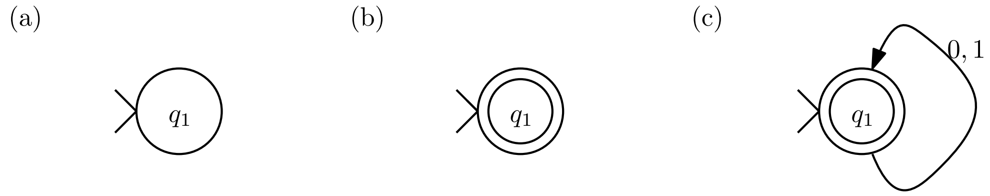
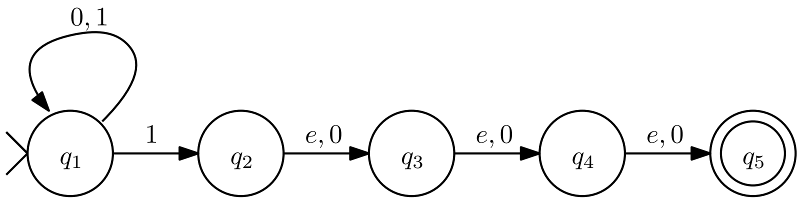
Theory of Computation, Fall 2023
Quiz 1&2 Solutions

Q1. (a) True. (b) True. (c) True. (d) True.

Q2. The following DFAs meet the requirements, respectively.



Q3. The following NFA meets the requirement.



Q4. Assume that NFA $M_A = (K_A, \Sigma, \delta_A, s_A, F_A)$ accepts A and NFA $M_B = (K_B, \Sigma, \delta_B, s_B, F_B)$ accpets B. Then, we construct the following NFA $M = (K, \Sigma, \delta, s, F)$ accepts L, thus L is regular. We find that L is composed of alternating elements in A and B, so we add a symbol $\mathcal{A}$ or $\mathcal{B}$ expanding the state to represent whether the currently string ends with a symbol in A or B, then δ can be constructed.

$$\begin{aligned} K &= K_A \times K_B \times \{\mathcal{A}, \mathcal{B}\} \\ s &= (s_A, s_B, \mathcal{A}) \\ F &= F_A \times F_B \times \{\mathcal{A}\} \cup \{s\} \end{aligned}$$

and

$$\begin{aligned} \delta((q_1, q_B, \mathcal{A}), a) &= (q_2, q_B, \mathcal{B}), \text{ if } \delta_A(q_1, a) = q_2, \text{where } q_1, q_2 \in K_A, q_B \in K_B, a \in \Sigma \\ \delta((q_A, q_1, \mathcal{B}), b) &= (q_A, q_2, \mathcal{A}), \text{ if } \delta_B(q_1, b) = q_2, \text{where } q_1, q_2 \in K_B, q_A \in K_A, b \in \Sigma \end{aligned}$$

Q5. L is not regular but context-free.

(a) Firstly, L is context-free because it can be generated by the following CFG.

$$\begin{aligned} S &\to ASA|a|b \\ A &\to a|b|c \end{aligned}$$

(b) Then, we prove L is not regular by the pumping theorem. Suppose, for the sake of contradiction, that this language is regular. Let p be the pumping length given by the pumping theorem. Consider the string $w = c^p a c^p \in L$. By pumping theorem, w can be written as $w = xyz$ such that the following holds.

1. $xy^iz \in L$ for any $i \geq 0$,
2. $|y| > 0$,
3. $|xy| \leq p$.

Since $|xy| \leq p$ implies that $y = c^k$ for some $k > 0$. Consider the string $xy^0z = c^{p-k}ac^p$. Clearly, this string does not belong to L. Therefore, L is not regular.

Q6. As the PDA reads the input, we use the stack as a unary counter to record the value of $2A - B$, where A and B are the number of a's and b's that have been read so far. After all the input symbols are consumed, if the stack is not empty (i.e., $2A \neq B$), the PDA will accept the input. We use the symbol $+$ to denote $+1$, and $-$ to denote -1. The PDA $P = (K, \Sigma, \Gamma, \Delta, s, F)$ is as follows.

$$\begin{aligned} K &= \{q_1, q_2, q_3\}, \\ \Sigma &= \{a, b\}, \\ \Gamma &= \{+, -\}, \\ s &= q_1, \\ F &= \{q_2, q_3\}, \end{aligned}$$

and Δ contains the following transitions.

(q, a, β)	(p, γ)
(q_1, a, e)	$(q_1, ++)$
$(q_1, a, -)$	$(q_1, +)$
$(q_1, a, --)$	(q_1, e)
(q_1, b, e)	$(q_1, -)$
$(q_1, b, +)$	(q_1, e)
$(q_1, e, +)$	(q_2, e)
$(q_2, e, +)$	(q_2, e)
$(q_1, e, -)$	(q_3, e)
$(q_3, e, -)$	(q_3, e)