In local search, if the optimization function has a constant value in a neighborhood, there will be a problem.

- [x] T
- [ ] F

答案正确：2 分 创建提问

For an optimization problem, given a neighborhood, if its local optimum is also a global optimum, one can reach an optimal solution with just one step of local improvements.

F

Consider a state-flipping algorithm for the Maximum-Cut problem. We say that partitions (A, B) and (A', B') are neighbors under the k-flip rule if (A', B') can be obtained from (A, B) by moving at most k nodes from one side of the partition to the other. If (A, B) is a local optimum under the p-flip rule, it is also a local optimum under the k-flip rule for every $k < p$.

- [x] T
- [ ] F

答案正确：2 分 创建提问

Consider the **Minimum Degree Spanning Tree** problem: Given a connected undirected graph $G(V, E)$, find a spanning tree T whose maximum degree over its vertices is minimized over all spanning trees of G. The problem can be shown to be NP-hard by reduction from the Hamiltonian Path Problem. On the other hand, we can use local search to design approximating algorithms. Denote $d(u)$ as the degree of vertex u on a tree T. Consider the following algorithm:

1. Find an arbitrary spanning tree T of G.
2. If there's some edge $e \in E(G) \setminus E(T)$ with endpoints u, v, and there's some other vertex w on the path between u, v on T such that $max\{d(u), d(v)\} + 1 < d(w)$, then we replace an edge e' incident to w on T with e, i.e. $T := T \cup \{e\} \setminus \{e'\}$.
3. Repeat Step (2) until there's no edge to replace.

It can be shown that this algorithm will terminate at a solution with maximum vertex degree $OPT + O(\log |V|)$. To show the algorithm will terminate in finite steps, a useful technique is to define a nonnegative potential function $\phi(T)$ and to show $\phi(T)$ is strictly decreasing after each step. Which of the following potential functions below satisfies the above requirements?

- [ ] A. $\phi(T) = \sum_{v \in V} d(v)$.
- [ ] B. $\phi(T) = \sum_{(u,v) \in E(T)} \max\{d(u), d(v)\}$.
- [ ] C. $\phi(T) = \sum_{u \in V} \sum_{v \in V, v \neq u} \sum_{w \in V, w \neq u,v} \max\{d(u), d(v), d(w)\}$.
- [x] D. $\phi(T) = \sum_{v \in V} 3^{d(v)}$.

答案正确：3 分 创建提问

** Load balancing problem: **
We have n jobs $j = 1, 2, \ldots, n$ each with processing time p_j being an integer number.
Our task is to find a schedule assigning n jobs to 10 identical machines so as to minimize the makespan (the maximum completion time over all the m

We adopt the following local search to solve the above load balancing problem.

**LocalSearch: **

Start with an arbitrary schedule.

Repeat the following until no job can be re-assigned:

- Let l be a job that finishes last.
- If there exists a machine i such that assigning job l to i allows l finish earlier, then re-assign l to be the last job on machine i.
- If such a machine is not unique, always select the one with the minimum completion time.

We claim the following four statements:

1. The algorithm LocalSearch finishes within polynomial time.
2. The Load-balancing problem is NP-hard.
3. Let OPT be the makespan of an optimal algorithm. Then the algorithm LocalSearch finds a schedule with the makespan at most of 1.8 OPT.
4. This algorithm finishes within $O(n^2)$.

How many statments are correct ?

- [ ] A. 0
- [ ] B. 1
- [ ] C. 2
- [x] D. 3

D

There are n jobs, and each job j has a processing time t_j. We will use a local search algorithm to partition the jobs into two groups A and B, where set A is assigned to machine M_1 and set B to M_2. The time needed to process all of the jobs on the two machines is $T_1 = \sum_{j \in A} t_j, T_2 = \sum_{j \in B} t_j$. The problem is to minimize $|T_1 - T_2|$.

Local search: Start by assigning jobs $1, \ldots, n/2$ to M_1, and the rest to M_2.
The local moves are to move a single job from one machine to the other, and we only move a job if the move decreases the absolute difference in the processing times. Which of the following statement is true?

- A. The problem is NP-hard and the local search algorithm will not terminate.
- B. When there are many candidate jobs that can be moved to reduce the absolute difference, if we always move the job j with maximum t_j, then the local search terminates in at most n moves.
- C. The local search algorithm always returns an optimal solution.
- D. The local search algorithm always returns a local solution with $\frac{1}{2}T_1 \leq T_2 \leq 2T_1$.

答案正确：3 分 创建提问