习题四

4. 解 令 $X_i = \begin{cases} 1, & \text{若第}i\text{次向右移动} \\ 0, & \text{若第}i\text{次向左移动.} \end{cases}$

则 $X_1, X_2, \cdots, X_n$ 相互独立，且 $P(X_i=1)=p$，$P(X_i=0)=1-p$.

因为 $\eta_n = X_1 + \cdots + X_n$

$$S_n = \text{前}n\text{次中向右移动次数} - \text{前}n\text{次中向左移动次数}$$

$$= \eta_n - (n-\eta_n) = 2\eta_n - n$$

利用期望的线性性：

$$E\eta_n = EX_1 + \cdots + EX_n = np$$

$$ES_n = 2E\eta_n - n = n(2p-1)$$

另解：$\because \eta_n \sim B(n,p)$，$\therefore E\eta_n = np$

又 $S_n = 2\eta_n - n$ $\therefore ES_n = 2E\eta_n - n = n(2p-1)$

6. 解：(1) $E(X)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x f(x,y)\,dx\,dy$

$=\int_0^{\infty}dx\int_0^{x} x\cdot\left(\frac{2}{x}e^{-2x}\right)dy$

$=\int_0^{\infty}2xe^{-2x}dx=\frac{1}{2}$

(2) $E(3X-1)=3E(X)-1=\frac{1}{2}$

(3) $E(XY)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy f(x,y)\,dx\,dy$

$=\int_0^{\infty}dx\int_0^{x}(xy)\cdot\left(\frac{2}{x}e^{-2x}\right)dy$

$=\int_0^{\infty}x^3e^{-2x}dx=-\frac{x^2}{2}e^{-2x}\Big|_0^{\infty}+\int_0^{\infty}xe^{-2x}dx=\frac{1}{4}$

7. 令q为Q点离绳子一端A距离.

(1) 令Y表示包含Q点的那段绳子长度，令X表示截点离A距离，则$X \sim U(0,1)$，且

$$Y=\begin{cases} X, & 若\ X>q; \\ 1-X, & 若\ X<q; \\ \min(q,1-q) & 若\ X=q \end{cases}$$

$$\therefore EY=\int_q^1 x\cdot 1\cdot dx+\int_0^q (1-x)\cdot 1\cdot dx=\frac{1}{2}+q(1-q)$$

(2). 令$f(q)=\frac{1}{2}+q(1-q)=\frac{3}{4}-(q-\frac{1}{2})^2$

$\therefore$当$q=\frac{1}{2}$时，$f(q)$最大

$\therefore$当Q点在绳子中点时，包含Q点绳子平均长度最大.

9. 第一种方法试验次数是500次.

对于第二种方法, 令X_i表示第i组化验次数, 则总化验次数 $X=X_1+\cdots+X_{\frac{500}{k}}$. 由于

$$X_i=\begin{cases}1, & 若第i组每人都无病\\ k+1, & 其它.\end{cases}$$

所以 $p(X_i=1)=0.8^k$. $p(X_i=k+1)=1-0.8^k$

$$E(X_i)=1\times p(X_i=1)+(k+1)p(X_i=k+1)=1+k(1-0.8^k)$$

$$\therefore E(X)=E(X_1)+\cdots+E(X_{\frac{500}{k}})=500\left[\frac{1}{k}+1-0.8^k\right]$$

因此当$0.8^k-\frac{1}{k}>0$, 第二种方法平均次数最少;

当$0.8^k-\frac{1}{k}=0$时, 两种方法平均次数一样;

当$0.8^k-\frac{1}{k}<0$时, 第一种方法平均次数最少.

11. (1) 解：(X,Y)的联合概率密度为：

$$f(x,y)=\begin{cases}\dfrac{1}{\pi r^2}, & 若\ x^2+y^2\le r^2;\\ 0, & 其它\end{cases}$$

$$\therefore E(X)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x f(x,y)\,dx\,dy$$

$$=\int_{-r}^{r}dx\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} x\cdot\frac{1}{\pi r^2}\,dy$$

$$=\int_{-r}^{r}\frac{2x\sqrt{r^2-x^2}}{\pi r^2}\,dx=0$$ （因为被积函数是奇函数）

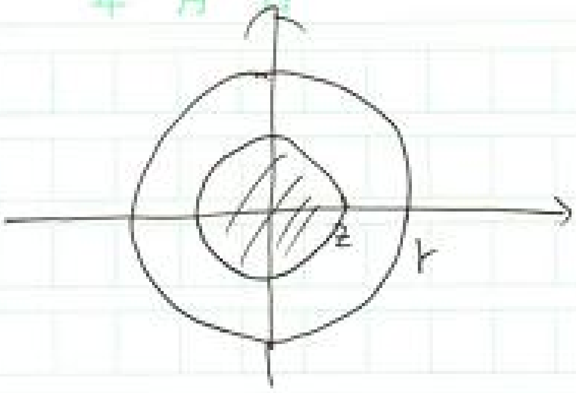
$$E(Y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} y f(x,y)\,dx\,dy.$$

$$=\int_{-r}^{r}dx\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} y\cdot\frac{1}{\pi r^2}\,dy=0$$ （因为被积函数是奇函数）

(2). 点A到$(0,0)$的距离是 $Z=\sqrt{X^2+Y^2}$.

对 $0<z\le r$ 有 $F_Z(z)=P(\sqrt{X^2+Y^2}\le z)=P(X^2+Y^2\le z^2)$

$$=\frac{\{(x,y):x^2+y^2\le z^2\}面积}{\{(x,y):x^2+y^2\le r^2\}面积}=\frac{\pi z^2}{\pi r^2}=\frac{z^2}{r^2}$$



$\therefore f_Z(z)=\frac{2z}{r^2}$

因此 $f_Z(z)=\begin{cases}\frac{2z}{r^2}, & \text{当 } 0<z\le r \text{ 时;}\\ 0, & \text{其它}\end{cases}$

$\therefore E(Z)=\int_{-\infty}^{+\infty} z f_Z(z)\,dz=\int_0^r z\frac{2z}{r^2}\,dz=\frac{2}{3}r$

另解：$E(Z)=E(\sqrt{X^2+Y^2})=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\sqrt{x^2+y^2}\,f(x,y)\,dx\,dy$

$=\int_{-r}^{r}dx\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}}\sqrt{x^2+y^2}\cdot\frac{1}{\pi r^2}\,dy$

作极坐标变换，令 $x=z\cos\theta$, $y=z\sin\theta$, $0\le\theta<2\pi$, $0<z\le r$,

则 $dx\,dy=\begin{vmatrix}\frac{\partial x}{\partial z} & \frac{\partial x}{\partial\theta}\\ \frac{\partial y}{\partial z} & \frac{\partial y}{\partial\theta}\end{vmatrix}dz\,d\theta=\begin{vmatrix}\cos\theta & -z\sin\theta\\ \sin\theta & z\cos\theta\end{vmatrix}dz\,d\theta=z\,dz\,d\theta$

$\therefore E(Z)=\int_0^r dz\int_0^{2\pi} z\cdot\frac{1}{\pi r^2}\cdot z\,d\theta=\int_0^r\frac{2z^2}{r^2}\,dz=\frac{2}{3}r$

13. 设这 n 个点为 $X_1, X_2, \cdots, X_n$，则 $X_1, \cdots, X_n$ 相互独立，且都服从 $U(0,1)$.

令 $M = \max\{X_1, \cdots, X_n\}$，$N = \min\{X_1, \cdots, X_n\}$

则相距最远的两点距离 $Z = M - N$.

对 $0<z<1$，$F_M(z) = P(M \le z) = P(X_1 \le z, \cdots, X_n \le z)$

$= P(X_1 \le z) \cdots P(X_n \le z) = z^n$

$1 - F_N(z) = P(N > z) = P(X_1 > z, \cdots, X_n > z)$

$= P(X_1 > z) \cdots P(X_n > z) = (1-z)^n$

$\therefore f_M(z) = \begin{cases} nz^{n-1}, & 0<z<1 \\ 0, & \text{其它} \end{cases}$，$f_N(z) = \begin{cases} n(1-z)^{n-1}, & 0<z<1 \\ 0, & \text{其它} \end{cases}$

$EM = \int_0^1 z \cdot nz^{n-1} dz = \frac{n}{n+1}$

$EN = \int_0^1 z \cdot n(1-z)^{n-1} dz \xlongequal{令 t=1-z} n\int_0^1 (1-t)t^{n-1} dt = \frac{1}{n+1}$

$\therefore E(Z) = E(M-N) = E(M) - E(N) = \frac{n}{n+1} - \frac{1}{n+1} = \frac{n-1}{n+1}$

14. 对k非负整数，

$$P(Y=k)=\sum_{n=k}^{\infty}P(X=n)P(Y=k|X=n)$$

$$=\sum_{n=k}^{\infty}e^{-\lambda}\frac{\lambda^n}{n!}C_n^k p^k(1-p)^{n-k}$$

$$=e^{-\lambda}\frac{(\lambda p)^k}{k!}\sum_{n=k}^{\infty}\frac{[\lambda(1-p)]^{n-k}}{(n-k)!}$$

$$\xlongequal{令m=n-k}e^{-\lambda}\frac{(\lambda p)^k}{k!}\sum_{m=0}^{\infty}\frac{[\lambda(1-p)]^m}{m!}$$

$$=e^{-\lambda}\frac{(\lambda p)^k}{k!}e^{\lambda(1-p)}=e^{-\lambda p}\frac{(\lambda p)^k}{k!}$$

即 $Y\sim P(\lambda p)$. $\therefore EY=\lambda p$.

16. 对 $\alpha>0$, $\Gamma(\alpha)=\int_0^{\infty} t^{\alpha-1} e^{-t}\,dt$.

$$E(X^k)=\int_{-\infty}^{+\infty} x^k f(x)\,dx=\int_0^{\infty}\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha+k-1}e^{-\lambda x}\,dx$$

$$\overset{令 t=\lambda x}{=\!=\!=}\int_0^{\infty}\frac{1}{\lambda^k\Gamma(\alpha)}t^{\alpha+k-1}e^{-t}\,dt=\frac{\Gamma(\alpha+k)}{\lambda^k\Gamma(\alpha)}$$

$$\begin{aligned}Var(X)&=E(X^2)-(EX)^2\\&=\frac{\Gamma(\alpha+2)}{\lambda^2\Gamma(\alpha)}-\left[\frac{\Gamma(\alpha+1)}{\lambda\Gamma(\alpha)}\right]^2\\&=\frac{\alpha(\alpha+1)}{\lambda^2}-\left(\frac{\alpha}{\lambda}\right)^2=\frac{\alpha}{\lambda^2}.\end{aligned}$$

(这里用到 $\Gamma(\alpha+1)=\alpha\Gamma(\alpha)$, 对 $\alpha>0$)

18. (1) 令X_i = $\begin{cases} 1, & \text{第}i\text{件产品正品;} \\ 0, & \text{其它} \end{cases}$

令X表示100件产品中正品数，则非正品数$Y=100-X$

~~令A,B,C分别表示各件机器~~

$$P(X_i=1)=0.7\times0.98+0.2\times0.9+0.1\times0.74=0.94$$

$\therefore EX_i=0.94 \quad Var(X_i)=0.94\times(1-0.94)=0.0564$

$$\therefore E(Y)=100-E(X)=100-E(X_1+\cdots+X_{100})$$

$$=100-(EX_1+\cdots+EX_{100})=100-94=6$$

$$Var(Y)=Var(100-X)=Var(X)=Var(X_1+\cdots+X_{100})$$

$$\xlongequal{X_1,\cdots,X_{100}\text{相互独立}} Var(X_1)+\cdots+Var(X_{100})=5.64$$

(2) 令A="这些产品都是正常制造".

则在A发生条件下，$X_1,\cdots,X_{100}$相互独立，且$P(X_i=1|A)=0.98$

$\therefore E(X_i|A)=0.98, \quad Var(X_i|A)=0.98\times0.02=0.0196$

$$E(X|A)=E(X_1+\cdots+X_{100}|A)=E(X_1|A)+\cdots+E(X_{100}|A)=98$$

$$Var(X|A)=Var(X_1|A)+\cdots+Var(X_{100}|A)=1.96$$

19. (1) $P(X+Y\geqslant 1)=1-P(X=0, Y=0)=1-P(X=0)P(Y=0)=\frac{3}{4}$

(2) $E(X\cdot(-1)^Y)=E(X)\,E[(-1)^Y]=\left[0\times\frac{1}{2}+1\times\frac{1}{2}\right]\left[(-1)^0\times\frac{1}{2}+(-1)^1\times\frac{1}{2}\right]=0$

$$Var(X\cdot(-1)^Y)=E\left[X^2(-1)^{2Y}\right]-\left(E\left[X\cdot(-1)^Y\right]\right)^2$$

$$=E(X^2)=0^2\times\frac{1}{2}+1^2\times\frac{1}{2}=\frac{1}{2}$$

30. 令X_i表示第i天的产煤量，则一月总产量

$$X=X_1+\cdots+X_{30}$$

且$X_1,\cdots,X_{30}$相互独立同分布，且都服从$N(1.5, 0.1^2)$.

$$X\sim N(30\times1.5,\ 30\times0.1^2)=N(45, 0.3)$$

$$P(X>46)=1-\Phi\left(\frac{46-45}{\sqrt{0.3}}\right)=1-\Phi\left(\sqrt{\frac{10}{3}}\right)=0.034$$

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29. (1) (X_1, X_2, X_3) 正态 $\Rightarrow X_1, X_2, X_3$ 都服从正态分布

由 a 和 B 知，$X_1 \sim N(0,1)$，$X_2 \sim N(0,16)$，$X_3 \sim N(1,4)$

(2) 由 B 知 $Cov(X_1, X_2) = 2 \neq 0 \Rightarrow X_1, X_2$ 不独立，相关

$Cov(X_1, X_3) = -1 \neq 0 \Rightarrow X_1, X_3$ 不独立，相关

$Cov(X_2, X_3) = 0 \Rightarrow X_2, X_3$ 独立，不相关.

(3) Y_1, Y_2 都是 (X_1, X_2, X_3) 线性组合，由正态分布的线性变换不变性知，(Y_1, Y_2) 是二元正态分布.

$E(Y_1) = EX_1 - EX_2 = 0$

$E(Y_2) = EX_3 - EX_1 = 1$

$D(Y_1) = DX_1 + DX_2 - 2Cov(X_1, X_2) = 1 + 16 - 2\times 2 = 13$

$D(Y_2) = DX_3 + DX_1 - 2Cov(X_3, X_1) = 4 + 1 - 2\times(-1) = 7$

$Cov(Y_1, Y_2) = Cov(X_1 - X_2, X_3 - X_1) = Cov(X_1, X_3) - Cov(X_2, X_3) - DX_1 + Cov(X_2, X_1) = -1 - 0 - 1 + 2 = 0$

$\therefore \rho_{Y_1 Y_2} = \dfrac{Cov(Y_1, Y_2)}{\sqrt{DY_1}\sqrt{DY_2}} = 0$

$\therefore (Y_1, Y_2)^T \sim N(0, 1, 13, 7, 0)$

或 $(Y_1, Y_2)^T \sim N(\mu, \Sigma)$，其中 $\mu = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$，$\Sigma = \begin{pmatrix} 13 & 0 \\ 0 & 7 \end{pmatrix}$.

17. $\frac{1}{2}e^{-|x|}$, $x^2\cdot\frac{1}{2}e^{-|x|}$ 为偶函数， $x\cdot\frac{1}{2}e^{-|x|}$ 为奇函数.

积分过程中利用函数奇偶性.

$f(x)=\frac{1}{2}e^{-|x|}$ ， $\because \int_{-\infty}^{0}\frac{1}{2}e^{x}dx+\int_{0}^{\infty}\frac{1}{2}e^{-x}dx=1$

即 $2\int_{0}^{\infty}\frac{1}{2}e^{-x}dx=1 \Rightarrow \int_{0}^{\infty}e^{-x}dx=1$

$EX=\int_{-\infty}^{0}x\cdot\frac{1}{2}e^{+x}dx+\int_{0}^{\infty}x\cdot\frac{1}{2}e^{-x}dx$

$\overset{\text{利用奇函数性质}}{=}$ 0

$E|X|=\int_{-\infty}^{0}-x\frac{1}{2}e^{x}dx+\int_{0}^{\infty}x\cdot\frac{1}{2}e^{-x}dx$

$\overset{\text{偶函数}}{=}\int_{0}^{\infty}xe^{-x}dx=-\int_{0}^{\infty}x\,de^{-x}$

$=-x\cdot e^{-x}\Big|_{0}^{\infty}+\int_{0}^{\infty}e^{-x}dx$

$=0+1 \quad =1$

$EX^2=\int_{-\infty}^{0}x^2\cdot\frac{1}{2}e^{x}dx+\int_{0}^{\infty}x^2\cdot\frac{1}{2}e^{-x}dx$

$\overset{\text{偶函数}}{=}\int_{0}^{\infty}x^2e^{-x}dx=-\int_{0}^{\infty}x^2de^{-x}$

$=-x^2e^{-x}\Big|_{0}^{\infty}+\int_{0}^{\infty}2xe^{-x}dx$

$=0-\int_{0}^{\infty}2x\,de^{-x}$

$=-2xe^{-x}\Big|_{0}^{\infty}+2\int_{0}^{\infty}e^{-x}dx$

$=0+2=2$

$E|X|^2=EX^2$

$DX=EX^2-(EX)^2=2-0=2$

$D|X|=E|X|^2-(E|X|)^2=2-1^2=1$

21. (1) $x|x|\cdot\frac{1}{2}e^{-x}$ 为奇函数

$E X|X| = \int_{-\infty}^{0} -x^2\cdot\frac{1}{2}e^{x}dx + \int_{0}^{\infty} x^2\cdot\frac{1}{2}e^{-x}dx = 0$

$\operatorname{cov}(X,|X|) = EX|X| - EXE|X|$

$= 0 - 0\cdot 1 = 0$

$\rho_{X|X|} = \frac{\operatorname{cov}(X,|X|)}{\sqrt{D(X)D(|X|)}} = \frac{0}{\sqrt{2}\cdot 1} = 0$

X与|X|不相关

(2) 令 $A=(0,1)$, $B=(2,3)$ $x\in(0,1)$则

$P(x\in(0,1), |x|\in(2,3)) = 0$ $(\because |x|\in(0,1))$

又 $\because P(x\in(0,1)) > 0$

$P(|x|\in(2,3)) > 0$

$\therefore P(x\in(0,1), |x|\in(2,3)) \neq P(x\in(0,1))\cdot P(|x|\in(2,3))$

(即证 $P(x\in A, |x|\in B) \neq P(x\in A)\cdot P(|x|\in B)$

找出一组A·B集合符合上述不等式即得证)

22. (1) $f(x,y)=\frac{1}{4}(1+xy)$ $|x|<1,\ |y|<1$

$f_X(x)=\int_{-1}^{1}\frac{1}{4}(1+xy)dy=\frac{1}{2}$ $|x|<1$ 同理 $f_Y(y)=\frac{1}{2}, |y|<1$

X.Y 为 $(-1,1)$ 上的均匀分布

$\therefore EX=EY=0$, $DX=DY=\frac{1}{3}$

$EXY=\int_{-1}^{1}\int_{-1}^{1}\frac{xy}{4}(1+xy)dxdy=\frac{1}{4}\int_{-1}^{1}\frac{2}{3}y^2dy=\frac{1}{9}$

$Cov(X,Y)=EXY-EX\cdot EY=\frac{1}{9}$, $\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{DX\cdot DY}}=\frac{1}{3}$

$\therefore$ X.Y 相关.

$f_X(x)f_Y(y)=\frac{1}{4}\neq f(x,y)$ $\therefore$ X.Y 不相互独立

$\therefore \rho_{XY}=\frac{1}{3}$, X.Y 相关且不独立.

当0<x, y<1时 (2) $P(X^2\le x, Y^2\le y)=\int_{-\sqrt{y}}^{\sqrt{y}}\int_{-\sqrt{x}}^{\sqrt{x}}\frac{1+xy}{4}dxdy=\sqrt{xy}$

当 $x.y\geq 0$ 时, $P(X^2\le x, Y^2\le y)=P(X^2\le \min(x,1), Y^2\le \min(y,1))$

$=\sqrt{\min(x,1)}\cdot\sqrt{\min(y,1)}$

$\therefore F_{X^2Y^2}=\begin{cases}\sqrt{\min(x,1)}\sqrt{\min(y,1)} & 当x, y\geq 0时\\ 0 & 其它\end{cases}$

$F_{X^2}(x)=F_{X^2Y^2}(x,\infty)=\begin{cases}\sqrt{\min(x,1)} & 当x\geq 0时\\ 0 & 其它\end{cases}$

$F_{Y^2}(y)=F_{X^2Y^2}(\infty,y)=\begin{cases}\sqrt{\min(y,1)} & 当y\geq 0时\\ 0 & 其它\end{cases}$

$\because F_{X^2Y^2}(x,y)=F_{X^2}(x)F_{Y^2}(y)$ $\therefore$ 相互独立

$\because$ 相互独立 $\therefore X^2 \cdot Y^2$ 不相关

26. (1). $P(\xi \le x) = P(X \le x, Y=1 \mid Y=1) \cdot P(Y=1) + P(X \ge -x, Y=-1 \mid Y=-1) \cdot P(Y=-1)$

由X·Y独立得 $= P(X \le x) \cdot P(Y=1) + P(X \ge -x) \cdot P(Y=-1)$

$= \Phi(x) \cdot p + \Phi(x) \cdot (1-p)$

$= \Phi(x)$

$\therefore \xi \sim N(0,1)$

(2). $\because$ X与Y独立 $\Rightarrow$ X^2与Y独立

$$\rho_{X\xi} = \frac{Cov(X,\xi)}{\sqrt{D(X)D(\xi)}} = Cov(X, XY)$$

$$= E(X^2Y) - E(X)E(XY)$$

$$= EX^2 \cdot EY - (E(X))^2 \cdot EY$$

$$= (EX^2 - (EX)^2) \cdot EY$$

$$= DX \cdot EY = EY = 2p-1$$

① $p = \frac{1}{2}$, $\rho_{X\xi} = 0$, X与ξ不相关

② $p > \frac{1}{2}$, $\rho_{X\xi} > 0$, X与ξ正相关

③ $0 < p < \frac{1}{2}$, $\rho_{X\xi} < 0$, X与ξ负相关.

独立性：举反例.

令 $A = (0,1)$, $B = (2,3)$

$P(X \in A, \xi \in B) = 0$ [$\because$ 若 $X \in A$, 则 $\xi \in (-1,1)$]

$\because P(X \in A) \ne 0$, $P(\xi \in B) \ne 0$

$\therefore P(X \in A, \xi \in B) \ne P(X \in A, \xi \in B)$

$\therefore$ X与ξ不相互独立.

27. (1) 从甲中抽取白球放入乙盒，则乙盒：3白 3黑

…… 黑 ……，∴ 乙盒：2白 4黑，

$P(X=0, Y=0) = \frac{3}{5} \times \frac{4}{6} = \frac{2}{5}$

$P(X=0, Y=1) = \frac{3}{5} \times \frac{2}{6} = \frac{1}{5}$

$P(X=1, Y=0) = \frac{2}{5} \times \frac{3}{6} = \frac{1}{5}$

$P(X=1, Y=1) = \frac{2}{5} \times \frac{3}{6} = \frac{1}{5}$

		Y = 0	Y = 1	$P(X=i)$
X	0	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{3}{5}$
	1	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
$P(Y=j)$		$\frac{3}{5}$	$\frac{2}{5}$	

$P(X=0, Y=0) = \frac{2}{5}$，$P(X=0) = \frac{3}{5}$，$P(Y=0) = \frac{3}{5}$

$\therefore P(X=0, Y=0) \neq P(X=0) \cdot P(Y=0) \Rightarrow X, Y$ 不独立

(2) $EX = 1 \times \frac{2}{5} = \frac{2}{5}$，$EY = 1 \times \frac{2}{5} = \frac{2}{5}$

$EXY = 1 \times 1 \times \frac{1}{5} = \frac{1}{5}$

$D(X) = EX^2 - (EX)^2 = \frac{2}{5} - \left(\frac{2}{5}\right)^2 = \frac{6}{25}$

$D(Y) = EY^2 - (EY)^2 = \frac{2}{5} - \left(\frac{2}{5}\right)^2 = \frac{6}{25}$

$Cov(X, Y) = EXY - EX \cdot EY = \frac{1}{5} - \frac{2}{5} \times \frac{2}{5} = \frac{1}{25}$

$\rho_{XY} = \dfrac{Cov(X, Y)}{\sqrt{D(X) D(Y)}} = \dfrac{\frac{1}{25}}{\frac{6}{25}} = \frac{1}{6}$

$\therefore$ X 与 Y 正相关.