

# Compiler Principle

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# 3 Parsing

## 3.3 LR Parsing

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# Bottom-up Parsing

- The **weakness** of  $LL(k)$  parsing techniques is that they must **predict** which production to use, having **seen only the first  $k$  tokens** of the right-hand side.
- $LR(k)$  parsing, is able to **postpone the decision** until it has seen **input tokens corresponding to the entire right-hand side of the production** in question.

# Bottom-up Parsing

- **Shift-reduce parsing**
  - ✓ Bottom-up parsing
  - ✓ LR(k)    **L**eft-to-right parse,  
              **R**ightmost derivation  
              **k**-token lookahead
- **More powerful than LL(k) parsers**

## **LALR variant:**


- ✓ The basis for parsers for most modern programming languages
- ✓ Implemented in tools such as Yacc

# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table





# Example


## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table




Input from lexer:

**yet to read**  
     
( id = num ; id = num ) EOF

State of parse so far:




# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table






Input from lexer:

$( \text{id} = \text{num} ; \text{id} = \text{num} ) \text{ EOF}$

State of parse so far:

$($

**SHIFT**


# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table



Input from lexer:

( id = num ; id = num ) EOF

**SHIFT**


State of parse so far: ( id

# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table






Input from lexer:

( id = num ; id = num ) EOF

yet to read

**SHIFT**

State of parse so far: ( id =

# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table




Input from lexer:

( id = num ; id = num ) EOF

yet to read



State of parse so far: ( id = num



**SHIFT**

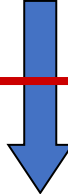
# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table



Input from lexer:

( id = num ; id = num ) EOF

State of parse so far:

( L

**REDUCE**

$S \rightarrow \text{id} = \text{num}$


$L \rightarrow S$

# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table




Input from lexer:

( id = num ; id = num ) EOF

yet to read

State of parse so far: ( L ;

**SHIFT**

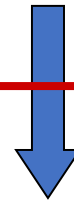
# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table




Input from lexer:

( id = num ; id = num ) EOF

yet to read



State of parse so far:

( L ; id = num


SHIFT  
SHIFT  
SHIFT

# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table






Input from lexer: ( id = num ; id = num ) EOF

yet to read



**REDUCE**

**$S \rightarrow \text{id} = \text{num}$**



State of parse so far: ( L ; S




# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table






Input from lexer:

( id = num ; id = num ) <sup>yet to read</sup> EOF

**REDUCE**  
 $S \rightarrow L ; S$


State of parse so far: ( L

# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table




---

Input from lexer:    ( id = num ; id = num ) EOF

State of parse so far:    ( L )

yet to read


**SHIFT**

# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table






Input from lexer: ( id = num ; id = num ) EOF

yet to read



State of parse so far:    S

**REDUCE**

**S  $\rightarrow$  ( L )**



# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table






Input from lexer: ( id = num ; id = num ) EOF

State of parse so far: A

**SHIFT**

**REDUCE**

**A  $\rightarrow$  S EOF**


**ACCEPT**

# Example

## Grammar:

$A \rightarrow S \text{ EOF}$      $L \rightarrow L ; S$      $S \rightarrow ( L )$   
 $L \rightarrow S$          $S \rightarrow \text{id} = \text{num}$

## Parsing Table




**Input from lexer:**    ( id = num ; id = num ) EOF

**State of parse so far:**    A

<b>A successful parse! Is this grammar LL(1)?</b>
---

# Bottom-up Parsing

- Parser **keeps track** of
  - ✓ **Position** in current input (what input to read next)
  - ✓ **A stack** of terminal & non-terminal symbols representing the “parse so far”
- Based on next input symbol & stack, **parser table indicates**
  - ✓ **shift**: push next input on to top of stack
  - ✓ **reduce R**:
    - top of stack should match RHS of rule
    - replace top of stack with LHS of rule
  - ✓ **error**
  - ✓ **accept** (we shift EOF & can reduce what remains on stack to start symbol)

# LR PARSING ENGINE

How does the LR parser know **when to shift and when to reduce?**

**Using a deterministic finite automaton !**

# LR PARSING ENGINE

## **Shift-reduce** algorithm (details)

- The parser summarizes the current “parse state” using **an integer**
  - ✓ The integer is actually **a state** in a finite automaton
  - ✓ The current parse state can be computed by **running the automaton over** the current **parse stack**



<b>a :Accept;</b>	<b><i>sn</i> :Shift into state <i>n</i>;</b>
<b>Error :</b>	<b><i>gn</i> :Goto state <i>n</i>;</b>
<b>(denoted by a blank entry in the table).</b>	<b><i>rk</i>: Reduce by rule <i>k</i>;</b>

	id	num	print	;	,	+	:=	(	)	\$	<i>S</i>	<i>E</i>	<i>L</i>
1	s4		s7								g2		
2				s3						a			
3	s4		s7								g5		
4							s6						
5				r1	r1					r1			
6	s20	s10						s8				g11	
7								s9					
8	s4		s7								g12		
9	s20	s10						s8				g15	g14
10				r5	r5	r5			r5	r5			
11				r2	r2	s16				r2			
12				s3	s18								
13				r3	r3					r3			
14					s19				s13				
15					r8				r8				
16	s20	s10						s8				g17	
17				r6	r6	s16			r6	r6			
18	s20	s10						s8				g21	
19	s20	s10						s8				g23	
20				r4	r4	r4			r4	r4			
21									s22				
22				r7	r7	r7			r7	r7			
23					r9	s16			r9				

# LR PARSING ENGINE

## Shift-reduce algorithm (details)

- **Algorithm:** Based on next input **symbol** & the parse **state** (as opposed to the entire stack), parser table indicates

***Shift( $n$ ):*** Advance input one token;  
push  $n$  on stack.

***Reduce( $k$ ):***

- ✓ Pop stack as many times as the number of symbols on the right-hand side of rule  $k$ ;
- ✓ Let  $X$  be the left-hand-side symbol of rule  $k$ ;
- ✓ In the state now on top of stack, **look up  $X$  to get "goto  $n$ "; Push  $n$  on top of stack.**

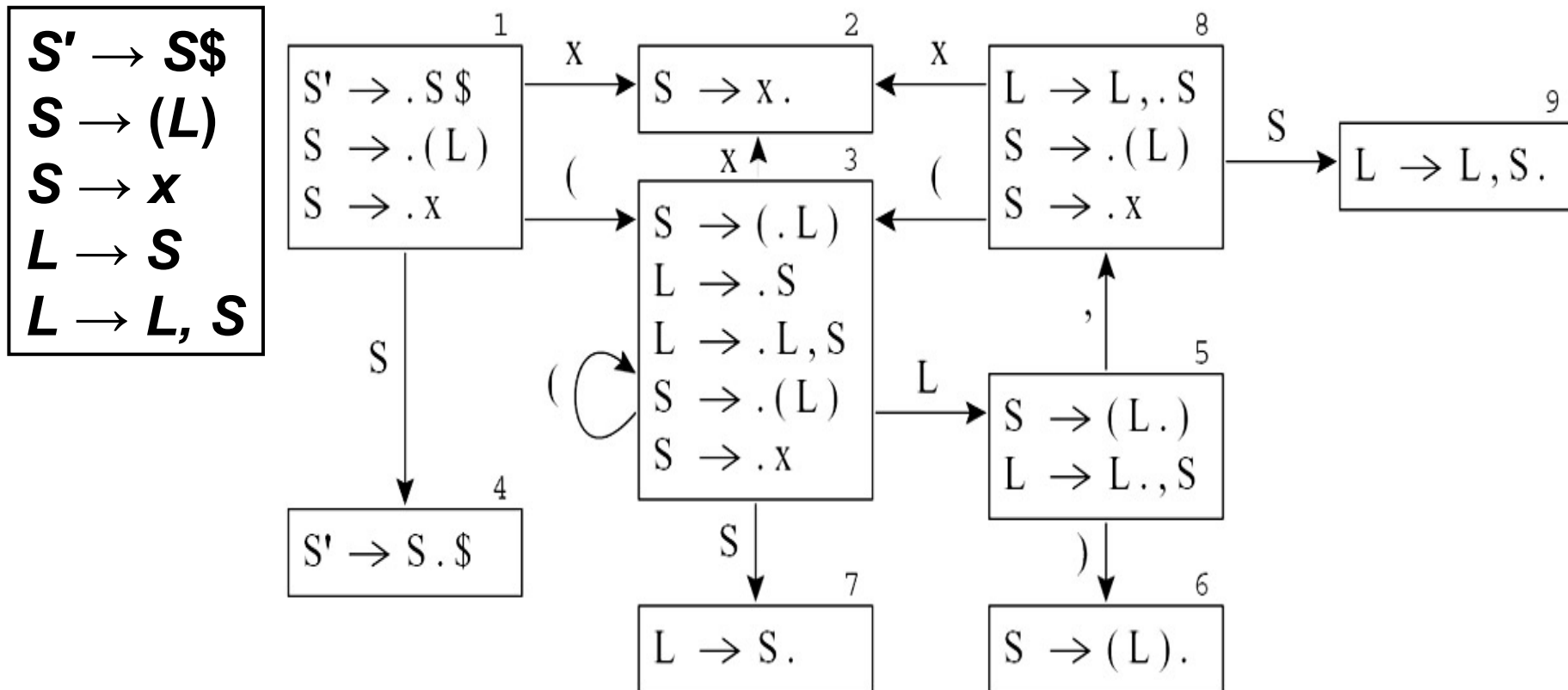
***Accept:*** Stop parsing, report success.

***Error:*** Stop parsing, report failure.

# LR(0) Parser Generation

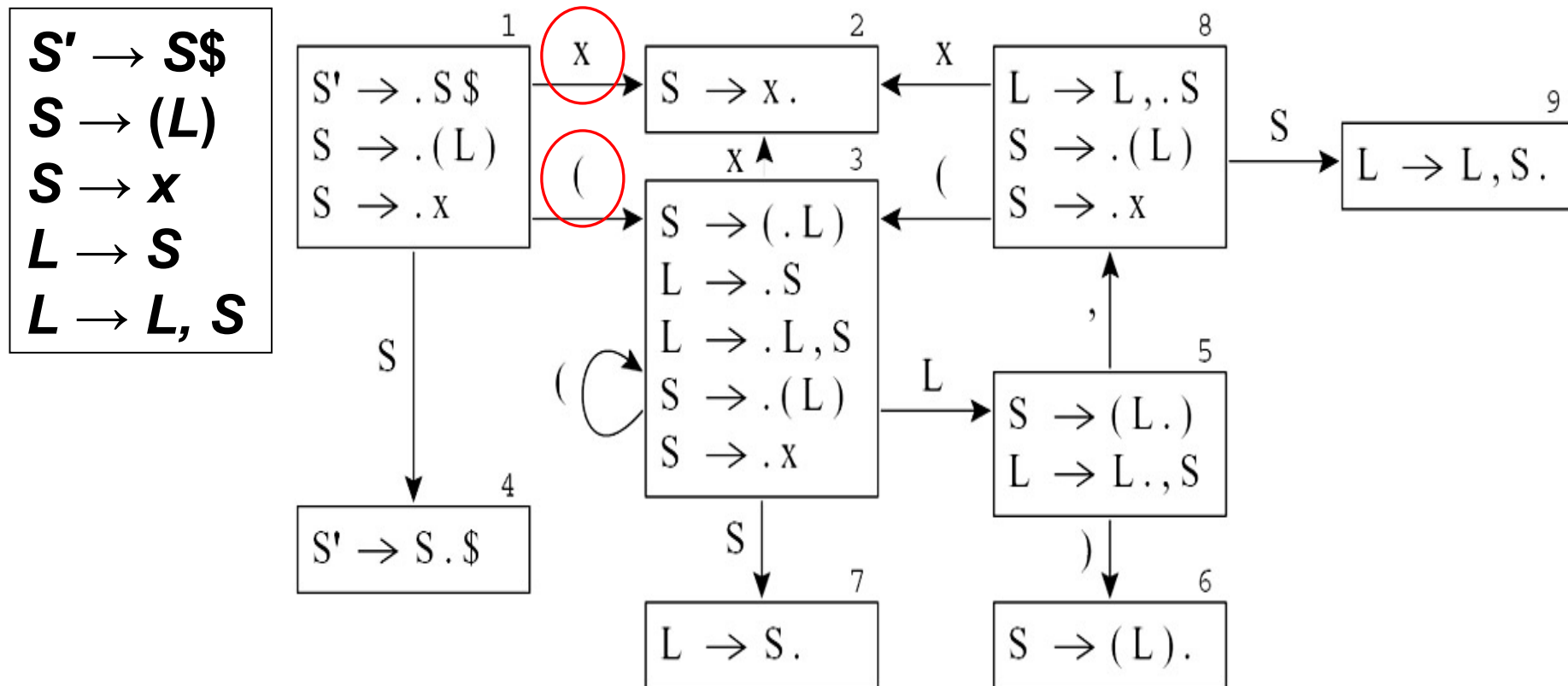
- Making shift/reduce decisions **without any lookahead.**

## Items and States



# LR(0) Parser Generation

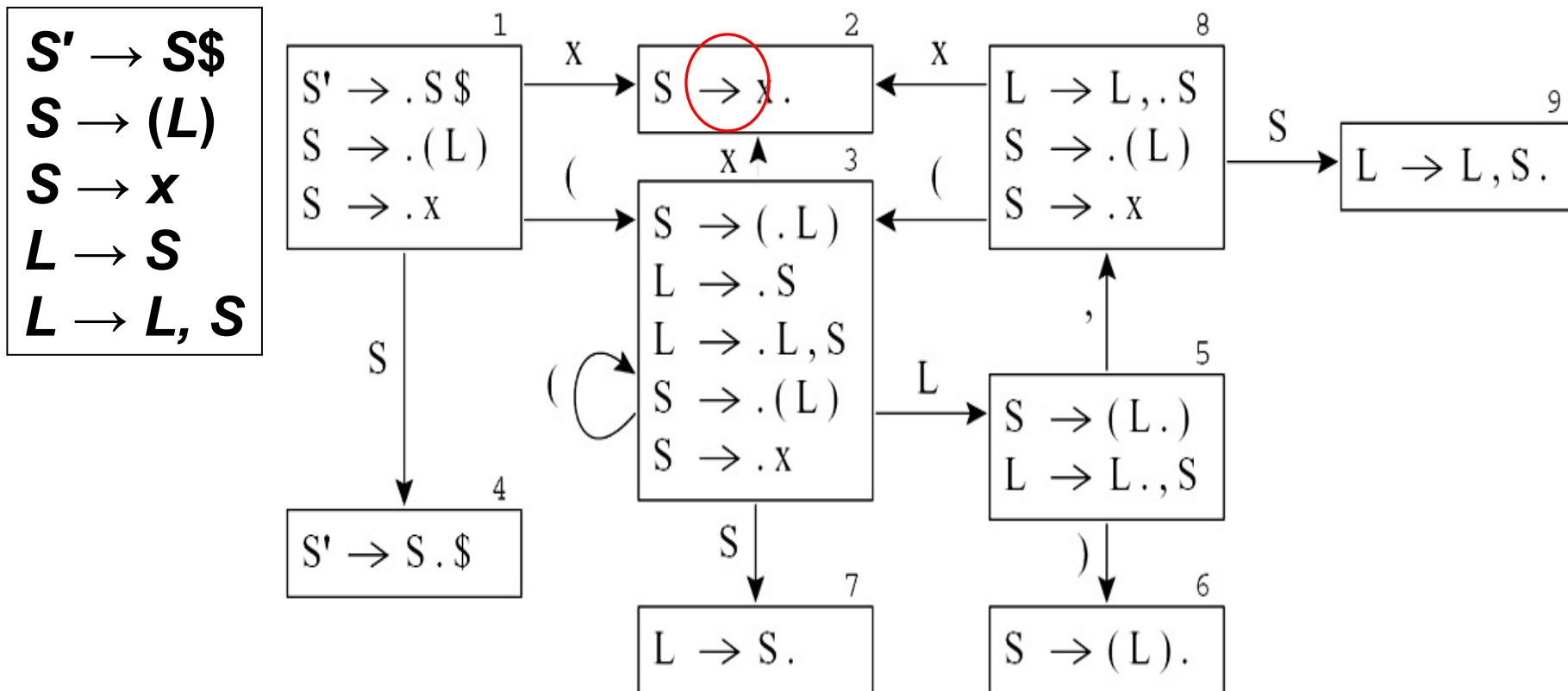
- Making shift/reduce decisions **without any lookahead.**



# LR(0) Parser Generation

- Making shift/reduce decisions **without any lookahead.**

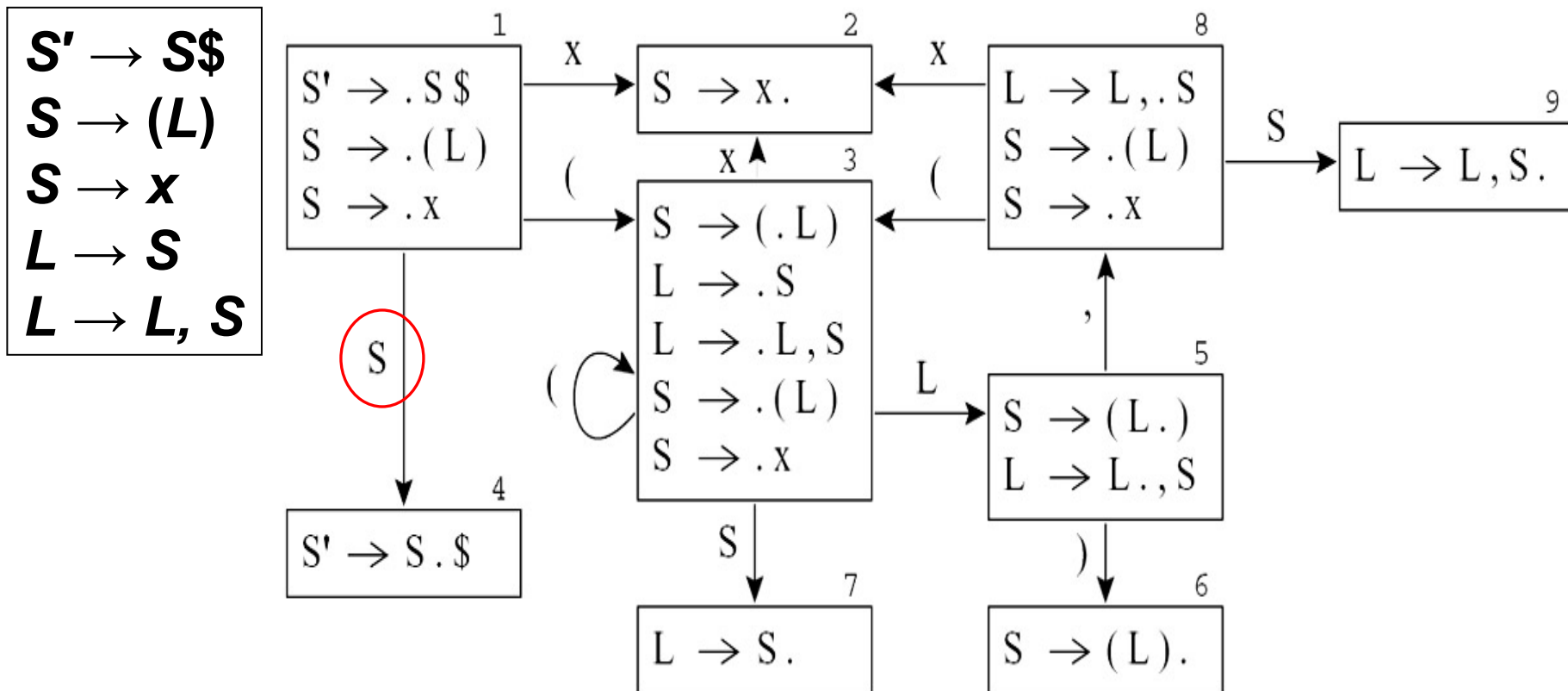
## Reduce actions



# LR(0) Parser Generation

- Making shift/reduce decisions **without any lookahead.**

## Goto actions



# LR(0) Parser Generation

**Closure( $I$ ) =**

repeat

for any item  $A \rightarrow \alpha.X\beta$  in  $I$

for any production  $X \rightarrow \gamma$

$I \leftarrow I \cup \{X \rightarrow .\gamma\}$

until  $I$  does not change.

return  $I$

	(	)	x	,	\$	\$	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

Items and States

**Goto( $I, X$ ) =**

set  $J$  to the empty set

for any item  $A \rightarrow \alpha.X\beta$  in  $I$

add  $A \rightarrow \alpha X.\beta$  to  $J$

return Closure( $J$ )

# LR(0) Parser Generation

## The algorithm for LR(0) parser construction.

- Augment the grammar with an auxiliary start production  $S' \rightarrow S\$$ .
- $T$  be the set of states seen so far
- **$E$  the set of (shift or goto) edges found so far.**

Initialize  $T$  to  $\{\text{Closure}(\{S' \rightarrow :S\})\}$

Initialize  **$E$**  to empty.

repeat

  for each state  $I$  in  $T$

    for each item  $A \rightarrow \alpha.X\beta$  in  $I$

      let  $J$  be  $\text{Goto}(I, X)$

$T \leftarrow T \cup \{J\}$

$E \leftarrow E \cup \{I \rightarrow^X J\}$

until  $E$  and  $T$  did not change in this iteration

	(	)	x	,	S	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

**S**<sub>n</sub> **g**<sub>k</sub>



# LR(0) Parser Generation

The algorithm for LR(0) parser construction.

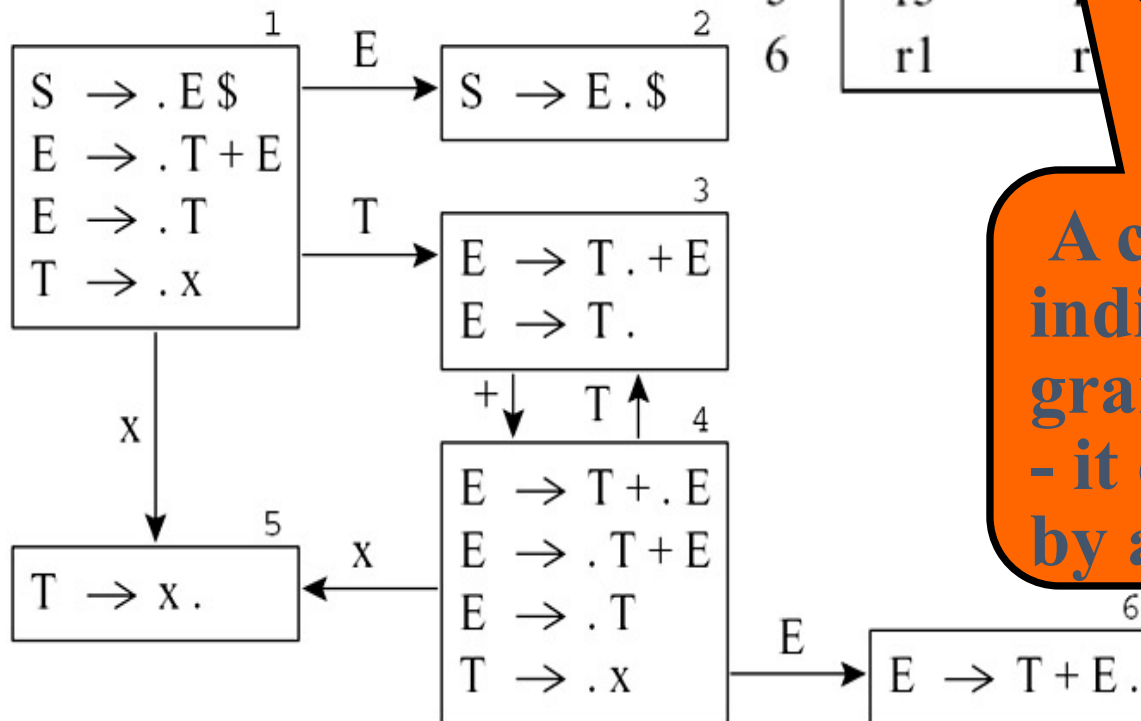
Compute set  $R$  of LR(0) **reduce actions**:

$R \leftarrow \{\}$   
for each state  $I$  in  $T$   
  for each item  $A \rightarrow \alpha$   
     $R \leftarrow R \cup \{(I, A \rightarrow \alpha)\}$

	(	)	x	,	\$	$S$	$L$
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

# SLR Parser Generation

$S \rightarrow E \$$   
 $E \rightarrow T + E$   
 $E \rightarrow T$   
 $T \rightarrow x$



	x	+	\$	E	T
1	s5			g2	g3
2			a		
3	r2	s4,r2	r2		
4	s5			g6	g3
5	r3		r3		
6	r1	r1	r1		

**A conflict and indicates that the grammar is not LR(0) - it cannot be parsed by an LR(0) parser.**

# SLR Parser Generation

- Put reduce actions into the table only where indicated **by the FOLLOW set**.

$R \leftarrow \{\}$   
for each state  $I$  in  $T$   
  for each item  $A \rightarrow \alpha.$  in  $I$   
    **for each token  $X$  in FOLLOW( $A$ )**  
       $R \leftarrow R \cup \{(I, X, A \rightarrow \alpha)\}$

	x	+	\$	$E$	$T$
1	s5			g2	g3
2			a		
3		s4	r2		
4	s5			g6	g3
5		r3	r3		
6			r1		

# LR(1) Items

## An LR(1) item

- Consists of **a grammar production**, a **right-hand-side position** (represented by the dot), and **a lookahead symbol**.

## An item ( $A \rightarrow \alpha.\beta$ ,

- $x$ ) Indicates that the sequence  $\alpha$  is on top of the stack, and at the head of the input is a string derivable from  $\beta x$ .

# Generate LR(1) Parsing Table

**Closure( $I$ ) =**

repeat

for any item  $(A \rightarrow \alpha.X\beta, z)$  in  $I$

for any production  $X \rightarrow \gamma$

for any  $w \in \text{FIRST}(\beta z)$

$I \leftarrow I \cup \{(X \rightarrow \cdot\gamma, w)\}$

until  $I$  does not change

return  $I$

The start state is the  
closure of the item  
 $(S' \rightarrow \cdot S \$, ?)$

$R \leftarrow \{\}$

for each state  $I$  in  $T$

for each item  $(A \rightarrow \alpha., z)$  in  $I$

$R \leftarrow R \cup \{(I, z, A \rightarrow \alpha)\}$

**Goto( $I, X$ ) =**

$J \leftarrow \{\}$

for any item  $(A \rightarrow \alpha.X\beta, z)$  in  $I$

add  $(A \rightarrow \alpha X.\beta, z)$  to  $J$

return Closure( $J$ ).

# Generte LR(1) Parsing Table

$S' \rightarrow . S \$$	?
$S \rightarrow . V = E$	\$
$S \rightarrow . E$	\$
$E \rightarrow . V$	\$
$V \rightarrow . x$	\$
$V \rightarrow . * E$	\$
$V \rightarrow . x$	=
$V \rightarrow . * E$	=

$S' \rightarrow . S \$$	?
$S \rightarrow . V = E$	\$
$S \rightarrow . E$	\$
$E \rightarrow . V$	\$
$V \rightarrow . x$	\$, =
$V \rightarrow . * E$	\$, =

**Closure(I) =**

repeat

for any item  $(A \rightarrow \alpha.X\beta, z)$  in  $I$

for any production  $X \rightarrow \gamma$

for any  $w \in \text{FIRST}(\beta z)$

$I \leftarrow I \cup \{(X \rightarrow .\gamma, w)\}$

until  $I$  does not change

return  $I$

1.  $S' \rightarrow S \$$

2.  $S \rightarrow V = E$

3.  $S \rightarrow E$

4.  $E \rightarrow V$

5.  $V \rightarrow x$

6.  $V \rightarrow * E$

# Generate LR(1) Parsing Table

**Goto(I, X) =**

$J \leftarrow \{$

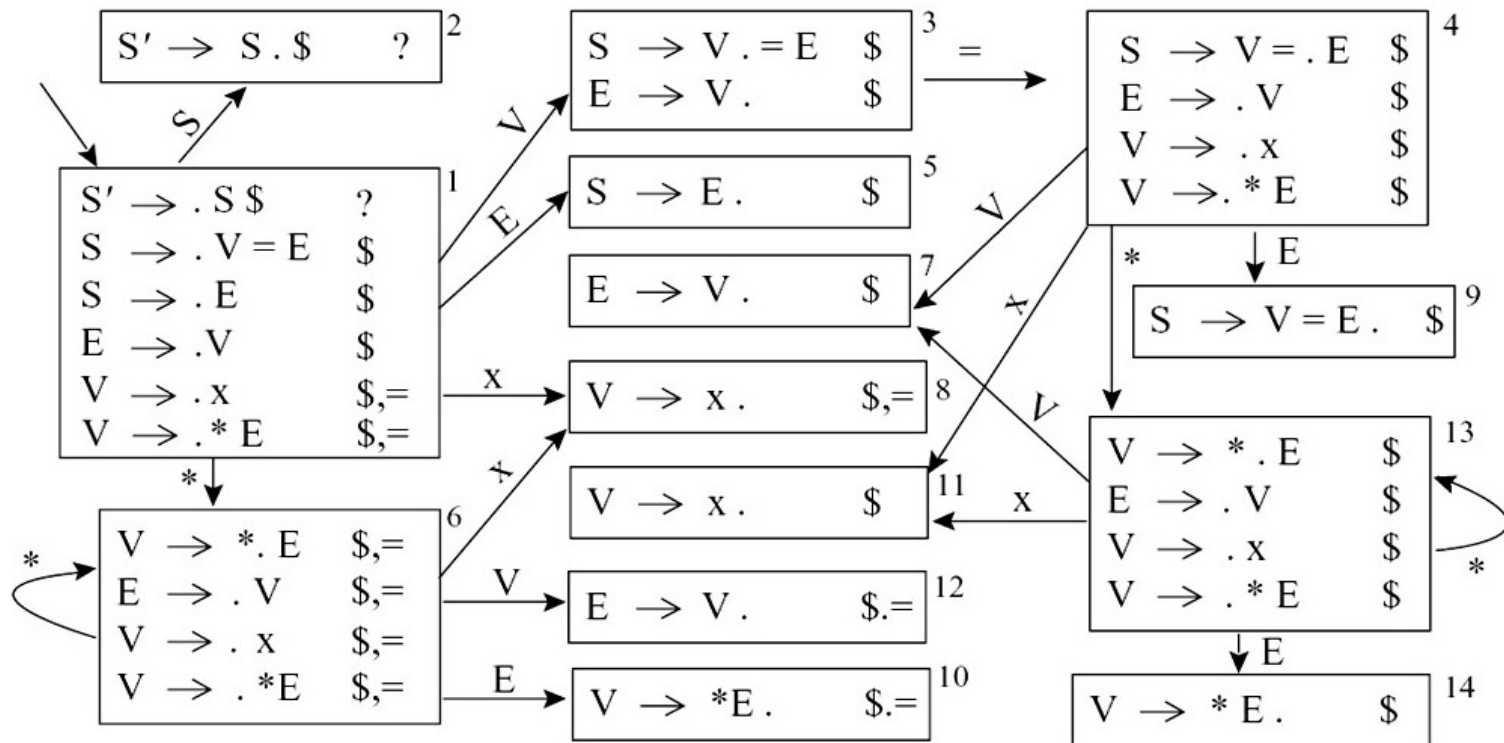
for any item  $(A \rightarrow \alpha.X\beta, z)$  in  $I$

add  $(A \rightarrow \alpha.X\beta, z)$  to  $J$

return  $\text{Closure}(J)$ .

	x	*	=	\$	S	E	V
1	s8	s6			g2	g5	g3
2				a			
3			s4	r3			
4	s11	s13				g9	g7
5				r2			
6	s8	s6				g10	g12
7				r3			
8			r4	r4			
9				r1			
10			r5	r5			
11				r4			
12			r3	r3			
13	s11	s13				g14	g7
14				r5			

(a) LR(1)



# Generate LR(1) Parsing Table

$R \leftarrow \{\}$

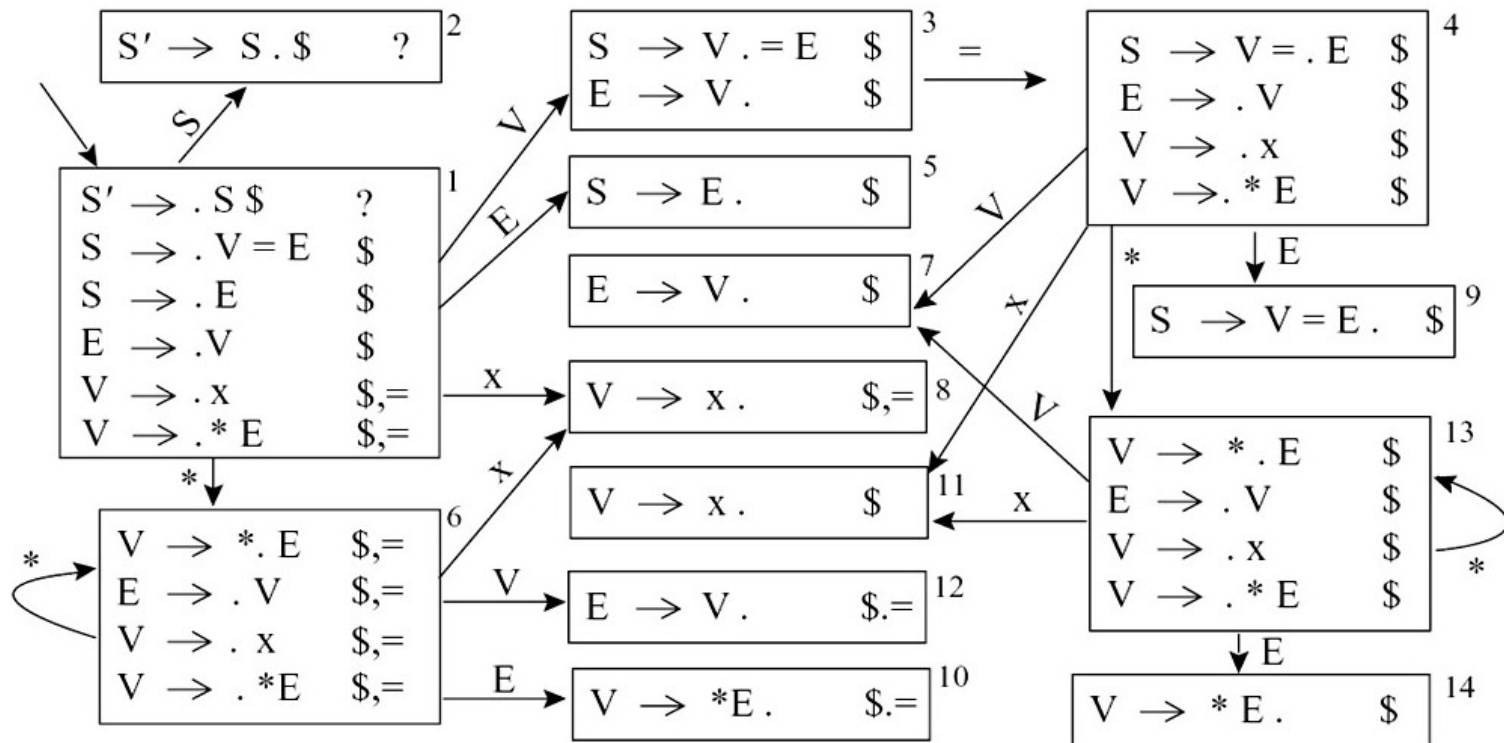
for each state  $I$  in  $T$

for each item  $(A \rightarrow \alpha., z)$  in  $I$

$R \leftarrow R \cup \{(I, z, A \rightarrow \alpha)\}$

	x	*	=	\$	S	E	V
1	s8	s6			g2	g5	g3
2				a			
3			s4	r3			
4	s11	s13				g9	g7
5				r2			
6	s8	s6				g10	g12
7				r3			
8			r4	r4			
9				r1			
10			r5	r5			
11				r4			
12			r3	r3			
13	s11	s13				g14	g7
14				r5			

(a) LR(1)





# An LR(1) Parsing Table

	$x$	$*$	$=$	$\$$	$S$	$E$	$V$
1	s8	s6			g2	g5	g3
2				a			
3			s4	r3			
4	s11	s13				g9	g7
5				r2			
6	s8	s6				g10	g12
7				r3			
8			r4	r4			
9				r1			
10			r5	r5			
11				r4			
12			r3	r3			
13	s11	s13				g14	g7
14				r5			

(a) LR(1)

# LALR(1) Parsing Tables

- LR(1) parsing tables can be **very large**, with many states.
- A smaller table can be made by **merging any two states whose items are identical except for lookahead sets**.
- The result parser is called an **LALR(1)** parser.

# LALR(1) Parsing Tables

	x	*	=	\$	S	E	V
1	s8	s6			g2	g5	g3
2				a			
3			s4	r3			
4	s11	s13				g9	g7
5				r2			
6	s8	s6				g10	g12
7				r3			
8			r4	r4			
9				r1			
10			r5	r5			
11				r4			
12			r3	r3			
13	s11	s13				g14	g7
14				r5			

(a) LR(1)

	x	*	=	\$	S	E	V
1	s8	s6			g2	g5	g3
2				a			
3			s4	r3			
4	s8	s6				g9	g7
5				r2			
6	s8	s6				g10	g7
7			r3	r3			
8			r4	r4			
9				r1			
10			r5	r5			

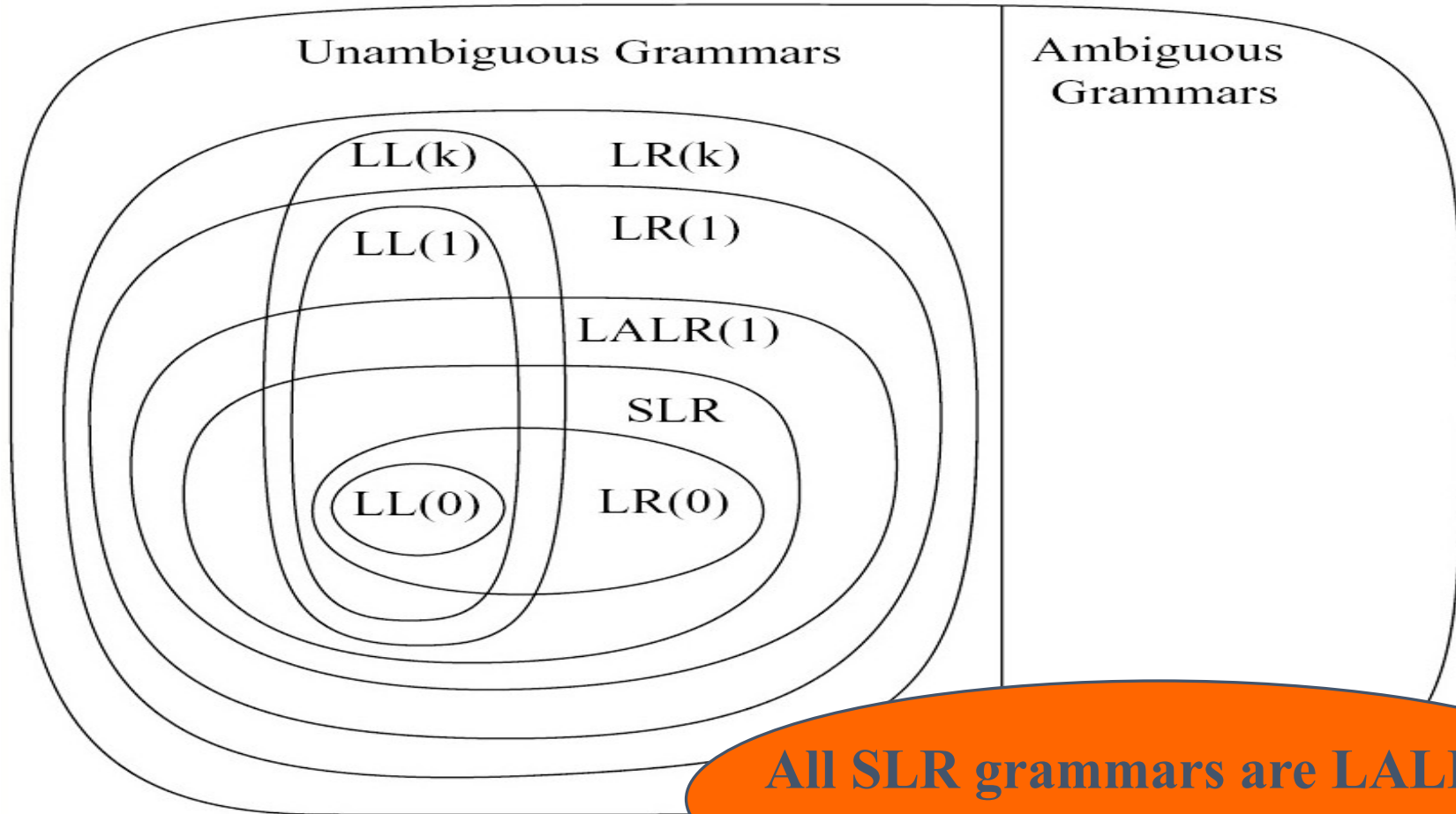
(b) LALR(1)

(6,13), (7,12), (8,11), (10,14): **merged**

- For some grammars, the LALR(1) table contains reduce-reduce conflicts where the LR(1) table has none.
- In practice the difference matters little.

# Hierarchy of Grammar Classes

# The relationship between several classes of grammars.



**All SLR grammars are LALR(1)  
but not vice versa.**

# LR Parsing of Ambiguous Grammars

**$S \rightarrow \text{if } E \text{ then } S \text{ else } S$**

**$S \rightarrow \text{if } E \text{ then } S$**

**$S \rightarrow \text{other}$**

if a then if b then s1 else s2 { (1) if a then { if b then s1 else s2 }  
(2) if a then { if b then s1 } else s2

**A shift-reduce conflict:**

$S \rightarrow \text{if } E \text{ then } S .$	else
$S \rightarrow \text{if } E \text{ then } S . \text{ else } S$	(any)

# LR Parsing of Ambiguous Grammars

- The grammar unchanged. In constructing the parsing table this conflict should be resolved by shifting( **prefer interpretation (1)**)
- The ambiguity can be eliminated by introducing auxiliary nonterminals  $M$

$S \rightarrow M$

$S \rightarrow U$

$M \rightarrow \text{if } E \text{ then } M \text{ else } M$

$M \rightarrow \text{other}$

$U \rightarrow \text{if } E \text{ then } S$

$U \rightarrow \text{if } E \text{ then } M \text{ else } U$

$M$  :for *matched statement*

$U$  :for *unmatched statement*

# The end of Chapter 3(3)

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