

the final exam of DM in 2020-2021 (memorized by  
YYF)

June 23, 2022

## 1 judgement question (2\*10)

1. If the statement  $P(n)$  is true for infinitely many positive integers  $n$  and  $P(n+1) \rightarrow P(n)$  if true for all positive integers  $n$ , then  $P(n)$  is true for all positive integers  $n$ .
2.  $\mathbf{C}$ , the set of complex numbers, has the same cardinality as  $\mathbf{R}$ , the set of real numbers. 考查实数集和复数集是否等势
3. The symmetric difference of  $A$  and  $B$ , denoted by  $A \oplus B$ , is the set containing those elements in either  $A$  or  $B$ , but not in both  $A$  and  $B$ . If  $A$ ,  $B$  and  $C$  are sets, and if  $A \oplus C = B \oplus C$ , then  $A = B$
4. Dijkstra's algorithm can be used to find the shortest path in simple graphs with any weight edges.
5. The proposition "If it is not sunny, then it is sunny." is equivalent to "It is sunny."
6. An undirected graph which has  $n$  vertices and  $n-1$  edges is a tree.
7. There is a unique minimal element and maximal element in the poset of positive integers under divisibility relations. 考察偏序集, 最大元与最小元
8. if  $f(x)$  is  $O(g(x))$ , then  $2^{f(x)}$  is  $2^{O(g(x))}$ .
9. the poset  $(\{1, 3, 6, 9, 12\}, |)$  is a lattice
10. I forgot it.

## 2 selection question (2\*10)

1. How many rooted tree with 4 vertexs()
2. a question about the change of domain in proposition (just like is  $\forall xP(x) \wedge \forall xQ(x)$  and  $\forall x(P(x) \wedge Q(x))$  equivalent? 考察量词辖域改变)
3. calculate a prefix form expression 考察前序表达式
4. if a graph has n vertexs and k component, how many edges can it has at most? 考察图的边数的计算
5. find the chromatic numbers of a graph 考察图的着色
6. find a pair of functions that have same order in the four choice above
7. if  $R_1$  and  $R_2$  are both equivalent relations, then which one above is equivalent relation too. ( $R_1 \cap R_2, R_1 \cup R_2, R_1 \oplus R_2, R_1 - R_2$ )
8. find the generating function of the sequence "0,1,0,0,0,1,0,0,0,1,0,0,0....." 考察生成函数
9. I forgot it
10. I forgot it

## 3 filling the bank

1. What's the max level of a full binary tree with 101 vertexs? 考察树的点数与层数的关系
2. find the smallest x that satisfies  $19x \equiv 4 \pmod{161}$  考察解同余方程
3. find the coefficient of the  $8^{th}$  term of  $\frac{1}{1-2x}$  级数
4. change natural language to a proposition 考察自然语言转命题

## 4 question need to reason out the details

1. turn a prefix form of a tree of proposition to a normal tree form, then turn it to the post form and finally prove it is a tautology. 一个前序命题树转回原树，并写出后序排列
2. find a conjecture of the even term in Fibonacci sequence and prove it by mathematical induction. 给出一个有关斐波那契数列的猜想（题目限定了关于什么的猜想），并用数学归纳法证明

3. suppose a tenary string is a string that only contains 0,1,2,find a **recurrence formula** of the tenary string that its length is  $n$  and doesn't contain consecutive 0 or consecutive 1.
4. 一个有向图的考察 (边的条数, 强联通子图个数, 欧拉哈密顿, 着色, 生成树)