# How to calculate covariance matrix for all state vectors?

The adaptive smoother can only give covariance matrix at each step. But how to calculate covariance matrix for all? This is important when doing Gibbs sampling and Q estimation.

Let

Let , and be the log-likelihood at step k. Then,

So, the gradient is:

And the block tri-diagonal hessian is:

(For robustness, use Fisher scoring in hessian, as shown in adaptive filtering)

Use the adaptive smoother to get the mean and then calculate the covariance matrix as .

# Gibbs Sampler

Assume . is the dimension of state vectors.

## Conditional Prior

1. is diagonal

The diagonal element of is .

Where and

1. No constraints on

Where and .

(To make the mean of loosely centered around )

## MCMC iteration:

1. Update

Use adaptive smoother (with or without window) to get the posterior mean and calculate/estimate the posterior covariance as above.

Then sample from the (approximate) full conditional distribution . To make best use of sparse structure (block tri-diagonal covariance), use Cholesky decomposition of : sample , then .

Here, I just show the idea. The hessian can be calculate along with adaptive filtering. So after adaptive filtering, we can get the Hessian immediately.

1. Update :
   1. is diagonal: update

Denote the row of as and element of as . Further, .

By conjugacy,

* 1. No constraints on

Let

By conjugacy,