Chapter 18

Valuation and Capital Budgeting for the Levered Firm
杠杆企业的估值与资本预算

Key Concepts and Skills

- Understand the effects of leverage on the value created by a project
- Be able to apply Adjusted Present Value (APV), the Flows to Equity (FTE) approach, and the WACC method for valuing projects with leverage

Adjusted Present Value Approach
调整净现值法

$$APV = NPV + NPVF$$

- The value of a project to the firm can be thought of as the value of the project to an unlevered firm (*NPV*) plus the present value of the financing side effects (*NPVF*).
- There are four side effects of financing:
  - The Tax Subsidy to Debt
  - The Costs of Issuing New Securities
  - The Costs of Financial Distress
  - Subsidies to Debt Financing

APV Example

Consider a project of the P. B. Singer Co. with the following characteristics:

Cash inflows: $500,000 per year for the indefinite future.

Cash costs: 72% of sales.

Initial investment: $475,000.

$t_C = 34\%$

$R_0 = 20\%$, where R_0 is the cost of capital for a project of an all-equity firm.

If both the project and the firm are financed with only equity, the project's cash flow is as follows:

Cash inflows	$500,000
Cash costs	−360,000
Operating income	140,000
Corporate tax (34% tax rate)	−47,600
Unlevered cash flow (UCF)	$ 92,400

APV Example

- NPV = \$92,400/0.20 - \$475,000 = -\$13,000;

  The project would be rejected by an all-equity firm: *NPV* < 0.

- Now imagine that the firm finances the project with debts so that the ratio of debt to the present value of the project under leverage is .25.

$$\text{Present value of levered project} = \text{Present value of unlevered project} + t_C \times B$$
$$V_{\text{With debt}} = \$462{,}000 + .34 \times .25 \times V_{\text{With debt}}$$

$$V_{\text{With debt}} \times (1 - .34 \times .25) = \$462{,}000$$
$$V_{\text{With debt}} = \$504{,}918$$

- Debt is .25 of value: \$126,229.50 = .25 * \$504,918.

APV Example

- Debt is \$126,229.50, so that the remaining investment of \$348,770.50 is financed with equity. The net present value of the project under leverage, which we call the adjusted present value, or the APV, is:

$$\text{APV} = \text{NPV} + t_C \times B$$
$$\$29{,}918 = -\$13{,}000 + .34 \times \$126{,}229.50$$

Flow to Equity Approach

- Discount the cash flow from the project to the equity holders of the levered firm at the cost of levered equity capital, R_S.
- There are three steps in the FTE Approach:
  - Step One: Calculate the levered cash flows (LCFs)
  - Step Two: Calculate R_S.
  - Step Three: Value the levered cash flows at R_S.

Step One: Levered Cash Flows

- Assuming an interest rate of 10 percent, the perpetual cash flow to equity holders in our P. B. Singer Co. example is:

Cash inflows	$500,000.00
Cash costs	−360,000.00
Interest (10% × $126,229.50)	−12,622.95
Income after interest	127,377.05
Corporate tax (34% tax rate)	−43,308.20
Levered cash flow (LCF)	$ 84,068.85

Step Two: Calculate R_S

$$R_S = R_0 + \frac{B}{S}(1 - T_C)(R_0 - R_B)$$

- The discount rate on unlevered equity, R_0 , is .20.
- The target debt-to-value ratio of 1/4 implies a target debt-to-equity ratio of 1/3.
- Assuming an interest rate of 10 percent

$$R_S = .222 = .20 + \frac{1}{3}(.66)(.20 - .10)$$

Step Three: Valuation

- The present value of the project's LCF is:

$$\frac{\text{LCF}}{R_S} = \frac{\$84{,}068.85}{.222} = \$378{,}688.50$$

- The initial investment is \$475,000 and \$126,229.50 is borrowed, the firm must advance the project \$348,770.50 out of its own cash reserves.

- NPV = \$378,688.50 - \$348,770.50 = \$29,918

WACC Method

$$R_{WACC} = \frac{S}{S+B} R_S + \frac{B}{S+B} R_B (1 - T_C)$$

- To find the value of the project, discount the unlevered cash flows at the weighted average cost of capital.
- The weight for equity, S /(S+B), and the weight for debt, B /(S+B), are target ratios. Target ratios are generally expressed in terms of market values, not accounting values.

$$\sum_{t=1}^{\infty} \frac{\text{UCF}_t}{(1 + R_{\text{WACC}})^t} - \text{Initial investment}$$

WACC Method

$$R_{\mathrm{WACC}} = \frac{3}{4} \times .222 + \frac{1}{4} \times .10 \times .66 = .183$$

- the UCF of the project to be \$92,400, implying that the present value of the project is:

$$\frac{\$92{,}400}{.183} = \$504{,}918$$

- the NPV of the project is:

$$\$504{,}918 - \$475{,}000 = \$29{,}918$$

A Comparison of the APV, FTE, and WACC Approaches

- All three approaches attempt the same task: valuation in the presence of debt financing.
- Guidelines:
  - Use *WACC* or *FTE* if the firm's target debt-to-value ratio applies to the project over the life of the project.
  - Use the *APV* if the project's level of debt is known over the life of the project.
- In the real world, the *WACC* is, by far, the most widely used.

Summary: APV, FTE, and WACC

	APV	WACC	FTE
Initial Investment	All	All	Equity Portion
Cash Flows	*UCF*	*UCF*	*LCF*
Discount Rates	R_0	R_{WACC}	R_S
PV of financing effects	Yes	No	No

Summary: APV, FTE, and WACC

1. Adjusted present value (APV) method:

$$\sum_{t=1}^{\infty} \frac{\text{UCF}_t}{(1 + R_0)^t} + \text{Additional effects of debt} - \text{Initial investment}$$

UCF_t = The project's cash flow at date t to the equityholders of an unlevered firm.

R_0 = Cost of capital for project in an unlevered firm.

2. Flow to equity (FTE) method:

= Capital Investment

$$\sum_{t=1}^{\infty} \frac{\text{LCF}_t}{(1 + R_S)^t} - (\text{Initial investment} - \text{Amount borrowed})$$

LCF_t = The project's cash flow at date t to the equityholders of a levered firm.

R_S = Cost of equity capital with leverage.

3. Weighted average cost of capital (WACC) method:

$$\sum_{t=1}^{\infty} \frac{\text{UCF}_t}{(1 + R_{\text{WACC}})^t} - \text{Initial investment}$$

R_{WACC} = Weighted average cost of capital.

Summary: APV, FTE, and WACC

Which approach is best?

- Use *APV* when the *level* of debt is constant
- Use *WACC* and *FTE* when the debt *ratio* is constant
  - *WACC* is by far the most common

Capital Budgeting When the Discount Rate Must Be Estimated

- World-Wide Enterprises (WWE) is a large conglomerate thinking of entering the widget business, where it plans to finance projects with a debt-to-value ratio of 25 percent. There is currently one firm in the widget industry, American Widgets (AW). This firm is financed with 40 percent debt and 60 percent equity. The beta of AW's equity is 1.5. AW, has a borrowing rate of 12 percent, and WWE expects to borrow for its widget venture at 10 percent. The corporate tax rate for both firms is .40, the market risk premium is 8.5 percent, and the riskless interest rate is 8 percent. What is the appropriate discount rate for WWE to use for its widget venture?

Capital Budgeting When the Discount Rate Must Be Estimated

- 1. Determining AW's cost of equity capital : First, we determine AW's cost of equity capital using the security market line (SML):

$$R_S = R_F + \beta \times (\bar{R}_M - R_F)$$
$$20.75\% = 8\% + 1.5 \times 8.5\%$$

- 2. Determining AW's hypothetical all-equity cost of capital

$$R_S = R_0 + \frac{B}{S}(1 - t_C)(R_0 - R_B)$$
$$20.75\% = R_0 + \frac{.4}{.6}(.60)(R_0 - 12\%)$$

Capital Budgeting When the Discount Rate Must Be Estimated

- 3. Determining R_S for WWE's widget venture:

$$R_S = R_0 + \frac{B}{S}(1 - t_C)(R_0 - R_B)$$

$$19.9\% = 18.25\% + \frac{1}{3}(.60)(18.25\% - 10\%)$$

- 4. Determining R_{WACC} for WWE's widget venture:

$$R_{WACC} = \frac{B}{S+B}R_B(1 - t_C) + \frac{S}{S+B}R_S$$

$$16.425\% = \frac{1}{4}10\%(.60) + \frac{3}{4}19.9\%$$

Beta and Leverage

- Recall that an asset beta would be of the form:

$$\beta_{\text{Asset}} = \frac{Cov(UCF, Market)}{\sigma^2_{\text{Market}}}$$

Beta and Leverage: No Corporate Taxes

- In a world without corporate taxes, and with **riskless** corporate debt ($b_{Debt} = 0$), it can be shown that the relationship between the beta of the unlevered firm and the beta of levered equity is:

$$\beta_{\text{Asset}} = \frac{\text{Equity}}{\text{Asset}} \times \beta_{\text{Equity}}$$

  - In a world without corporate taxes, and with **risky** corporate debt, it can be shown that the relationship between the beta of the unlevered firm and the beta of levered equity is:

$$\beta_{\text{Asset}} = \frac{\text{Debt}}{\text{Asset}} \times \beta_{\text{Debt}} + \frac{\text{Equity}}{\text{Asset}} \times \beta_{\text{Equity}}$$

Beta and Leverage: With Corporate Taxes

- In a world with corporate taxes, and riskless debt, it can be shown that the relationship between the beta of the unlevered firm and the beta of levered equity is:

$$\beta_{\text{Equity}} = \left(1 + \frac{\text{Debt}}{\text{Equity}} \times (1 - T_C)\right) \beta_{\text{Unlevered firm}}$$

- Since $\left(1 + \frac{\text{Debt}}{\text{Equity}} \times (1 - T_C)\right)$ must be more than 1 for a levered firm, it follows that $\beta_{\text{Equity}} > \beta_{\text{Unlevered firm}}$

Beta and Leverage: With Corporate Taxes

$$V_U + t_C B = V_L = B + S$$

$$\frac{B}{B+S} \times \beta_B + \frac{S}{B+S} \times \beta_S \qquad \frac{B}{V_L} \times \beta_B + \frac{S}{V_L} \times \beta_S$$

- The beta of the levered firm can also be expressed as a weighted average of the beta of the unlevered firm and the beta of the tax shield:

$$\frac{V_U}{V_U + t_C B} \times \beta_U + \frac{t_C B}{V_U + t_C B} \times \beta_B$$

Beta and Leverage: With Corporate Taxes

- If the beta of the debt is zero, then:

$$\beta_{\text{Equity}} = \left(1 + \frac{\text{Debt}}{\text{Equity}} \times (1 - T_C)\right)\beta_{\text{Unlevered firm}}$$

- If the beta of the debt is non-zero, then:

$$\beta_{\text{Equity}} = \beta_{\text{Unlevered firm}} + (1 - T_C)(\beta_{\text{Unlevered firm}} - \beta_{\text{Debt}}) \times \frac{B}{S_L}$$

Beta and Leverage

$$V_U + t_C B = V_L = B + S$$

$$\frac{B}{V_L} \times \beta_B + \frac{S}{V_L} \times \beta_S \qquad \frac{V_U}{V_U + t_C B} \times \beta_U + \frac{t_C B}{V_U + t_C B} \times \beta_B$$

Beta and Leverage

- The J. Lowes Corporation, which currently manufactures staples, is considering a $1 million investment in a project in the aircraft adhesives industry. The corporation estimates unlevered aftertax cash flows (UCF) of $300,000 per year into perpetuity from the project. The firm will finance the project with a debt-to-value ratio of .5 (or, equivalently, a debt-to-equity ratio of 1.0). The three competitors in this new industry are currently unlevered, with betas of 1.2, 1.3, and 1.4. Assuming a risk-free rate of 5 percent, a market risk premium of 9 percent, and a corporate tax rate of 34 percent, what is the net present value of the project?

Beta and Leverage

- 1. Calculating the average unlevered beta in the industry: The average unlevered beta across all three existing competitors in the aircraft adhesives industry is:

$$\frac{1.2 + 1.3 + 1.4}{3} = 1.3$$

- 2. Calculating the levered beta for J. Lowes's new project : Assuming the same unlevered beta for this new project as for the existing competitors:

$$\beta_{\text{Equity}} = \left(1 + \frac{(1 - t_C)\ \text{Debt}}{\text{Equity}}\right)\beta_{\text{Unlevered firm}}$$

$$2.16 = \left(1 + \frac{.66 \times 1}{1}\right) \times 1.3$$

Beta and Leverage

- 3. Calculating the cost of levered equity for the new project: We calculate the discount rate from the security market line (SML) as follows:

$$R_S = R_F + \beta \times [\bar{R}_M - R_F]$$
$$.244 = .05 + 2.16 \times .09$$

- 4. Calculating the WACC for the new project : The formula for determining the weighted average cost of capital is:

$$R_{\text{WACC}} = \frac{B}{V} R_B (1 - t_C) + \frac{S}{V} R_S$$
$$.139 = \frac{1}{2} \times .05 \times .66 + \frac{1}{2} \times .244$$

Beta and Leverage

- 5. Determining the project's value: Because the cash flows are perpetual, the NPV of the project is:

$$\frac{\text{Unlevered cash flows (UCF)}}{R_{\text{WACC}}} - \text{Initial investment}$$

$$\frac{\$300{,}000}{.139} - \$1 \text{ million} = \$1.16 \text{ million}$$

Summary

1. The APV formula can be written as:

$$APV = \sum_{t=1}^{\infty} \frac{UCF_t}{(1+R_0)^t} + \begin{matrix}\text{Additional} \\ \text{effects of} \\ \text{debt}\end{matrix} - \begin{matrix}\text{Initial} \\ \text{investment}\end{matrix}$$

2. The FTE formula can be written as:

$$FTE = \sum_{t=1}^{\infty} \frac{LCF_t}{(1+R_S)^t} - \left(\begin{matrix}\text{Initial} \\ \text{investment}\end{matrix} - \begin{matrix}\text{Amount} \\ \text{borrowed}\end{matrix} \right)$$

3. The WACC formula can be written as

$$NPV_{WACC} = \sum_{t=1}^{\infty} \frac{UCF_t}{(1+R_{WACC})^t} - \begin{matrix}\text{Initial} \\ \text{investment}\end{matrix}$$