Parallel Segmented Merge and Its Applications to Two Sparse Matrix Kernels

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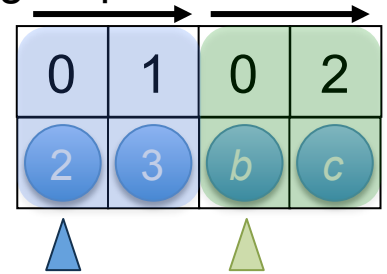
Background

1. Merge (sort) and its use
2. A definition of segmented merge
3. Merge in parallel

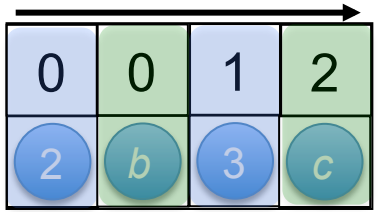
Merge (sort)

- We consider to merge two sorted key-value ascending/descending sequences into one. A serial algorithm first creates two pointers running along the two sequences, compares the entries pointed by them, and saves the smaller/larger one to the resulting sequence.

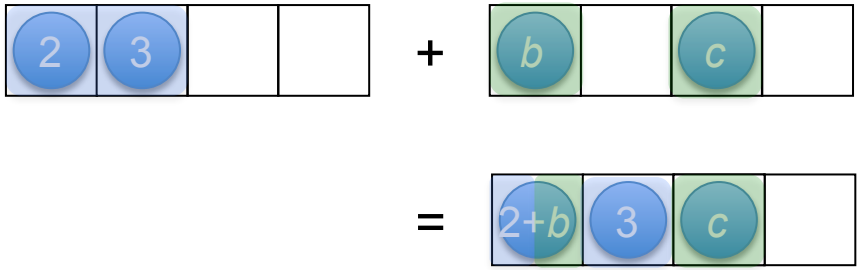
Two sorted sequences



One sorted resulting sequence

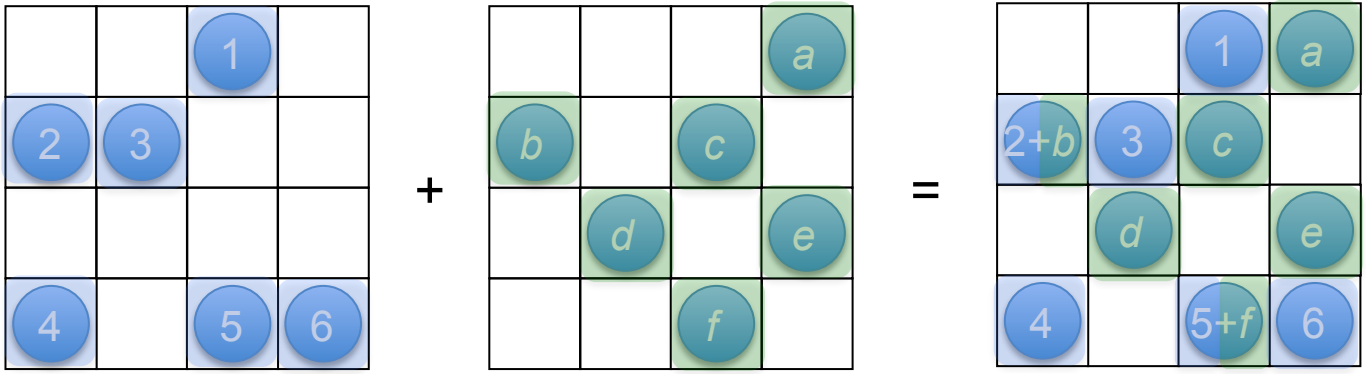


Applying the left merging to sparse vector-vector addition:



Use case 1: sparse matrix-matrix addition

- Add a sparse matrix *A* by a sparse matrix *B*, and obtain a resulting sparse matrix *C*.



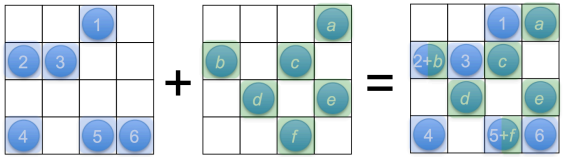
A (4x4) sparse, 6 nonzeros

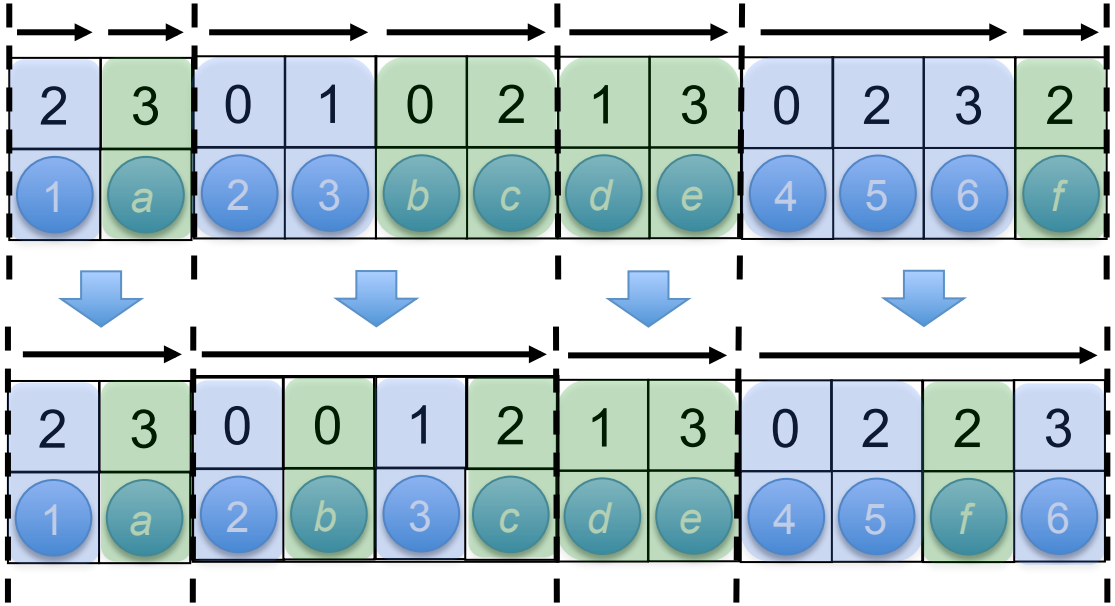
B (4x4) sparse, 6 nonzeros

C (4x4) sparse, 10 nonzeros

Use case 1: sparse matrix-matrix addition

- Each resulting row is merged from two rows.





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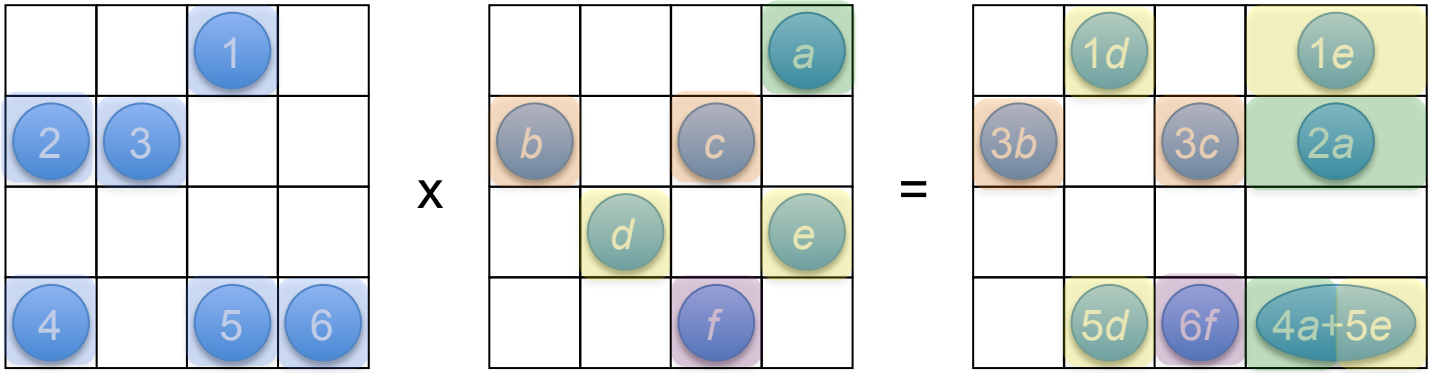
Use case 2: sparse matrix-matrix multiplication

- Multiply a sparse matrix *A* by a sparse matrix *B*, and obtain a resulting sparse matrix *C*.



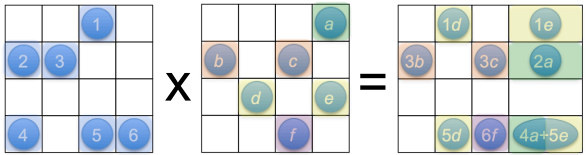
A (4x4) sparse, 6 nonzeros

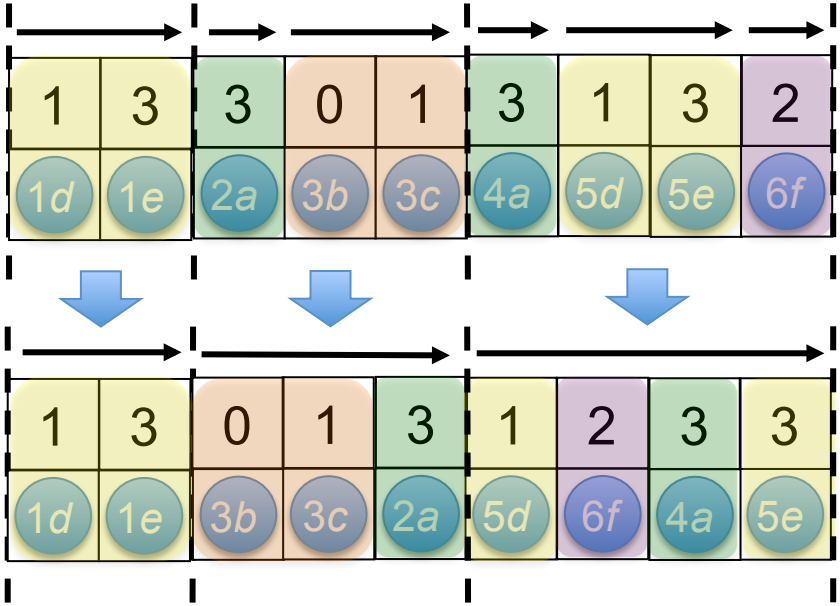
B (4x4) sparse, 6 nonzeros

C (4x4) sparse, 8 nonzeros

Use case 2: sparse matrix-matrix multiplication

- Each resulting row is merged from multiple rows.





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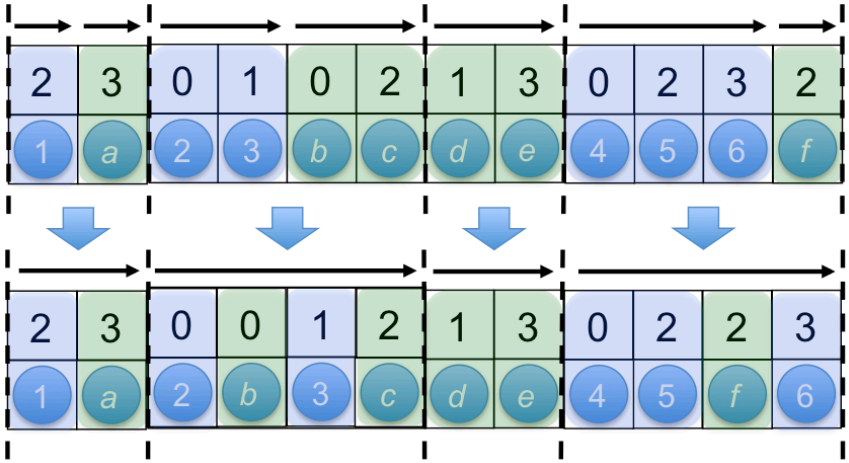
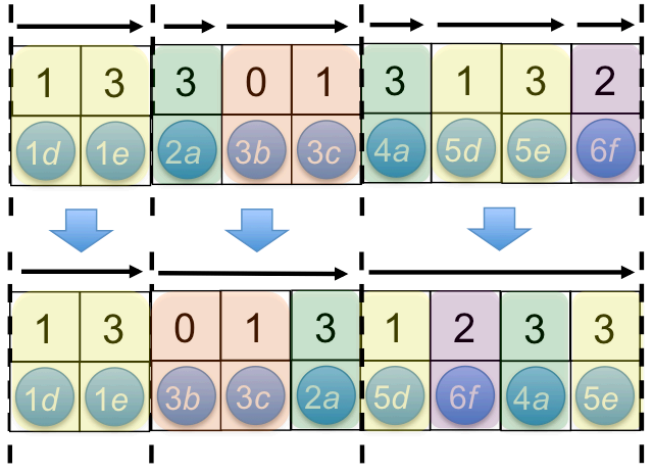
2. A definition of segmented merge
3. Merge in parallel

A definition of segmented merge (segmerge)

- **Segmented merge** operation merges *q* sub-segments into *p* segments.



q = 7, *p* = 4



q = 6, *p* = 3

Background

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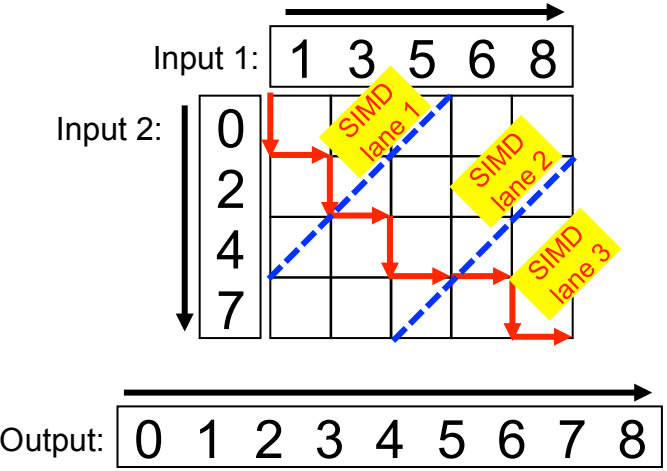
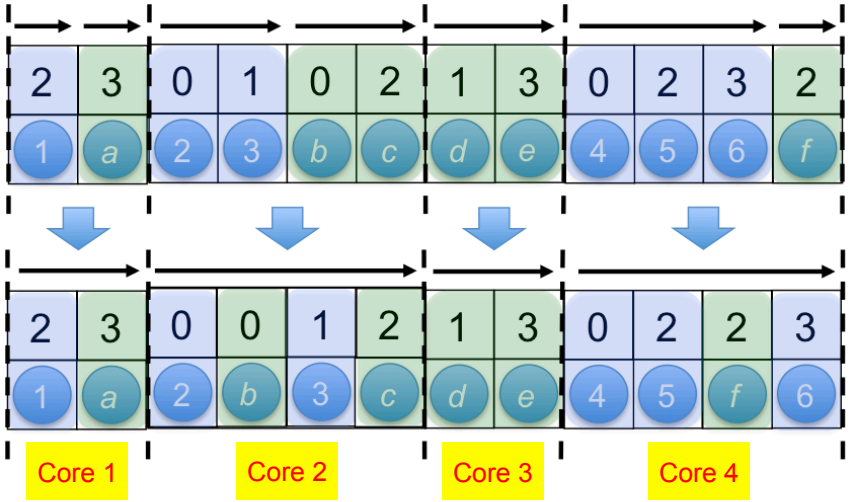
Merge in parallel

- There are two parallel merge methods:
  - (1) **mergeing two sub-segments**: merge-path works in a vectorized and balanced way,
  - (2) **mergeing multiple segments**: assigning each segment to a core





Though each segment can be merged in a balanced way, cores may have imbalanced workload due to very diverse segment sizes.

This motivates our segmerge algorithm.

Green, McColl, Bader. GPU Merge Path: a GPU Merging Algorithm. ICS '12.

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Parallel Segmented Merge

1. Algorithm description
2. Experiments: SpTRANS and SpGEMM
3. Micro-benchmarks

Overview of our segmerge

Step 1. create level-1 and level-2 pointers

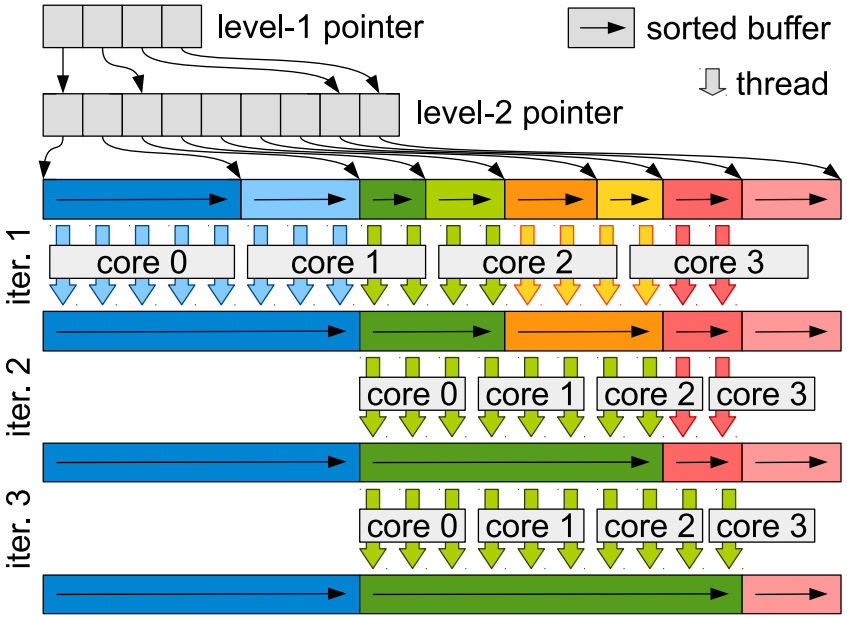
Step 2. fix each thread's workload (#entries) and compute #threads

Step 3. sub-segments scatter info to threads

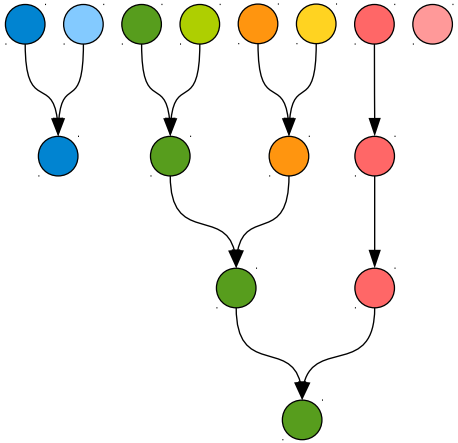
Step 4. threads gather info to find workload (similar to merge path)

Step 5. threads are evenly assigned to cores

Step 6. iterate steps 2- 5 until finish



Merge tree of the left segments

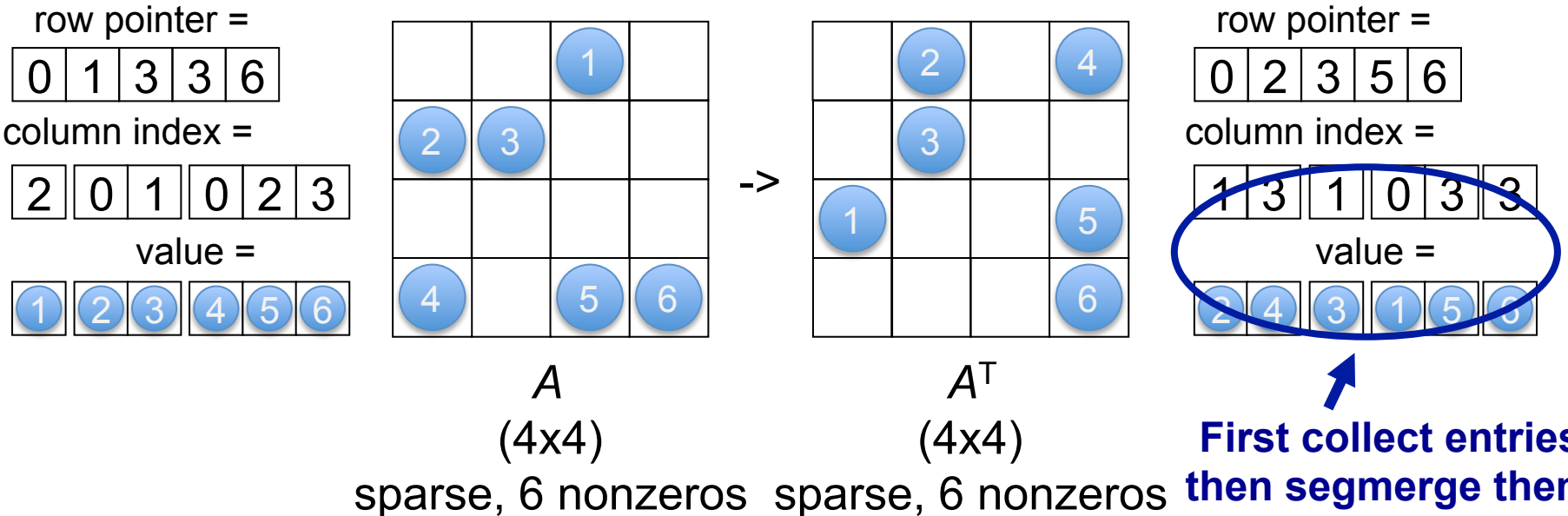


Parallel Segmented Merge

1. Algorithm description
2. Experiments: SpTRANS and SpGEMM
3. Micro-benchmarks

Experiment 1: sparse matrix transposition

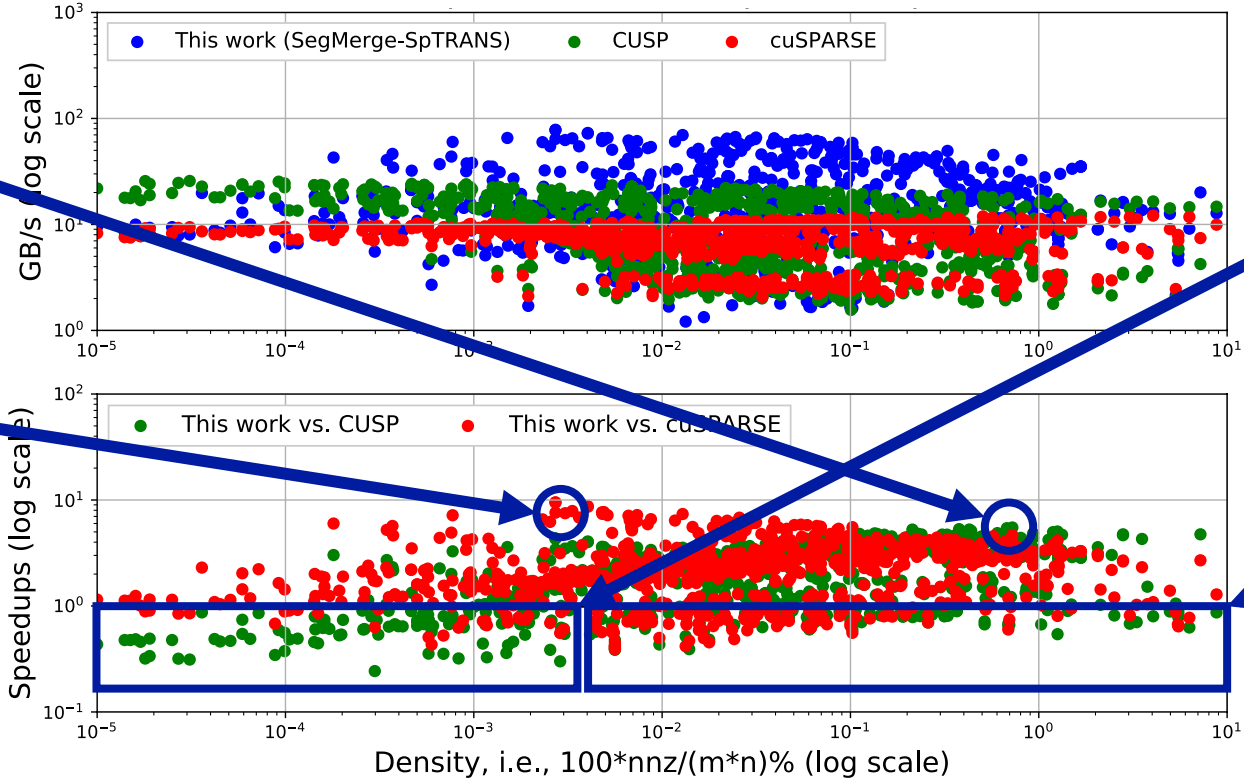
- Transpose a sparse matrix *A* (in CSR) to a sparse matrix *B* (i.e., A^T, in CSR). This is equivalent to a conversion from CSR to CSC, or vice versa.



Experiment 1: sparse matrix transposition

~7x over CUSP

~10x over cuSPARSE



CUSP behaves better when very sparse

In some cases slower than CUSP and cuSPARSE

GPU: Titan X

#matrices: 956

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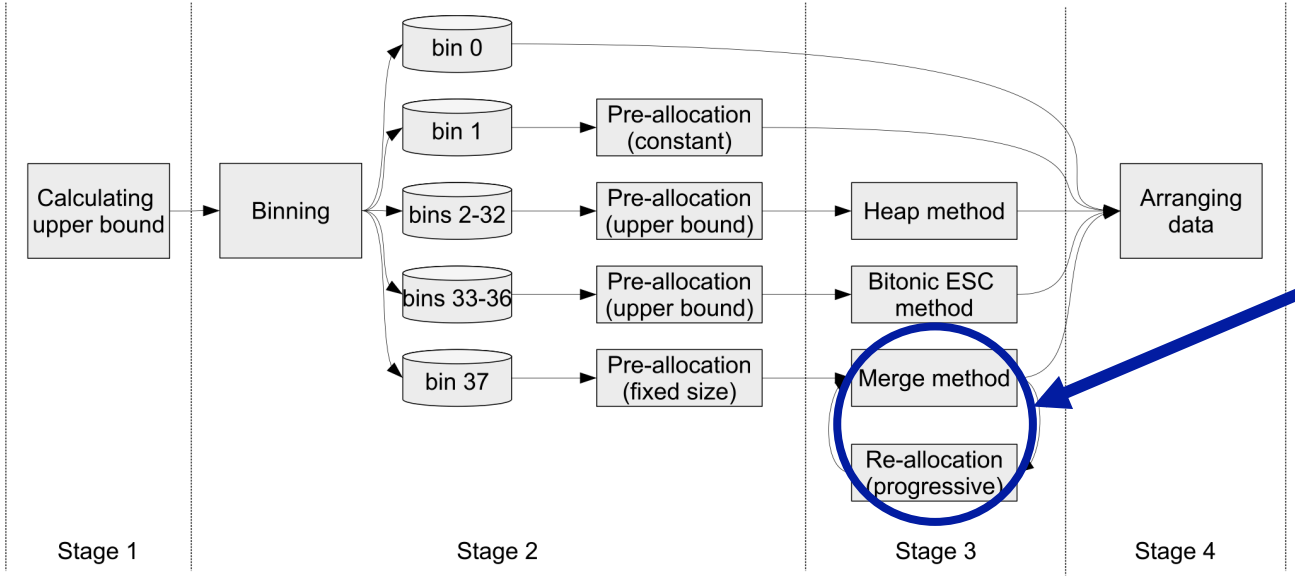
Experiment 2: sparse matrix-matrix multiplication



Only use segmerge for `long' rows

Weifeng Liu, Brian Vinter. **A Framework for General Sparse Matrix-Matrix Multiplication on GPUs and Heterogeneous Processors**. *JPDC. 2015.* (extended from *IPDPS14*)

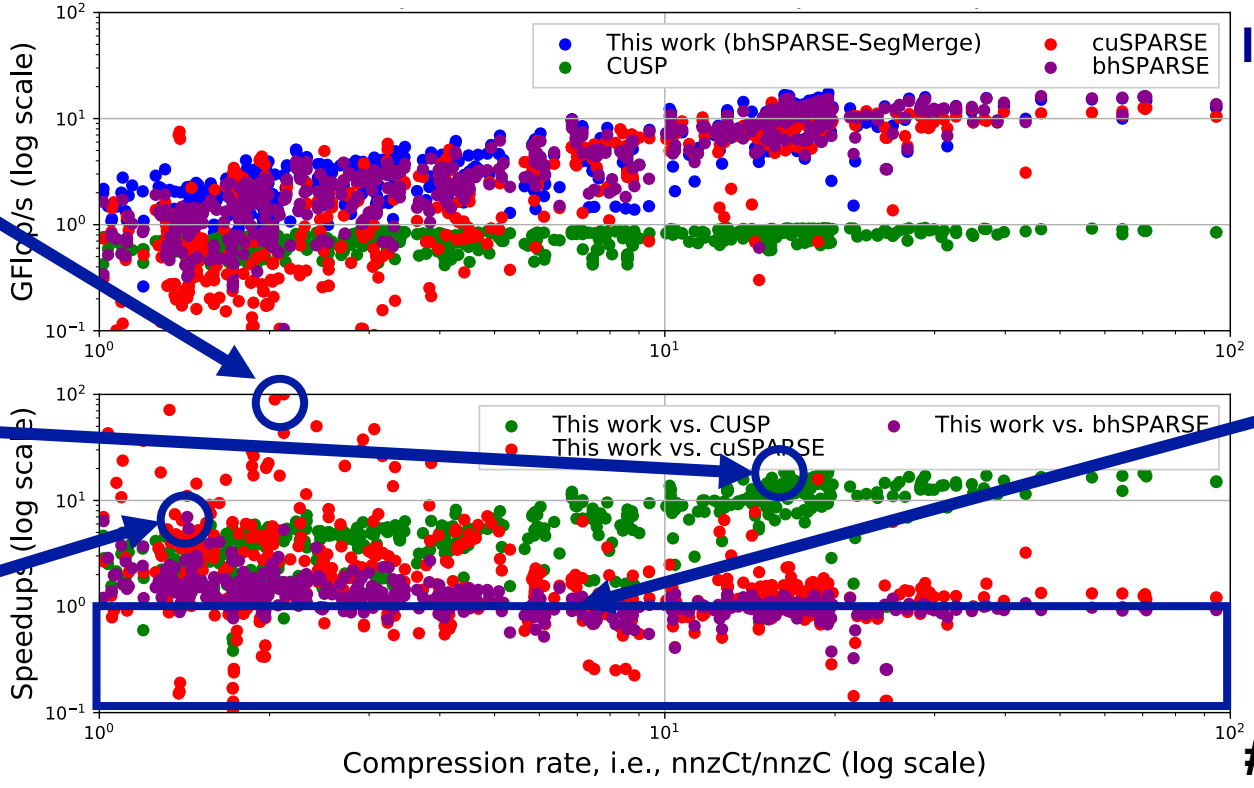
Weifeng Liu, Brian Vinter. **An Efficient GPU General Sparse Matrix-Matrix Multiplication for Irregular Data**. *IPDPS14*.

Experiment 2: sparse matrix-matrix multiplication

~100x over cuSPARSE

~20x over CUSP

~7x over bhSPARSE



In some cases, cuSPARSE is faster, and bhSPARSE rarely gives higher perf.

GPU: Titan X

#matrices: 956

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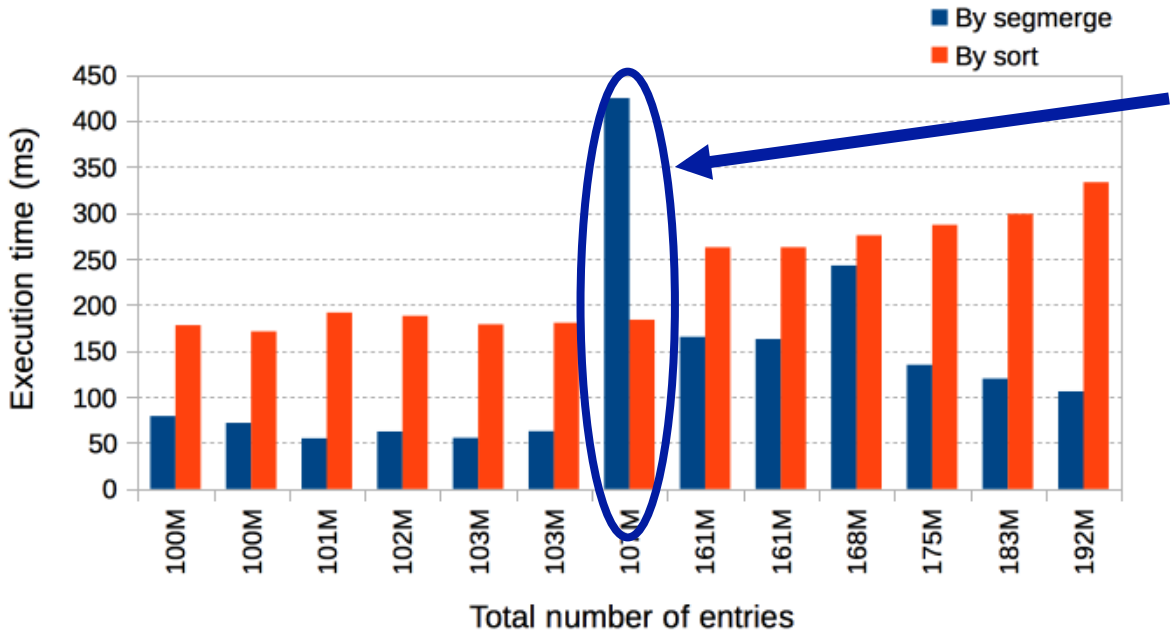
Parallel Segmented Merge

1. Algorithm description
2. Experiments: SpTRANS and SpGEMM
3. Micro-benchmarks

Micro-benchmark 1: segmerge vs sort

- In the SpGEMM test, if we replace segmerge with CUDA thrust-sort:

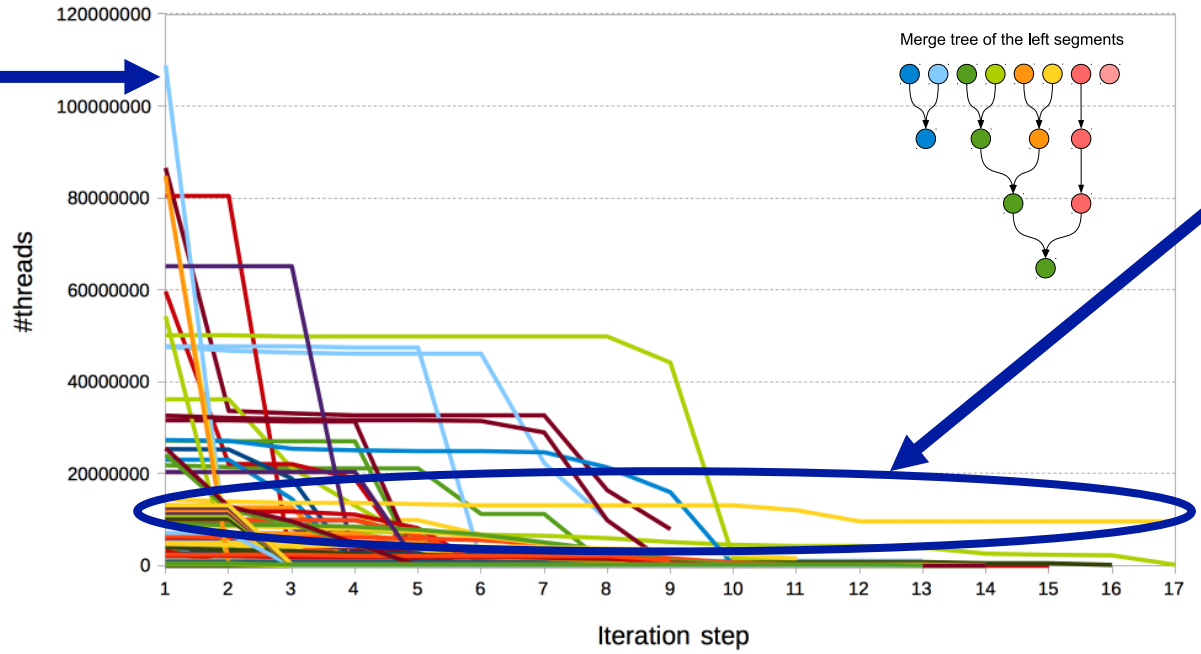


In this case, sort is faster than segmerge

Micro-benchmark 2: #threads in iterations

- In the SpGEMM test, we record #threads issued in each iteration step:

108813822
552638
547690
515739
349187
32396
12215
6863
5987
5971
5963
5959
5956



14406767
13862538
13592550
13520724
13306832
13139154
13130214
13125706
13123376
13121288
11973924
9579396
9579268
9579188
9579132
9579116
9579088

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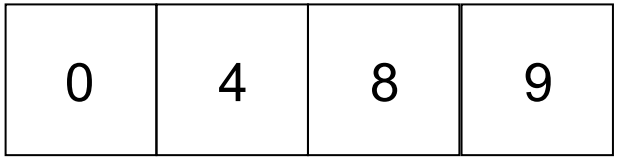
Conclusion

- Though some parallel segmented operations (segsum[Blelloch TR93, Liu PARCO15], segsort [Hou ICS17], etc.) exist, parallel segmerge has been largely ignored.
- Our segmerge algorithm always evenly distributes entries to threads, thus runs in a very balanced way on GPU.
- SpTRANS and SpGEMM are largely accelerated by using segmerge at proper stages.
- According to micro-benchmarks, there are still some optimizations can be done.

T k u !



A y Q s n s ?