浙江大学 2018－2019 学年 春夏 学期

《离散数学及其应用》课程期末考试试卷

课程号：211B0010 ，开课学院： 计算机

考试试卷：√A 卷、B 卷（请在选定项上打√）

考试形式：√闭、开卷（请在选定项上打√），允许带___________入场

考试日期：2019 年 07 月 04 日，考试时间：120 分钟

诚信考试，沉着应考，杜绝违纪。

考生姓名：________学号:__________任课教师:_________所属院系:_________

题序	一	二	三	四	五	六	七	总 分
得分								
评卷人								

1. (20 marks) Determine whether the following statements are true or false. If it is true write a √ otherwise a × in the blank before the statement.

1) (×) "This statement is false." is a proposition.

2) (×) If a relation R on a nonempty set A is transitive then $R^2 = R$.

3) (√) The wheel W_n is not a bipartite graph for every n>=3.

4) (√) P(A) = P(B), if and only if A = B , where P(X) is the power set of X.

5) (×) A weakly connected directed graph with $deg^+(v)=deg^-(v)$ for all vertices v is not always strongly connected.

6) (√) The Hasse diagram for the partial ordering ({1, 2, 3, 4, 5, 6, 7, 8, 9}, |) is not a tree.

7) (×) $\left\lceil \frac{x}{2} \right\rceil = \left\lfloor \frac{x+1}{2} \right\rfloor$ for all real number x.

8) (×) There is not any countable infinite set A with a bijection: A→A×A.

9) (√) Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Then $a_n \leq 3^n$ for all positive integers.

10) (√) If $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

2. (33 marks) Filling in the blanks.

1) If T is a full 3-ary tree with 10 vertices, its minimum and maximum heights are__2，3______.

2) Use Huffman coding to encode these symbols with given frequencies: A: 0.10, B: 0.20, C: 0.05, D: 0.15, E: 0.30, F: 0.12, G: 0.08. The average number of bits required to encode a symbol is ___2.63___.

3) If G is a planar connected graph with 10 vertices, each of degree 4, then G has _____12___ regions.

4) The full disjunctive normal form of $\neg r \vee (p \leftrightarrow q)$ is

_ m0 ∨ m1 ∨ m2 ∨ m4 ∨ m6 ∨ m7______.

5) Let A={a, b, c, d, e}， the Hasse diagram of a partial relation R on A is illustrated in Fig. 1

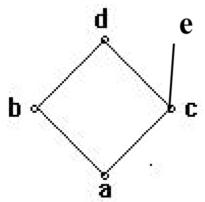


Fig.1

Then |R| = _12

{(a,a),(a,b),(a,c),(a,d),(a,e),(b,b),(b,d),(c,c),(c,d),(c,e),(d,d),(e,e)}___.

6) There are ___9____ non-isomorphic rooted trees with 5 vertices.

7) There is a binary tree. Its postorder traversal is DEBFCA，and its inorder traversal is DBEACF. Its preorder is________ABDECF___________.

8) Suppose A={1,2,3}, there are __8__ relations which are reflexive and symmetric on the set A; there are ___5___ equivalence relations on the set A; there are__19____ partial orderings on the set A.

9) Suppose that S= {a, b}. How many ordered pairs (A, B) are there such that A and B are subsets of S with A⊆B? __9___.

10) Suppose W is a weighted graph (See Fig. 2), the length of the shortest path between a and z is___*15*_____.

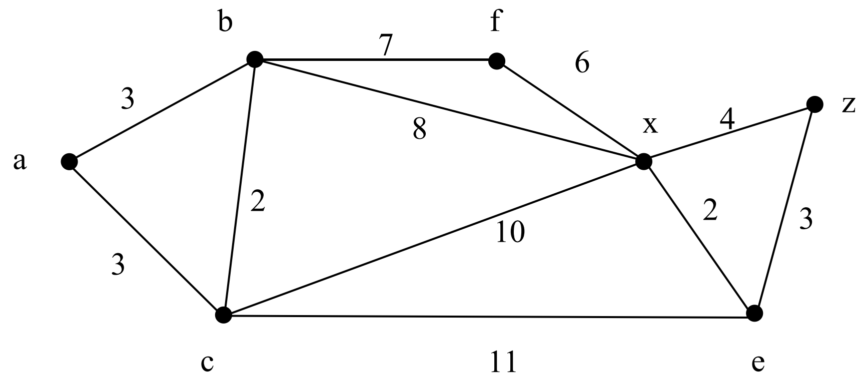


Fig. 2

3. (12 marks) How many different ways can you put 9 coins in 9 boxes which are labeled $B_1,.., B_9$ on them

(1) if the coins are all different and no box is empty?

(2) if the coins are all different and only two boxes B_1 and B_9 are empty?

(3) if the coins are all different and exactly four boxes are not empty?

(4) if the coins are all different and each box is either empty or contains exactly three coins?

(5) if the coins are identical?

(6) if the coins are identical and exactly six boxes are empty?

评分标准：列出式子即可，不必算最后数字。式子正确，即使最后数字算错也给全部分。

(1) （2分）P(9,9) = 362880

(2) （2分）7^9-C(7,1)*6^9+C(7,2)* 5^9-C(7,3)*4^9+C(7,4)* 3^9- C(7,5)* 2^9+ C(7,6)

= 40353607–70543872+41015625–9175040+688905–10752+7 = 2328480

（解法2 ： 7! * S(9,7)）

（解法3：C(9,3) * 7! + C(9,2) * C(7,2) * 7!/2 ）

（解法4：C(7,2) * 9! / (2! *2!) + C(7,1) * 9!/3! ）

（解法5：C(9,3) * 7! + C(7,5) * P(9,5) * C(4,2) ）

（解法6：C(9,3) * 7! + C(7,2) * C(9,2) * C(7,2) * 5! ）

注：思路正确，有所遗漏（例如解法3第二个式子漏除2）给半对。

(3) （2分）(4^9-C(4,1)* 3^9+ C(4,2)* 2^9-C(4,3)） *C(9,4) = 186480* 126 = 23496480

（解法2 ： 4! * S(9,4) * C(9,4)）

（解法3 ： C(9,4) * （穷举六种情况组合：6111,5211,4311,4221,3321,3222)）

注：解成5个盒子的给半对。

(4) （2分）9!/(3!*3!*3!) * C(9,3) = 141120

（解法2 ： C(9,3)* C(9,3)* C(6,3)）

(5) （2分）C(9+9-1，9) = 24310

(6) （2分）C(3+6-1,6) * C(9,3) = 2352

注：解成C(3+9-1,9)* C(9,3)的给半对；列出C(3+6-1,6)给半对

注：一个半对给1分；两个或三个半对：都合起来给2分。

4. (8 marks)

(1) Find the smallest partial ordering on {1, 2, 3} that contains (1,1), (3,2), (1,3).

(2) Find the smallest equivalent relation on {1, 2, 3} that contains (1,1), (3,2), (1,3).

(1) {(1,1),(2,2),(3,3),(3,2),(1,3),(1,2) （4 marks）

(2) {(1,1), (1,2), (1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)} （4 marks）

评分标准：

（1） 每小题4分，满分为正确枚举出所有有序对或者写出0-1矩阵

（2） 概念清楚，但漏掉小部分有序对，酌情扣分。

5. (8 marks) Let a_n be the number of strings of length n consisting of the characters 0, 1, 2 with no consecutive 0's.

(1) Find a recurrence relation for a_n and give the necessary initial condition(s).

(2) Find an explicit formula for a_n by solving the recurrence relation in part (1).

（1）共4分

$a_n = 2a_{n-1} + 2a_{n-2}$

或者 $a_n = 2a_{n-1} + 2a_{n-2}$ （3分）

$a_0 = 1$ $a_1 = 3$ OR $a_1 = 3$ $a_2 = 8$ （1分）

（2）共4分

x2-2x-2=0 x_1=1+sqrt(3) x_2=1-sqrt(3) （1分）

$a_n = c1 * x_1^n + c2 * x_2^n$ （2分）

1 = c1 + c2

3 = c1 *(1+sqrt(3))+ c2 * （1-sqrt(3)）

c1 = 0.5 + sqrt(3)/3

c2 = 0.5 - sqrt(3)/3 （1分）

a_n = （0.5 + sqrt(3)/3） * （1+sqrt(3))^n + (0.5 - sqrt(3)/3) * (（1-sqrt(3)) ^{n}

6. (10 marks) G is a directed graph (See Fig. 3).

(1) Find the number of different paths of length 3.

(2) Determine whether G is strongly connected or weakly connected.

(3) Is the underlying undirected graph of G a Hamilton graph? Justify your answer.

(4) Find the chromatic number of the underlying undirected graph of G.

(5) Find the spanning tree for the underlying undirected graph of G. Choose V_4 as the

root of the spanning tree.

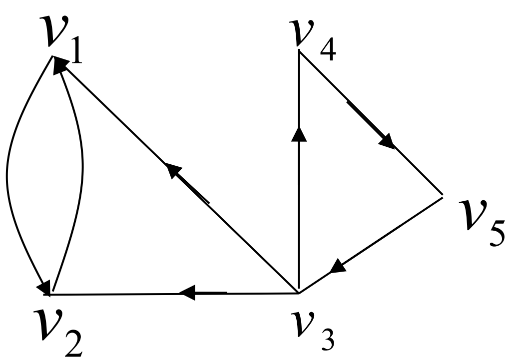


Fig. 3

(1) 共2分

M = $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ **1分**

Calculate M^3

$$M^2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$M^3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Total 11 1分

注：给出结论，无过程也得满分。

(2) 2分

weakly connected, not strongly connected

注：给出结论，即得满分

(3) 2 分

No. delete V3 make 2 connected components

注：结论1分，原因1分

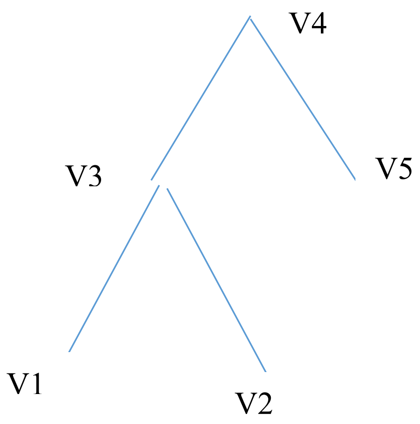
(4) 2分

3

注：给出结论即得满分。

(5) 2分

Multi solutions. Example



注：给出一个以V_4为根的树即得满分，若给出的不是带根树，扣1分。

7. (9 marks) Let G be a connected planar simple graph with at least 3 vertices containing no triangles, let ***e*** and ***v*** be the number of edges and the number of vertices of G, respectively. Prove that:

(1) $e \leq 2v-4$.

(2) G has a vertex of degree at most 3.

(3) $\chi(G) \leq 4$. Where $\chi(G)$ is the chromatic number (色数) of G. (**You cannot use "the four color theorem" in your proof.**)

每小题3分。

(1) $2e = \sum \deg(R_i) \geq 4r$（1分） $4r \leq 2e$ $e-v+2=r \leq 0.5e$（1分）$0.5e \leq v-2$ $e <= 2v-4$

（1分）

(2)

So we have

$$\sum_{v \in V(G)} \deg(v) = 2|E(G)| \leq 4n - 8,$$

（1分）

which by the pigeonhole principle implies that there is a vertex $v \in V(G)$ with $\deg(v) \leq 3$.

（2分）

(3) We prove it by induction on the number of vertices of G. If $|V(G)| \leq 4$, there is nothing to prove. （1分）

Suppose the statement holds for all the graphs with $n-1$ vertices, we prove it for the graph G on n vertices.

Now by the induction hypothesis, since the graph $G - v$ is also triangle-free, we have $\chi(G - v) \leqslant 4$. So by coloring the vertex v by a color different from its neighbors, we get a valid 4-coloring of G. （2分）