Question 3 (Total: 10 marks)

a) (i) & (ii)

Southwest Baking Company is part of the baking products manufacturing industry in the United States which was founded in 2006. The objective of the company is to provide and deliver limited yet high quality baked goods such as 9-Grain Wheat Bread, 9-Grain Honey Oat Bread, FlatBread, Italian Bread and also the Italian Herbs and Cheese Bread to Subway stores in different states.

As the Covid-19 pandemic occurred during the start of 2020, the whole manufacturing industry has been significantly impacted where the food manufacturing industry is one of them. This pandemic has affected the economy of the country and also resulted in supply chain disruptions and mass production shutdowns. Due to the Movement Control Order (MCO) enforced by the government, operating hours of a business and movement of consumers have been restricted. Therefore, retailers no longer purchase a large quantity of breads as before. This has caused a drop in the sales of Southwest Baking Company. In addition, the order cancellations by retailers has caused the breads produced to reach their expiry date and become moldy. The company is forced to throw away the bread which leads to revenue loss. Because of the decrement in sales revenue, the quantity of raw materials purchase will need to be taken into consideration by the company. During the period of the MCO, other baking companies also faced the same problem like Southwest Baking Company. Therefore, the companies have applied many strategies to increase their sales. For example, they held many promotional activities and released many new products in this period. So, the significant reduction in the sales has deeply affected the financial and operation of the Southwest Baking company.

The main objective of this pure integer programming model is to help the company to make a better decision on the production of the bread so that the company can increase the sales of the company and earn the maximum profit in this competitive frozen bread industry. From this model, the optimal product mix and maximum profit that can be earned will be calculated. This resulted in more concentration on the production of that particular product.

In order to acquire more accurate and precise data, we have done a lot of related research including the basic bread recipe ingredients, golden ratio of basic bread ingredients weight and also the resemblance amount of ingredients needed for a Subway bread. Next, we also modify the profit prices of these breads, ingredients needed as well as their constraints. The tables below indicate the profit price of the breads, the amount of ingredients needed by these breads and the cost of each raw material and their availability per week.

Product	9-Grain Wheat Bread	9-Grain Honey Oat Bread	FlatBread	Italian Bread	Italian Herbs and Cheese Bread
Profit Price (RM)	7	9	10	8	12

Table 1 The profit price of the breads

Product	Amount of ingredients needed (g)				
	9-Grain Wheat Bread	9-Grain Honey Oat Bread	FlatBread	Italian Bread	Italian Herbs and Cheese Bread
Wheat Flour	450	600	550	500	700
Yeast	30	40	30	30	50
Water	300	300	250	300	250
Milk	0	0	150	0	150
Salt	15	25	10	10	25
Sugar	30	50	20	20	50

Table 2 The amount of ingredients needed by these breads

Raw Material	Cost per kilogram (RM)	Availability per week (g) (approx.)	
Wheat Flour	15	635200	
Yeast	10	42000	
Water	3	332000	

Milk	7	65000
Salt	7	20000
Sugar	6	38000

Table 3 The cost of each raw material and their availability per week

Product	Ordered Quantity
9-Grain Wheat Bread	300
9-Grain Honey Oat Bread	200
FlatBread	150
Italian Bread	250
Italian Herbs and Cheese Bread	100

Table 4 The fixed ordered quantity of bread of the customer in every week

From the Table 1, we know that Southwest Baking Company can make a profit of RM 7 of a 9-Grain Wheat Bread, RM 9 of a 9-Grain Honey Oat Bread, RM 10 of a FlatBread, RM 8 of a Italian Bread and RM 12 of a Italian Herbs and Cheese Bread.

The stated data above enabled us to obtain the objective function as below:

Maximum Profit,
$$P = 7x1 + 9x2 + 10x3 + 8x4 + 12x5$$

where P = Total profit gained (RM)

x1 = Number of 9-Grain Wheat Bread to be produced

x2 = Number of 9-Grain Honey Oat Bread to be produced

x3 = Number of FlatBread to be produced

x4 = Number of Italian Bread to be produced

x5 = Number of Italian Herbs and Cheese Bread to be produced

The equations based on the ingredients needed, availability of the ingredients and the ordered quantity of each bread are as below:

```
1. 450x1 + 600x2 + 550x3 + 500x4 + 700x5
                                                     635200 (wheat flour)
                                              \leq
2. 30x1 + 40x2 + 30x3 + 30x4 + 50x5
                                              \leq
                                                     42000 (yeast)
3. 300x1 + 300x2 + 250x3 + 300x4 + 250x5
                                                     332000 (water)
                                              \leq
4. 150x3 + 150x5
                                              \leq
                                                     65000 (milk)
5. 15x1 + 25x2 + 10x3 + 10x4 + 25x5
                                              \leq
                                                     20000 (salt)
6. 30x1 + 50x2 + 20x3 + 20x4 + 50x5
                                              \leq
                                                     38000 (sugar)
                 300 (minimum number of 9-Grain Wheat Bread to be produced)
7. x1
                 200 (minimum number of 9-Grain Honey Oat Bread to be produced)
8. x2
9. x3
                 150 (minimum number of FlatBread to be produced)
          \geq
10. x4
                 250 (minimum number of Italian Bread to be produced)
          >
11. x5
                 100 (minimum number of Italian Herbs and Cheese Bread to be produced)
12. x1, x2, x3, x4, x5 \ge 0; x1, x2, x3, x4, x5 int (non-negativity)
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(iii)

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Console

    ■ Example1.mod 
    □ Example1.run

AMPL
                                                                    var x1 >= 0 integer;
ampl: include Example1.mod
                                                                    var x2 >= 0 integer;
CPLEX 20.1.0.0: optimal integer solution; objective 10444
                                                                    var x3 >= 0 integer;
3 MIP simplex iterations
                                                                    var x4 >= 0 integer;
0 branch-and-bound nodes
                                                                     var x5 >= 0 integer;
x1 = 300
x2 = 200
                                                                     maximize p: 7*x1 + 9*x2 + 10*x3 + 8*x4 + 12*x5;
x3 = 296
x4 = 250
                                                                    s.t. C1: 450*x1 + 600*x2 + 550*x3 + 500*x4 + 700*x5 <= 635200;
x5 = 132
                                                                    s.t. C2: 30*x1 + 40*x2 + 30*x3 + 30*x4 + 50*x5 <= 42000;
p = 10444
                                                                    s.t. C3: 300*x1 + 300*x2 + 250*x3 + 300*x4 + 250*x5 <= 332000;
                                                                    s.t. C4: 150*x3 + 150*x5 <= 65000;
ampl:
                                                                     s.t. C5: 15*x1 + 25*x2 + 10*x3 + 10*x4 + 25*x5 <= 20000;
                                                                     s.t. C6: 30*x1 + 50*x2 + 20*x3 + 20*x4 + 50*x5 <= 38000;
                                                                     s.t. C7: x1 >= 300;
                                                                    s.t. C8: x2 >= 200:
                                                                    s.t. C9: x3 >= 150:
                                                                    s.t. C10: x4 >= 250:
                                                                    s.t. C11: x5 >= 100;
                                                                     option solver cplex;
                                                                    display x1, x2, x3, x4, x5, p;
```

We have successfully determined the optimal solution for the Southwest Baking Company production by using a pure integer programming method. From the result generated by the AMPL, it is profitable and desirable for the company to concentrate more on the production of FlatBread and Italian Herbs and Cheese Bread in one week after producing the fixed ordered quantity of bread of the customer. This is because the increase of the number of production of these two breads is able to help the company to earn the maximum profit of RM10444.