

# CHALMERS



TITLE OF PROJECT

*Master's Thesis in ...*

ANDREAS BÜLLING

Department of Mathematical Sciences  
CHALMERS UNIVERSITY OF TECHNOLOGY  
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## **Abstract**

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## Acknowledgements

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# 1

## Introduction

Short introductory text here...

### **1.1 Section**

Something more here...

### **1.2 Outline**

How is this report structured?



# 2

## Electrohydrodynamics in microchannels

In this chapter some fundamental physics behind electrokinetic flow, important for later discussions, will be presented. Also a modelling approach based on the coupling of Navier-Stokes, Nernst-Planck and Poission's equations is given.

### 2.1 Electrical double layers

### 2.2 Pressure-driven electrokinetic flow

### 2.3 Physical model

# 3

## Model benchmarks

llala

### 3.1 Advection-Diffusion

Before the implementation of the Nernst-Planck part of the model is tested, a special case is considered, i.e. when the electrical potential in the domain is constant. This makes the source term including the electrical potential in eq. (??) vanish and we have to solve only for advection and diffusion.

Introducing characteristic scales for the concentration ( $C_0$ ), advective velocity ( $u_0$ ) and length ( $l_0$ ) respectively, gives the non-dimensional advection-diffusion equation for incompressible flow:

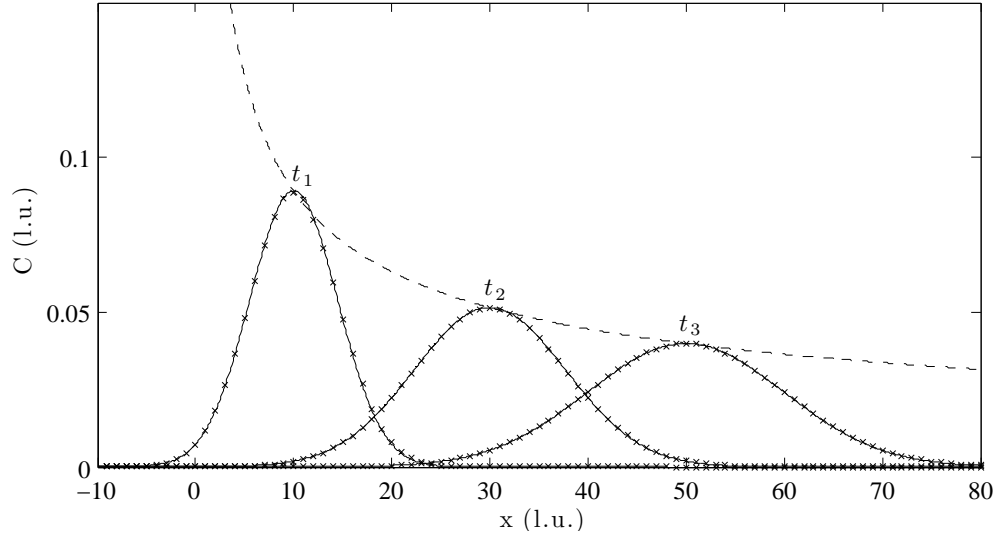
$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = \frac{D}{u_0 l_0} \nabla^2 C. \quad (3.1)$$

All variables in (3.1) are non-dimensional. The quantity  $Pe = u_0 l_0 / D$  is often referred to as the Péclet number. It determines the relation between contributions to the dynamics from advection and diffusion respectively. For  $Pe \gg 1$  the dynamics is dominated by advection and for  $Pe \ll 1$  by diffusion.

The LB model described in section ?? was tested by studying the evolution in time and space of a point mass in one dimension. The analytical solution of eq. (3.1) in one dimension with initial conditions  $C(x, t = 0) = \delta(x)$  on an infinite domain is:

$$C(x, t) = \sqrt{\frac{Pe}{4\pi t}} \exp\left(-\frac{(x - ut)^2 Pe}{4t}\right). \quad (3.2)$$

In the numerical computations the parameters  $Pe = 10$  and  $|\mathbf{u}| = 0.1$  were used. The domain consisted of 200 lattice nodes and three snapshots in time at  $t = 100, 200, 300$  were compared to the analytical solution. The result is presented in fig. 3.1.



**Figure 3.1:** Obtained solutions ( $\times$ ) of the advection-diffusion equation for a point mass evolving in time and space. Three different times ( $t_n = 100n$ ) are compared to analytical solutions (solid). The Amplitude of the solutions as function of time has also been plotted (dashed). The advecting velocity,  $u_0 = 0.1$  and the Peclet number,  $Pe = 10$ . All units are in lattice units.

## 3.2 Nernst-Planck equation