Theory of Computation, Fall 2023
Assignment 5 (Due November 1 Wednesday 10:00 am)

Only part I will be graded.

1 Part I

Q1. Show that if L is context-free, so is

$$L^R = \{w^R : w \in L\}.$$

Q2. Show that the following language is not context free.

$$L = \{w \in \{1,2,3,4\}^* : \text{the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s}\}$$

2 Part II

Q3. Let A be a context-free language. Let B be a regular language. Prove that $A \cap B$ is context-free. You may assume that A and B are defined over the same alphabet Σ. (Hint: let P_A be a PDA accepting A. Let M_B be an NFA accepting B. Construct a PDA $P_\cap$ that conceptually runs P_A and M_B in parallel.)

Q4. Let $A = \{w \in \{a,b,c\}^* : w$ has same number of a's, b's, and c's$\}$.

(a) Prove that A is not context-free. (Hint: It is not necessary to use pumping theorem. You may try the conclusion of Q3.)

(b) Show that $\overline{A}$ is context-free. (Hint: it suffices to show that $\overline{A}$ is a union of several context-free languages.)