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	0.0	1.1	2.1	3.1	4.1	5.1
1.4	0.5	-0.4	-1.2	-1.8	-2.0	-2.0	-2.0	-2.0	0.0	1.0	2.0	2.0	2.0	2.0
1.8	1.0	0.2	-0.4	-0.8	-1.0	-0.8	-0.4	0.2	1.0	1.8	2.7	3.6	4.5	5.5
2.4	1.7	1.0	0.5	0.1	0.0	0.1	0.5	1.0	1.7	2.4	3.2	4.1	4.9	5.8
3.1	2.4	1.8	1.4	1.1	1.0	1.1	1.4	1.8	2.4	3.1	3.8	4.6	5.4	6.3
3.8	3.2	2.7	2.3	2.1	2.0	2.1	2.3	2.7	3.2	3.8	4.5	5.2	6.0	6.8
4.6	4.1	3.6	3.3	3.1	3.0	3.1	3.3	3.6	4.1	4.6	5.2	5.9	6.6	7.4
5.4	4.9	4.5	4.2	4.1	4.0	4.1	4.2	4.5	4.9	5.4	6.0	6.6	7.3	8.0
6.3	5.8	5.5	5.2	5.1	5.0	5.1	5.2	5.5	5.8	6.3	6.8	7.4	8.0	8.7

2.1	1.4	0.8	0.4	0.1	0.0	0.1	0.4	0.8	1.4	2.1	2.8	3.6	4.0	4.0
1.4	0.7	0.0	-0.5	-0.9	-1.0	-0.9	-0.5	0.0	0.7	1.4	2.2	3.0	3.0	3.0
0.8	0.0	-0.8	-0.6	-0.2	0.0	-0.2	-0.6	-0.8	0.0	0.8	1.7	2.0	2.0	2.0
0.4	-0.5	-0.6	0.2	0.8	1.1	0.8	0.2	-0.6	-0.5	0.4	1.0	1.0	1.0	1.0
0.1	-0.9	-0.2	0.8	1.6	2.0	1.6	0.8	-0.2	-0.9	0.0	0.0	0.0	0.0	0.0
0.0	-1.0	0.0	1.0	2.0	3.0	2.0	1.0	0.0	-1.0	-1.0	-1.0	0.0	0.0	0.0
0.1	-0.9	-0.2	0.8	1.6	2.0	1.6	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4	-0.5	-0.6	0.2	0.8	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.8	0.0	-0.8	-0.6	-0.2	0.0	1.0	2.0	2.0	1.0	0.0	-1.0	0.0	0.0	0.0
1.4	0.7	0.0	-0.5	-1.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2.1	1.4	0.8	0.0	-1.0	-2.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
2.8	2.0	1.0	0.0	-1.0	-2.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
3.0	2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.0	2.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

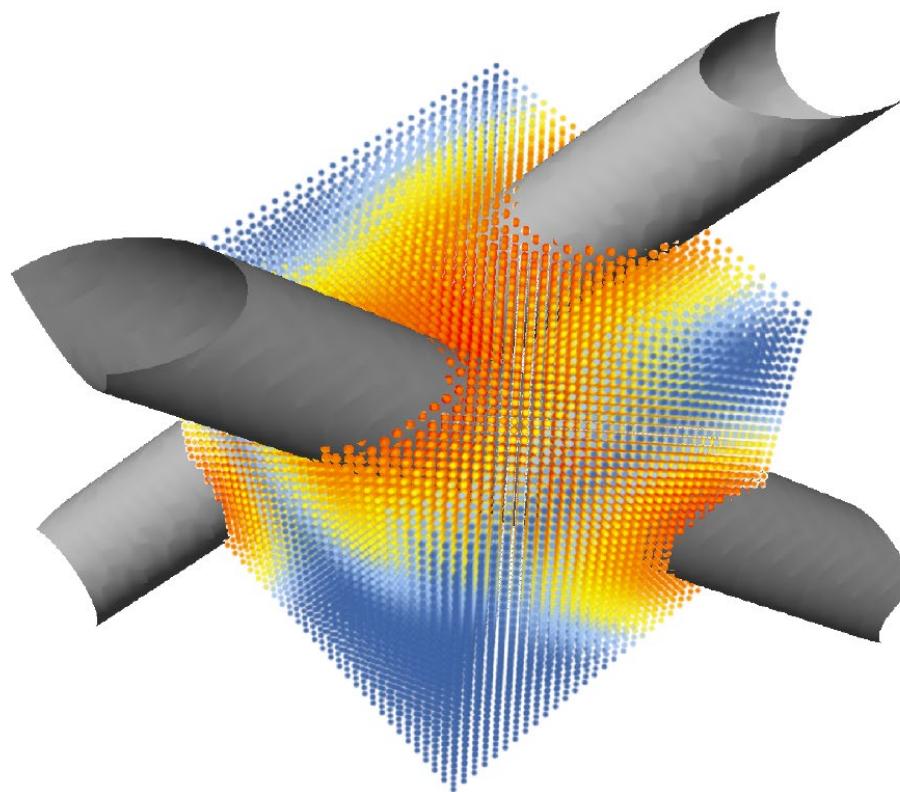
5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
40	30	30	30	30	30	30	30	30	30	30	30	30	30	30
4.0	3.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
4.0	3.0	2.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
4.0	3.0	2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	2.0	3.0	4.0	5.0
4.0	3.0	2.0	1.0	0.0	-1.0	-1.0	-1.0	-0.8	0.1	1.1	2.1	3.1	4.1	5.1
4.0	3.0	2.0	1.0	0.0	-1.0	-1.8	-12	-0.4	0.5	1.4	2.3	3.3	4.2	5.2
4.0	3.0	2.0	1.0	0.0	-1.0	-0.8	-0.4	0.2	1.0	1.8	2.7	3.6	4.5	5.5
4.0	3.0	2.0	1.0	0.1	0.0	0.1	0.5	1.0	1.7	2.4	3.2	4.1	4.9	5.8
4.0	3.0	2.0	1.0	1.1	1.0	1.1	1.4	1.8	2.4	3.1	3.8	4.6	5.6	6.3
4.0	3.2	2.7	2.3	2.1	2.0	2.1	2.3	2.7	3.2	3.8	4.5	5.2	6.0	6.8
46	4.1	3.6	3.3	3.1	3.0	3.1	3.3	3.6	4.1	4.6	5.2	5.9	6.6	7.4
54	4.9	4.5	4.2	4.1	4.0	4.1	4.2	4.5	4.9	5.4	6.0	6.6	7.3	8.0
4.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0

2.6	2.1	1.7	1.3	1.0	1.0	1.3	1.7	2.1	2.6	3.1	3.5	4.0	4.3	
1.8	1.3	0.8	0.4	0.1	-0.1	0.1	0.8	1.3	1.8	2.3	2.8	3.2	3.5	
1.4	0.6	0.0	-0.5	-0.9	-1.1	-0.9	-0.5	0.0	0.6	1.1	1.6	2.0	2.4	2.7
1.1	0.2	-0.7	-1.3	-1.8	-2.1	-1.8	-1.3	-0.7	-0.2	0.2	0.6	0.8	1.8	
0.9	-0.1	-1.1	-2.0	-2.7	-3.1	-2.7	-2.0	-1.3	-0.7	-0.2	0.2	0.6	0.8	1.8
0.8	-0.2	-1.2	-2.2	-3.2	-4.1	-3.2	-2.5	-1.8	-1.3	-0.8	-0.5	-0.2	0.8	1.8
0.9	-0.1	-1.1	-2.0	-2.9	-3.5	-3.2	-2.7	-2.2	-1.8	-1.4	-1.2	-0.2	0.8	1.8
1.1	0.2	-0.7	-1.6	-2.3	-3.1	-2.6	-2.3	-2.1	-1.1	-0.1	0.8	1.8		
1.4	0.6	-0.3	-1.0	-1.7	-2.3	-2.7	-3.3	-3.2	-2.1	-1.1	-0.1	0.		

Regular grid, 2D

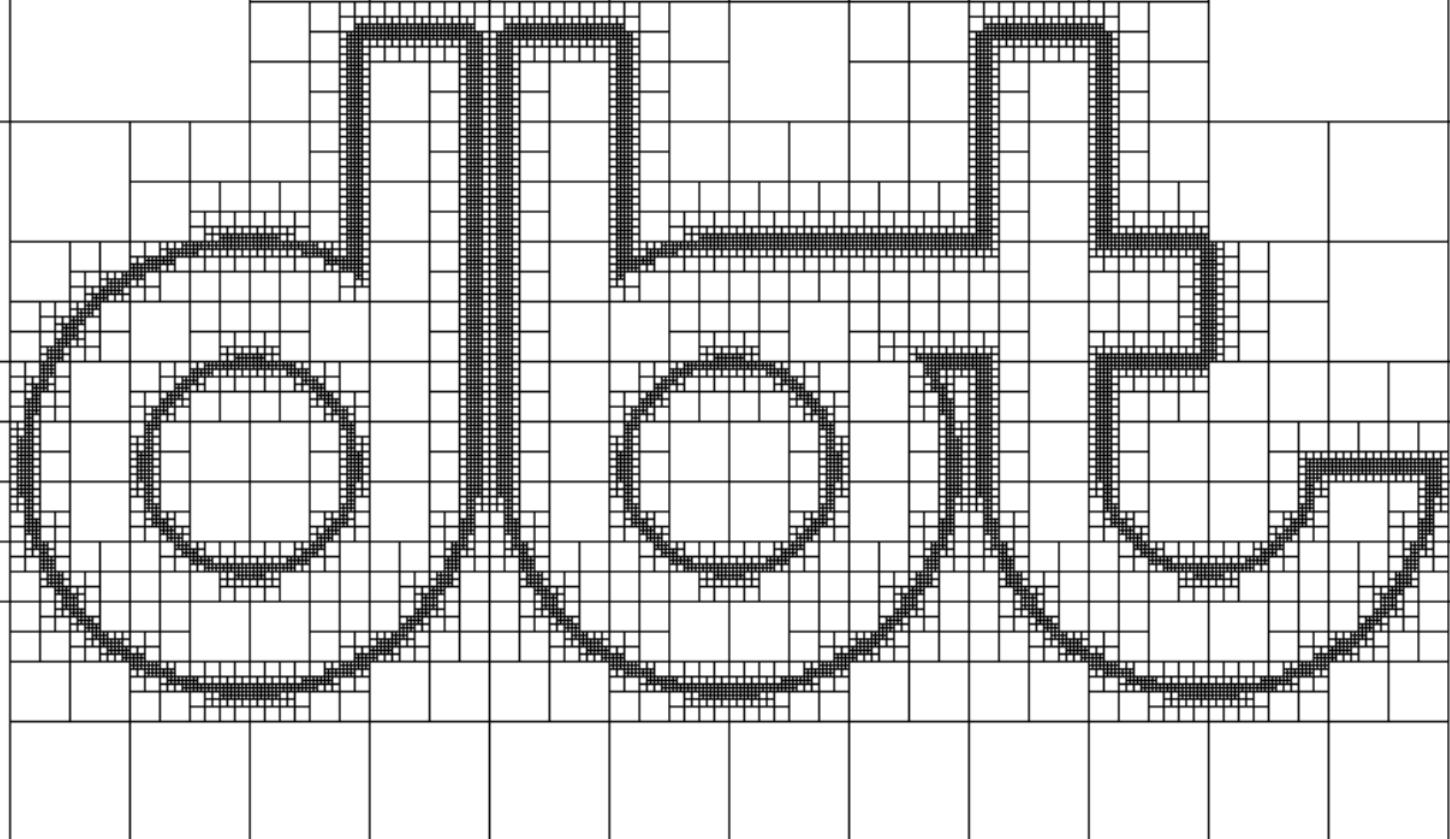


Regular grid, 3D



dot

Octree



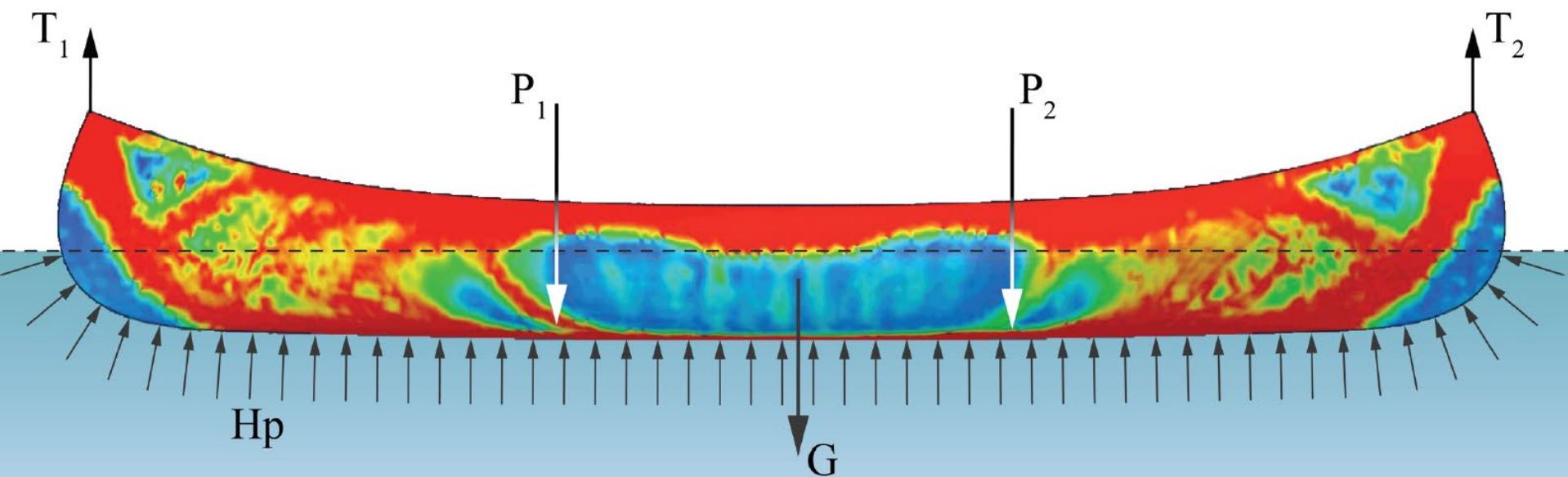
Applications

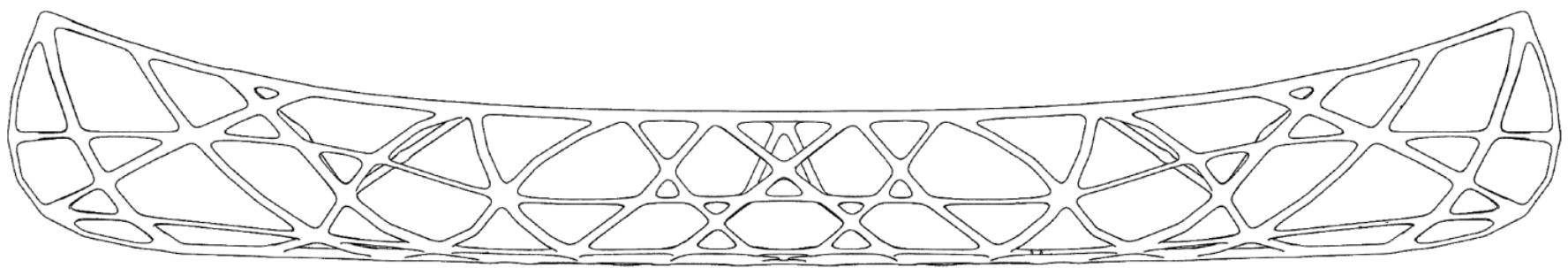
Jipa, A., Bernhard, M., Ruffray, N., Wangler, T., Flatt, R., & Dillenburger, B. (2017). *skeleTHon Formwork*. In SIGraDi 2017, XXI Congreso de la Sociedad Ibero-americana de Gráfica Digital. Concepción, Chile.

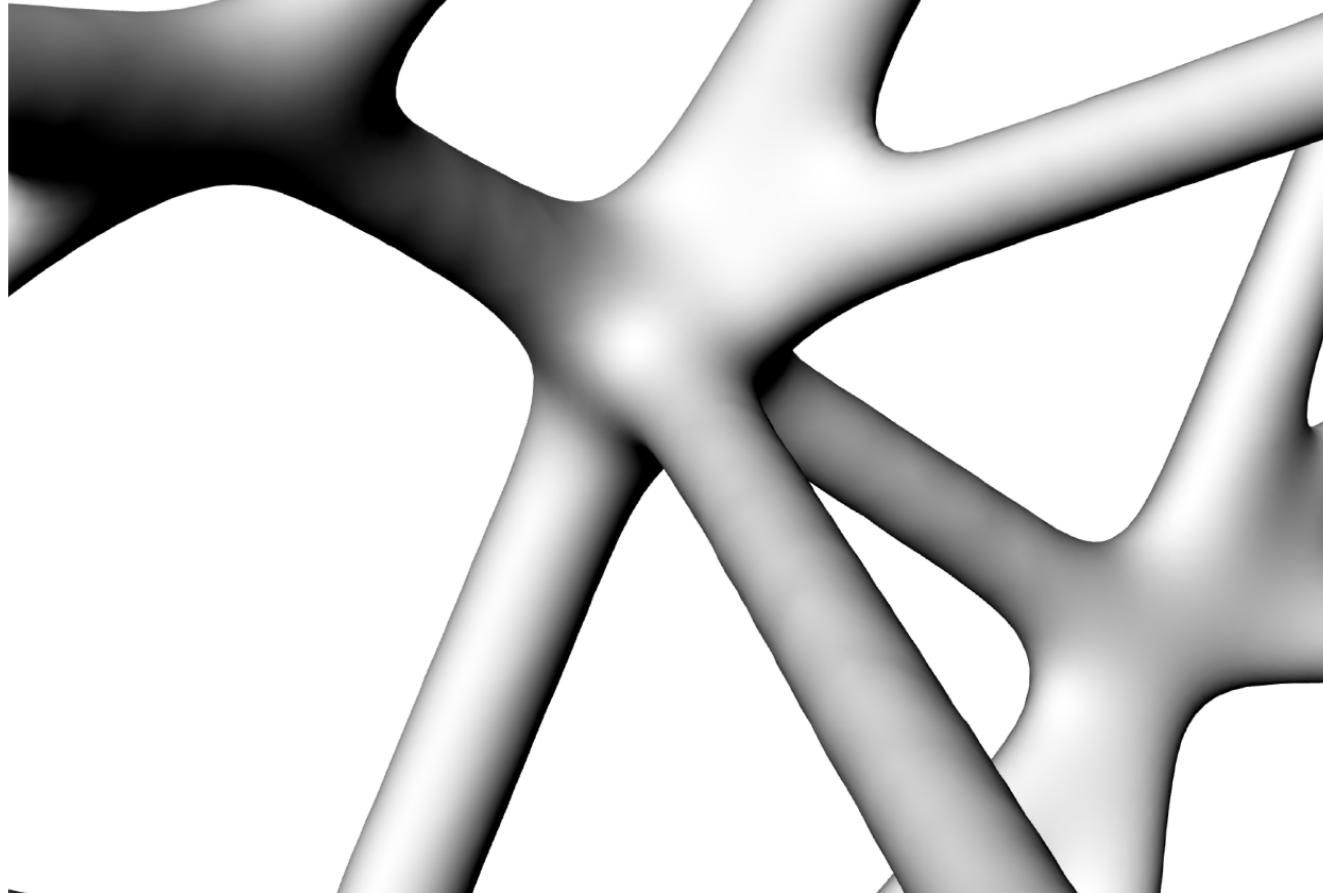
Jipa, A., Bernhard, M., Ruffray, N., Wangler, T., Flatt, R., & Dillenburger, B. (2017). *skeleTHon Formwork 3D Printed Plastic Formwork for Load-Bearing Concrete Structures*. In Blucher Design Proceedings (pp. 345–352). São Paulo

Jipa, A., Bernhard, M., & Dillenburger, B. (2017). *Submillimeter Formwork*. 2017 TxA Emerging Design + Technology. Austin, Texas, USA.

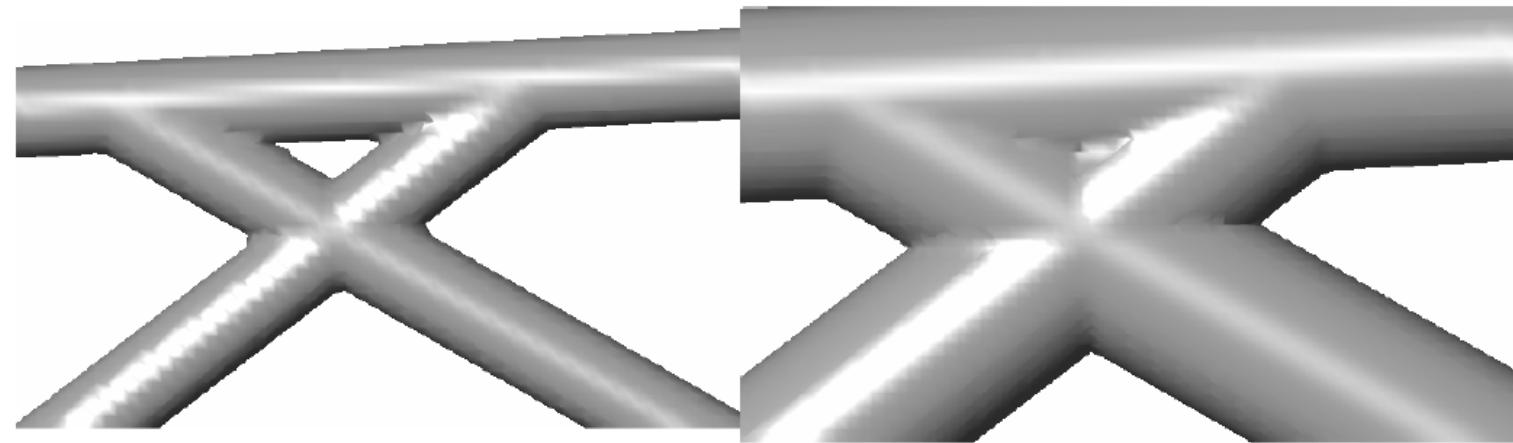












MAS 17/18



Angela Yoo, Jetana Ruangjun
Frank Lin, Jun Su



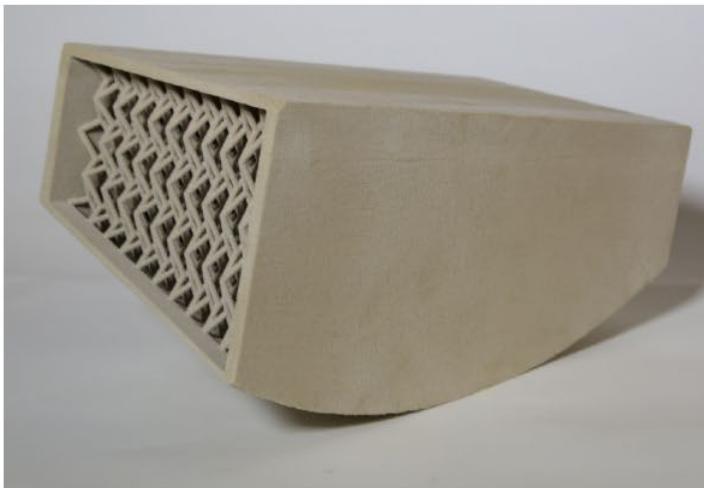
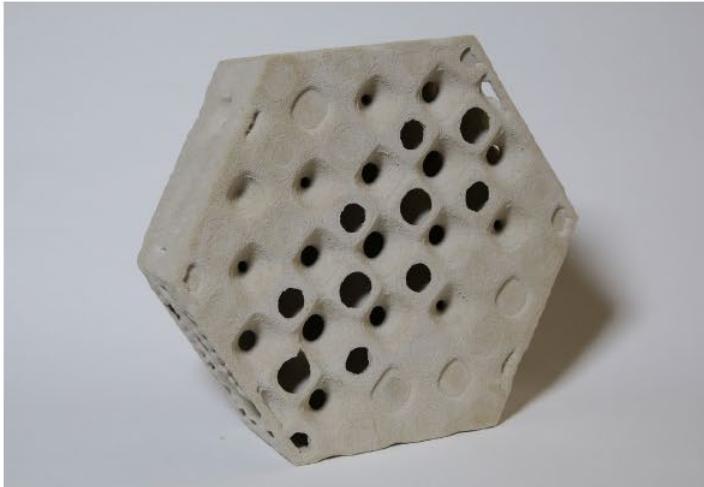
dbt



Sahar Barzani, Fernando Cena, Haruna Okawa
Rafael Pastrana, Francisco Regalado



ETH zürich



top left: Noor Khader and Yoana Taseva

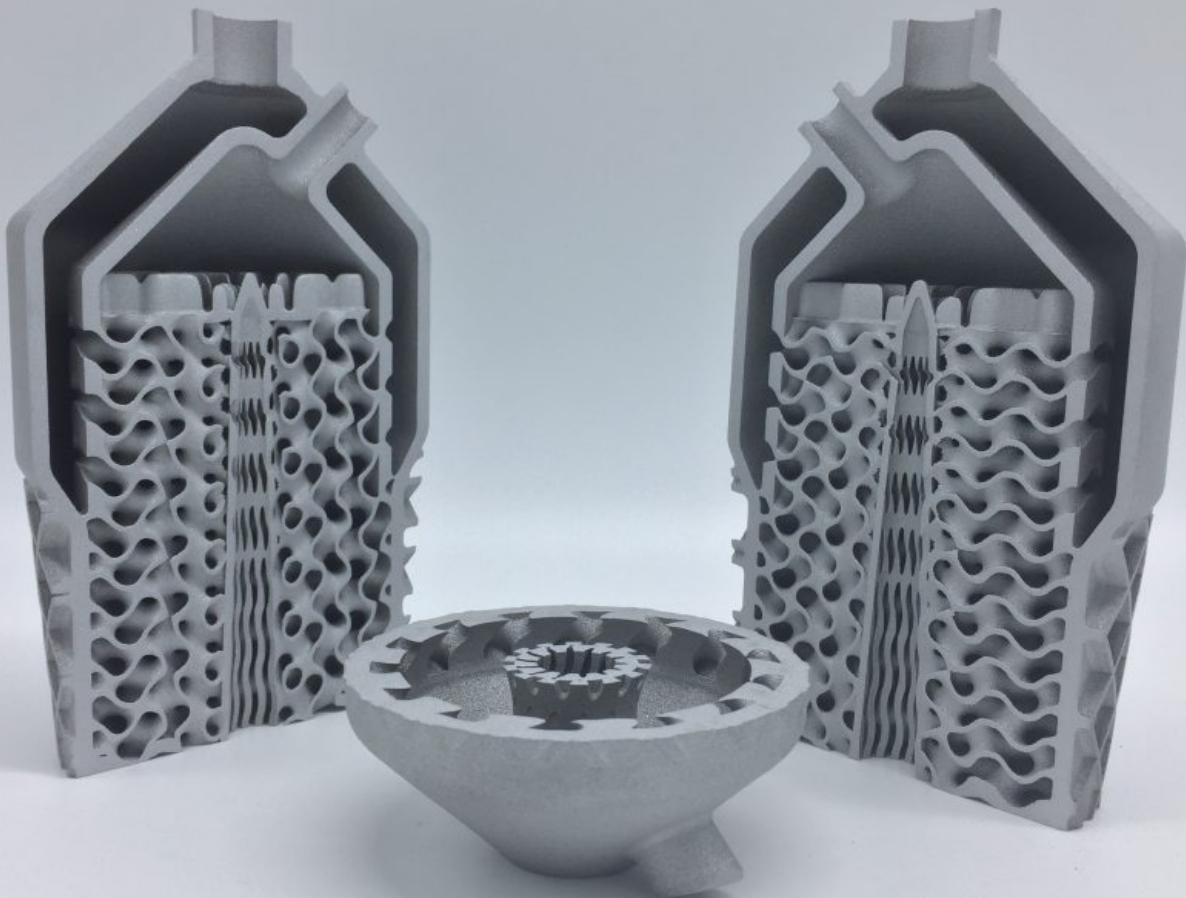
top right: Nik Eftekhar Olivo and Wenqian Yang

bottom left: Antonio Barney and Nicolas Feihl

bottom right: Keerthana Udaykumar and Eleni Skevaki

photos: Tom Mundy

NTopology





Fermi Paradox by Mercury, Demoscene 2016

Source code:

https://www.shadertoy.com/view/1dScDh?fbclid=IwAR1OgEUqqAjcWVDvfKdc_AKm6nttKRaX7QX1lwPeMB6zGsZeLkPMzLlugkw

Tutorial:

<https://www.youtube.com/watch?v=-pdSjBPH3zM>

Iniqo Quiles, Greek temple

Summary

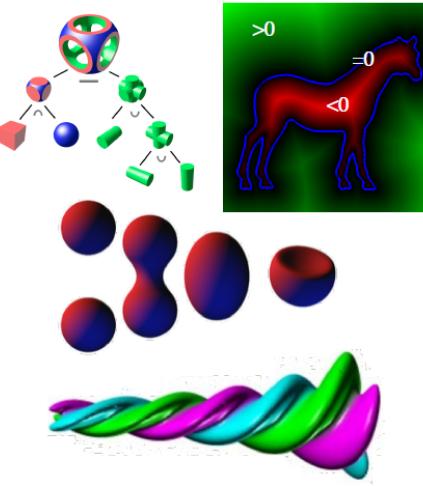
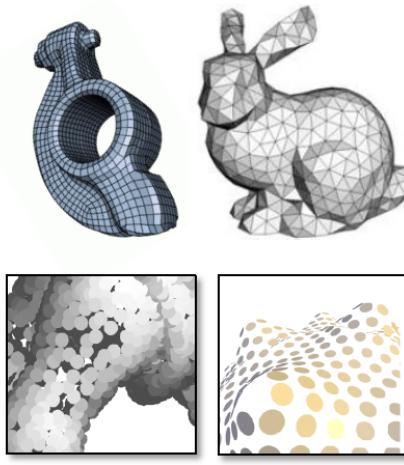
Implicit	Discrete/Sampled
 <ul style="list-style-type: none">• Metaballs/blobs• Distance fields	 <ul style="list-style-type: none">• Meshes• Point set surfaces

Illustration from igl lab, ETH

Comparison of geometry representation methods

Very compact and lightweight description

implicit

discrete / sampled

Comparison of geometry representation methods

Easy to find points that lie exactly on the surface

implicit

Very compact and lightweight description

discrete / sampled

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Easy to determine if a point is inside
or outside of the surface

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Easy to handle changes in topology

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Thank you