Register No. ____________________ Name: ____________________ Score: ____________________

1. (20 points) Determine whether the following statements are true or false. If it is true write a √ otherwise a × in the blank before the statement.

( ) (1) The proposition "if it is not raining, then it is raining" is equivalent to "it is raining".

( ) (2) The proposition $((p \to \neg q) \wedge q) \to \neg p$ is a tautology.

( ) (3) The following set of propositions is consistent.

Whenever the system software is being upgraded, users cannot access the file system.
If users can access the file system, then they can save new files.
If users cannot save new files, then the system software is not being upgraded."

( ) (4) Assume that $\exists y \forall x P(x, y)$ is true and that the domain of discourse is nonempty. Then the statement $\forall x \exists y P(x, y)$ must also be true.

( ) (5) The set of irrational numbers between $\sqrt{2}$ and $\pi/2$ is uncountable.

If users cannot save new files, then the system software is not being upgraded.

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( ) (6) Let $F=\{f \mid f: N \rightarrow \{0,1,2,3,4,5,6,7,8,9\}\}$, Where N is the set of natural numbers, then F is uncountable.

( ) (7) If $P(A) \in P(B)$, then $A \in B$. ($P(S)$ is the power set of S).

( ) (8) $A \oplus A = A$.

( ) (9) The function $f(n) = 3\lfloor n/3 \rfloor$ from Z to Z is a one-to-one function (injection), where Z is the set of integers.

( ) (10) There exists a one-to-one function from R to $Z \times Z$, where R is the set of real numbers and Z is the set of integers.

is the set of integers.

2. (20 points) Fill in the blanks.

(1) Suppose $A=\{a, b\}$ and $B=\{a, b, c, \{d\}, \{e\}\}$, then $|P(A\times B)|=$________________________.

(2) Write English statement using the following predicates and any needed quantifiers.

Suppose the variable x represents students and y represent courses, and :

$U(y)$: y is an upper-level course $F(x)$: x is a freshman

$A(x)$: x is a part-time student $T(x, y)$: student x is taking course y.

Every part-time freshman is taking some upper-level course.

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(3) Give a recursive definition of the set of integer 1, 111, 11111, 1111111, . . . (Include initial conditions and assume that the sequences begin with a_1).

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(4) $|\overline{A} \cap \overline{B} \cap \overline{C}|=$ ____

(5) $|\{\varnothing,\{\varnothing\}\}\oplus\{\varnothing\}| =$ ____

3. (4 points) Use the definition of big-oh to prove that $\dfrac{6n+4n^5-4}{7n^2-3}$ is $O(n^3)$.

4.(10 points) Convert the following formula into logically equivalent formula in full disjunctive normal form. Determine whether it is tautology, contradiction or contingence. Find the assignments of p, q and r for which the formula is true.

$$(\neg r \vee (q \rightarrow p)) \rightarrow (p \rightarrow (q \vee r))$$

5. (6 points) Assuming the domain consists of all persons.

Let $p(x)$: x in this class , $q(x)$: x enjoys whale watching, $r(x)$: cares about ocean pollution.

Show that the following argument is valid.

Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution.