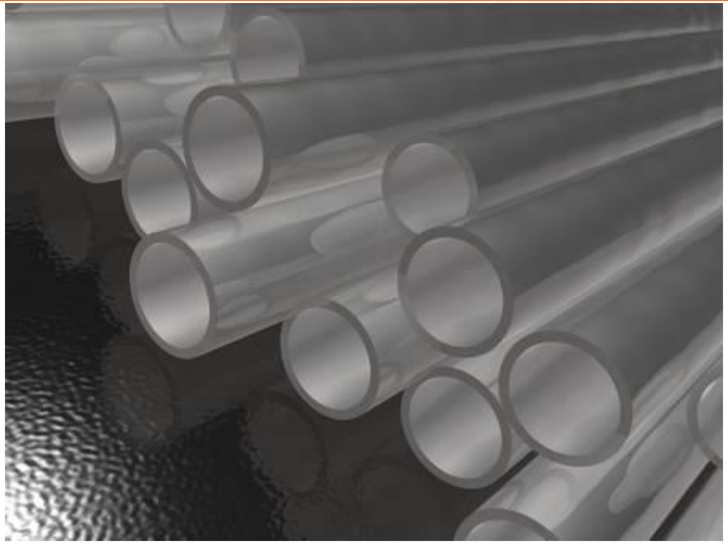
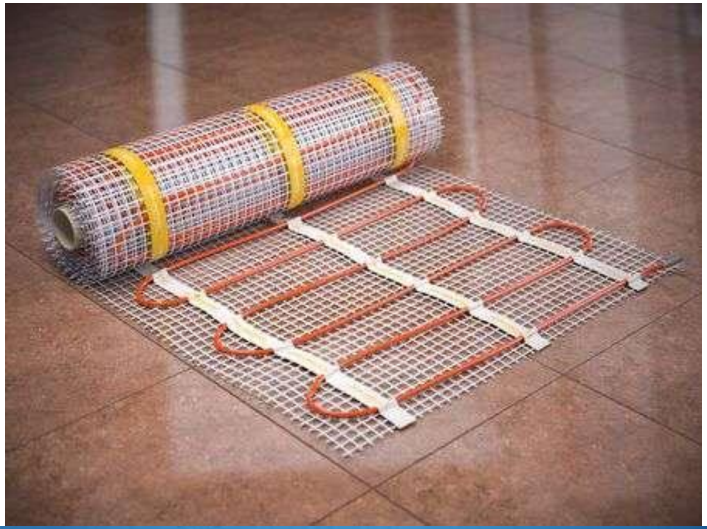
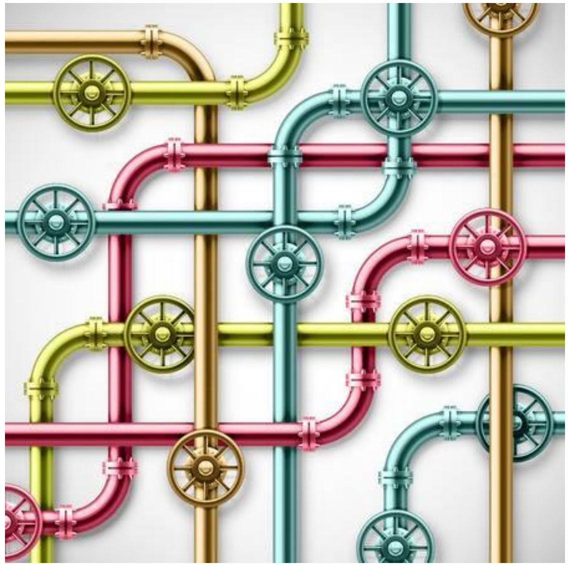
第7讲（第14章）

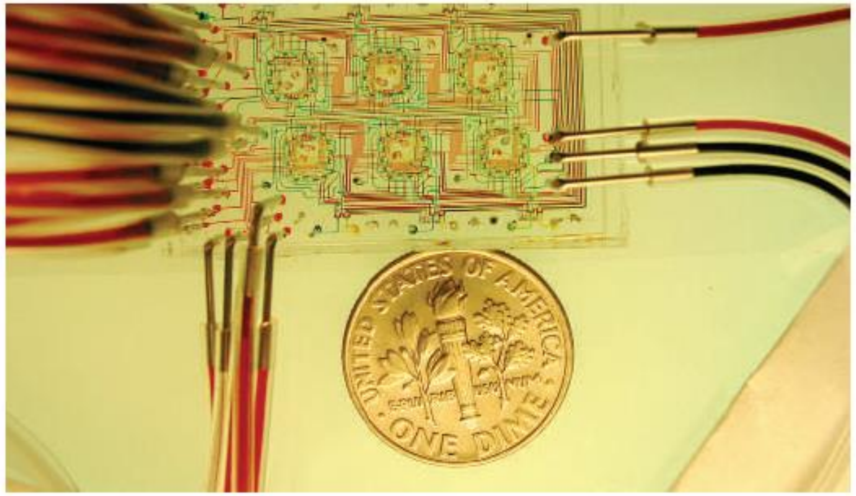
封闭管道内的流动

各式各样的管道









封闭管道 (closed conduit)　　明渠 (open channel)

管道流动的量纲分析

水平放置的等截面直圆管中的不可压缩流动涉及的变量及量纲

Variable	Symbol	Dimension
Pressure drop	ΔP	M/Lt^2
Velocity	v	L/t
Pipe diameter	D	L
Pipe length	L	L
Pipe roughness	e	L
Fluid viscosity	μ	M/Lt
Fluid density	ρ	M/L^3

粗糙度 (roughness)：表征管壁突起高度的量

根据白金汉Pi理论（第5讲内容），变量可以组成4个独立的无量纲参数，将*v*、*D*、*ρ*作为主变量，有

$$\pi_1 = v^a D^b \rho^c \Delta P$$
$$\pi_2 = v^d D^e \rho^f L$$
$$\pi_3 = v^g D^h \rho^i e$$
$$\pi_4 = v^j D^k \rho^l \mu$$

解一个指数相等的方程组，得到

$$\pi_1 = \frac{\Delta P}{\rho v^2}$$

$\Delta P/\rho$ 用 gh_L 代替，有 $\pi_1 = \frac{h_L}{v^2/g}$

$$\pi_2 = \frac{L}{D}$$
$$\pi_3 = \frac{e}{D}$$
$$\pi_4 = \frac{vD\rho}{\mu}$$

$$\frac{h_L}{v^2/g} = \phi_1\left(\frac{L}{D}, \frac{e}{D}, \mathrm{Re}\right)$$

h_L称为压头(水头)损失，head loss

压力降可以表达成

$$\frac{h_L}{v^2/g} = \phi_1\left(\frac{L}{D}, \frac{e}{D}, \mathrm{Re}\right) = \frac{L}{D}\phi_2\left(\frac{e}{D}, \mathrm{Re}\right)$$

用摩擦系数来表达上式，有

$$h_L = 2f_f \frac{L}{D}\frac{v^2}{g}$$ 或者 $$h_L = f_D \frac{L}{D}\frac{v^2}{2g}$$

f_f称为范宁（**Fanning**）摩擦系数，f_D称为达西（**Darcy**）摩擦系数

$$\frac{F}{A} \equiv C_f \frac{\rho v_\infty^2}{2}$$

摩擦系数：圆管中充分发展的层流

层流　　**Hagen-Poiseuille方程**

$$-\frac{dP}{dx}=32\frac{\mu v_{\text{avg}}}{D^2}$$

沿管道长度积分

$$-\int_{P_0}^{P}dP=32\frac{\mu v_{\text{avg}}}{D^2}\int_0^L dx$$

得到

$$\Delta P=32\frac{\mu v_{\text{avg}}L}{D^2}$$

用水头损失表示，有

$$h_L=32\frac{\mu v_{\text{avg}}L}{g\rho D^2}=2f_f\frac{L}{D}\frac{v^2}{g}$$

其中

$$f_f=16\frac{\mu}{Dv_{\text{avg}}\rho}=\frac{16}{\text{Re}}$$

和雷诺数成反比，Re<2300

例1：对应临界Re数的流速有多大？

直径5 cm的圆管，达到临界Re数2300的流速是多少？

$$(a)\ \text{Air:}\qquad \frac{\rho V d}{\mu} = \frac{(1.205\ \text{kg/m}^3)V(0.05\ \text{m})}{1.80\ \text{E-5 kg/(m}\cdot\text{s)}} = 2300 \qquad \text{or} \qquad V \approx 0.7\ \frac{\text{m}}{\text{s}}$$

$$(b)\ \text{Water:}\qquad \frac{\rho V d}{\mu} = \frac{(998\ \text{kg/m}^3)V(0.05\ \text{m})}{0.001\ \text{kg/(m}\cdot\text{s)}} = 2300 \qquad \text{or} \qquad V = 0.046\ \frac{\text{m}}{\text{s}}$$

日常碰上的流动，多数是湍流

摩擦系数：圆管中的湍流

光滑壁面

湍流核心区，有 $v^+ = 5.5 + 2.5\ln y^+$

$$v^+ \equiv \frac{\bar{v}}{\sqrt{\tau_0/\rho}} \qquad y^+ \equiv \frac{\sqrt{\tau_0/\rho}}{v}y$$

方程13-16、13-18、13-21

求平均速度

$$v_{\text{avg}} = \frac{\int_0^A \bar{v}\,dA}{A} = \frac{\sqrt{\tau_0/\rho}\int_0^R\left(2.5\ln\left\{\frac{\sqrt{\tau_0/\rho}y}{v}\right\}+5.5\right)2\pi r\,dr}{\pi R^2}$$

$$\frac{F}{A} \equiv C_f\frac{\rho v_\infty^2}{2}$$

在圆管中 $y = R - r$，积分可得

$$v_{\text{avg}} = 2.5\sqrt{\tau_0/\rho}\ln\left\{\frac{\sqrt{\tau_0/\rho}R}{v}\right\} + 1.75\sqrt{\tau_0/\rho} \qquad (1)$$

由于C_f和f_f等价，有

$$\frac{v_{\text{avg}}}{\sqrt{\tau_0/\rho}} = \frac{1}{\sqrt{f_f/2}}$$

代入公式（1）变为

$$\frac{1}{\sqrt{f_f/2}} = 2.5\ln\left\{\frac{R}{v}v_{\text{avg}}\sqrt{f_f/2}\right\} + 1.75$$

重新写成含雷诺数形式，有

$$\frac{1}{\sqrt{f_f}} = 4.06\log_{10}\{\text{Re}\sqrt{f_f}\} - 0.60$$

von Karman

实验数据所得的公式

$$\frac{1}{\sqrt{f_f}} = 4.0\log_{10}\{\text{Re}\sqrt{f_f}\} - 0.40$$

Nikuradse

粗糙壁面

理论推导

$$\frac{1}{\sqrt{f_f}} = 4.06\ \log_{10}\frac{D}{e} + 2.16$$

实验推导

$$\frac{1}{\sqrt{f_f}} = 4.0\ \log_{10}\frac{D}{e} + 2.28$$

注意，此时摩擦系数与Re无关

总结

For laminar flow ($\mathrm{Re} < 2300$)

$$f_f = \frac{16}{\mathrm{Re}}$$

For turbulent flow (smooth pipe, $\mathrm{Re} > 3000$)

$$\frac{1}{\sqrt{f_f}} = 4.0\ \log_{10}\{\mathrm{Re}\sqrt{f_f}\} - 0.40$$

For turbulent flow (rough pipe, ($\mathrm{Re} > 3000$, $D/e)/(\mathrm{Re}\sqrt{f_f}) < 0.01$)

粗糙到什么程度才叫粗糙？

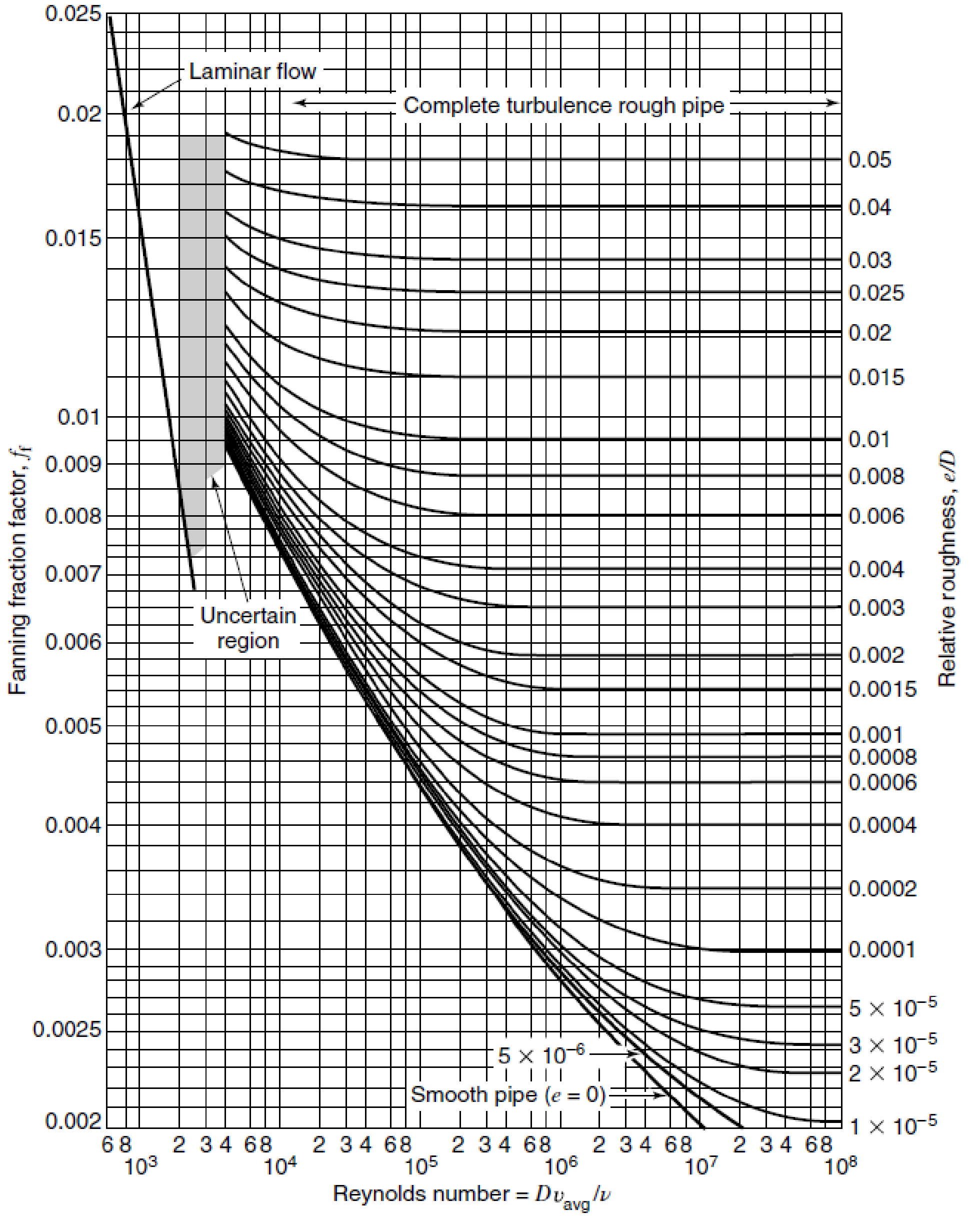
$$\frac{1}{\sqrt{f_f}} = 4.0\ \log_{10}\frac{D}{e} + 2.28$$

And for transition flow

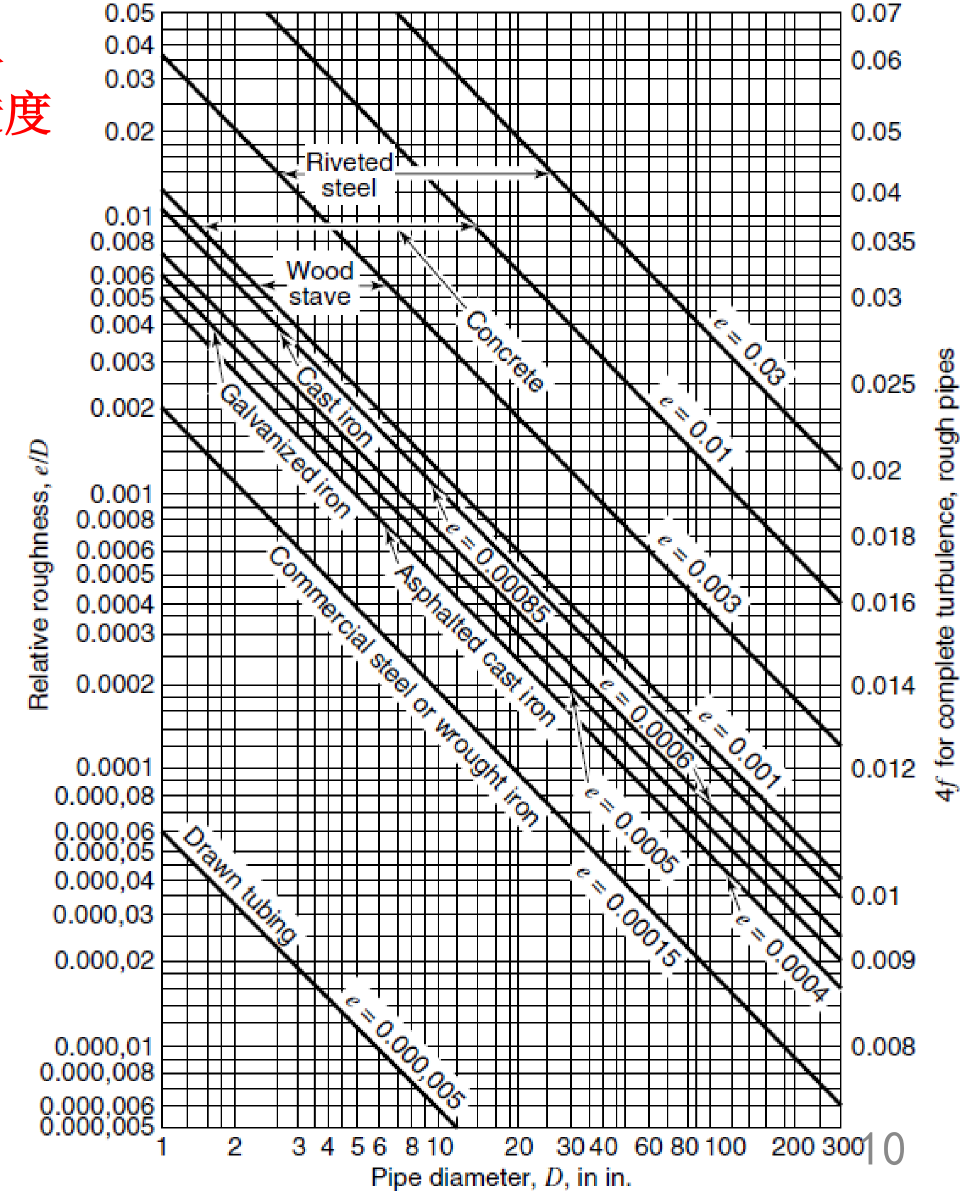
$$\frac{1}{\sqrt{f_f}} = 4\ \log_{10}\frac{D}{e} + 2.28 - 4\ \log_{10}\left(4.67\frac{D/e}{Re\sqrt{f_f}} + 1\right)$$

管流的摩擦系数

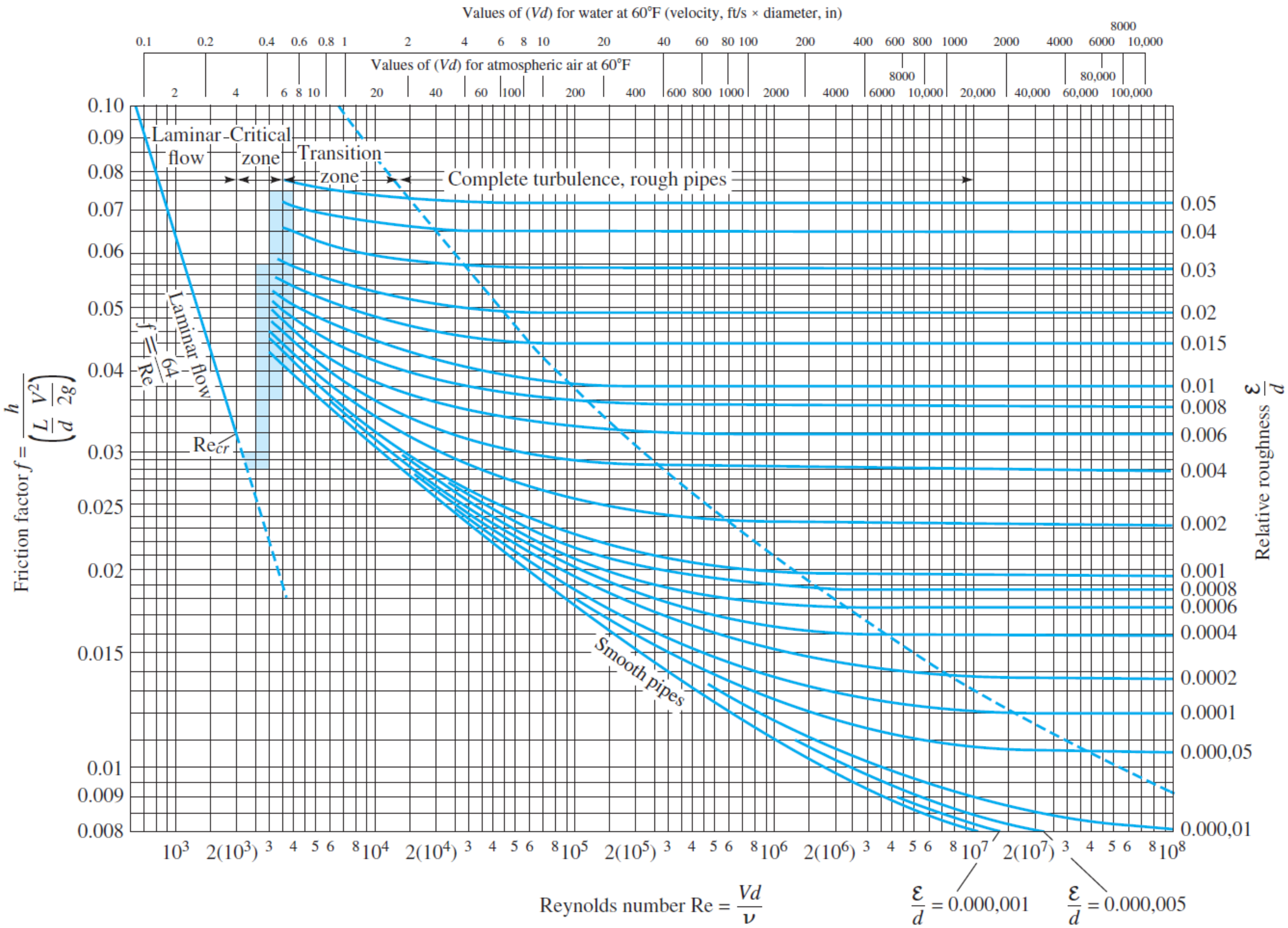
摩擦系数
Moody chart



各种材料
壁面粗糙度



摩擦系数的Moody图



各种常用材料的壁面粗糙度

		ε		
Material	**Condition**	**ft**	**mm**	**Uncertainty, %**
Steel	Sheet metal, new	0.00016	0.05	±60
	Stainless, new	0.000007	0.002	±50
	Commercial, new	0.00015	0.046	±30
	Riveted	0.01	3.0	±70
	Rusted	0.007	2.0	±50
Iron	Cast, new	0.00085	0.26	±50
	Wrought, new	0.00015	0.046	±20
	Galvanized, new	0.0005	0.15	±40
	Asphalted cast	0.0004	0.12	±50
Brass	Drawn, new	0.000007	0.002	±50
Plastic	Drawn tubing	0.000005	0.0015	±60
Glass	—	Smooth	Smooth	
Concrete	Smoothed	0.00013	0.04	±60
	Rough	0.007	2.0	±50
Rubber	Smoothed	0.000033	0.01	±60
Wood	Stave	0.0016	0.5	±40

压头损失（head loss）

压头损失 $$h_L = 2 f_f \frac{L}{D} \frac{v^2}{g}$$

For laminar flow (Re < 2300)

$$f_f = \frac{16}{\mathrm{Re}}$$

For turbulent flow (smooth pipe, Re > 3000)

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\{\mathrm{Re}\sqrt{f_f}\} - 0.40$$

For turbulent flow (rough pipe, (Re > 3000, D/e)/(Re$\sqrt{f_f}$) < 0.01)

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\frac{D}{e} + 2.28$$

And for transition flow

$$\frac{1}{\sqrt{f_f}} = 4 \log_{10}\frac{D}{e} + 2.28 - 4 \log_{10}\left(4.67\frac{D/e}{Re\sqrt{f_f}} + 1\right)$$

求解摩擦系数的显式公式

$$\frac{1}{\sqrt{f_f}} = -3.6 \log_{10}\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{e}{3.7D}\right)^{10/9}\right]$$

误差<1.5%

管件、阀门等造成的压头损失

各种部件会引起额外的压头损失 $$h_L = \frac{\Delta P}{\rho} = K\frac{v^2}{2g}$$ **K为不同部件的系数**

也可以折合成等价长度 L_{eq} $$h_L = 2\, f_f \frac{L_{eq}}{D}\frac{v^2}{g}$$

	Fitting	K	L_{eq}/D
球阀	Globe valve, wide open	7.5	350
角阀	Angle valve, wide open	3.8	170
	Gate valve, wide open	0.15	7
阀门	Gate valve, $\frac{3}{4}$ open	0.85	40
	Gate valve, $\frac{1}{2}$ open	4.4	200
	Gate valve, $\frac{1}{4}$ open	20	900
标准90度弯头	Standard 90° elbow	0.7	32
小半径90度弯头	Short-radius 90° elbow	0.9	41
大半径90度弯头	Long-radius 90° elbow	0.4	20
标准45度弯头	Standard 45° elbow	0.35	15
T型三通，侧面	Tee, through side outlet	1.5	67
T型三通，直流	Tee, straight through	0.4	20
180度弯头	180° Bend	1.6	75

当量直径

当量直径

$$D_{\mathrm{eq}} = 4\,\frac{\text{cross-sectional area of flow}}{\text{wetted perimeter}}$$

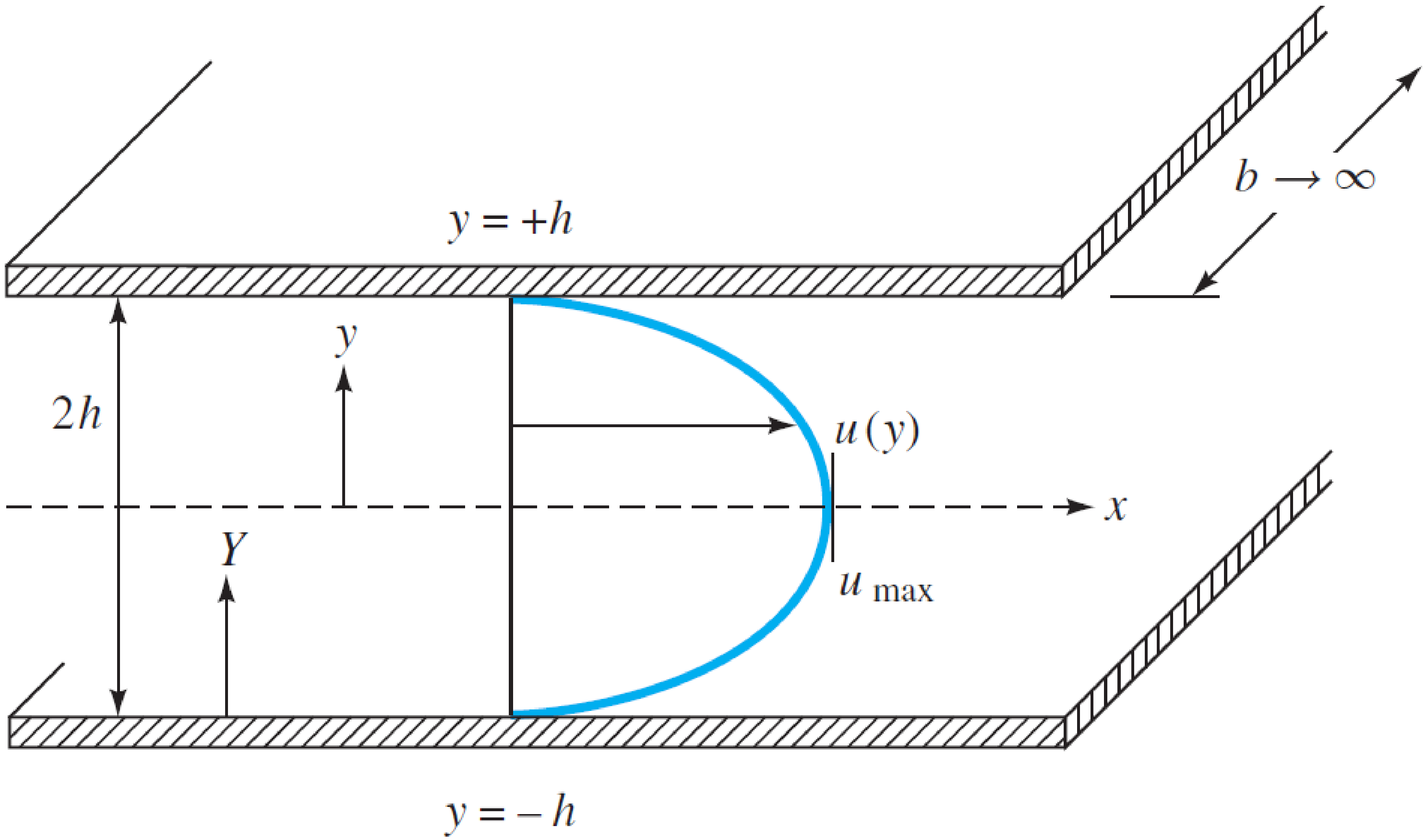
例子：环形管道的当量直径

$$\text{Cross-sectional area} = \frac{\pi}{4}(D_0^2 - D_i^2)$$

$$\text{Wetted perimeter} = \pi(D_0 + D_i)$$

$$D_{\mathrm{eq}} = 4\,\frac{\pi/4}{\pi}\,\frac{(D_0^2 - D_i^2)}{(D_0 + D_i)} = D_0 - D_i$$

当量直径是否准确？



无穷大平板间的层流

当量直径

$$D_h = \frac{4A}{\mathcal{P}} = \lim_{b\to\infty} \frac{4(2bh)}{2b + 4h} = 4h$$

$$u = u_{\max}\left(1 - \frac{y^2}{h^2}\right) \qquad u_{\max} = \frac{h^2}{2\mu}\frac{\Delta p}{L}$$

$$h_L = 2 f_f \frac{L}{D}\frac{v^2}{g}$$

平均速度 $$V = \frac{2}{3}u_{\max}$$

壁面剪切力 $$\tau_w = \mu\left|\frac{du}{dy}\right|_{y=h} = h\frac{\Delta p}{L} = \frac{3\mu V}{h}$$

近似摩擦系数为

$$f_f = \frac{16}{Re}$$

水头损失 $$h_L = \frac{\Delta p}{\rho g} = \frac{3\mu L V}{\rho g h^2}$$

低估 33 %！

得到摩擦系数

$$f_f = \frac{h_L D g}{2LV^2} = \frac{Dg \times 3\mu L V/\rho g \left(\frac{D}{4}\right)^2}{2LV^2} = \frac{24}{Re}$$

使用当量直径会带来误差

例2：倾斜直管道流动需要的推动功率

Water at 59°F flows through a straight section of a 6-in.-ID cast-iron pipe with an average velocity of 4 fps. The pipe is 120 ft long, and there is an increase in elevation of 2 ft from the inlet of the pipe to its exit.

Find the power required to produce this flow rate for the specified conditions.

能量守恒

$$\frac{\delta Q}{dt} - \frac{\delta W_s}{dt} - \frac{\delta W_\mu}{dt} = \iint_{\text{c.s.}} \rho\left(e + \frac{P}{\rho}\right)(\mathbf{v}\cdot\mathbf{n})\, dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} \rho e\, dV$$

$$\frac{\delta Q}{dt} = 0 \qquad \frac{\delta W_s}{dt} = W \qquad \frac{\delta W_\mu}{dt} = 0$$

各项具体为

$$\iint_{\text{c.s.}} \rho\left(e + \frac{P}{\rho}\right)(\mathbf{v}\cdot\mathbf{n})\, dA = \rho A v_{\text{avg}}\left(\frac{v_2^2}{2} + gy_2 + \frac{P_2}{\rho} + u_2 - \frac{v_1^2}{2} - gy_1 - \frac{P_1}{\rho} - u_1\right)$$

$$\frac{\partial}{\partial t}\iiint_{\text{c}\cdot\text{v}} \rho e\, dV = 0$$

整理后

$$\dot{W}/\dot{m} = \cancel{\frac{v_1^2 - v_2^2}{2}} + g(y_1 - y_2) + \cancel{\frac{P_1 - P_2}{\rho}} + u_1 - u_2$$

变为

$$\dot{W}/\dot{m} = g(y_1 - y_2) - gh_L$$

已知参数

$$\mathrm{Re} = \frac{\left(\frac{1}{2}\right)(4)}{1.22 \times 10^{-5}} = 164{,}000 \qquad \frac{e}{D} = 0.0017$$

根据公式

$$\frac{1}{\sqrt{f_f}} = -3.6 \log_{10}\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{e}{3.7D}\right)^{10/9}\right]$$

得到

$$f_f = 0.0059$$

可以求得水头损失

$$h_L = \frac{2(0.0059)(120\ \mathrm{ft})(16\ \mathrm{ft}^2/\mathrm{s}^2)}{(0.5\ \mathrm{ft})(32.2\ \mathrm{ft}/\mathrm{s}^2)} = 1.401\ \mathrm{ft}$$

$$h_L = 2f_f \frac{L}{D}\frac{v^2}{g}$$

$$\dot{W}/\dot{m} = g(y_1 - y_2) - gh_L$$

最后代入公式计算得到所需功率

$$\dot{W} = \frac{-g\left((-2\ \mathrm{ft}) - 1.401\ \mathrm{ft}\right)}{550\ \mathrm{ft\ lb_f/hp-s}}\left[\frac{62.3\ \mathrm{lb_m/ft^3}}{32.2\ \mathrm{lb_m ft/s^2\ lb_f}}\left(\frac{\pi}{4}\right)\left(\frac{1}{2}\mathrm{ft}\right)^2\left(4\frac{\mathrm{ft}}{\mathrm{s}}\right)\right]$$
$$= 0.300\ \mathrm{hp}$$

例2：求一定操作条件下的匹配管径

A heat exchanger is required, which will be able to handle 0.0567 m^3/s of water through a smooth pipe with an equivalent length of 122 m. The total pressure drop is 103,000 Pa. What size pipe is required for this application?

能量守恒

$$\frac{\delta Q}{dt} - \frac{\delta W_s}{dt} - \frac{\delta W_\mu}{dt} = \iint_{\text{c.s.}} \rho\left(e + \frac{P}{\rho}\right)(\mathbf{v}\cdot\mathbf{n})\quad dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} \rho e\, dV$$

$$\frac{\delta Q}{dt} = 0 \quad \frac{\delta W_s}{dt} = 0 \quad \frac{\delta W_\mu}{dt} = 0$$

各项具体为

$$\iint_{\text{c.s.}} \rho\left(e + \frac{P}{\rho}\right)(\mathbf{v}\cdot\mathbf{n})\, dA = \rho A v_{\text{avg}}\left(\frac{v_2^2}{2} + g y_2 + \frac{P_2}{\rho} + u_2 - \frac{v_1^2}{2} - g y_1 - \frac{P_1}{\rho} - u_1\right)$$

$$\frac{\partial}{\partial t}\iiint_{\text{c}\cdot\text{v}} \rho e\, dV = 0$$

整理后

$$0 = \frac{P_2 - P_1}{\rho} + gh_L$$

代入参数

$$0 = -\frac{103{,}000\ \mathrm{Pa}}{1000\ \mathrm{kg/m^3}} + 2f_f\left(\frac{0.0567}{\pi D^2/4}\right)^2 \frac{\mathrm{m^2}}{\mathrm{s^2}} \cdot \frac{122}{D}\frac{\mathrm{m}}{\mathrm{m}}\frac{g}{g}$$

$$0 = -103 + 1.27\frac{f_f}{D^5}$$

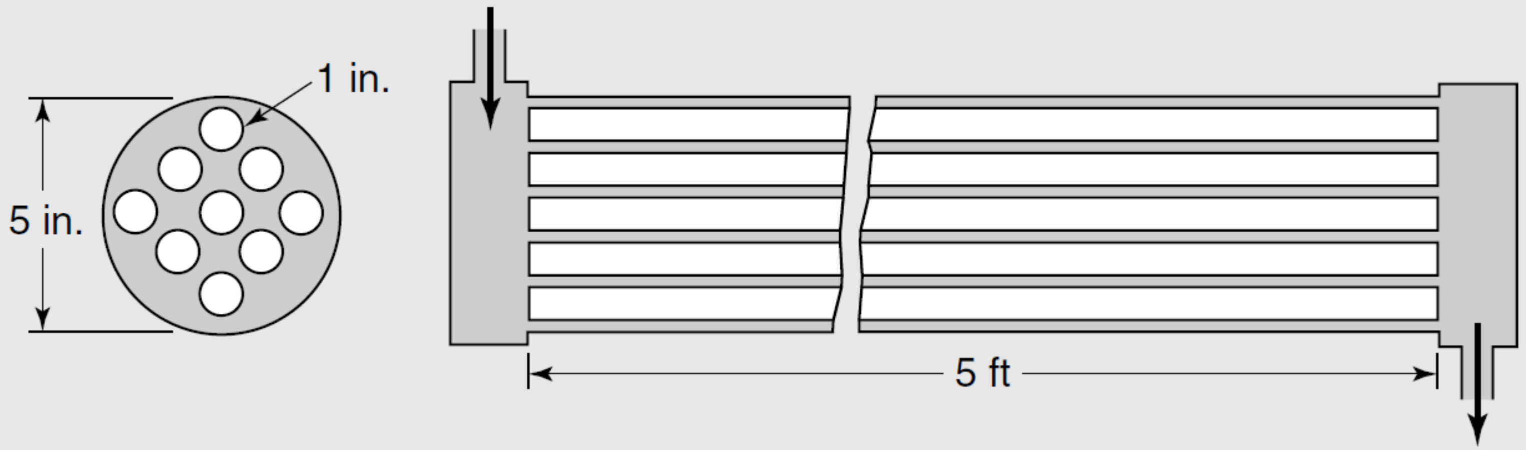
摩擦系数依赖于直径D

1. Assume a value for f_f.
2. Using this f_f, solve the above equation for D.
3. Calculate Re with this D.
4. Using e/D and the calculated Re, check the assumed value of f_f.
5. Repeat this procedure until the assumed and calculated friction-factor values agree.

迭代求得 D=0.132 m

例3：求已知压降下复杂横截面通道的流量

An existing heat exchanger has a cross section as shown in Figure 13.3 with nine 1-in.-OD tubes inside a 5-in.-ID pipe. For a 5-ft length of heat exchanger, what flow rate of water at 60°F can be achieved in the shell side of this unit for a pressure drop of 3 psi?



和例2类似，有 $$0 = \frac{P_2 - P_1}{\rho} + gh_L$$

求当量直径

$$\text{Flow area} = \frac{\pi}{4}(25 - 9) = 4\pi \text{ in.}^2$$

$$\text{Wetted perimeter} = \pi(5 + 9) = 14\pi \text{ in.}$$

$$D_{eq} = 4\frac{4\pi}{14\pi} = 1.142 \text{ in.}$$

代入参数

$$0 = -\frac{3\ \mathrm{lbf/in.^2}(144\ \mathrm{in.^2/ft^2})}{1.94\ \mathrm{slugs/ft^3}} + 2f_f v_{\mathrm{avg}}^2\ \mathrm{ft^2/s^2}\frac{5\ \mathrm{ft}}{(1.142/12)\ \mathrm{ft}}\frac{g}{g}$$

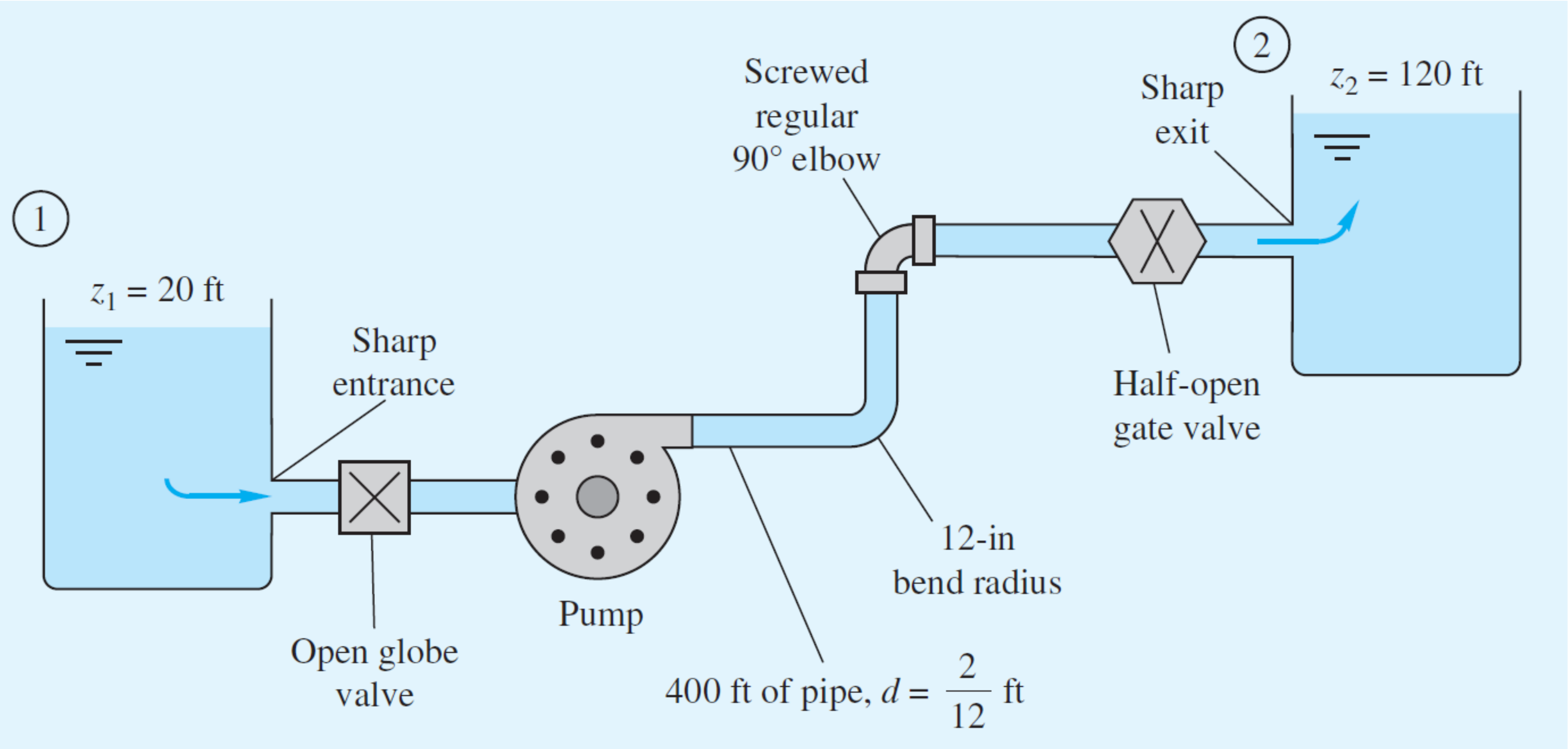
$$0 = -223 + 105 f_f v_{\mathrm{avg}}^2$$

摩擦系数依赖于流速

1. Assume a value for f_f.
2. Calculate v_{avg} from the above expression.
3. Evaluate Re from this value of v_{avg}.
4. Check the assumed value of f_f
5. If the assumed and calculated values for f_f do not agree, repeat this procedure until they do.

迭代求得 v_{avg}=23.6 ft/s，流量为 2.06 ft^3/min

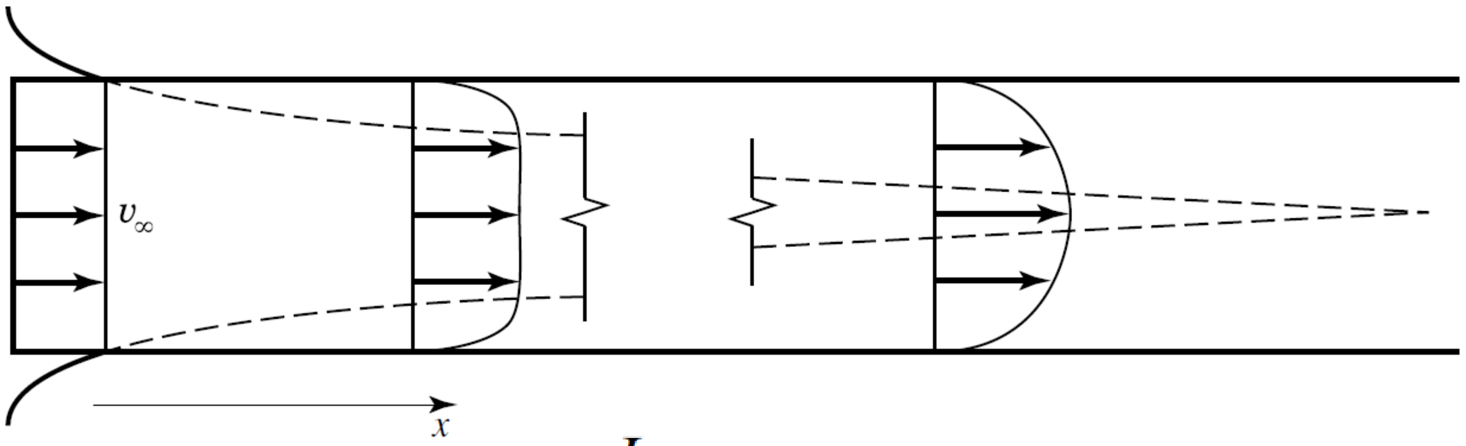
例4：真实管路系统



$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) + h_f + \sum h_m - h_p$$

泵的功率 $$P = \rho g Q h_p$$

圆管流动入口端的摩擦系数



入口效应
Entrance effect

充分发展的层流
流向速度剖面不变

$$\frac{L_e}{D} = 0.0575\ \mathrm{Re}$$

充分发展的湍流　　教材认为无解析公式 $L_e > 50D$

但最近的计算流体力学（CFD）结果提出了经验公式 $\frac{L_e}{d} \approx 1.6\ \mathrm{Re}_d^{1/4} \quad \text{for} \quad \mathrm{Re}_d \le 10^7$

Re_d	4000	10^4	10^5	10^6	10^7
L_e/d	13	16	28	51	90

利用入口效应

d=0.5 in, Q= 5 gal/min, L= 60 ft，
求入口区域占总长比例

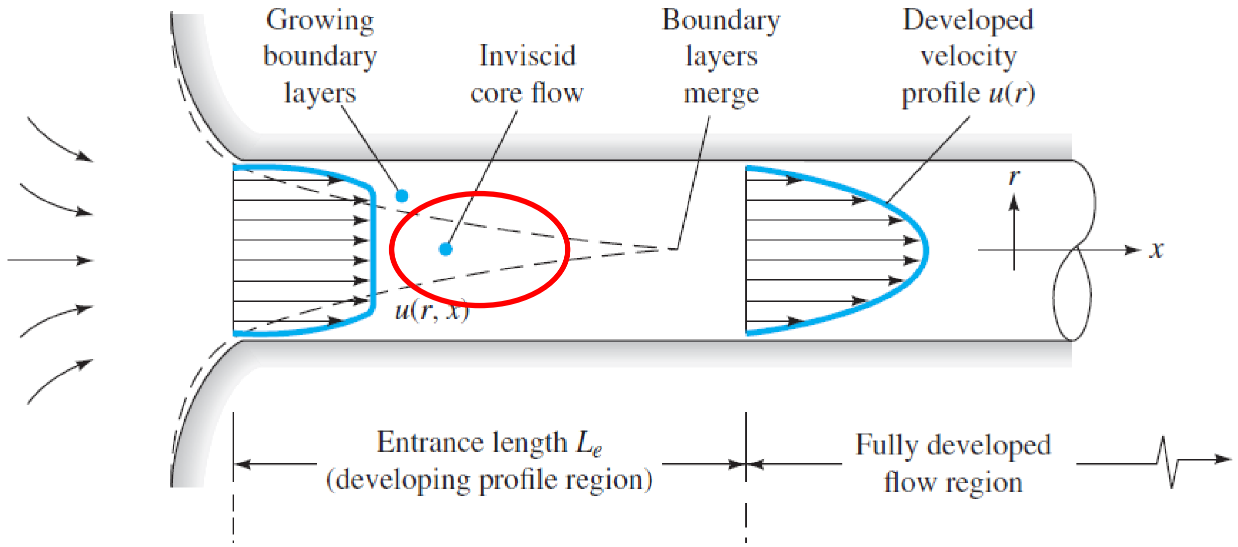
$$V = \frac{Q}{A} = \frac{0.0111\ \text{ft}^3/\text{s}}{(\pi/4)[(\frac{1}{2}/12)\ \text{ft}]^2} = 8.17\ \text{ft/s}$$

$$\text{Re}_d = \frac{Vd}{\nu} = \frac{(8.17\ \text{ft/s})[(\frac{1}{2}/12)\ \text{ft}]}{1.09 \times 10^{-5}\ \text{ft}^2/\text{s}} = 31{,}300$$

$$\frac{L_e}{d} \approx 1.6\,\text{Re}_d^{1/4} = (1.6)(31{,}300)^{1/4} = 21$$

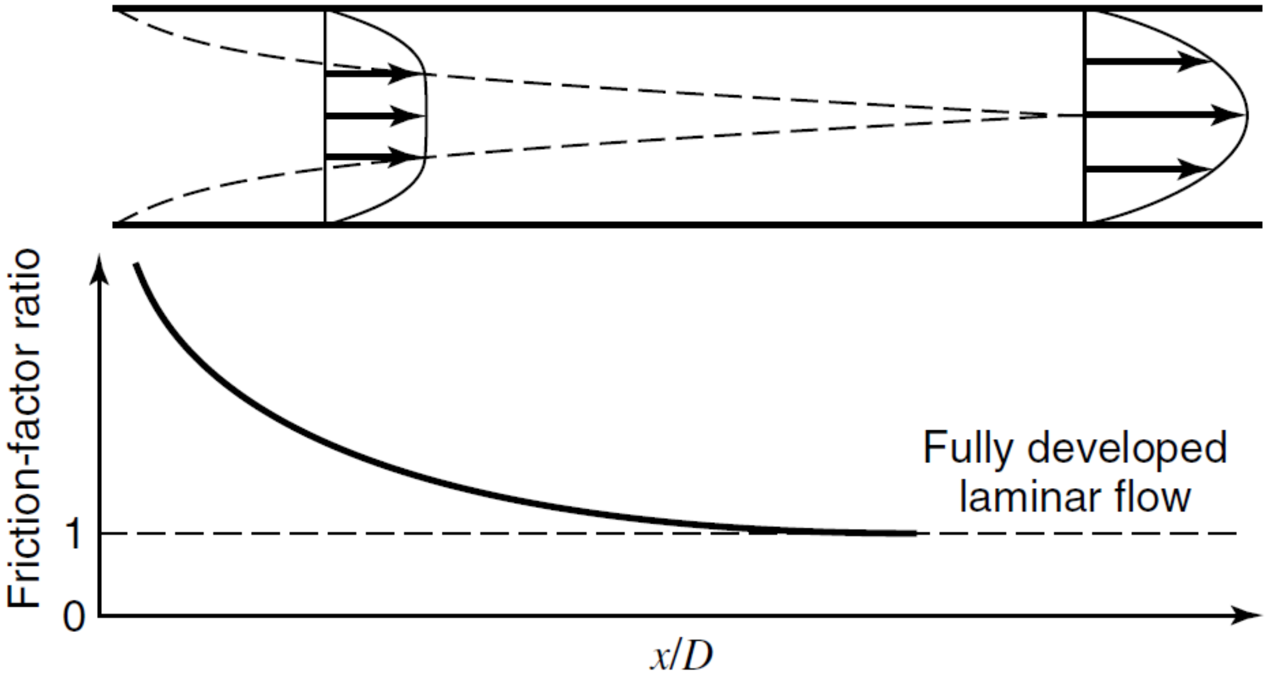
***Le*= 10.5 in**

10.5/720 = 1.5 %



对风洞来说，反而不希望在充分发展的流动中进行实验，因为壁面效应违背了自由飞行（free-flight）条件

层流入口摩擦系数的变化规律

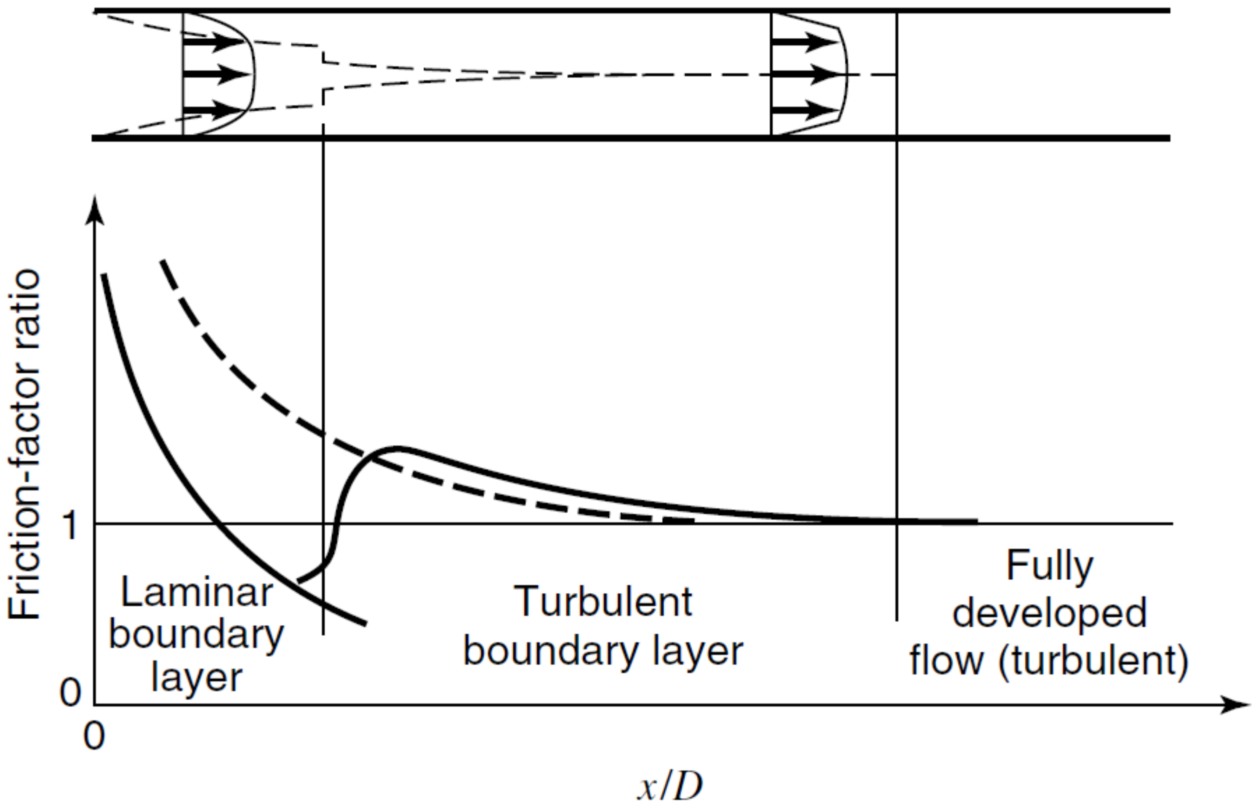


入口附近层流发展和摩擦系数分布

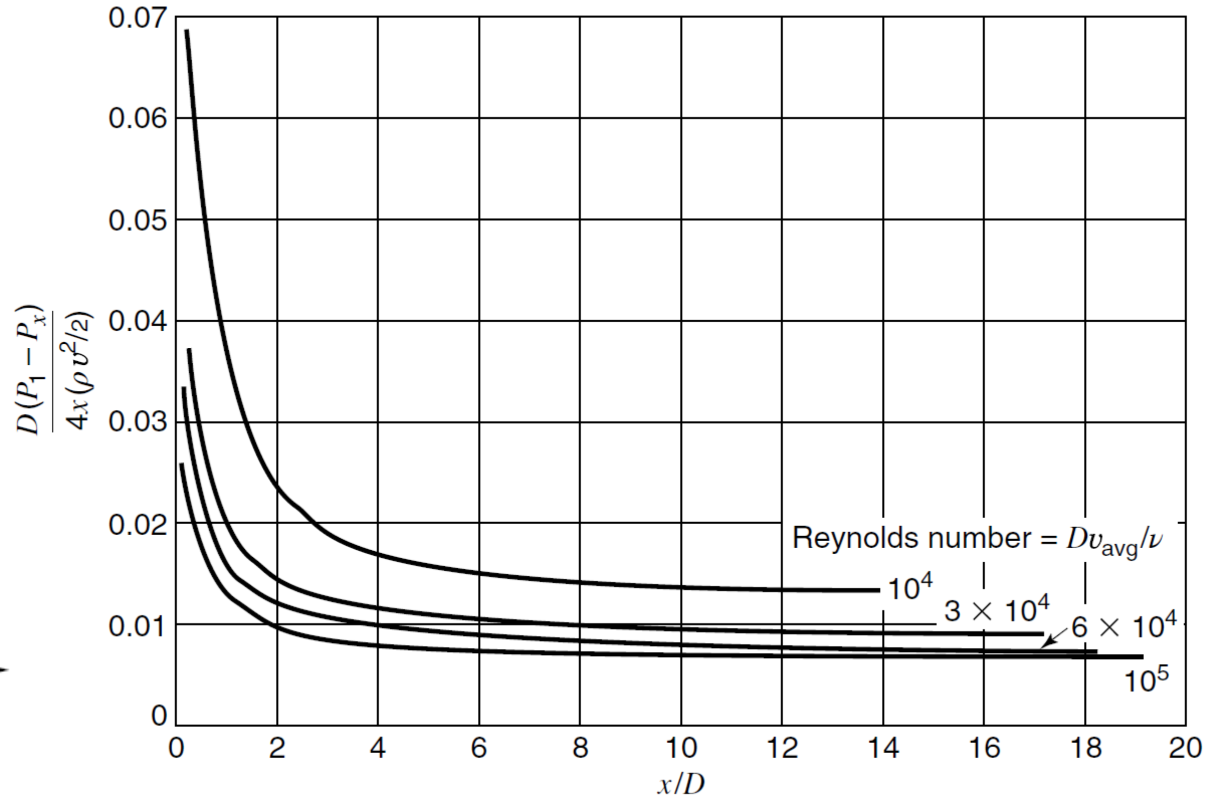
$\dfrac{x/D}{\mathrm{Re}}$	$f_f\left(\dfrac{x}{D}\right)$
0.000205	0.0530
0.000830	0.0965
0.001805	0.1413
0.003575	0.2075
0.00535	0.2605
0.00838	0.340
0.01373	0.461
0.01788	0.547
0.02368	0.659
0.0341	0.845
0.0449	1.028
0.0620	1.308
0.0760	1.538

从入口到下游不同位置的
平均摩擦系数

湍流入口摩擦系数的变化规律



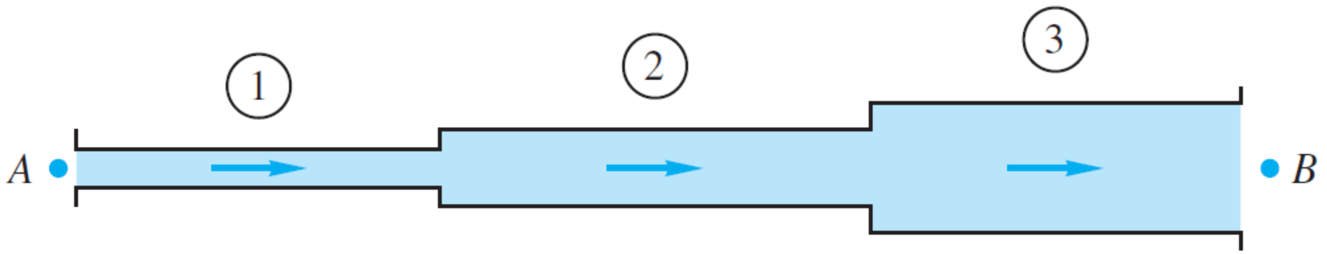
入口附近湍流发展和摩擦系数分布



入口附近静压降分布

在很多情况下，流动来不及达到充分发展状态，因此实际摩擦系数大于充分发展流动前提获得的预测值！

复杂管路系统：串联管道



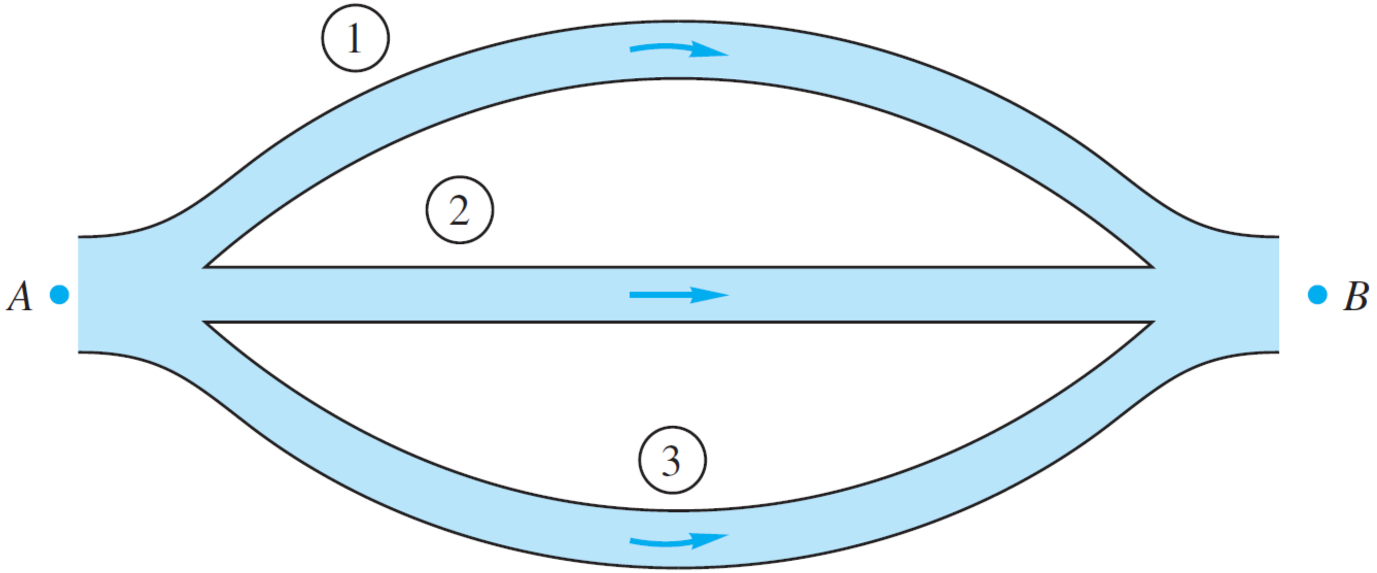
(a)

$$Q_1 = Q_2 = Q_3 = \text{const}$$

$$V_1 d_1^2 = V_2 d_2^2 = V_3 d_3^2$$

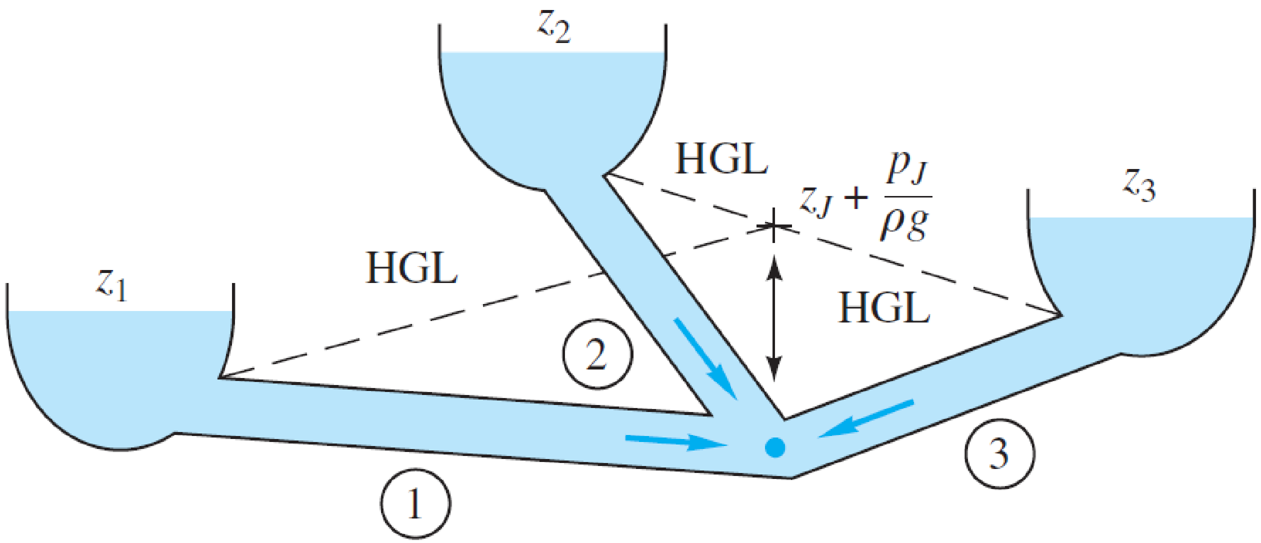
$$\Delta h_{A \to B} = \Delta h_1 + \Delta h_2 + \Delta h_3$$

复杂管路系统：并联管道



$$\Delta h_{A\to B} = \Delta h_1 = \Delta h_2 = \Delta h_3$$
$$Q = Q_1 + Q_2 + Q_3$$

复杂管路系统：汇聚结构管道



$$Q_1 + Q_2 + Q_3 = 0$$

$$\Delta h_1 = \frac{V_1^2}{2g}\frac{f_1 L_1}{d_1} = z_1 - h_J$$

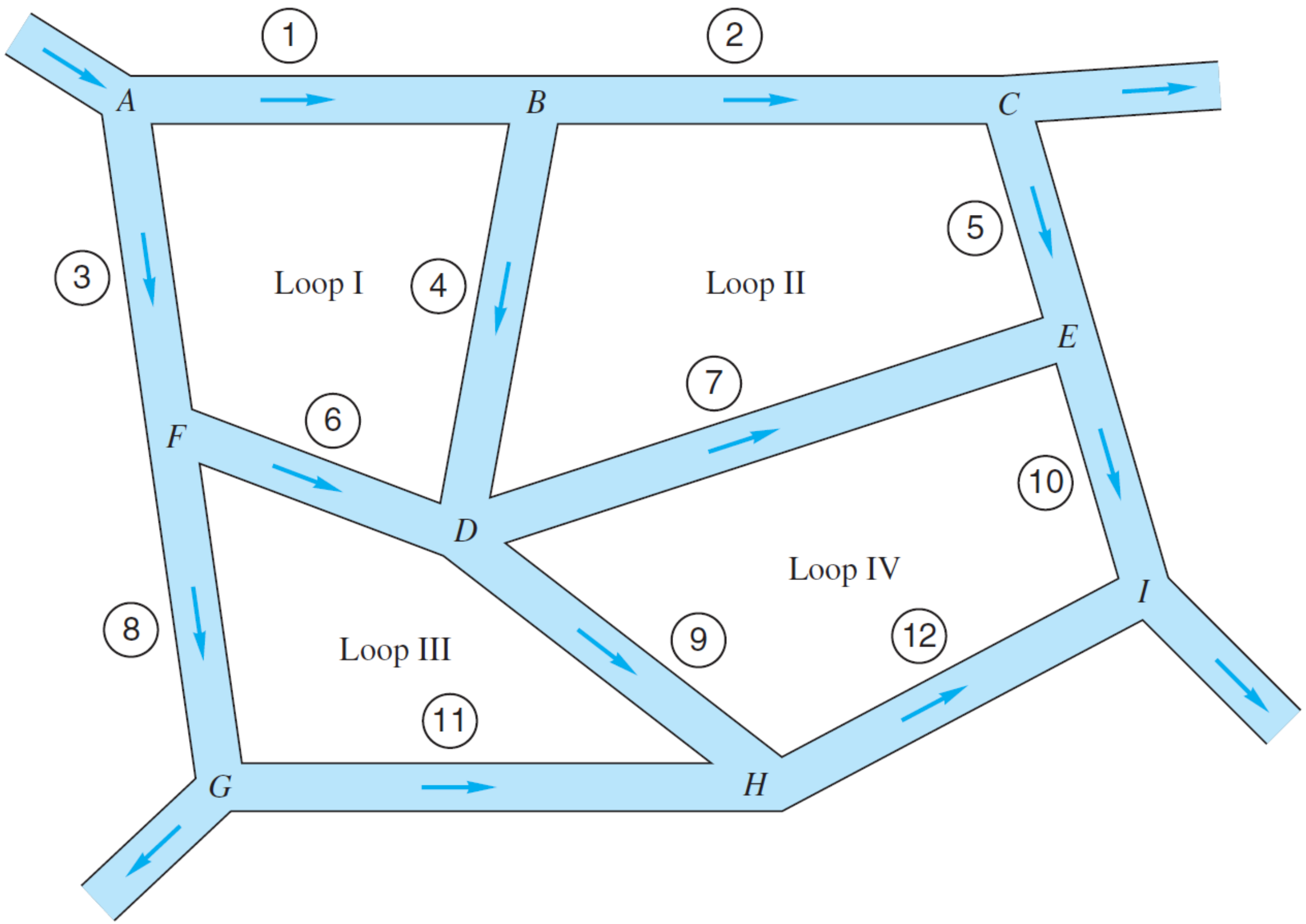
$$\Delta h_2 = \frac{V_2^2}{2g}\frac{f_2 L_2}{d_2} = z_2 - h_J$$

$$\Delta h_3 = \frac{V_3^2}{2g}\frac{f_3 L_3}{d_3} = z_3 - h_J$$

$$h_J = z_J + \frac{p_J}{\rho g}$$

管道流动与电路具有相似性！

复杂管路系统



- 节点净流量为零
- 封闭回路没有净压力变化为零
- 压力变化满足基本规律

会产生一组代数方程

课后作业

14.5、14.15