

# Calcback

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## 1 What is this?

This is a script to get the complex refractive index  $n = n * ik$  from the ellipsometric parameters  $\Delta$  and  $\Psi$  I got from a simulation. The result for

300nm SiO<sub>2</sub> should look like this:

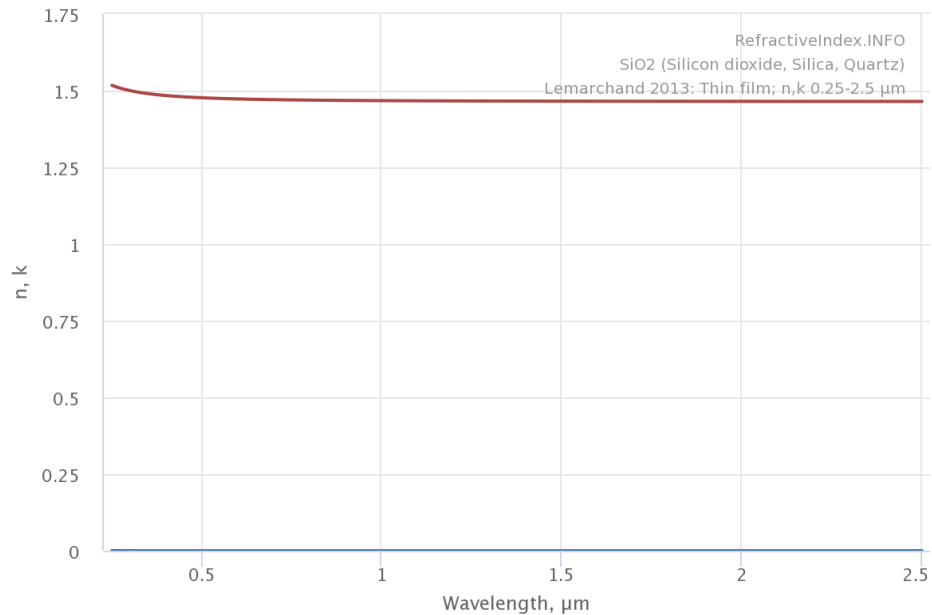


Figure 1: Refractive index should look like this

## 2 List of Todos:

2.1 TODO Write a loop for all wavelengths after it works for one.

2.2 TODO Then take even more wavelengths (rows)

## 3 Imports:

```
import numpy as np
import matplotlib
matplotlib.use('Agg')
import matplotlib.pyplot as plt
```

## 4 Defining some variables:

Defining some variables for later use:

```
CSVFILE = "head300nmSiO2.csv" # head = only 10 rows of data
phi_i = 70 * np.pi / 180 # converting incident angle from deg (first number) to rad
d_L = 300 # thickness of layer in nm
n_air = 1 # refractive index of air
rerange = 5 # upper limit for real part
imrange = 1 # upper limit for imaginary part
i = 0 # only look at one wavelength (row in csv)
```

## 5 Read .csv-file:

Read the values into a two dimensional numpy array as `[[lambda,Psi,Delta,ns,ks],...]` (Skip columns 3 and 4)

```
csv = np.loadtxt(CSVFILE, usecols=(0,1,2,5,6), delimiter=",", skiprows=1)
```

The array looks like this:

csv

```
[[300.      55.2217535  84.37228319  2.6726      3.0375      ]
 [303.      50.11187439  93.3085011   2.7346      3.0381      ]
 [306.      46.35824553  98.43681392  2.7967      3.0368      ]
 [309.      43.50539341 101.18051798  2.8588      3.0334      ]
 [312.      41.29392865 102.19236832  2.9206      3.0279      ]
 [315.      39.48751217 101.93002    2.9822      3.0205      ]
 [318.      37.90308303 100.64846104  3.0435      3.0109      ]
 [321.      36.47640803  98.54577151  3.1042      2.9994      ]
 [324.      35.12615859  95.72242205  3.1644      2.9858     ]]
```

## 6 Calculate $\rho$

### 6.1 Create a matrix containing every possible refractive index ( $n+ik$ ):

Change the last number in the "linspace" to adjust the resolution.

```

lsp_re = np.linspace(1, rerange, 1001)
lsp_im = np.linspace(0.01, imrange, 1001)
re, im = np.meshgrid(lsp_re, lsp_im, copy=False)
n_L = 1j * np.round(im,6) + np.round(re,6)
n_L = n_L.flatten() # create onedimensional array

```

This gives the following matrix:

`n_L`

```

[1.  +0.01j 1.004+0.01j 1.008+0.01j ... 4.992+1.j  4.996+1.j
 5.  +1.j   ]

```

## 6.2 Calculate $\rho$ :

### 6.2.1 First we define some functions:

1. Snell's Law to calculate the refractive angles:  $\Phi$  is the incident angle for the layer,  $n_1$  and  $n_2$  are refractive indices of first and second medium. Returns the angle of refraction.

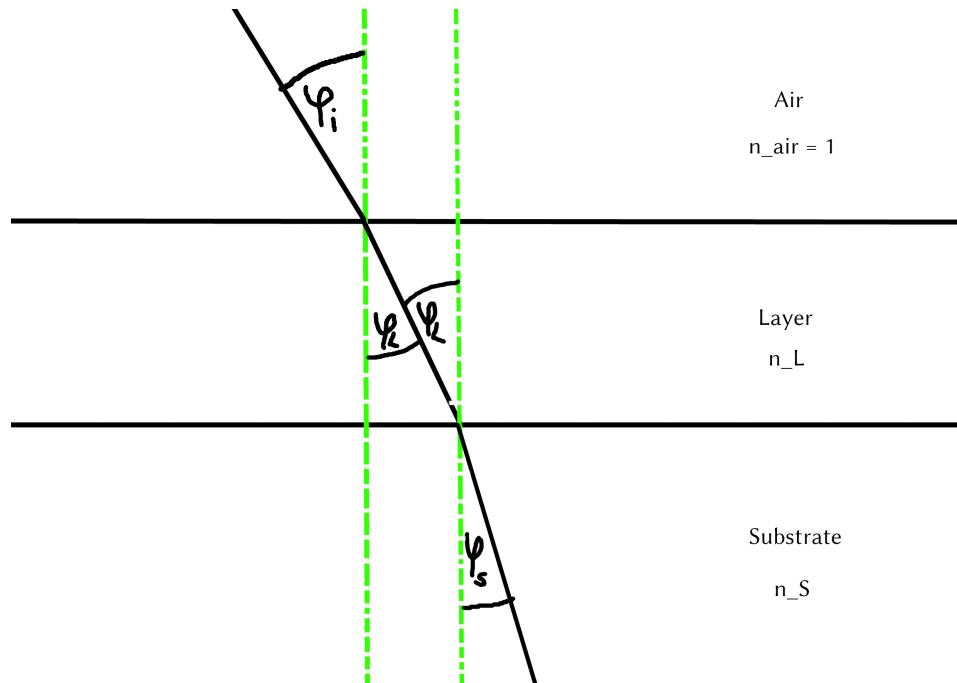


Figure 2: Snell's Law

```
def snell(phi, n1, n2):
    phi_ref = np.arcsin((np.sin(phi) * n1) / n2)
    return phi_ref
```

2. Calculate  $r_p$  and  $r_s$  with Fresnel equations:

```
def fresnel(n1, phi1, n2, phi2):
    """Takes refractive indices and angles of two layers to calculate the amplitudes
    rs = (n1 * np.cos(phi1) - n2 * np.cos(phi2)) / (n1 * np.cos(phi1) + n2 * np.cos(phi2))
    rp = (n2 * np.cos(phi1) - n1 * np.cos(phi2)) / (n2 * np.cos(phi1) + n1 * np.cos(phi2))
    return rs, rp
```

3. Calculate  $\rho$  for the layer with eq. 5.2 in Spectroscopic Ellipsometry  
fujiwara2009spectroscopic:

```
def calc_rho(rs_al, rp_al, rs_ls, rp_ls, d_L, n_L, lambda_vac):
    beta = 2 * np.pi * d_L * n_L * np.cos(phi_L) / lambda_vac
    rp_L = (rp_al + rp_ls * np.exp(-2*j*beta)) / (1 + rp_al * rp_ls * np.exp(-2*j*beta))
    rs_L = (rs_al + rs_ls * np.exp(-2*j*beta)) / (1 + rs_al * rs_ls * np.exp(-2*j*beta))
    rho = rp_L / rs_L
    return rho
```

### 6.2.2 Then we call these functions one after another to calculate

$\rho$ :

Get refractive index of the substrate ( $n_S$ ) and lambda from the csv:

```
lambda_vac = csv[i, 0]
n_S = (csv[i, 3] + 1j * csv[i, 4])
```

Then call the above defined functions

```
phi_L = snell(phi_i, n_air, n_L)
phi_S = snell(phi_L, n_L, n_S)
# Fresnel equations:
# air/layer:
rs_al, rp_al = fresnel(n_air, phi_i, n_L, phi_L)
# layer/substrate:
rs_ls, rp_ls = fresnel(n_L, phi_L, n_S, phi_S)

rho_L = calc_rho(rs_al, rp_al, rs_ls, rp_ls, d_L, n_L, lambda_vac)
```

### 6.2.3 Identify the best fitting rho with $\rho = \tan(\psi) * e^{i\Delta}$ :

```
# psi is in our csv-file at index 1, delta at index 2 at row "i" for lambda
psi = csv[i][1]
delta = csv[i][2]
rho = np.tan(psi) * np.exp(1j * delta)
diff = abs(rho - rho_L) # magnitude of complex number
idx = np.argmin(diff) # index of the minimum
minimum = diff[idx]
n = n_L[idx]
print("The layer has the refractive index n_L = ", n)
```

The layer has the refractive index n\_L = (4.008+0.15157j)

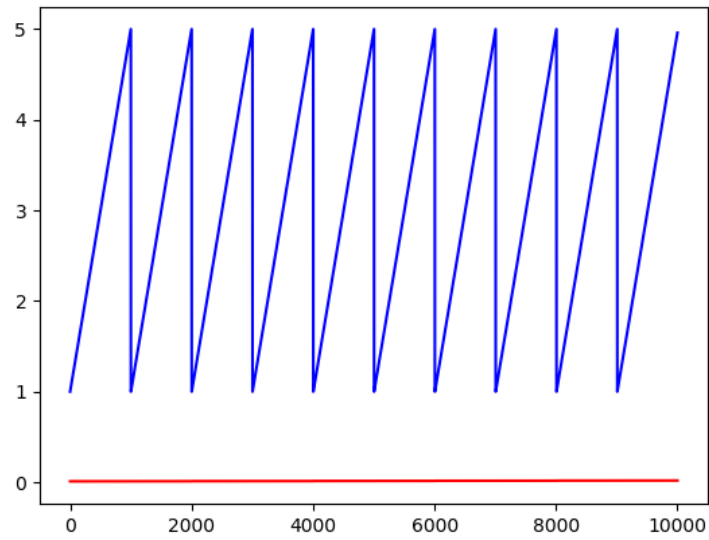
## 7 Plot some things for checking results:

If we use a high resolution, those plots are not showing much, thats why they are only showing the first 10000 values.

### 7.1 Plot real and imaginary part of the created n<sub>L</sub> matrix:

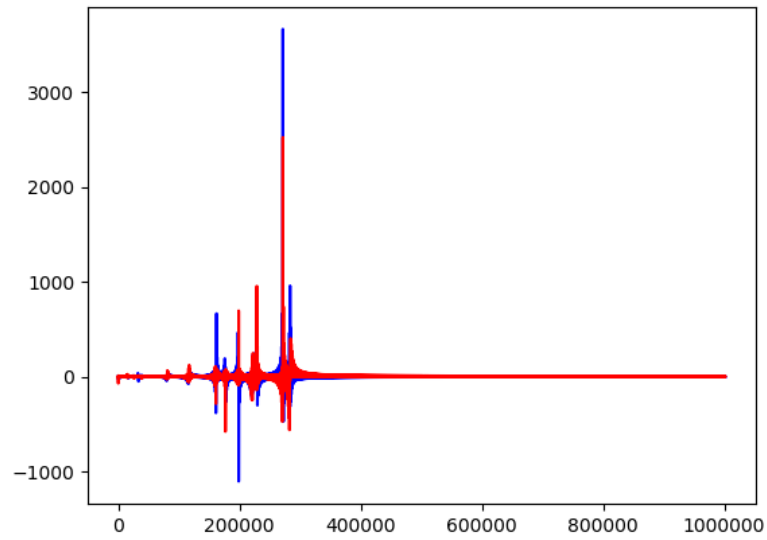
Real part is blue, imaginary is red.

```
fig = plt.figure()
plt.plot(np.real(n_L[:10000]), c='b')
plt.plot(np.imag(n_L[:10000]), c="r")
plt.savefig('n_L.png')
'./n_L.png'
```



## 7.2 Plot real and imaginary part of $\rho_L$

```
fig = plt.figure()
plt.plot(np.real(rho_L), c='b')
plt.plot(np.imag(rho_L), c='r')
plt.savefig('rho_L.png')
"./rho_L.png"
```

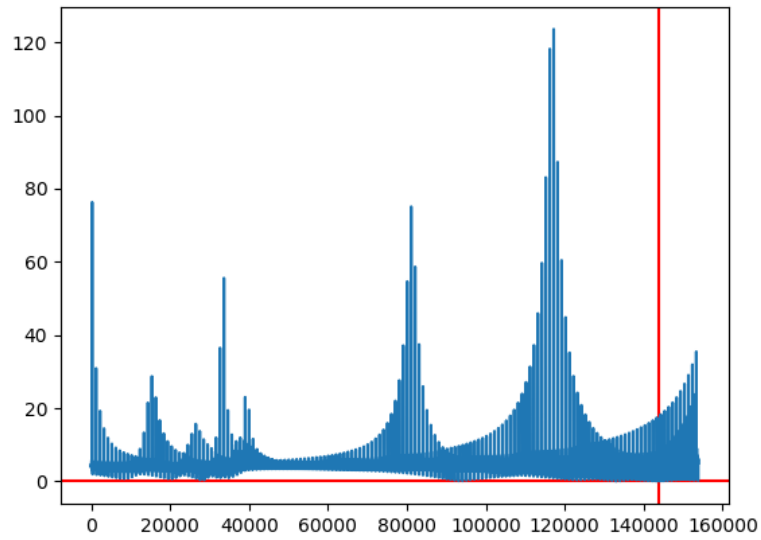


### 7.3 Plot of the difference between $\rho_L$ and the given $\rho$ and determined minimum:

The difference is shown in blue, the red lines show the minimum.

```
fig = plt.figure()
plt.axvline(idx, c='r')
plt.axhline(minimum, c='r')
plt.plot(diff[:idx+10000])
plt.savefig('diff.png')
"./diff.png"
```





#### 7.4 Plot refractive angle $\phi_L$ and $n_L$ :

$n_L$  is shown in green, real part of  $\phi_L$  in blue, imaginary in red. A relation between these should be visible.

```
fig = plt.figure()
plt.plot(np.real(phi_L[:5000]), 'b')
plt.plot(np.imag(phi_L[:5000]), 'r')
plt.plot(np.real(n_L[:5000]), c='g')
plt.savefig('phi_L.png')
"phi_L.png"
```

